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Wave propagation of bi-directional porous FG beams using Touratier's higher-order shear deformation beam theory

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Abstract. This work presents an analytical approach to investigate wave propagation in bi-directional functionally graded cantilever porous beam. The formulations are based on Touratier's higher-order shear deformation beam theory. The physical properties of the porous functionally graded material beam are graded through the width and thickness using a power law distribution. Two porosities models approximating the even and uneven porosity distributions are considered. The governing equations of the wave propagation in the porous functionally graded beam are derived by employing the Hamilton's principle. Closed-form solutions for various parameters and porosity types are obtained, and the numerical results are compared with those available in the literature. The numerical results show the power law index, number of wave, geometrical parameters and porosity distribution models affect the dynamic of the FG beam significantly.

Keywords: bi-directional FG beam; frequencies; porosity types; Touratier's higher-order shear deformation; wave propagation

1. Introduction

Functionally graded material (FGM), this mineral is getting attention many researchers for its advanced nature eristic. This is due to its distinctive texture by the continuous contrast between metallic and ceramic constituents through the beam's thickness direction with a specific volume fraction index, functionally graded constructions are adopted because of their advantages and very high mechanical performance (rigidity), increase the bond strength, and reduce the thermal stresses. FGMs is developed for specific applications in aircraft engines, nuclear power, military industries, nuclear reactors, civil structures, and in some medical materials, etc. (Reddy and Chin 1998, Reddy 2000, Muller *et al.* 2003, Aliaga and Reddy 2004, Lu *et al.* 2009, Houari *et al.* 2013,

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Sayyad and Ghugal 2017, Slimane *et al.* 2021). Many studies have been performed on FG beams and FG plates to analyse the static, dynamic and thermo-mechanical behaviors. Ding *et al.* (2007) obtained elasticity solutions for anisotropic FG beams using the Airy stress function. Zhong *et al.* (2007) developed an analytical method for FG cantilever beams subjected to various types of mechanical loadings. In Daouadji *et al.* (2013) developed elasticity solutions to analyse functionally graded fixed-free beams subjected to different loads using the semi inverse method. Bouremana *et al.* (2013) presented an efficient a new first shear deformation FG beam theory based on neutral surface position. Xu *et al.* (2014) presented the two-dimensional elasticity solutions of FG beams with varying thickness. Sayyad and Ghugal (2017) developed a theory involving shear deformation to bending functionally graded beams and plates.

The higher order shear deformation theory has been applied to study the dynamic behavior of functionally graded beams, plates and Nano-plates (Aydogdu and Taskin 2007, Sina *et al.* 2009, Koochaki 2011, Larbi *et al.* 2015, Hadji *et al.* 2016, Sayyad and Ghugal 2017b, Benadouda *et al.* 2017, Shahsavari *et al.* 2018, Hadji *et al.* 2022, Adıyaman 2022.).

Alshorbagy *et al.* (2011) studied free vibration characteristics of a FG beams by a finite element technique based on Euler-Bernoulli beam theory. Bouzidi *et al.* (2020) studied the vibrational behavior of FG rotary blade system, material properties gradation of the blade and the shaft is described using the power law distribution, and a new expression of power law distribution is developed to express the gradation of the material properties of the blade in the direction of the thickness and display direction at the same time. Using the same mechanical distribution of the material, Hassaine *et al.* (2022), analysed the effect of transverse cracks on the natural frequencies of Euler-Bernoulli FG beam via classical finite elements method. Saimi *et al.* (2023) investigated the dynamic and buckling response of bi-directional graded material beams (BDFB) with transverse cracks, using the differential quadrature finite elements method with considering different boundary conditions.

The thermos-mechanical behavior of FG beams and plates has attracted attention many researches in various engineering, (Abdelhak *et al.* 2015, El-Sayed Habib *et al.* (2019), Lan (2020), Roodgar and Suphanut (2023), Eiadtrong *et al.* (2023).

Some previous studies have included the problem of porosity and its effect on the structures behavior, as porosities-model may appear within functionally graded structures during the production progress. Da Chen *et al.* (2016) investigated the free and forced vibrations of shear deformable porous FG beams. Benadouda *et al.* (2017) has investigated the influence of many parameters on the wave propagation of a FG beam having porosities. The governing equations of the wave propagation in the FG beam are derived by using the Hamilton’s principle, and the analytic dispersion relations of the FG beam are obtained by solving an eigenvalue problem. Belaid Batou *et al.* (2019), studied wave propagations in sigmoid functionally graded (S-FG) plates, using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. The material properties of sigmoid functionally graded are assumed to vary through thickness according sigmoid model. The (S-FG) plates are supposed to be imperfect, which means that they have a porous distribution (even and uneven) through the thickness of these plates. Extensive results are presented to check the efficient of present methods to predict wave dispersion and velocity wave in S-FG plates. An analytical analysis for the study of vibratory behavior and wave propagation of FG plates is presented by Bennai *et al.* (2019), based on a high order shear deformation theory. The field of displacement of theory is present of indeterminate integral variables. Equations of motion are derived by the principle of minimization of energies. Analytical solutions of free vibration and wave propagation are obtained for FGM plates simply supported by integrating the analytic dispersion relation. Illustrative examples are given to show the effects porosity parameter, material graduation, thickness-to-length ratio and porosity distribution on the vibration and wave propagation of FG plates.

Hadji *et al.* (2022) considered the effect of evenly and unevenly distributed porosity on the bending and free vibration of porous FG beams resting on elastic foundations. The hyperbolic shear deformation theory is applied for the kinematic relations, the study showed that the effect of porosity on the bending of imperfect beams with even porosity distribution and the uneven distribution of porosity is very obvious, and the even porosity distribution gives the highest shear stress value compared to the other distributions porosity. Several models-porosity have been introduced recently with different distributions (Gökhan Adıyaman 2022, Amoozgar and Gelman 2022, Al-Itbi and Noori 2023, Saffari 2023, Wattanasakulpong *et al.* 2023, Medjdoubi *et al.* 2023, Dahmane *et al.* 2023, Mellal *et al.* 2023).

It is clear from the literature discussed above that there is a lack of studies on wave propagation in the FGMs beam containing porosities and with different distribution in thickness and width. The objective of this work is to investigate the wave propagation of bidirectional porous FG beam using Touratier's Higher-order shear deformation theory, according to the power law distribution and wave number. The material properties of the beam are supposed to be dependent on the gradation pattern through the width and thickness directions via power-law form. The distribution of porosity through the cross sections to two distribution functions, namely Even and Uneven-O. The governing equations of the wave propagation in the bidirectional porous FG cantilevered-beam are derived by using the Hamilton's principle. The numerical results are evaluated with those from earlier studies. several studies cases were done to examine the influence of the power-law gradation index, porosity models, geometrical parameters and wave number on the first natural frequencies of FG fixed-free beam.

2. Mathematical formulation and theories

Consider a porous FG beam with a width b, thickness h, and length L; a porous functionally graded beam, composed of metal (SUS304) and ceramic (Si3N4) materials, is considered. In this investigation, two forms of porosity are considered, evenly distribution as seen in Fig. 1, whereas the other one are characterized by an Unevenly-O distribution, see the Fig. 2.

The general material properties of an FGM beam with evenly (uniformly) distributed porosity can be written as (Wattanasakulpong *et al.* 2014, Cong *et al.* 2018, Dahmane *et al.* 2023)

$$P(z)=P_m\left(V_m(z)-\frac{\alpha}{2}f_p\right)+P_c\left(V_c(z)-\frac{\alpha}{2}f_p(z)\right) \tag{1}$$

f_p is a function that shows the distribution of the porosities through the cross-section of the beam, $f_p = 1$ for even-porous FG beam.

$$P(z)=P_m\left(V_m(z)-\frac{\alpha}{2}\right)+P_c\left(V_c(z)-\frac{\alpha}{2}\right) \tag{2}$$

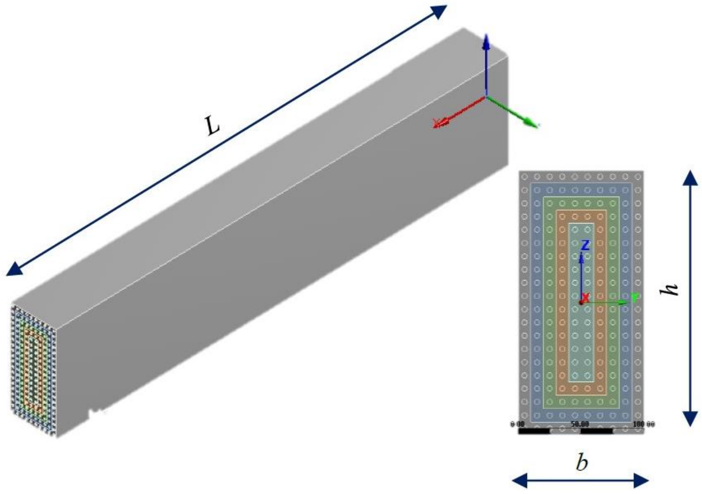

Fig. 1 Bidirectional even-porous FG beam (Dahmane *et al.* 2023)

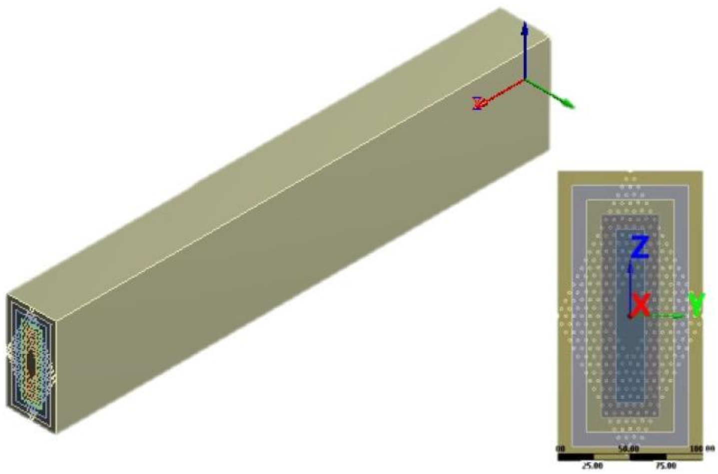

Fig. 2 Bidirectional uneven-porous FG beam

The material properties P of FG beam can be written as vary continuously along the width *y*-axis and the thickness *z*-axis according to the following relation (Hassaine *et al.* 2022)

$$P(y,z)=P_mV_m(y,z)+P_cV_c(y,z) \tag{3}$$

In our study, the material properties P of even-porous FG beam can be written as (Dahmane *et al.* 2023)

$$P(y,z)=P_m\left(V_m(y,z)-\frac{\alpha}{2}f_p\right)+P_c\left(V_c(y,z)-\frac{\alpha}{2}f_p(y,z)\right) \tag{4}$$

Where $P(y,z)$ denotes either the density $\rho(y,z)$ and the Young's modulus $E(y,z)$.

The volume fractions of ceramic $V_c(y,z)$ and metal $V_m(y,z)$ are considered to be distributed along the width and thickness as follows

$$V_c = \left(\frac{y}{b}+\frac{1}{2}\right)^k \left(\frac{z}{h}+\frac{1}{2}\right)^n \tag{5}$$

$$V_m = 1-\left(\frac{y}{b}+\frac{1}{2}\right)^k \left(\frac{z}{h}+\frac{1}{2}\right)^n \tag{6}$$

Where,
k: the FG power-law index along the width direction *y*-axis, $0 \le k \le \infty$.
n: the FG power-law index along the width direction *z*-axis, $0 \le n \le \infty$.
Properties of the even-porous FG beam can be obtained as

$$P(y,z) = (P_c - P_m)\left(\frac{y}{b}+\frac{1}{2}\right)^k \left(\frac{z}{h}+\frac{1}{2}\right)^n + P_m - (P_c - P_m)\frac{\alpha}{2} f_p(y,z) \tag{7}$$

The Young's modulus and material density can be obtained from Eq. (7) as

$$E(y,z) = (E_c - E_m)\left(\frac{y}{b}+\frac{1}{2}\right)^k \left(\frac{z}{h}+\frac{1}{2}\right)^n + E_m - (E_c - E_m)\frac{\alpha}{2} f_p(y,z) \tag{8}$$

$$\rho(y,z) = (\rho_c - \rho_m)\left(\frac{y}{b}+\frac{1}{2}\right)^k \left(\frac{z}{h}+\frac{1}{2}\right)^n + \rho_m - (\rho_c - \rho_m)\frac{\alpha}{2} f_p(y,z) \tag{9}$$

The beam is described by two-dimensional plane stress problem, the kinematic strain-displacement relations are described (Dahmane *et al.* 2023)

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \varepsilon_{xz} \end{Bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \end{bmatrix} = \begin{Bmatrix} u(x,z,t) \\ w(x,z,t) \end{Bmatrix} \tag{10}$$

In which *u*, *w* are the displacements in *x* and *z* directions (Mellal *et al.* 2023)

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{11}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{12}$$

Where u_0 is the mid-plane displacement of the beam in the x direction, w_b and w_s are the bending and shear components of transverse displacement, respectively.

The various expressions of the unidirectional and bi-directional uneven porosity distribution are presented in the following functions, respectively (Benadouda *et al.* 2017)

$$f_p(z) = \left(1 - \frac{2|z|}{h}\right) \tag{13}$$

$$f_p(y,z) = \left(1 - \frac{2|y|}{b}\right)\left(1 - \frac{2|z|}{h}\right) \tag{14}$$

Our choice of shear shape function is determined based on sinusoidal shear deformation theory (SSDT) of Touratier's (1991)

$$f(z) = z - \frac{h}{\pi} sin\left(\frac{\pi z}{h}\right) \tag{15}$$

And

$$g(z) = cos\left(\frac{\pi z}{h}\right) \tag{16}$$

2.1 Kinematics and constitutive equations

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Mellal *et al.* 2023)

$$\int_{t_1}^{t_2} (\delta U - \delta K)\, dt = 0 \tag{17}$$

Where δU is the variation of strain energy; and δK is the variation of kinetic energy.
The constitutive stress-strain equation, in a case of FG porous beam, can be represented as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} \frac{E(y,z)}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \varepsilon_{xz} \end{bmatrix} \tag{18}$$

The non-zero strains components are obtained as (Dahmane *et al.* 2023)

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + z k_{xx}^b + f(z) k_{xx}^s \,,\ \gamma_{xz} = g(z)\gamma_{xz}^0 \tag{19}$$

The stress resultants are obtained as

$$\begin{Bmatrix} N_{xx} \\ M_{xx}^b \\ M_{xx}^s \end{Bmatrix} \begin{bmatrix} A_{11} & B_{11} & B_{11}^s \\ B_{11} & D_{11} & D_{11}^s \\ B_{11}^s & D_{11}^s & H_{11}^s \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^0 \\ k_{xx}^b \\ k_{xx}^s \end{bmatrix} \tag{20}$$

Where the cross-sectional stiffness is expressed as

$$\{A_{11} \quad B_{11} \quad D_{11} \quad B_{11}^s \quad D_{11}^s \quad H_{11}^s\} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{E(y,z)}{1-\nu^2} (1, z, z^2, f(z), zf(z), f(z)^2) dz \tag{21}$$

And

$$A_{55}^{s}=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^{2}dz \tag{22}$$

The equations of motion can be expressed in terms of displacements (u_0, w_b, w_s) and the appropriate equations take the form

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}-B_{11}\frac{\partial^3 w_b}{\partial x^3}-B_{11}^{s}\frac{\partial^3 w_s}{\partial x^3}=J_0\ddot{u}_0-J_1\frac{\partial \ddot{w}_b}{\partial x}-J_2\frac{\partial \ddot{w}_s}{\partial x} \tag{23}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}-D_{11}\frac{\partial^4 w_b}{\partial x^4}-D_{11}^{s}\frac{\partial^4 w_s}{\partial x^4}=J_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\frac{\partial \ddot{u}_0}{\partial x}-J_3\nabla^2\ddot{w}_b-J_4\nabla^2\ddot{w}_s \tag{24}$$

The inertia coefficients are defined as follows

$$\left(J_0,J_1,J_2,J_3,J_4,J_5\right)=\int_{-h/2}^{h/2}\left(1,z,f,z^2,z\,f,f^2\right)\rho(z)dz \tag{25}$$

2.2 Analytical solution

We assume solutions for u_0, w_b, and w_s representing propagating waves in the *x*-direction with the form

$$\begin{Bmatrix} u_0(x,t) \\ w_b(x,y,t) \\ w_s(x,y,t) \end{Bmatrix}=\begin{Bmatrix} U\,exp\left[i(\lambda x-\omega t)\right] \\ W_b\,exp\left[i(\lambda x-\omega t)\right] \\ W_s\,exp\left[i(\lambda x-\omega t)\right] \end{Bmatrix} \tag{26}$$

Where U, W_b, and W_s are the coefficients of the wave amplitude, λ is the wave number of wave propagation along *x*-axis direction, ω is the Eigen-frequency. The analytical solutions can be obtained by

$$\left(\left[\mathrm{K}\right]-\omega^2\left[\mathrm{M}\right]\right)\begin{Bmatrix} U \\ W_b \\ W_s \end{Bmatrix}=\{0\} \tag{27}$$

Where

$$\left[K\right]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},\ \left[M\right]=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{28}$$

Where

$$a_{11}=-A_{11}\lambda^2,\ a_{12}=i\,\lambda^3 B_{11},\ a_{21}=-i\,\lambda^3 B_{11},\ a_{13}=i\,B_{11}^{s}\lambda^3,\ a_{31}=-i\,B_{11}^{s}\lambda^3,\ a_{22}=-D_{11}\lambda^4,\ a_{23}=-D_{11}^{s}\lambda^4,$$
$$a_{33}=-\left(H_{11}^{s}K_1^4+A_{55}^{s}K_1^2\right) \tag{29}$$

$$m_{11}=-J_0,\ m_{12}=i\,J_1\lambda,\ m_{21}=-i\,J_1\lambda,\ m_{22}=-J_0-J_3\lambda^2,\ m_{13}=i\,J_2\lambda,\ m_{31}=-i\,J_4\lambda\ \ m_{23}=m_{32}=-J_0-J_4\lambda^2,$$
$$m_{33}=-J_0-J_5\lambda^2 \tag{30}$$

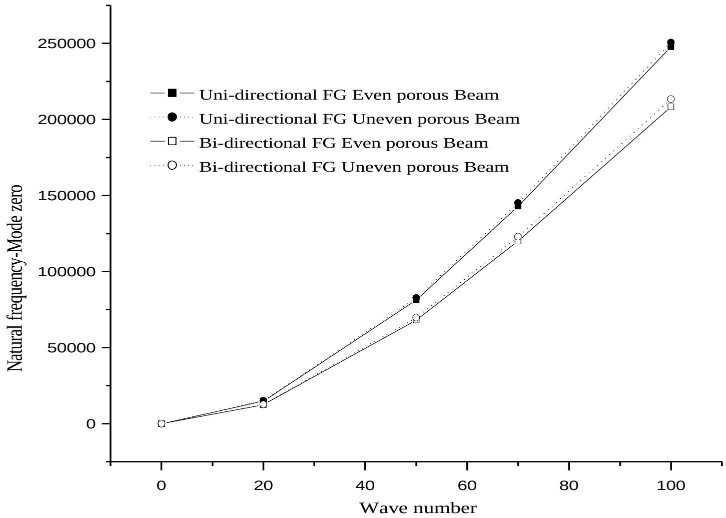

Fig. 3 First natural frequency of porous FG beam according to the wave number for *α*=0.1, and the power index *n*=*k*=2

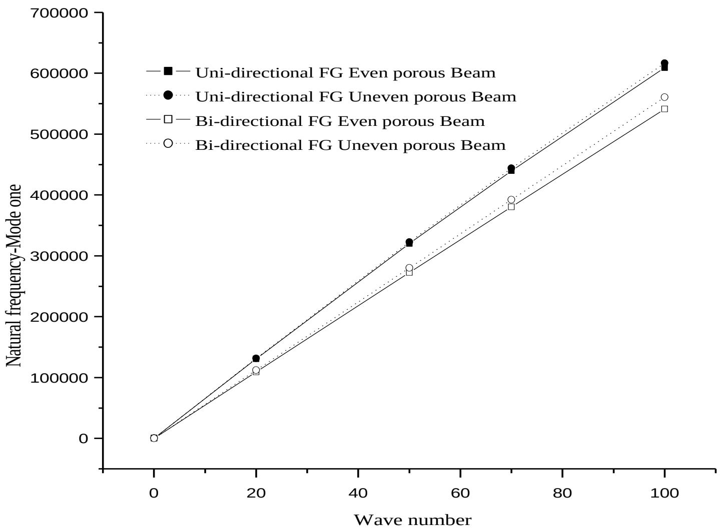

Fig. 4 Second natural frequency of porous FG beam according to the wave number for *α*=0.1, and the power index *n*=*k*=2

For non-trivial solutions of Eq. (27) the following determinants should be zero

$$\left| [K] - \omega^2 [M] \right| = 0 \tag{31}$$

The roots of Eq. (29) can be expressed as (Benadouda *et al.* 2017)

$$\omega_1 = W_1(\lambda),\ \omega_2 = W_2(\lambda),\ \omega_3 = W_3(\lambda) \tag{32}$$

They correspond to the wave modes M_0, M_1 and M_2 respectively, Benadouda *et al.* (2017).

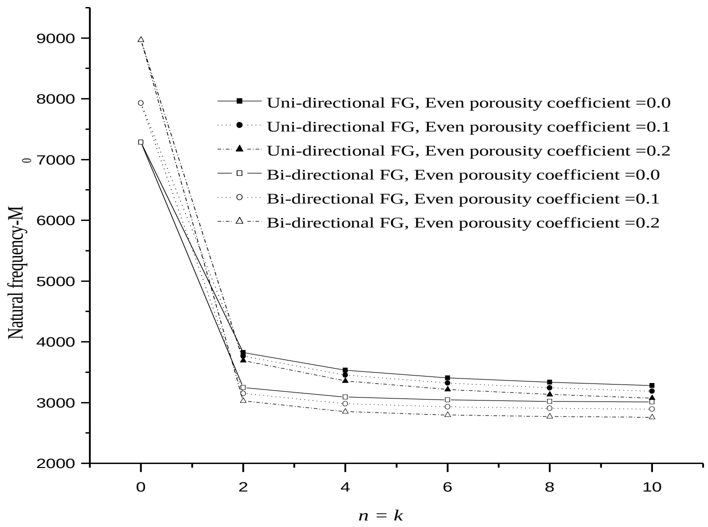

Fig. 5 First natural frequency of porous FG beam according to the power index for different coefficient porosity, wave number is 10

The phase velocity of wave propagation in the FG beam can be expressed as (Batou *et al.* 2019)

$$C_i = \frac{W_i(\lambda)}{\lambda}, \quad (i=1,2,3) \tag{33}$$

2.3 Numerical results and discussion

The material characteristics of the two constitutes phases, which form the FG beam are given as:

Ceramic (Si3N4): Young's modulus is 348.43 GPa, the density is 2370 (kg m-3);

Metal (SUS304): Young's modulus is 201.04 GPa, the density is 8166 (kg m-3);

Poisson coefficient is 0.3 for both.

Firstly, the second natural frequencies of even and uneven porosity FG beam for various wave number propagation, and for power indices is 2, are determined and compared with Benadouda *et al.* (2017) for the unidirectional FG fixed-free beam, using an analytical solution based on a Reddy higher order shear theory, where the porosity coefficient is 0.1. The very good agreement is observed, see Figs. 3 and 4. The biggest change in values is 15.91% for uniform porosity, while it is 14.81% for the second type of porosity, these changes in values correspond to the first mode.

The results illustrate that the porosity has a very important effect on the frequency values, an increase in the porosity coefficient in solids leads to a decrease in their rigidity, and that is through a decrease in the value of the natural frequencies. The results show that the porosity of the uniform distribution has the greatest effect in reducing the frequencies compared to the irregular distribution, especially with the increase in wave number. The results illustrate that the second adopted method (bi-directional) shows a wider difference between uniform and no-uniform porosity values, especially in the mode two, where the value of the largest change is estimated at a 3.57%.

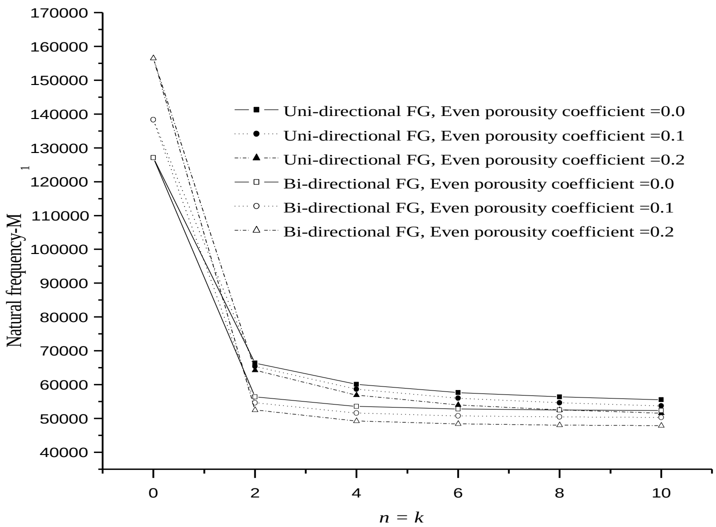

Fig. 6 Second natural frequency of porous FG beam according to the power index for different coefficient porosity, wave number is 10

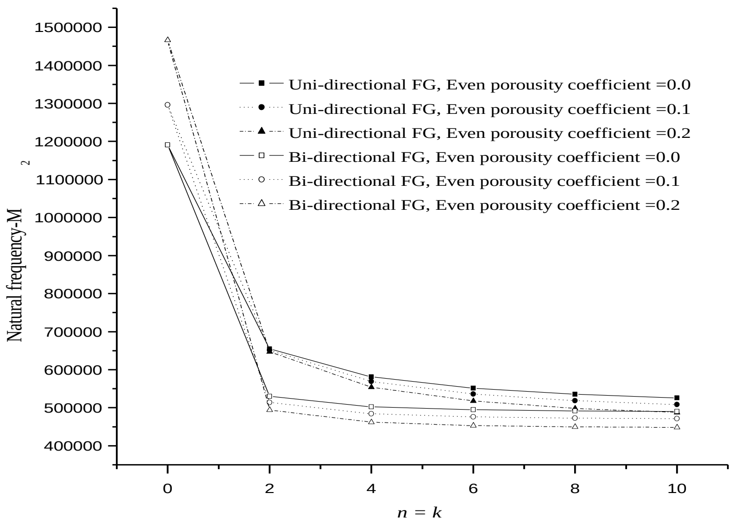

Fig. 7 Natural frequency (mode two) of porous FG beam according to the power index for different coefficient porosity, wave number is 10

Figs. 5, 6, and 7 represents the variation of the natural frequencies of the first three modes for unidirectional and bi-directional even and uneven-porous FG beam as function to the power law index for different coefficient porosity, where the wave number is 10. It is clear that both porosities, coefficient porosity and power law index affect the frequency values. These figures demonstrate that the fundamental frequencies of the FG beam decrease when the power indices increase, the decrease is due to the passage from alumina (Si3N4) which has a higher elastic modulus with lower density to steel (SUS304) which has lower elastic modulus with higher density. There is a consistent contrast in the results between the unidirectional results and

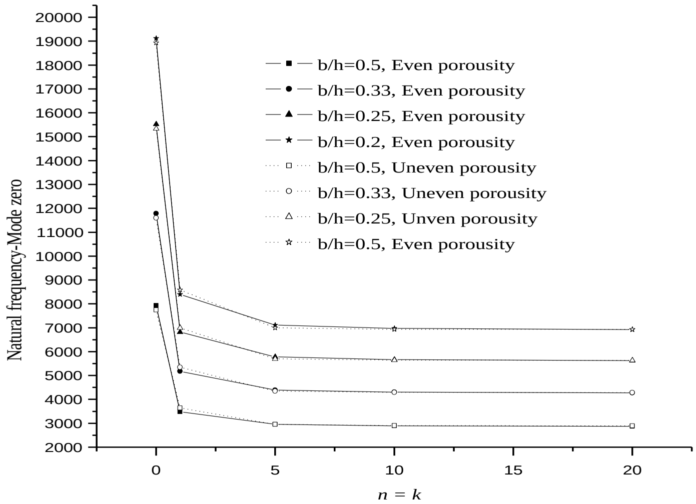

Fig. 8 Natural frequency (mode zero) of porous FG beam according to the power index for different geometrical parameters, wave number is 10

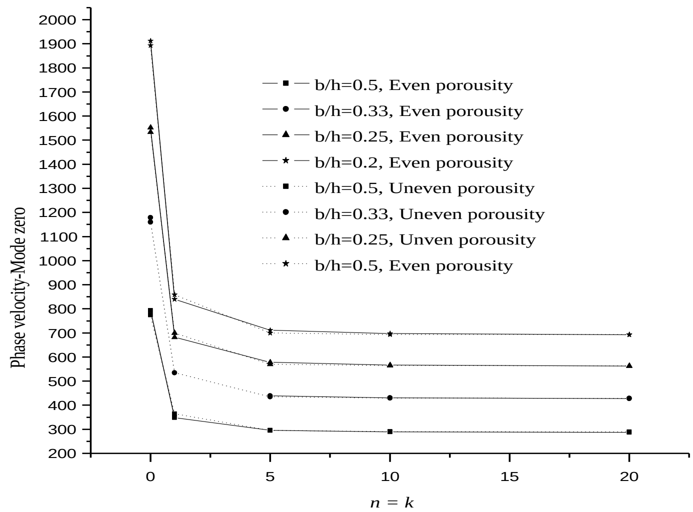

Fig. 9 Phase velocity (mode zero) of porous FG beam according to the power index for different geometrical parameters, wave number is 10

bidirectional results.

The Figs. 8-9 represent a comparison of the first frequency variation and the associated wave velocity (mode zero) of bi-directional FG porous beam, in term of power law index and for different geometrical parameters *b/h*, where wave number 10. The results showed that the frequency values decrease with decreasing cross-sectional area, and the values do not differ significantly in the presence of both types of porosity in this topic.

Figs. 10-12 show the variation of the first three frequencies of bi-directional FG porous beam with even distribution, and from 13-18 with Uneven-O distribution as a function of the wave number for various power-law index and coefficient porosity. The results show that there is a large

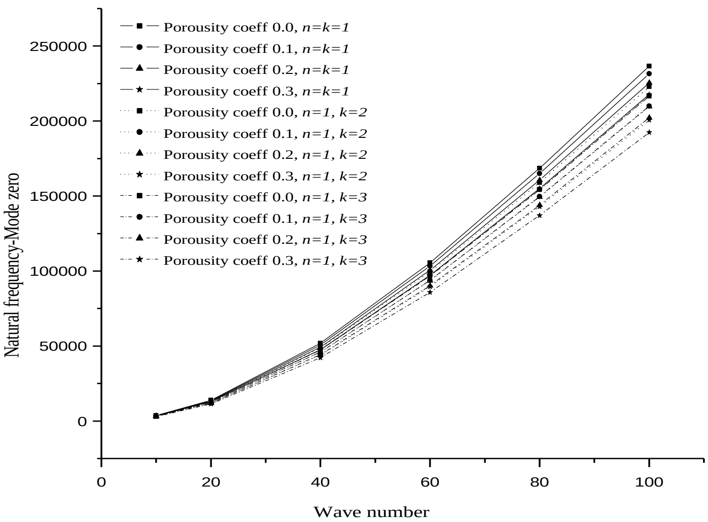

Fig. 10 Natural frequency-M_0 of uniform-porous FG beam according to the wave number propagation for different power index, and different coefficient porosity where wave number is 10

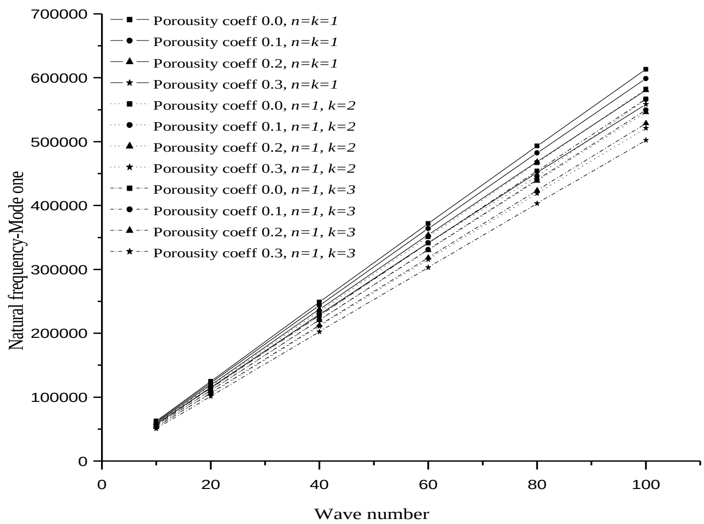

Fig. 11 Natural frequency-M_1 of uniform-porous FG beam according to the wave number propagation for different power index, and with different coefficient porosity

variation in the values, especially the last mode. Also, all values take the same upward trend and decrease in the same pattern. The results show that the concentration of the irregular substance distribution in both axes affects the values of the expected frequencies.

5. Conclusions

In the present work, the wave propagation issue in porous, bi-directional functionally graded

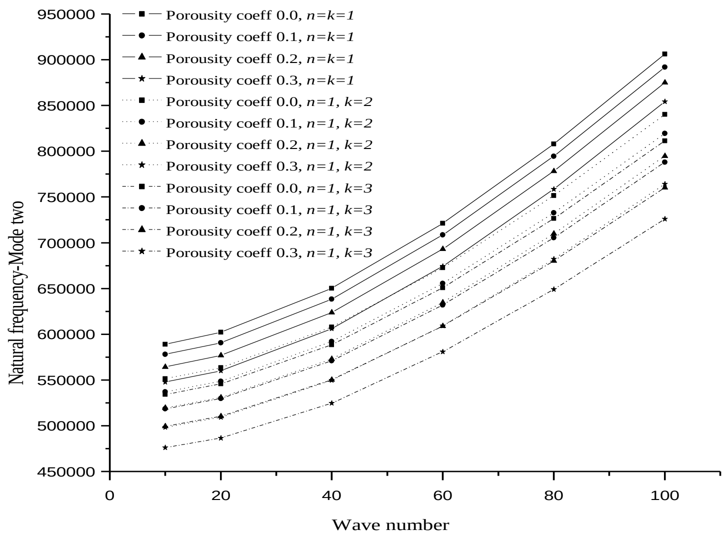

Fig. 12 Natural frequency-M_2 of uniform-porous FG beam according to the wave number propagation for different power index, and with different coefficient porosity where

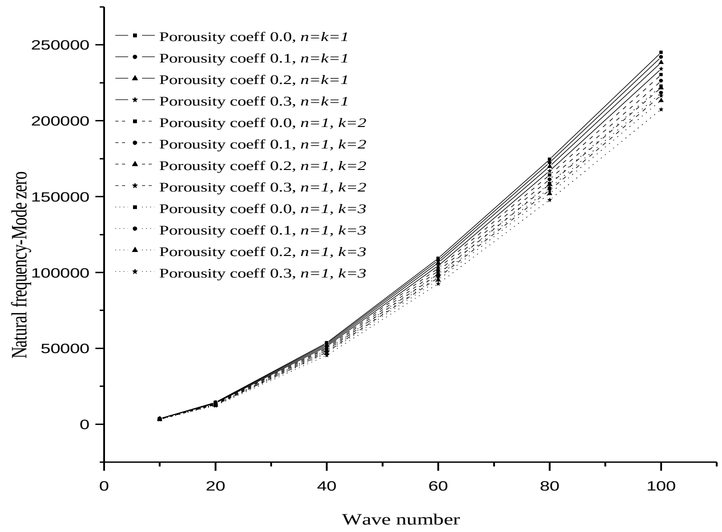

Fig. 13 Natural frequency-M_0 of uneven-porous FG beam according to the wave number propagation for different power index, and with different coefficient porosity

cantilevered beam was studied using Touratier's higher-order shear theory. Two models of porosity were considered, uniform distribution called even and no-uniform distribution called uneven. According to the power law index, the material's characteristics are dependent change gradually through the width y-direction and thickness z-direction. The governing equations of the wave propagation dispersion in the porous FG beam are derived by employing the Hamilton's principle. The analytical solution of the porous FG cantilever beam is obtained by solving an eigenvalue problem. the natural frequencies of the first three modes, phase velocity were obtained. The results

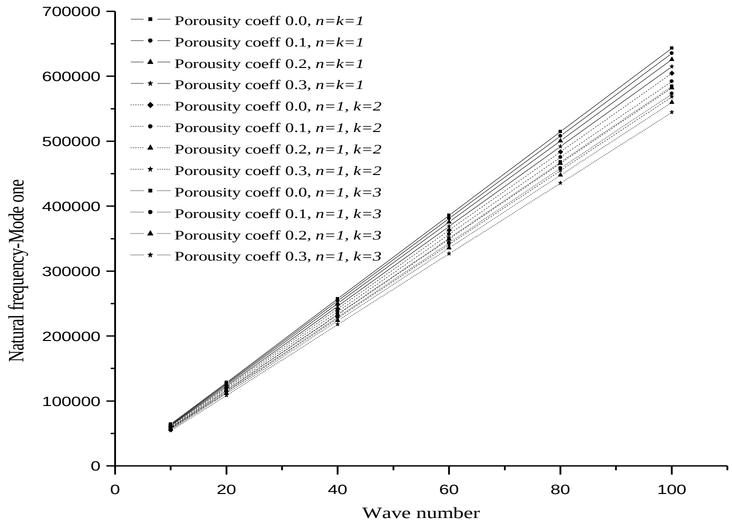

Fig. 14 Natural frequency-M_1 of uneven-porous FG beam according to the wave number propagation for different power index, and with different coefficient porosity

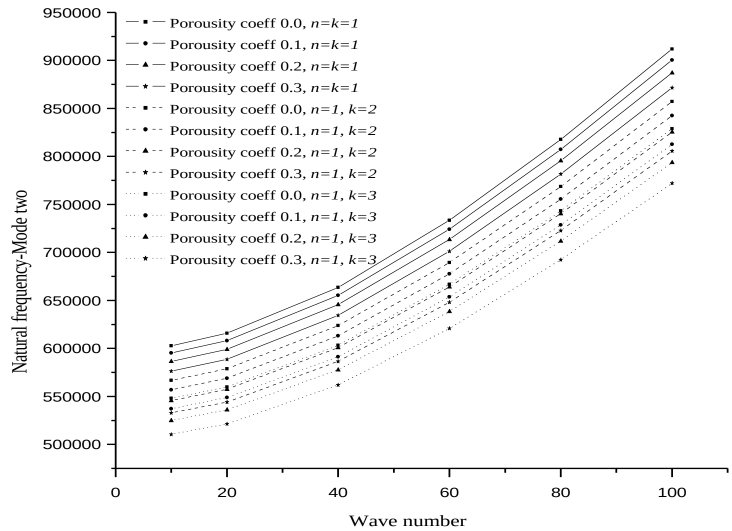

Fig. 15 Natural frequency-M_2 of uneven-porous FG beam according to the wave number propagation for different power index, and with different coefficient porosity

obtained for bi-directional porous FG beam were compared with those reported in the literature, and the results were very agreement. The effects of the volume fraction distributions, the wave number of propagation, and the porosity types on dynamic of FG beam are discussed in details:

- The power index, wave number, and porosity types affect the wave propagation of the beam significantly;
- The porosity has a very important effect on the frequency values, an increase in the porosity coefficient in solids leads to a decrease in their stiffness, and that is through a decrease in the

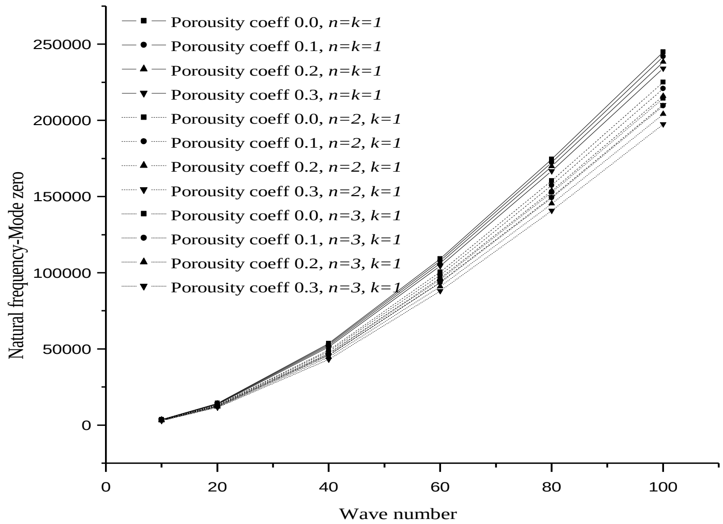

Fig. 16 First natural frequency-M_0 of uneven-porous FG beam according to the wave number propagation for different power index, and different coefficient porosity

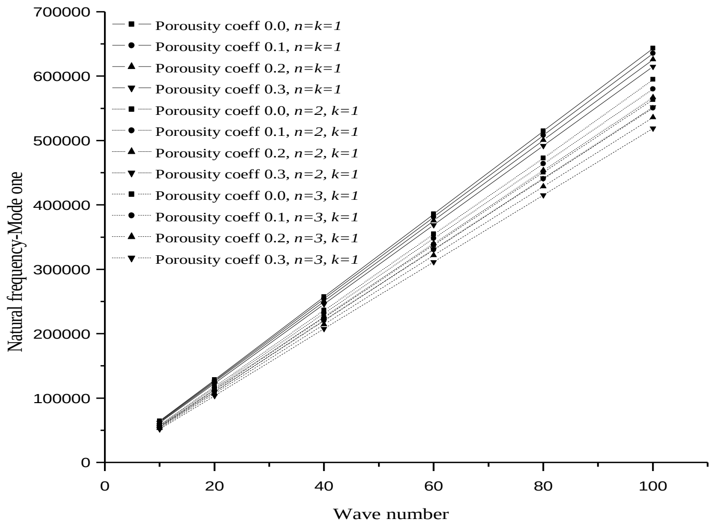

Fig. 17 Second natural frequency-M_1 of uneven-porous FG beam according to the wave number propagation for different power index, and different coefficient porosity

value of the fundamental frequencies;

• The porosity of the even distribution has the greatest effect in reducing the natural frequencies compared to the uneven distribution;

• The uneven porosity predicts higher fundamental frequencies than the even porosity;

• The concentration of the asymmetrical substance distribution of FG in both axes affects the values of the expected frequencies;

• The current model presented bi-directional porous FG beam is more appropriate analytical approach for using then unidirectional porous FG beam.

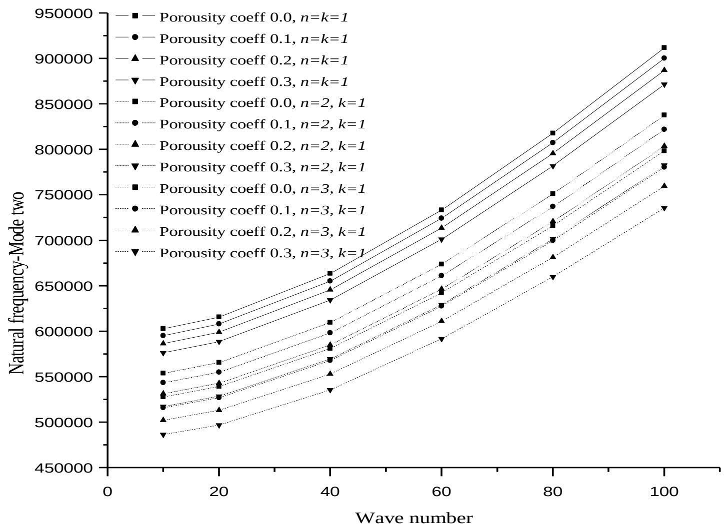

Fig. 18 Third natural frequency-M_2 of uneven-porous FG beam according to the wave number propagation for different power index, and with different coefficient porosity

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