

# Deformation in a homogeneous isotropic thermoelastic solid with multi-dual-phase-lag heat & two temperature using modified couple stress theory

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**Abstract.** The objective of this paper is to study the deformation in a homogeneous isotropic thermoelastic solid using modified couple stress theory subjected to inclined load with two temperatures with multi-dual-phase-lag heat transfer. Uniformly distributed and linearly distributed forces have been applied to find the functionality of the problem. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, conductive temperature, stress components and couple stress are obtained in the transformed domain. A numerical inversion technique has been used to obtain the solutions in the physical domain. The effect of two temperature and inclined load is depicted graphically on the resulted quantities.

**Keywords:** modified couple stress theory; two temperature; isotropic solid; inclined load; Laplace and Fourier transform; couple stress moment tensor

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## 1. Introduction

Classical continuum mechanics predicts the behavior of structures under loads at macro scale, but careful experiments have shown that it deviates in capturing behavior of materials at micro/nano scale.

In classical elasticity theory, forces are transmitted at an infinitesimal element surface as tractions or more specifically force tractions. On the other hand, in size dependent theories, moments are transmitted on an infinitesimal element surface as moment or couple tractions in addition to force tractions. These force and moment tractions can then be represented by tensorial (force) stresses and couple stresses on infinitesimal element. Correspondingly new measures of deformation, such as curvatures, are presented in addition to strains. First mathematical model to examine the material with couple stresses was presented by Cosserat and Cosserat (1909). Displacements and independent rotations, known as microrotations, were used as the kinematical quantities. Their work was further revived by Mindlin (1964), Eringen (1999), Nowacki (1986) and Chen and Wang (2001). Another branch of theories, known as second gradient or strain gradient theories were developed by Mindlin and Eshel (1968), Yang *et al.* (2002) and Lazar *et al.* (2005). These involve gradients of strains,

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- Southampton, Boston, U.S.A., 295-308.
- Tzou, D.Y. (1995), "A unified field approach for heat conduction from macro to micro scales", *J. Heat Transfer*, **117**(1), 8-16. <https://doi.org/10.1115/1.2822329>.
- Vlase, S., Marin, M., Öchsner, A. and Scutaru, M.L. (2019), "Motion equation for a flexible one-dimensional element used in the dynamical analysis of a multibody system", *Continuum Mechanics and Thermodynamics*, **31**(3), 715-724. <https://doi.org/10.1007/s00161-018-0722-y>.
- Voigt, W. (1887), *Theoretische Studien über die Elasticitätsverhältnisse der Krystalle (Theoretical studies on the elasticity relationships of crystals)*, Abhandlungen der Königlich-Gesellschaft der Wissenschaften in Göttingen, Dieterichsche Verlags-Buchhandlung.
- Yang, F., Chong, A.C.M., Lam, D.C.C. and Tong, P. (2002), "Couple stress based strain gradient theory for elasticity", *Int. J. Solids. Struct.*, **39**(10), 2731-2743. [https://doi.org/10.1016/S0020-7683\(02\)00152-X](https://doi.org/10.1016/S0020-7683(02)00152-X).
- Yang, Z. and He, D. (2017) "Vibration and buckling of orthotropic functionally graded micro-plates on the basis of a re-modified couple stress theory", *Results Phys.*, **7**, 3778-3787. <https://doi.org/10.1016/j.rinp.2017.09.026>.
- Zenkour A.M. (2020), "Magneto-thermal shock for a fiber-reinforced anisotropic half-space due to a refined multi-dual-phase-lag model", *J. Phys. Chem. Solids*, **137**, 109213. <https://doi.org/10.1016/j.jpcs.2019.109213>.
- Zihao, Y. and He, D. (2019), "A microstructure-dependent plate model for orthotropic functionally graded micro-plates", *Mech. Adv. Mater. Struct.*, **26**(14), 26, 1218-1225. <https://doi.org/10.1080/15376494.2018.1432794>.