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Ultrasonic waves in a single walled armchair carbon nanotube resting on nonlinear foundation subjected to thermal and in plane magnetic fields

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Abstract. The present paper is concerned with the study of nonlinear ultrasonic waves in a magneto thermo (MT) elastic armchair single-walled carbon nanotube (ASWCNT) resting on polymer matrix. The analytical formulation is developed based on Eringen's nonlocal elasticity theory to account small scale effect. After developing the formal solution of the mathematical model consisting of partial differential equations, the frequency equations have been analyzed numerically by using the nonlinear foundations supported by Winkler-Pasternak model. The solution is obtained by ultrasonic wave dispersion relations. Parametric work is carried out to scrutinize the influence of the non local scaling, magneto-mechanical loadings, foundation parameters, various boundary condition and length on the dimensionless frequency of nanotube. It is noticed that the boundary conditions, nonlocal parameter, and tube geometrical parameters have significant effects on dimensionless frequency of nano tubes. The results presented in this study can provide mechanism for the study and design of the nano devices like component of nano oscillators, micro wave absorbing, nano-electron technology and nano-electro- magneto-mechanical systems (NEMMS) that make use of the wave propagation properties of armchair single-walled carbon nanotubes embedded on polymer matrix.

Keywords: nonlocal elasticity; armchair; CNT; Euler-beam theory; NEMS

1. Introduction

Magneto thermo elastic armchair single-walled carbon nanotube (ASWCNT) have been extensively employed in many structural applications. Owing to unique properties, these materials are proper substitutes for traditional nano materials. Using MT based nano materials in polymer matrix as surrounding medium to generate better efficiency of embedded nano structures has been attracting great research attention in recent years Ebrahimi and Dabbagh (2018). Due to the arrival of nonlocal continuum theory, the nonlocal Euler- Bernoulli and Timoshenko beam models enable the assessment of scaling effect on a CNT's dispersion relations (Wang 2005, Heirechea *et al*. 2008). Erignen (2002, 1972, 1972) elaborated the nonlocal continuum field theories and validated in different nano materials. Wang *et al.* (2006) carried out the nonlocal continuum models to investigate

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the small scale effect on elastic buckling of carbon nanotubes and referred the impact of small scale effect on vibration modes.

Fang (2013) have discussed the nonlinear free vibration of double walled carbon nanotubes based on the nonlocal elasticity theory. They found that the surrounding elastic medium plays an important role in the nonlinear propagation and the amplitudes development. Saadatnia and Esmailzadeh (2017) investigated the nonlinear harmonic vibration of a piezoelectric-layered nanotube conveying fluid flow and they concluded that the effects small scale parameter is quite considerable in the frequency responses of the system in the presence of fluid environment. Askari and Esmailzadeh (2017) exposed the forced vibration of fluid conveying carbon nanotubes considering thermal effect and nonlinear foundations. Gheshlaghi and Hasheminejad (2011) studied the surface effects on the nonlinear vibrational behavior of homogenous Nano beams.

Sadeghi-Goughari *et al.* (2017) studied the vibrational behavior of a CNT conveying magnetic fluid subjected to a longitudinal magnetic field. Zhena *et al.* (2019) verified the free vibration of viscoelastic nanotube under longitudinal magnetic field and indicated the fact that the first natural frequency increases slightly with the increase of the nonlocal parameter, while higher natural frequencies decrease significantly with the increase of the nonlocal parameter. Dai *et al*. (2018) explored the exact modes for postbuckling characteristics of nonlocal nanobeams in a longitudinal magnetic field. Ebrahimi and Barati (2018) investigated the propagation of waves in nonlocal porous multi-phase nano crystalline nanobeams via longitudinal magnetic field effect. They pointed out the concept that the wave frequencies and phase velocities may increase or decrease with the reduction in the inhomogeneity magnitudes. Li *et al.* (2016) illustrated the wave propagation in viscoelastic single-walled carbon nanotubes with surface effect under magnetic field based on nonlocal strain gradient theory and concluded with the importance of damping coefficient. Arani *et al*. (2016) discussed the longitudinal magnetic field effect on wave propagation of fluid-conveyed SWCNT using Knudsen number and surface considerations. Zhang *et al.* (2016) studied the vibration analysis of horn-shaped single-walled carbon nanotubes embedded in viscoelastic medium via a longitudinal magnetic field. A two scale coefficient model is developed to study the propagation of longitudinal stress waves under a longitudinal magnetic field by Güven (2015) via a unified nonlocal elasticity theory. Wang (2006) validated the non-local elastic shell model for studying longitudinal waves in single-walled carbon nanotubes and found that the microstructure and the coupling of the longitudinal wave and radial motion play a vital role in the dispersion of waves. Ebrahimi and Dabbagh (2018) assessed the magnetic field effects on thermally affected propagation of acoustical waves in rotary double nanobeam systems and it is intimated that the higher magnetic field intensity reveals amplified phase speeds in small wave numbers. Elsayed *et al*. (2019) elaborated the effect of Thomson and initial stress in a thermo-porous elastic solid under G-N electromagnetic theory. They showcased that the significant contribution of initial stress, Thomson coefficient effect, and magnetic field in the physical variant. Azarboni (2019) explored the magneto-thermal primary frequency response analysis of carbon nanotube considering surface effect under different boundary conditions. They inferred that the increasing of longitudinal magnetic field leads to shifting the backward jumping at higher excitation amplitude values for different boundary conditions. Basutkar *et al*. (2019) stated the static analysis of flexoelectric nanobeams incorporating surface effects using element free Galerkin method. Pradhan and Phadikar (2019) utilized the nonlocal continuum models to analyse the small scale effect on vibration of embedded multi-layered graphene sheets. Moreno-Navarro *et.al* (2018) have developed the novelty of such a comprehensive formulation in providing the basis for the analysis of practical cases going beyond the classical ones such as piezoelectric, thermo-elastic or magnetostrictive towards more complex combinations of the coupled fields.

Hadzalic *et al*. (2020) have studied 3D thermo-hydro-mechanical coupled discrete beam lattice model of saturated pork-plastic medium. Calin-Itu *et al.* (2019) has developed the improved rigidity of composite circular plates through radial ribs. Rukavina *et al.* (2019) have introduced the ED-FEM multi-scale computation procedure for localized failure. Vlase *et al*. (2019) have Analyzed a Motion equation for a flexible one-dimensional element used in the dynamical analysis of a multibody system.

Thermal buckling properties of zigzag single-walled carbon nanotubes using a refined nonlocal model are investigated by Semmah *et al*. (2014) and they inferred that the thermal buckling properties of SWCNTs are strongly dependent on the scale effect and additionally on the chirality of zigzag carbon nanotube. Naceri *et al*. (2011) presented the wave propagation in armchair single-walled carbon nanotubes (SWCNTs) under thermal environment. Baghdadi *et al.* (2014) noted the thermal effect on vibration characteristics of armchair and zigzag single-walled carbon nanotubes using nonlocal parabolic beam theory. They indicated the significant dependence of natural frequencies on the temperature change as well as the chirality of armchair and zigzag carbon nanotube. The thermal effect on vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory is investigated by Benzair *et al*. (2008) it is concluded that the effects lead to a decrease in frequencies when compared with those obtained by the Euler beam model. Besseghier *et al*. (2011) presented the thermal effect on wave propagation in double-walled carbon nanotubes embedded in a polymer matrix using nonlocal elasticity. Zhang *et al.* (2007) studied the thermal effects on interfacial stress transfer characteristics of single-/multi-walled carbon nanotubes/polymer composite systems under thermal loading by means of thermoelastic theory and conventional fiber pullout models. Lata and Kaur (2019a, 2019) despite of this several researchers worked on different theory of thermoelasticity. Lata *et al.* (2016) and Kumar *et al*. (2016, 2017) studied the deformation in transversely isotropic material using thermoelasticity.

Narendar *et al*. (2011) predicted the nonlocal scaling parameter for armchair and zigzag single-walled carbon nanotubes through molecular structural mechanics, nonlocal elasticity and wave propagation technique. Arani *et al.* (2013) employed nonlocal shear deformable shell model for electro-thermo-torsional buckling of an embedded armchair DWBNNT. Adda Bedia *et al.* (2015) contributed the study on the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium. Zidour *et al.* (2014) developed the buckling analysis of chiral single-walled carbon nanotubes by using the nonlocal Timoshenko beam theory. Bensattalah *et al.* (2016) investigated the thermal and chirality effects on vibration of single-walled carbon nanotubes embedded in a polymeric matrix using nonlocal elasticity theories. Hsu *et al*. (2008) presented the resonance frequency of Chiral single walled carbon nanotubes using Timoshenko beam theory. Aydogdu (2012) studied the axial vibration of single walled carbon nanotube embedded in an elastic medium and concluded with fact that the axial vibration frequencies of SWCNT embedded in an elastic medium highly over estimated by the classical rod model because of ignoring the effect of small length scale. Ansari *et al.* (2012) pointed out the dynamic stability of embedded single-walled carbon nanotubes including thermal environment effects and inferred the result that the difference between instability regions predicted by local and nonlocal beam theories is significant for nanotubes with lower aspect ratios. This present research makes the first attempt to develop the nonlinear ultrasonic waves in a magneto-thermo (MT) elastic armchair single-walled carbon nanotube (ASWCNT) resting on polymer matrix. To the best of the author's knowledge, there has been no record regarding the nonlinear ultrasonic waves in a magneto-thermo (MT) elastic armchair single-walled carbon nanotube (ASWCNT) resting on polymer matrix via Eringen's nonlocal elasticity theory. Therefore, there is a strong scientific need to understand the nonlinear ultrasonic vibration behavior of the armchair single-walled carbon nanotube (ASWCNT) resting on polymer matrix in magneto- thermo (MT) elastic environment.

The nonlinear magneto- thermo elastic waves in an armchair single-walled carbon nanotube (ASWCNT) resting on polymer matrix is studied via Euler beam theory. The analytical formulation is developed based on Eringen's nonlocal elasticity theory to account small scale effect. The solution is obtained by ultrasonic wave dispersion relations. Parametric studies are conducted to scrutinize the influence of the magneto-electro-mechanical loadings, nonlocal parameter, and aspect ratio on the deflection characteristics of nanotube. The influence of each parameter is highlighted through a group of diagrams and tables.

2. Mathematical formulations

2.1 Eringen nonlocal theory of elasticity

This theory assumes that stress state at a reference point x in the body is regarded to be dependent not only on the strain state at x but also on the strain states at the all other points X' of the body. The general form of the constitutive equations in the non-local form of elasticity contains an integral over the entire region of interest. The integral contains a non-local kernel function, which describes the relative influences of the strains at various locations on the stress at a given location. The constitutive equations of linear, homogeneous, isotropic, non-local elastic solid with zero body forces are given by Eringen (1972, 1983, 2002) as follows

$$\sigma_{ij} = -\rho(f_j - \ddot{u}_j), \tag{1}$$

$$\sigma_{ij}(X) = \int_v \pi(|X - X'|), \tau)\sigma_{ij}{}^c(X')dv(X'), \tag{2}$$

$$\sigma_{ij}{}^c = C_{ijkl}\varepsilon_{kl}, \tag{3}$$

$$e_{ij}(X') = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{4}$$

Eq. (1) is the equilibrium equation, where $\sigma_{ij,i}, \rho, f_j, u_j$ are the stress tensor, mass density, body force density and displacement vector at a reference point x in the body, respectively, at the time t, Eq. (3) is the classical constitutive relation where $\sigma^c{}_{ij}(X')$ is the classical stress tensor at any point X' in the body, which is related to the linear strain tensor $e_{ij}(X')$ at the same point. Eq. (4) is the classical strain displacement relationship. The kernel function $\pi(|X - X'|, \tau)$ is the attenuation function which incorporated the nonlocal effect in the constitutive equations. The volume integral in Eq. (2) is over the region v occupied the body. It is clear that, the only difference between Eqs. (1)-(4) and the corresponding equations of classical elasticity in Eq. (2) replaces the Hooke's law in Eq. (3) by Eq. (2). Eq. (2) consists the parameters which correspond to the non-local modulus has dimensions of $(lengh)^{-3}$ and so it depends on a characteristic length (lattice parameter, size of grain, granular distance, etc.) and "l." is an external characteristic length of the system (wavelength, crack length, size or dimensions of sample, etc.). Therefore the non-local modulus can be written in the following form

$$\pi = \pi(|X - X'|, \tau), \tau = \frac{e_o a}{l}, \tag{5}$$

Where $e_0 a$ is a constant corresponding to the material's and has to be determined for each materials independently. Then, the integro-partial differential Eq. (2) of non-local elasticity can be simplifies to partial differential equation as follows τ

$$(1 - \tau^2 l^2 \nabla^2)\sigma_{ij}(X) = \sigma_{ij}^{\ c}(X) = C_{ijkl} e_{kl}(X). \tag{6}$$

where C_{ijkl} is the elastic modulus tensor of classical isotropic elasticity and e_{ij} is the strain tensor. Where ∇^2 denotes the second-order spatial gradient applied on the stress tensor σ_{ij} and $\tau = e_o a / l$. Eringen proposed e_0=0.39 by the matching of the dispersion curves via non-local theory for place wave and born-Karman model of lattice dynamics at the end of the Brillouin zone $(ka = \pi)$, where a is the distance between atoms is and k is the wavenumber in the phonon analysis. On the order hand, Eringen proposed e_0=0.31 in his study Eringen (1972) for Rayleigh surface wave via non-local continuum mechanics and lattice dynamics.

2.2 Atomic structure of carbon nanotube

Carbon nanotubes are considered to be tubes formed by rolling a grapheme sheet about the $\vec{T}$ vector. A vector perpendicular to the $\vec{T}$ is the chiral vector denoted by $\vec{C_h}$. The chiral vector and corresponding chiral angle define the type of CNT. ie zigzag, armchair and chiral $\vec{C_h}$ .can be expressed with respect to two base vector $\vec{a_1}$ and $\vec{a_2}$ as under

$$\vec{C_h} = n\vec{a_1} + m\vec{a_2}. \tag{7}$$

Where n and m are the indices of translation which decide the structure around the circumference Fig. 1 descripts the lattice of transition (n,m) along with the base vectors $\vec{a_1}$ and $\vec{a_2}$. If the indices of translation are such that m=0 and $m=n$ then the corresponding CNT are categorized as zigzag and armchair, respectively. Considering the chirality diameter and the chiral angle of the CNT be calculated by the chiral vector for each nanostructures. The diameter of armchair single-walled carbon nanotube for ($n=m$) is given by Tokio (1995)

$$d = \frac{3na}{\pi}. \tag{8}$$

Based on the link between molecular mechanics and solid mechanics, Wu *et al.* (2006) developed an energy-equivalent model for studying the mechanical properties of single-walled carbon nanotube. Using the same method, the equivalent Young's modulus of armchair nanotube are expressed as

$$E_{SWCNT} = \frac{4\sqrt{3KC}}{9Ct + 4Ka^2 t({\gamma^2}_{21} + {\gamma^2}_{22})}, \tag{9}$$

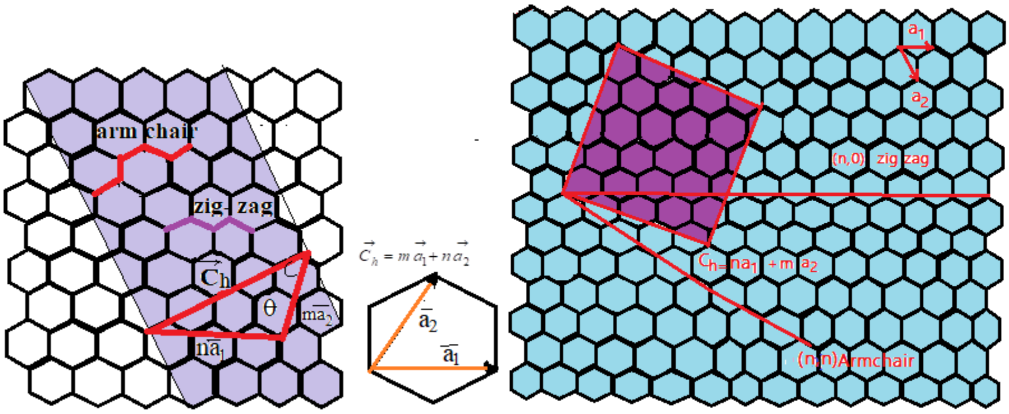

Fig. 1 (a) Geometric properties of SWCNT (b) Graphene structure of carbon nanotube

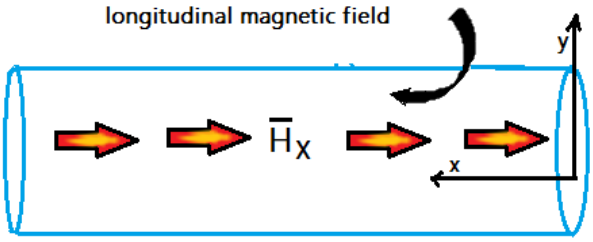

Fig. 2 Single-walled carbon nanotube subjected to axial magnetic field

where K and C are the force constants. t is the thickness of the parameters γ_{21} and γ_{22} are given by:

$$\gamma_{21}=\frac{-3\sqrt{4-3\cos^2(\pi/2n)}\cos(\pi/2n)}{8\sqrt{3}-2\sqrt{3}\cos^2(\pi/2n)},\qquad \gamma_{22}=\frac{12-9\cos^2(\pi/2n)}{16\sqrt{3}-4\sqrt{3}\cos^2(\pi/2n)},$$

Letting $n\to\infty$ the expressions of Young's modulus of a graphite sheet is given by

$$E_g=\frac{16\sqrt{3}Kt}{18Ct+Ka^2t}. \tag{10}$$

2.3 Basic equations of magnetic field force

The CNTs appear in a hollow structure formed by covalently bonded carbon atoms, which can be imagined as a rectangular graphite sheet rolled from one side of its longest edge to form a cylindrical tube (shown in Fig. 2). A cylindrical coordinate system (X, θ,Z) is shown in Fig. 3. Here X expresses the longitudinal direction of shell, θ expresses the circumferential direction of shell and Z expresses the radial direction, the surface defined by Z=0 is set on the middle surface of the shell.

Assuming that the magnetic permeability, η of CNTs equals the magnetic permeability of the medium around it, Maxwell equation are given by

$$f=\nabla\times\bar{s},\nabla\times\varepsilon=-\eta.\frac{\partial\bar{s}}{\partial t},\qquad div\bar{s}=0,\ \bar{s}=\nabla\times(\bar{U}\times\bar{H}),\ \varepsilon=-\eta\left(\frac{\partial\bar{s}}{\partial t}\times\bar{H}\right), \tag{11}$$

f, $\bar{s}$, ε and $\bar{U}$ represent the current density, strength vectors of electric field, disturbing vectors of magnetic field and the vector of displacement respectively, ∇ is the Hamilton arithmetic operators of shell $\nabla = \left(\frac{\partial}{\partial X}\vec{i} + \frac{1}{R} + \frac{\partial}{\partial \theta}\vec{j} \right)$. Appling a longitudinal magnetic field vector $\bar{H}(H_{x,} 0,0)$ exerted on the *i* layer carbon nanotube with the cylindrical coordinate (*R*,*θ*,*Z*) and the displacement vector $\bar{U} = (W_i, V_{i,} Y_i)$ of the *i* layer carbon nanotube to Eq. (11), yields

$$\bar{s} = \nabla \times (\bar{U} \times \bar{H}) = \left(-\frac{H_x}{R_i} \cdot \frac{\partial V_i}{\partial \theta}, \quad H_x \cdot \frac{\partial V_i}{\partial X}, \qquad H_x \cdot \frac{\partial Y_i}{\partial X} \right), \tag{12a}$$

$$f = \nabla \times \bar{s} = \left(\frac{H_x}{R_i} \cdot \frac{\partial^2 y_i}{\partial x \partial \theta}, \quad -H_x \cdot \frac{\partial^2 y_i}{\partial x^2}, \quad \frac{H_x}{R_i^{\,2}} \cdot \frac{\partial^2 v_i}{\partial \theta^2} + H_x \cdot \frac{\partial^2 v_i}{\partial x^2} \right), \tag{12b}$$

The Lorentz force q induced by the longitudinal magnetic field can be written as

$$q = (\bar{q}_x, \bar{q}_\theta, \bar{q}_z) = f \times B = f \times \eta \bar{H} = \eta \left(0, \frac{H_x^{\,2}}{R_i^{\,2}} \frac{\partial^2 V_i}{\partial X^2} + H_x^{\,2} \cdot \frac{\partial^2 V_i}{\partial X^2}, \; H_x^{\,2} \cdot \frac{\partial^2 Y_i}{\partial X^2} \right), \tag{12c}$$

where q_X, q_θ and q_Z express the Lorentz force along the *X*, *θ* and *Z* direction as follows (Fig. 3)

$$\bar{q}_x = 0, \tag{12d}$$

$$\bar{q}_\theta = \eta \left(\frac{H_x^{\,2}}{R^2} \cdot \frac{\partial^2 v_i}{\partial x^2} + H_x^{\,2} \cdot \frac{\partial^2 v_i}{\partial x^2} \right), \tag{12e}$$

$$\bar{q}_z = \eta \left(H_x^{\,2} \cdot \frac{\partial^2 Y}{\partial X^2} \right), \tag{12f}$$

The external force q_{mag} consists of the Lorentz force $\bar{q}_Z$ due to the longitudinal magnetic field and the distributed transverse force F_s due to the effect of surface tension, That is

$$q_{(mag)} = \bar{q}_Z(x) + F_s, \tag{13}$$

where the Lorentz force $\bar{q}_Z$ is defined in Eq. (12f), **(**Li *et al*. 2016), and the distributed transverse force F_s can be defined as (Lei 2012)

$$F_s = \left(H_s \cdot \frac{\partial^2 Y}{\partial X^2} \right), \tag{14}$$

Here η is the magnetic permeability, H_x is the component of the longitudinal magnetic field vector exerted on the SWCNTs in the *x* direction, and H_s is a constant, defined as

$$H_s = 2\mu(d + h), \tag{15}$$

where μ denotes the residual surface tension. The term $\bar{q}_Z$ is the magnetic force per unit length

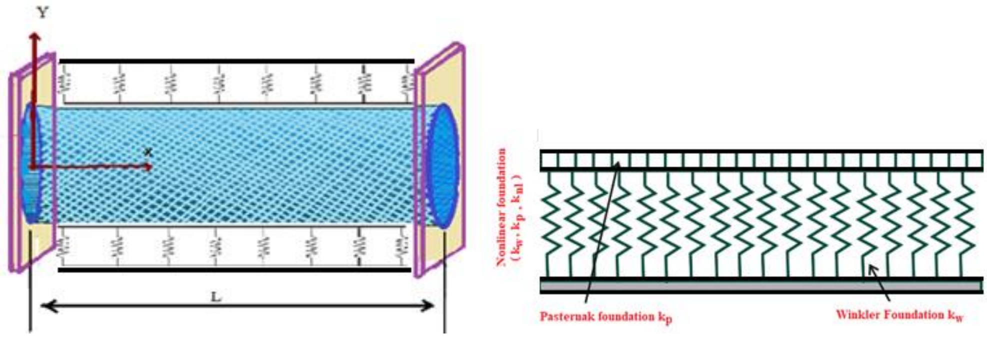

Fig. 3 (a) Geometry of SWCNT in polymer matrix (b) geometry of SWCNT in nonlinear

due to Lorentz force exerted on the tube in *Z*-direction considered form when considering the effect of surface elasticity the effect bending stiffness *EI* should be modified as (Lei 2012, Yan and Jiang 2011)

$$EI^* = EI + Q_s E_s. \tag{16}$$

Where $Q_s = \frac{\pi}{8}(d+h)^3$, Here E_s denotes the surface Young's modulus, *h* is the effective thickness of SWCNTs, and *d* is already denoted in Eq. (8).

3. Euler Bernoulli beam theory (EBT) based on nonlocal relations

The partial differential equation which governs of free vibration of nanotube under the influence of thermal and Lorentz force can be expressed as

$$\frac{\partial \Pi}{\partial X} + N_t \frac{\partial^2 Y}{\partial X^2} + q_{(mag)} + f(x) = \rho A \frac{\partial^2 Y}{\partial t^2}\ . \tag{17}$$

where *f*(*x*) is the interaction pressure per unit axial length between the nanotube and the surrounding elastic medium. *A* is the cross section of CNT. The resultant shear force Π on the cross section of the nanotube is defined in the following equilibrium equation.

$$\Pi = \frac{\partial M}{\partial X}. \tag{18}$$

N_t denotes the temperature dependent axial force with thermal expansion coefficient *a*. This constant force is defined as (Narendar *et al*. 2011)

$$N_t = -EA\alpha T. \tag{19}$$

where *A* is the cross section of nanotube and *T* is the temperature change. The longitudinal magnetic flux due to Lorentz force exerted on the tube in *Z* direction is represented by the term q_Z and is read from (Murmu *et al*. 2012)

$$q_z = \eta A H_x^{\ 2} \frac{\partial^2 Y}{\partial X^2} - EA\alpha T. \tag{20}$$

where H_x is the magnetic field strength and η is the magnetic permeability. For the Euler -beam theory the resultant bending moment M in Eq. (18) can be taken as follows

$$M = \int_A z\sigma_{xx} dA, \tag{21}$$

where σ_{xx} is the nonlocal axial stress defined by nonlocal continuum theory. The constitutive Eq. (6) of a homogeneous isotropic elastic solid in non-local form for one-dimensional nanotube is taken as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial X^2} = E\varepsilon_{xx}, \tag{22}$$

where E is the Young's modulus of the tube, ε_{xx} is the axial strain, $(e_0 a)$ is a nonlocal parameter which represents the impact of nonlocal scale effect on the structure. a is an internal characteristic length. The nonlocal relations in Eq. (22) can be written with temperature environment as follows

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial X^2} = E\varepsilon_{xx} - E\alpha T. \tag{23}$$

In the context of Euler -Bernoulli beam model, the axial strain ε_{xx} for small deflection is defined as

$$\varepsilon_{xx} = -z \frac{\partial^2 Y}{\partial X^2}, \tag{24}$$

where z is the transverse co-ordinate in the positive direction of deflection. By using Eqs. (23)-(24), in Eq. (21), the bending moment M can be expressed as

$$M - (e_0 a)^2 \left[\frac{\partial^2 M}{\partial X^2} \right] = EI^* \frac{\partial^2 Y}{\partial X^2}, \tag{25}$$

where $I = \int_A z^2 dA$ is the moment of inertia. By substituting Eq. (25) into Eq. (17), the nonlocal bending moment M and shear force Π can be expressed as follows

$$M - (e_0 a)^2 \left[(\rho A) \frac{\partial^2 Y}{\partial t^2} + q_{(mag)} - f(x) + EA\alpha T \right] = EI^* \frac{\partial^2 Y}{\partial X^2}, \tag{26}$$

$$\Pi - (e_0 a)^2 \left[(\rho A) \frac{\partial^3 Y}{\partial X^2 \partial t^2} + \frac{\partial^2 q_{(mag)}}{\partial X^2} - \frac{\partial f(x)}{\partial X} + EA\alpha T \right] = EI^* \frac{\partial^3 Y}{\partial X^3}, \tag{27}$$

For the transverse vibration, the equation of motion (17) can be expressed under distributed pressure and thermal interaction with surrounding polymer elastic medium as

$$\begin{aligned} f(x) &= EI^* \frac{\partial^4 Y}{\partial X^4} + EA\alpha T \frac{\partial^2 Y}{\partial x^2} + (\rho A) \frac{\partial^2 Y}{\partial t^2} + q_{(mag)} \frac{\partial^2 Y}{\partial x^2} + F_s \frac{\partial^2 Y}{\partial X^2} \\ &- \left((e_0 a)^2 \left(EA\alpha T \frac{\partial^4 Y}{\partial X^4} + q_{(mag)} \frac{\partial^4 Y}{\partial X^4} + F_s \frac{\partial^4 Y}{\partial X^4} - \frac{\partial^2 f(x)}{\partial X^2} \right) \right). \end{aligned} \tag{28}$$

The pressure per unit axial length acting on the surface of the tube due to the surrounding elastic medium, can be described by a Winkler-Pasternak type model (Barati 2018)

$$f(x) = -(-K_w + K_p + K_{nl}). \tag{29}$$

where K_w, K_p, and K_{nl} are Winkler, Pasternak and nonlinear constant foundation. Introduction of Eq. (29) into Eq. (28) yields

$$\begin{aligned} & EI^* \frac{\partial^4 Y}{\partial X^4} + EA\alpha T \frac{\partial^2 Y}{\partial X^2} + \rho A \frac{\partial^2 Y}{\partial t^2} + \left(\eta A H_x^{\ 2} \frac{\partial^2 Y}{\partial X^2} + H_x \frac{\partial^2 Y}{\partial X^2} \right) \\ & -\left(e_0 a\right)^2 \left(EA\alpha T \frac{\partial^4 Y}{\partial X^4} + (\rho A + (K_w - K_p - K_{nl})) \frac{\partial^2 Y}{\partial X^2} + \left(\eta H_x^{\ 2} . \frac{\partial^4 y}{\partial X^4} - H_x \frac{\partial^4 Y}{\partial X^4} \right) \right) = (K_w - K_p - K_{nl}). \end{aligned} \tag{30}$$

4. Ultrasonic wave solution

Eq. (28) can be transformed in to frequency domain using Fourier transformation Narender (2010)

$$Y(x,t) = \sum_{n=1}^{N} \hat{Y}(x) e^{-j(kn-\omega_n t)}. \tag{31}$$

where $\hat{Y}$ is the amplitude of the wave motion, $j = \sqrt{-1}$, k is the wave number ω_n the circular frequency of sampling point and N is the Nyquist frequency. The sampling rate and the number of sampling points should be sufficiently large to have relatively good resolution of both high and low frequencies respectively. Substitution of Eq. (31) into Eq. (29), we get

$$\begin{aligned} & \sum_{n=1}^{N} \left[\left(EI^* \frac{\partial^4 \hat{Y}}{\partial X^4} + EA\alpha T \frac{\partial^2 \hat{Y}}{\partial X^2} + \rho A \frac{\partial^2 \hat{Y}}{\partial t^2} + \left(\eta A H_x^{\ 2} \frac{\partial^2 \hat{Y}}{\partial X^2} + H_x \frac{\partial^2 \hat{Y}}{\partial X^2} \right) \right. \right. \\ & \left. \left. -\left(e_0 a\right)^2 \left(EA\alpha T \frac{\partial^2 \hat{Y}}{\partial X^2} + (\rho A + (K_w - K_p - K_{nl})) \frac{\partial^2 \hat{Y}}{\partial X^2} - \left(\eta A H_x^{\ 2} \frac{\partial^2 \hat{Y}}{\partial X^2} + H_x \frac{\partial^2 \hat{Y}}{\partial X^2} \right) \right) \right) + (K_w - K_p - K_{nl}) \right] e^{i\omega_n t} = 0, \end{aligned} \tag{32}$$

This equation must be satisfied for each N and hence can be written as the ordinary differential equation in single variable X as

$$\begin{aligned} & \sum_{n=1}^{N} \left[\left(EI^* \frac{\partial^4 \hat{Y}}{\partial X^4} + EA\alpha T \frac{\partial^4 \hat{Y}}{\partial X^4} + \rho A \frac{\partial^2 \hat{Y}}{\partial t^2} + \left(\eta A H_x^{\ 2} \frac{\partial^2 \hat{Y}}{\partial X^2} + H_x \frac{\partial^2 \hat{Y}}{\partial X^2} \right) \right. \right. \\ & \left. \left. -\left(e_0 a\right)^2 \left(\begin{matrix} (\rho A + (K_w - K_p - K_{nl})) \dfrac{\partial^2 \hat{Y}}{\partial X^2} \\ -\left(\eta A H_x^{\ 2} \dfrac{\partial^2 \hat{Y}}{\partial X^2} + H_x \dfrac{\partial^2 \hat{Y}}{\partial X^2} \right) + EA\alpha T \dfrac{\partial^2 \hat{Y}}{\partial X^2} \end{matrix} \right) \right) + (K_w - K_p - K_{nl}) \right] e^{i\omega_n x} = 0, \end{aligned} \tag{33}$$

The above equation must full filled for each values for small n and can be written in the following with single variable X. Eq. (32) can be reduced as

$$\begin{bmatrix} EI^{*}\dfrac{\partial^4 \hat{Y}}{\partial X^4}+EA\alpha T\dfrac{\partial^4 \hat{Y}}{\partial X^4}+\left(\eta AH_x^{\,2}\dfrac{\partial^2 \hat{Y}}{\partial X^2}+H_x\dfrac{\partial^2 \hat{Y}}{\partial X^2}\right)-\dfrac{\partial^2}{\partial t^2}(\rho A) \\ \left[\left(\rho A+(K_w-K_p-K_{nl})+EA\alpha T\right)\left(\eta AH_x^{\,2}\dfrac{\partial^2 \hat{Y}}{\partial X^2}-H_x\dfrac{\partial^2 \hat{Y}}{\partial X^2}\right)(e_0a)^2\dfrac{\partial^2 \hat{Y}}{\partial X^2}\right]+(K_w-K_p-K_{nl}) \end{bmatrix}=0, \tag{34}$$

The dimensionless variables are defined as

$$\frac{X}{L}=x,\quad \frac{Y}{L}=y,\quad \alpha_i=\frac{I_i}{l},\quad \tau=\frac{e_0a}{l},\quad K_w=\frac{k_wL^4}{EI^*},$$
$$K_p=\frac{k_pL^4}{EI^*},\quad K_{nl}=\frac{k_{nl}L^4}{EI^*},\quad \eta=\frac{1}{\left(1+EA\alpha T\right)},\quad \bar{N}_T=\frac{N_tL^2}{EI^*}, \tag{35}$$

substituting $\hat{Y}(x)=\hat{Y}e^{-i\omega x}$ into Eq. (35) employing Eq. (34) yields

$$\left(1+EA\alpha T-\eta AH_x^{\,2}\right)\frac{\partial^4 \hat{Y}}{\partial x^4}+\left[EA\alpha T-\eta AH_x^{\,2}-(k_w+k_p+k_{NL})+\tau^2\right]\frac{\partial^2 \hat{Y}}{\partial x^2}$$
$$+2i\rho A\frac{\partial \hat{Y}}{\partial x}-[\rho A+(k_w+k_p+k_{nl})]=0. \tag{36}$$

For non-trivial solution of the wave amplitude $\bar{Y}$ implies that,

$$\bar{Y}(x,t)=ye^{-ik_nx}. \tag{37}$$

Substituting Eq. (37) into Eq. (36), the following equation is obtained for non-trivial solution of the wave amplitude y

$$\left(1+EA\alpha T-\eta AH_x^{\,2}\right)k_n^{\,4}+\left[EA\alpha T-\eta aH_x^{\,2}+(k_w-k_p-k_{nl})+\tau^2\right]k_n^{\,2}$$
$$+\left(2i\rho A\right)k_n-[\rho A+(k_w-k_p-k_{nl})]=0. \tag{38}$$

which represents the characteristic equation for a continuum structure (ECS) coupled with surrounding medium of an SWCNT

5. Boundary conditions

Here, an analytical solution of the governing equations for vibration of nanobeam having simply-supported (S-S) and clamped-clamped (C-C) boundary condition is presented which they are given as:

5.1 Simply supported SWCNT

The boundary conditions for the simply supported problem are $(X)=(0,\ L)$.

$$Y(x)|_{X=0}=0\ ,$$

$$M(X)=\left(-EI^{*}\frac{\partial^2 Y(X)}{\partial X^2}+(e_0a)^2\left[(\rho A)\frac{\partial^2 Y(X)}{\partial t^2}+q_{(mag)}\frac{\partial^2 Y(X)}{\partial X^2}-f(x)\frac{\partial^2 Y(X)}{\partial X^2}+EA\alpha T\frac{\partial^2 Y(X)}{\partial X^2}\right]\right)_{X=0}=0,$$

$$Y(x)|_{X=L}=0\ ,$$

$$M(X)=\left(-EI^{*}\frac{\partial^2 Y(X)}{\partial X^2}+(e_0a)^2\left[(\rho A)\frac{\partial^2 Y(X)}{\partial t^2}+q_{(mag)}\frac{\partial^2 Y(X)}{\partial X^2}-f(x)\frac{\partial^2 Y(X)}{\partial X^2}+EA\alpha T\frac{\partial^2 Y(X)}{\partial X^2}\right]\right)_{X=L}=0,$$

$$Y(x)|_{X=0}=0\ ,$$

$$\Pi(X)=\left(-\left(EI^{*}\ \frac{\partial^3 Y}{\partial X^3}\right)+(e_0a)^2\left[(\rho A)\frac{\partial^3 Y}{\partial X^2\partial t^2}+\frac{\partial^2 q_{(mag)}}{\partial X^2}-\frac{\partial f(x)}{\partial X}+EA\alpha T\right]\right)_{X=0}=0\,,$$

$$Y(x)|_{X=L}=0\ ,$$

$$\Pi(X)=\left(-\left(EI^{*}\frac{\partial^3 Y}{\partial X^3}\right)+(e_0a)^2\left[(\rho A)\frac{\partial^3 Y}{\partial X^2\partial t^2}+\frac{\partial^2 q_{(mag)}}{\partial X^2}-\frac{\partial f(x)}{\partial X}+EA\alpha T\right]\right)_{X=L}=0\,. \tag{39}$$

5.2 Clamped-clamped SWCNT

Assume the case where both the ends of the beam are clamped and are subjected to axial compressive load. The boundary conditions for this case are given as

$$Y(x)|_{X=0}=0\ ,\quad \frac{\partial Y(X)}{\partial X}|_{X=0}=0\ ,\quad Y(x)|_{X=L}=0\ ,\qquad \frac{\partial Y(X)}{\partial X}|_{X=L}=0\ , \tag{40}$$

6. Numerical results and discussion

In this section, the ASWCNT embedded in a polymer matrix subjected to magneto -thermo elastic forces is considered as an example for the nonlinear vibration analysis. The geometrical and material parameter taken for the numerical verification is shown in Table 1. Tables 2-4 exhibit the numerical results of the natural frequencies for different Non local constants and values of various foundations parameters. From these tables it is observed that the frequencies are falling when the non local scale values rises in both C-C and S-S boundary conditions. The result also show that as the foundation stiffness increases the magnitude of frequency also increases. The natural frequency of simply supported and clamped CNT is calculated at different thermal parameter and nonlocal values in Table 5. From these results it is observed that as the thermal parameter grows the frequency also increases but the small scale effects reduces the values of frequency in both boundary conditions. Further the non local boundary rises the frequency values while the local boundary amplifies the natural frequency via thermal environment. In Table 6, the natural frequency values are reported for different densities and Poisson ratios. As shown the density and Poisson ratio variation affects the natural frequency with below 2%. Table 7 present the comparative study between the numerical results of maximum transverse deflection of C-C CNT with and without surface effect. Results predict the reasonable agreement with the literature.

Table 1 Material properties Lee and Chang (2009), Bessegheir *et al*. (2011)

Materials	PZT
EI	$1.1122\times10^{-25}\ N\,m^9$
α^0	$-1.5\times10^{-6}\ C^{-1}$
ρ	$2.3\ \ g/cm^3$
e_0	$0.31\ \ nm$
a	$0.142 N/m$
E_s	$35.3 N/m$
μ	$4\pi\times10^{-7} N/m$
H_x	$2\times10^8 A/m$

Table 2 Nonlocal constant and Winkler foundation effects on frequency of nanotube (L/h=10) (K_p=0), (K_{nl}=0)

	K_w=25	K_w=25	K_w=25	K_w=25
τ	C-C	C-C	S-S	S-S
0.5	0.5142	1.5609	1.3239	1.3701
1	1.2204	1.2310	1.0926	1.0970
1.5	0.9665	0.9721	0.8442	0.8663
2	0.6416	0.6776	0.5919	0.6116

Table 3 Nonlocal constant and Pasternak foundation effects on frequency of nanotube (L/h=10) (K_w=0), (K_{nl}=0)

	K_p=25	K_p=50	K_p=25	K_p=50
τ	C-C	C-C	S-S	S-S
0.5	1.5651	1.6725	1.9367	1.9583
1	1.3495	1.3576	1.8640	1.9356
1.5	1.1019	1.1117	0.7296	0.8345
2	0.9543	0.9656	0.7296	0.7712
2.5	0.7792	0.7930	0.7149	0.7188

Table 4 Non local constant and nonlinear foundation effects on frequency nanotube (L/h=10) (K_w=0), (K_p=0)

	K_{nl}=25	K_{nl}=50	K_{nl}=25	K_{nl}=50
τ	C-C	C-C	S-S	S-S
0.5	1.2321	1.2541	1.1512	1.2349
1	1.1881	1.2101	0.9926	1.0704
1.5	1.2221	1.1441	0.8458	0.9177
2	1.0121	1.0141	0.7469	0.7671
2.5	0.9681	0.9901	0.6404	0.6475

Table 5 Natural frequency (THz) of a CNT in both local and nonlocal boundary conditions

	α^0=−1.5×10^{-6} C^{-1}		α^0=−2.0×10^{-6} C^{-1}		α^0=−2.5×10^{-6} C^{-1}	
τ	LBC	NBLC	LBC	NBLC	LBC	NBLC
0	0.0180	0.0178	0.0176	0.0173	0.0256	0.0259
0.5	0.0145	0.0142	0.0169	0.0167	0.0131	0.0229
1.	0.0134	0.0131	0.0150	0.0148	0.0212	0.0214
1.5	0.0039	0.0027	0.0116	0.0113	0.0207	0.0205

Table 6 Natural frequency (THz) of a S-S CNT for different density poisson ratios

	Density			Poisson's ratio		
	L/R	ρ_1=2.16	ρ_2=2.3	ρ_3=2.7	v_1	v_2
1	1.1634	1.0575	0.8226	1.0618	1.0604	1.0527
2	0.7977	0.7821	0.7664	0.7593	0.7554	0.7546
3	0.4114	0.4087	0.4002	0.4964	0.4924	0.4918
4	0.3498	0.3382	0.3157	0.3454	0.3453	0.3396
5	0.2539	0.2459	0.2391	0.2377	0.2374	0.2362
6	0.1913	0.1852	0.1790	0.1947	0.1931	0.1927
7	0.1513	0.1435	0.1353	0.1295	0.1287	0.1268
8	0.1435	0.1353	0.1265	0.0904	0.0863	0.0804
9	0.1171	0.1069	0.0956	0.0690	0.0633	0.0629
10	0.0976	0.0869	0.0777	0.0438	0.0432	0.0425
15	0.0828	0.0676	0.0478	0.0216	0.0213	0.0199

Table 7 Comparison of maximum transverse deflection in C-C nanotube incorporating Surface effects (Basutkar *et al.* 2019)

	Basutkar *et al.*		Author	
(L/h)	($S. E$)	($S. E$≠0)	($S. E$=0)	($S. E$≠0)
10	0.6343	0.6334	0.6396	0.6363
15	0.9472	0.9459	0.9450	0.9432
20	1.2550	1.2530	1.2934	1.2914

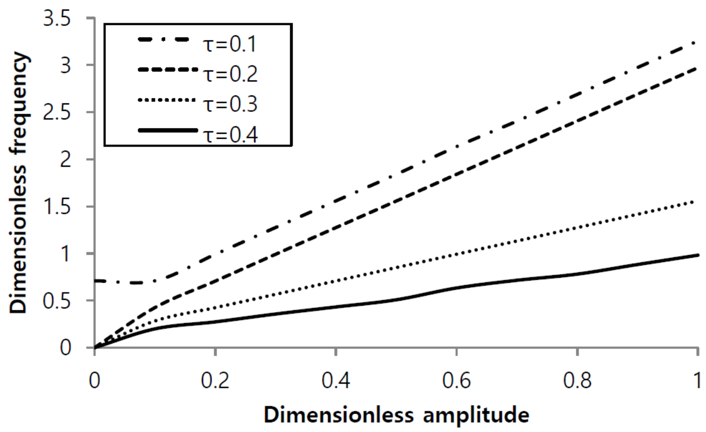

Fig. 4 Graph of dimensionless frequency versus dimensionless amplitude for various non-local parameter (L/h=10, V=0, K_w= K_p=20, ΔT=20)

Figs. 4-5 are investigated the effect of dimensionless frequency versus non dimensional amplitude for various non-local parameters through L/h=10-20, V=0, K_p=20, ΔT=20, respectively and it is found that increasing value of amplitude caused increase of dimensionless frequency. When nonlocal parameter arise the magnitude of frequency decreases, and we understand from this subject that nonlocal values has significant role under frequency. But a softening behavior is observed in frequency due to the rise in slenderness in Fig. 5. The variations of the dimensionless frequency of

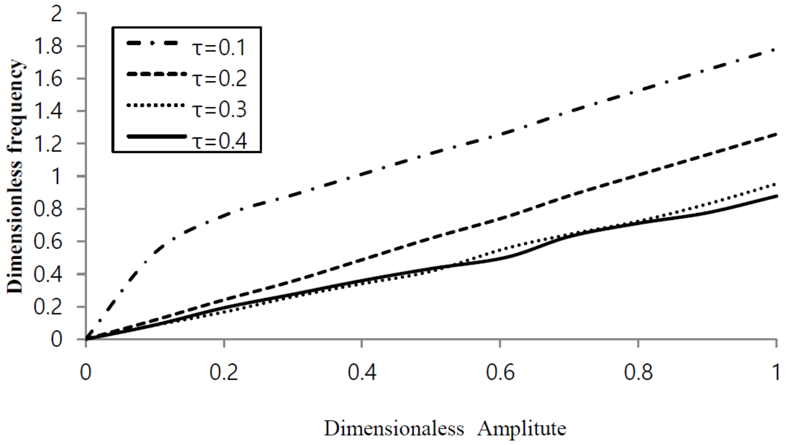

Fig. 5 Graph of dimensionless frequency versus dimensionless amplitude for various non-local parameter (L/h=20, V=0, K_w=K_p=20, ΔT=20)

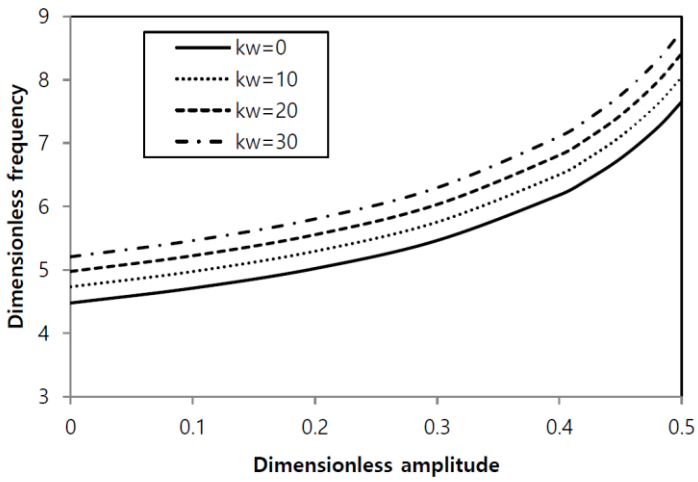

Fig. 6 Graph of dimensionless frequency versus dimensionless amplitude for various (L/h=1 0, V=5, K_p=20, ΔT=20)

nanotube versus the non dimensional amplitude via various Winkler parameters for L/h=10, V=0, K_p=20, ΔT=20 and 30, are shown in Figs. 6 and 7, respectively. It is found from this figure that regardless of the magnitude of Winkler's parameter, the dimensionless frequency increase with the increase of non dimensional amplitude, from this it is referred that the stiffness of the nanotube is hardening through this increment. It must be mentioned that at a constant electric voltage the increase of dimensionless frequency with non dimensionless amplitude measurement with a higher rate of temperature.

Variation of dimensionless frequency of nanotube with respect to non-dimensional amplitude via various voltage values with L/h=10-20, K_p=K_w=20-30 and ΔT=20, are presented in Figs. 8-9. It is found; rise in non-dimensional amplitude caused that increasing of dimensionless frequency of nanotube through various voltage values. So the axial tensile and compressive forces produced in the nanotubes via the constructed positive and negative voltages, respectively. In addition, it is

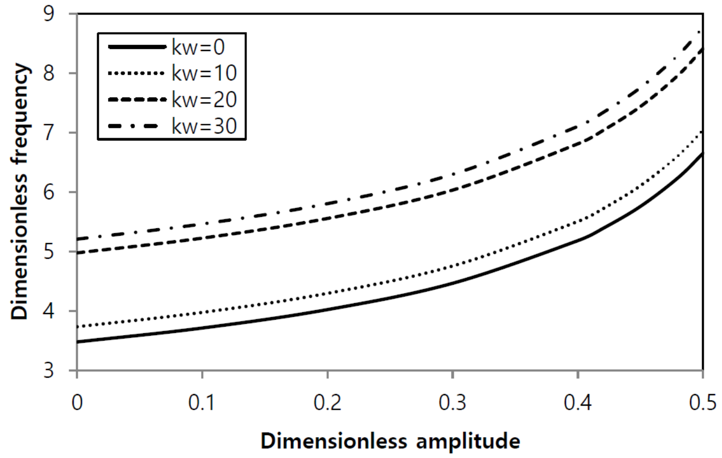

Fig. 7 Graph of dimensionless frequency versus dimensionless amplitude for various for various Pasternak foundation values (L/h=10, V=5, K_p=20, ΔT=30)

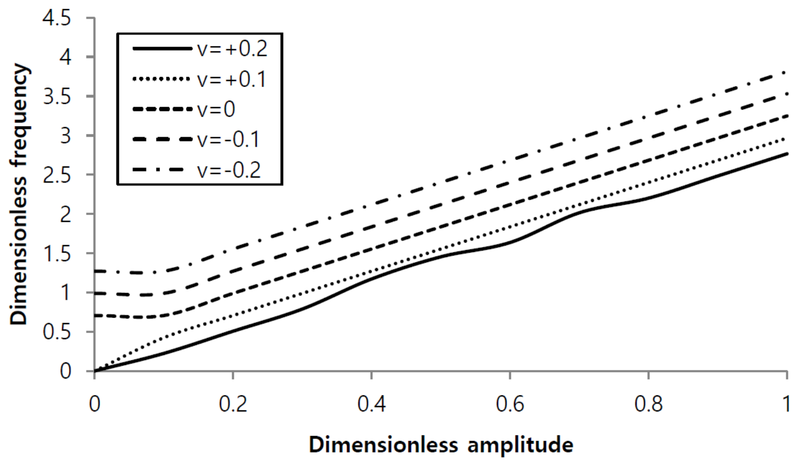

Fig. 8 Graph of dimensionless frequency versus dimensionless amplitude for various for Various electric voltages (L/h=10, K_w=K_p=20, ΔT=20)

lightly observed that the dimensionless frequency is lightly dependent of slenderness ratio and foundation parameter for constant temperature value (ΔT=20). The effect of the dimensionless frequency versus the non-dimensional amplitude of nanotube for various values of nonlinear foundation parameter is verified in Figs. 10-11 for the variant L/h=10-20, K_p=K_w=20-30 and ΔT=20. One can observe that the dimensionless frequency rises in a wave propagation trend through increasing non dimensional amplitude when L/h=10, K_p=K_w=20 but in the increasing values of slenderness and foundation parameters the dimensionless frequency is increasing with oscillation and crossing energy exchange trend. It is found to the fact that an increase foundation parameters and slenderness yields increase of magnitude in the stiffness of the nanotube. Figs. 12-13, demonstrates the variation of bending moment along the first three modes versus length of the nanotube for L/h=10, V=0, K_w=K_p=20, ΔT=20, ΔH=10. It is referred from these figures that as the length grows the bending moment is attaining tensile and compressive nature while growing through

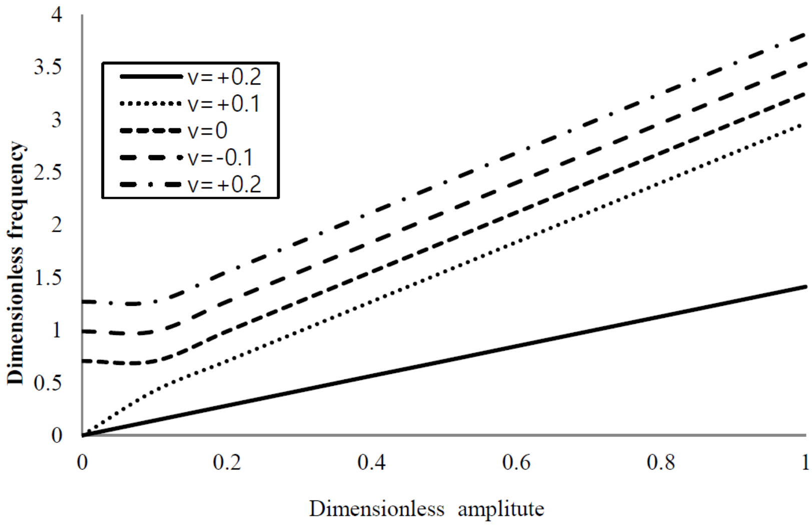

Fig. 9 Graph of dimensionless frequency versus dimensionless amplitude for various for various electric voltages (L/h=20, K_w=K_p=30, ΔT=20)

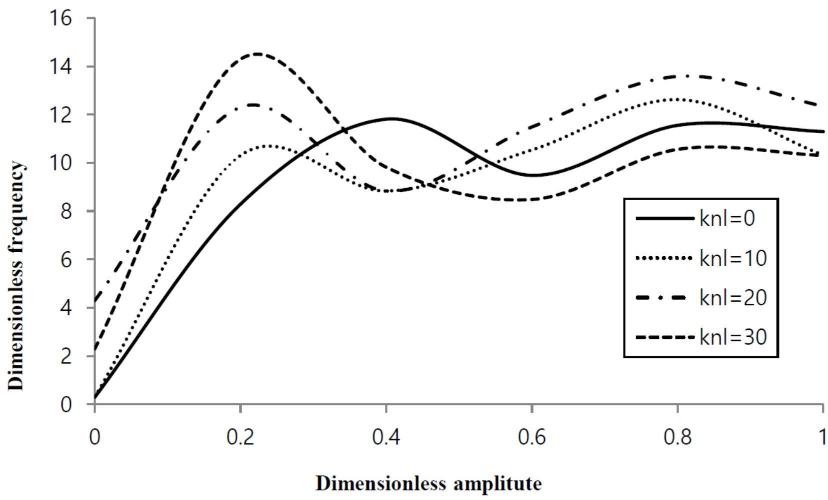

Fig. 10 Graph of dimensionless frequency versus dimensionless amplitude for various for various nonlinear foundation values (L/h=10, V=0, K_w=K_p=10, ΔT=10)

higher modes. Also, the influence of magnetic field effect is observed in the wave trend.

7. Conclusions

This paper presents the study of nonlinear magneto-thermo elastic waves in an armchair single-walled carbon nanotube resting on polymer matrix via Euler beam theory. The analytical formulation is developed based on Eringen's nonlocal elasticity theory to account small scale effect. This medium is embedded in a nonlinear foundation supported by Winkler-Pasternak model. The solution

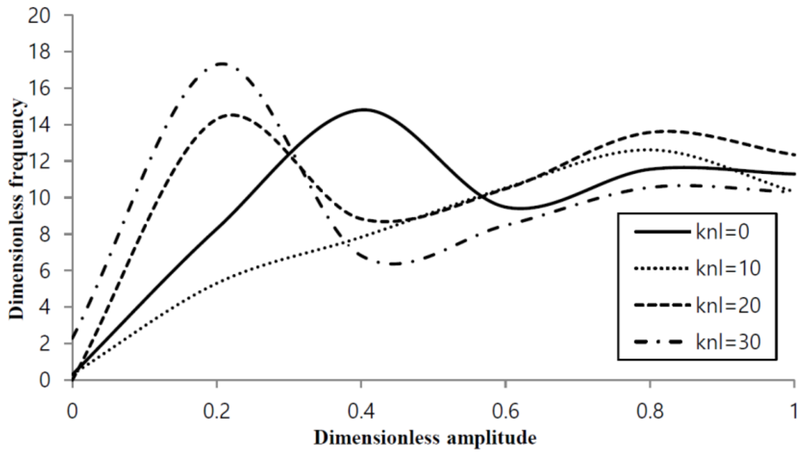

Fig. 11 Graph of dimensionless frequency versus dimensionless amplitude for various for various for various nonlinear foundation (*L/h*=20, *V*=0, K_w=K_p=20, Δ*T*=20)

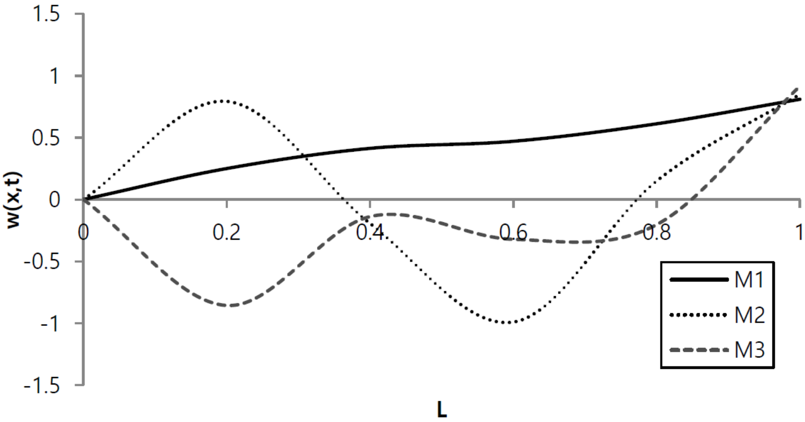

Fig. 12 Graph of bending moment versus length for various modes (*L/h*=10, *V*=0, K_w=K_p=20, Δ*T*=20, Δ*H*=10)

is obtained by ultrasonic wave dispersion relations. Parametric work is introduced to scrutinize the influence of the magneto-electro-mechanical loadings, nonlocal parameter, and aspect ratio on the deflection characteristics of nanotube. The findings are summarized as

- It is found that the non local scaling constant amplifies the frequency
- Further it is observed that the increase in the foundation constants raises the stiffness of the medium.
- The result also show that as the dimensionless amplitude enhances the non dimensional frequency
- The structure is able to attain higher frequency once the edge condition is C-C followed by S-S.
- It was observed that embedding the structure on an elastic foundation is able to improve the dynamic behavior of the armchair nano tube. Moreover, it was understood that the Pasternak parameter can affect the structure more than the Winkler parameter.

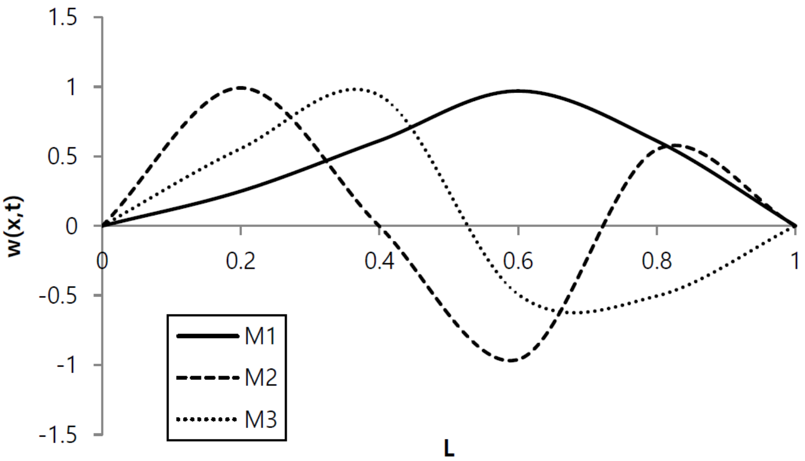

Fig. 13 Graph of bending moment versus length for various modes (*L/h*=10, *V*=0, K_w= K_p=20, Δ*T*=20, Δ*H*=20)

• It is noticed that the natural frequency is arrived below 1% in both local and non local boundary conditions in the presence of temperature coefficients.
• It is found that, the density and Poisson ratio variation affects the natural frequency with below 2%.
• The results presented in this study can provide mechanism for the study and design of the nano devices like component of nano oscillators, micro wave absorbing, nano-electron technology and nano-electro- magneto-mechanical systems (NEMMS) that make use of the wave propagation properties of armchair single-walled carbon nanotubes embedded on polymer matrix.

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