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Stress analysis of laminated composite and sandwich cylindrical shells using a generalized shell theory

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Abstract. In this article, stress analysis of laminated composite and sandwich cylindrical shells is presented using equivalent single layer higher-order shell theories. A theoretical unification of the several shell theories is presented using a generalized shell theory. A theory is independent of the choice of shape function associated with the transverse shear stress. The present theory satisfies traction free conditions on the top and bottom surfaces of the shell. The principle of virtual work is employed to formulate governing equations and boundary conditions. Closed-formed analytical solutions are obtained using the Navier's solution technique. Numerical results are obtained for simply supported laminated composite and sandwich cylindrical shells.

Keywords: a generalized shell theory; laminated; sandwich; cylindrical shells; stress analysis

1. Introduction

Shell type structures carries loads and moments by a combined membrane and bending actions. Therefore, they are widely used in many engineering applications such as aerospace, automotive, civil, mechanical, etc. The stress analysis of laminated composite and sandwich cylindrical shells is of general interest to the researchers. The laminated shells made up of fibrous composite materials have high strength-to-weight and stiffness-to-weight ratios. Due to low transverse shear moduli of fibrous composite materials, transverse shear deformation is more significant in the kinematics of laminated and sandwich shells. Equivalent single layer (ESL) theories are widely used for the analysis of laminated composite beams, plates and shells. ESL beam, plate and shell theories are mainly classified into classical theory (Kirchhoff 1850), first order shear deformation theory (Mindlin 1951) and higher order shear deformation theories (Sayyad and Ghugal 2015, 2017a, Liew *et al*. 2011, Qatu and Asadi 2012). Classical shell theory and first first order shear deformation theory are inaccurate to predict correct bending behaviour of thick shells made up of composite materials in which transverse shear deformation is more significant. Therefore, researchers have developed higher order shear deformation theories to predict accurate bending behaviour of thick laminated

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composite and sandwich shells. Higher order ESL theories are classified into 1) parabolic shear deformation theories (Bhimaraddi 1984, Kant and Khare 1997, Khare *et al*. 2005, Reddy 1984a, b, Ghumare and Sayyad 2019, Naik and Sayyad 2019) 2) trigonometric shear deformation theories (Levy 1877, Touratier 1991, Ghugal and Sayyad 2013, Sayyad and Ghugal 2013, 2014a, b, 2017b, Mantari *et al*. 2012a, Neves *et al*. 2012a) 3) hyperbolic shear deformation theories (Soldatos 1992, Akavci 2010, Neves *et al*. 2012b) 4) exponential shear deformation theories (Karama *et al*. 2009, Aydogdu 2009, Sayyad 2013, Sayyad and Ghugal 2014c) 5) mixed shear deformation theories (Thai *et al*. 2014) etc.

Soldatos and Timarci (1993) and Timarci and Soldatos (1995) have presented static and vibration analysis of laminated composite cylindrical shells using various shear deformation theories. Zenkour and Fares (2000) presented modified first order shear deformation theory for the thermal analysis of laminated composite cylindrical shells. Alibeigloo (2009) presented static and free vibration analysis of angle-ply laminated cylindrical shells using the three-dimensional theory of elasticity by making use of state space differential quadrature method. Khdeir (2011) presented static and free vibration analysis of cross-ply laminated cylindrical panels and circular cylindrical shells using first order shear deformation theory. Solutions are obtained for different sets of boundary conditions using the state space method. Asadi *et al*. (2012) presented static and free vibration analysis of laminated cylindrical shells using first order shear deformation theory. Khalili *et al*. (2012) have developed higher order shear and normal deformation theory for the free vibration analysis of homogenous isotropic circular cylindrical shells. Mantari *et al*. (2011, 2012) and, Mantari and Soares (2012, 2014) presented static and free vibration analysis of laminated composite plates and shells using higher-order shear deformation theories. Viola *et al*. (2013) presented static analysis of doubly-curved shells using various higher order shear deformation theories. Tornabene *et al*. (2012a, b) and Tornabene and Viola (2013) presented a new procedure to obtain through-the-thickness distributions of strains and stresses in laminated composite and sandwich singly-curved and doubly-curved shells. Tornabene *et al*. (2014) presented static flexural analysis of doubly curved anisotropic shell panels using differential quadrature method based on Carrera's unified formulation. Tornabene *et al*. (2015a, b, c, 2016, 2017) presented a new procedure to recover inter-laminar stresses in singly-curved and doubly-curved laminated composite and functionally graded shells.

Carrera and Brischetto (2008, 2009) have presented analysis of laminated composite and sandwich shells using various kinematic models. Carrera *et al*. (2011) presented a comparison of various shell theories for the free vibration analysis of multi-layered, orthotropic cylindrical shells using Carrera's unified formulation. Carrera *et al*. (2013, 2015) have presented mechanical and thermal analysis of cylindrical and doubly-curved laminated composite and sandwich shells using Carrera's unified formulation. Recently, Sayyad and Ghugal (2019) presented static and free vibration analysis of laminated composite and sandwich spherical shells using a generalized shell theory.

1.1 Objectives of present study

A generalized shell theory is presented in this study with the following objectives.

1) To present a theoretical unification of equivalent single layer higher order shell theories using a generalized shell theory. The parabolic shear deformation shell theory (PSDST) of Reddy (1984), trigonometric shear deformation shell theory (TSDST) of Levy (1877), hyperbolic shear deformation shell theory (HSDST) of Soldatos (1992), exponential shear deformation shell theory (ESDST) of Karama *et al*. (2009), first order shear deformation shell theory (FSDST) of Mindlin

(1951) and classical shell theory (CST) of Kirchhoff (1850) are recovered from the present generalized shell theory.

2) To present transverse deflection and through-the-thickness distributions of stresses of laminated composite and sandwich cylindrical shells.

3) To recover transverse shear stresses from 3D stress equilibrium equations of elasticity to ascertain continuity at layer interface/s.

Governing equilibrium equations of the present theory are derived using the principle of virtual work. Closed-formed analytical solutions are obtained using the Navier's solution technique for simply supported boundary conditions of cylindrical shells. A computer code is developed in Fortran 77 to determine displacemnts and stresses. All numerical results are obtained for isotropic, laminated composite and sandwich cylindrical shells and are presented in non-dimensional form. Numerical results for the plates are compared with 3D elasticity solutions presented by Pagano (1970) whereas numerical results for the cylindrical shells are compared with previously published results in the literature.

2. Theoretical formulation

2.1 Cylindrical shell under consideration

A cylindrical shell of rectangular planform of width a, length b, thickness h shown in Fig. 1 is used in the theoretical formulation. x, y, z represent laminate axes and 1, 2, 3 represent material axes. $\boldsymbol{R}$ denotes the principal radius of curvature of the middle plane along the chordwise (i.e., x) direction. The planform is bounded by the region $0 \le x \le a,\ 0 \le y \le b$ and $-h/2 \le z \le h/2$. Laminated composite shell is made up of orthotropic composite material and composed of a $\boldsymbol{N}$ number of layers. All layers are assumed to be perfectly bonded together. Laminated shell is subjected to uniform load.

2.2 Kinematics of the present unified shell theory

A generalized shell theory is developed for the stress analysis of laminated composite and

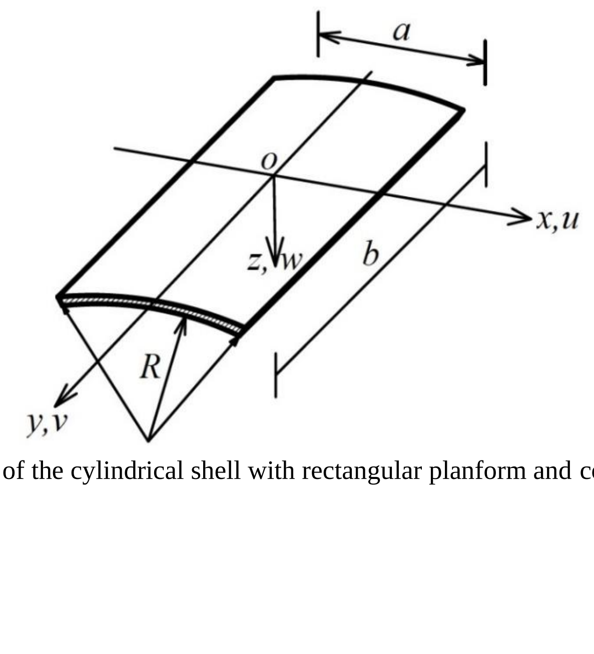

Fig. 1 Geometry of the cylindrical shell with rectangular planform and coordinate systems

sandwich cylindrical shells. In-plane displacement field uses extension, bending and shear components. Transverse displacement is assumed to be function of ***x*** and ***y*** coordinates only i.e., effect of transverse normal strain is neglected. Polynomial, trigonometric, hyperbolic and exponential type transverse shear strain functions are used to account for the effect of transverse shear deformation. Hence, the displacement field of the present generalized shell theory is

$$
\begin{aligned}
u(x,y,z) &= \left(1+\frac{z}{R}\right)u_0(x,y) - z\frac{\partial w_0(x,y)}{\partial x} + \zeta(z)\phi(x,y) \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0(x,y)}{\partial y} + \zeta(z)\psi(x,y) \\
w(x,y) &= w_0(x,y)
\end{aligned}
\tag{1}
$$

Here ***u***, ***v*** and ***w*** are the displacements of any point of shell in the *x*-, *y*- and *z*-directions, respectively; u_0, v_0, w_0 are the displacements of any point on the middle plane of the shell in the *x*-, *y*- and *z*-directions, respectively; $\zeta(z)$ represent shape functions associated with the transverse shear strain along the shell thickness. Following shape functions are used in Eq. (1) to recover classical and higher-order shell theories such as PSDST, TSDST, HSDST, ESDST, FSDST and CST.

PSDST: $\zeta(z) = z\left[1-(4/3)(\bar{z})^2\right],\ \bar{z} = z/h$

TSDST: $\zeta(z) = (h/\pi)\sin(\pi\bar{z})$

HSDST: $\zeta(z) = z\cosh(1/2) - h\sinh(\bar{z})$

ESDST: $\zeta(z) = z\, e^{-2(\bar{z})^2}$

FSDST: $\zeta(z) = z$

CST: $\zeta(z) = 0$

Normal and shear strain components $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz})$ of the present displacement field stated in Eq. (1) are derived using following strain-displacement relations.

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} + \frac{w}{R}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \\
\gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} - \frac{u_0}{R}, \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}
\end{aligned}
\tag{2}
$$

Following are the strain components obtained using Eqs. (1) and (2)

$$
\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + z\varepsilon_x^1 + \zeta(z)\varepsilon_x^2 \\
\varepsilon_y &= \varepsilon_y^0 + z\varepsilon_y^1 + \zeta(z)\varepsilon_y^2 \\
\gamma_{xy} &= \gamma_{xy}^0 + z\gamma_{xy}^1 + \zeta(z)\gamma_{xy}^2 \\
\gamma_{xz} &= \zeta'(z)\gamma_{xz}^0 \\
\gamma_{yz} &= \zeta'(z)\gamma_{yz}^0
\end{aligned}
\tag{3}
$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x} + \frac{w_0}{R}, \quad \varepsilon_x^1 = -\frac{\partial^2 w_0}{\partial x^2}, \quad \varepsilon_x^2 = \frac{\partial \phi}{\partial x}, \quad \varepsilon_y^0 = \frac{\partial v_0}{\partial y},$$

$$\varepsilon_y^1 = -\frac{\partial^2 w_0}{\partial y^2}, \quad \varepsilon_y^2 = \frac{\partial \psi}{\partial y}, \quad \gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}, \tag{4}$$

$$\gamma_{xy}^1 = -2\frac{\partial^2 w_0}{\partial x \partial y}, \quad \gamma_{xy}^2 = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}, \quad \gamma_{xz}^0 = \phi, \quad \gamma_{yz}^0 = \psi$$

2.3 Constitutive relations

The following constitutive relations are used to obtain stress components of the k^{th} layer of the laminated composite and sandwich shells composed of ***N*** number of layers.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^{(k)} \tag{5}$$

where Q_{ij} are the stiffness coefficients and expressed in-terms of engineering constants as follows

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{12} = \frac{\mu_{21}E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \tag{6}$$

$$Q_{66} = G_{12}, \quad Q_{55} = G_{13}, \quad Q_{44} = G_{23}$$

2.4 Governing equations

The principle of virtual work stated in Eq. (7) is used to formulate governing equations.

$$\int_{dV} \sigma_{ij} \delta\varepsilon_{ij}\, dV - \int_{dA} q\,\delta w\, dA = 0 \tag{7}$$

where δ is the variational operator. Eq. (7) can be written in following form

$$\int_V \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{zx}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz} + \tau_{xy}\delta\gamma_{xy}\right) dV - \int_A q(x,y)\,\delta w\, dA = 0 \tag{8}$$

Substituting strain components $\left(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}\right)$ from Eq. (3) into the Eq. (8), one can get Eq. (9).

$$\begin{aligned} \int_{dV} \{ \; & \sigma_x\left[\varepsilon_x^0 + z\varepsilon_x^1 + \zeta(z)\varepsilon_x^2\right] + \sigma_y\left[\varepsilon_y^0 + z\varepsilon_y^1 + \zeta(z)\varepsilon_y^2\right] + \tau_{xy}\left[\gamma_{xy}^0 + z\gamma_{xy}^1 + \zeta(z)\gamma_{xy}^2\right] \\ & + \tau_{xz}\zeta'(z)\gamma_{xz}^0 + \tau_{yz}\zeta'(z)\gamma_{yz}^0 \; \} dV - \int_A q(x,y)\,\delta w\, dA = 0 \end{aligned} \tag{9}$$

After performing integrations with respect to ***z*** coordinate and, introducing the force and moment resultants into Eq. (9), one can get

$$\int_0^b\int_0^a \Big\{ N_x\delta\varepsilon_x^0 + M_x^b\delta\varepsilon_x^1 + M_x^s\delta\varepsilon_x^2 + N_y\delta\varepsilon_y^0 + M_y^b\delta\varepsilon_y^1 + M_y^s\delta\varepsilon_y^2 + N_{xy}\delta\gamma_{xy}^0 + M_{xy}^b\delta\gamma_{xy}^1 + M_{xy}^s\delta\gamma_{xy}^2 + Q_x\delta\gamma_{xz}^0 + Q_y\delta\gamma_{yz}^0 - q(x,y)\delta w_0 \Big\} = 0 \tag{10}$$

where expressions for force and moment resultants can be obtained using following relations. Superscript ***b*** is used for the resultants due to bending whereas superscript ***s*** is used for the resultants due to shear deformation.

$$\begin{aligned}
\{N_x \quad M_x^b \quad M_x^s\}^T &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \{1 \quad z \quad \zeta(z)\}^T \sigma_x^k\, dz \\
\{N_y \quad M_y^b \quad M_y^s\}^T &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \{1 \quad z \quad \zeta(z)\}^T \sigma_y^k\, dz \\
\{N_{xy} \quad M_{xy}^b \quad M_{xy}^s\}^T &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \{1 \quad z \quad \zeta(z)\}^T \tau_{xy}^k\, dz \\
\{Q_x \quad Q_y\}^T &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \{\tau_{xz} \quad \tau_{yz}\}^T \zeta'(z)\, dz
\end{aligned} \tag{11}$$

By substituting the stresses from Eq. (5) into Eq. (11), the following expressions of the force and moment resultants can be obtain

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix}
A_{11} & B_{11} & As_{11} & A_{12} & B_{12} & As_{12} & 0 & 0 & 0 \\
A_{12} & B_{12} & As_{12} & A_{22} & B_{22} & As_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & A_{66} & B_{66} & As_{66} \\
B_{11} & D_{11} & Bs_{11} & B_{12} & D_{12} & Bs_{12} & 0 & 0 & 0 \\
B_{12} & D_{12} & Bs_{12} & B_{22} & D_{22} & Bs_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & B_{66} & D_{66} & Bs_{66} \\
As_{11} & Bs_{11} & Ass_{11} & As_{12} & Bs_{12} & Ass_{12} & 0 & 0 & 0 \\
As_{12} & Bs_{12} & Ass_{12} & As_{22} & Bs_{22} & Ass_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & As_{66} & Bs_{66} & Ass_{66}
\end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^1 \\ \varepsilon_x^2 \\ \varepsilon_y^0 \\ \varepsilon_y^1 \\ \varepsilon_y^2 \\ \gamma_{xy}^0 \\ \gamma_{xy}^1 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{12}$$

and

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} Acc_{55} & 0 \\ 0 & Acc_{44} \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix} \tag{13}$$

where $A_{ij}, B_{ij}, D_{ij}, As_{ij}, Bs_{ij}, Ass_{ij}, Acc_{ij}$ are the cylindrical shell stiffnesses, defined as follows

$$\begin{aligned}
\{A_{ij} \quad B_{ij} \quad D_{ij}\} &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} Q_{ij}^k \{1 \quad z \quad z^2\}\, dz, \qquad (i,j=1,2,3,6), \\
\{As_{ij} \quad Bs_{ij} \quad Ass_{ij}\} &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} Q_{ij}^k\, \zeta(z)\{1 \quad z \quad \zeta(z)\}\, dz, \quad (i,j=1,2,3,6), \\
Acc_{ij} &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} Q_{ij}^k \left[\zeta'(z)\right]^2 dz \qquad (i,j=4,5)
\end{aligned} \tag{14}$$

Following governing equations of the present generalized shell theory are obtained by integrating the Eq. (10) by parts and setting the coefficients of $\delta u_0, \delta v_0, \delta w_0, \delta\phi$ and $\delta\psi$ equal to zero.

$$\begin{aligned}
&\delta u_0: \quad \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \\
&\delta v_0: \quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0 \\
&\delta w_0: \quad \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x \partial y}+\frac{\partial^2 M_y^b}{\partial y}-\frac{N_x}{R}+q=0 \\
&\delta\phi: \quad \frac{\partial M_x^s}{\partial x}+\frac{\partial M_{xy}^b}{\partial y}-Q_x=0 \\
&\delta\psi: \quad \frac{\partial M_y^s}{\partial y}+\frac{\partial M_{xy}^b}{\partial x}-Q_y=0
\end{aligned} \tag{15}$$

The boundary conditions along edges x=0 and x=a obtained are of the following form

$$\begin{aligned}
&N_x=0 && \text{or} && u_0 \text{ is specified} \\
&N_{xy}=0 && \text{or} && v_0 \text{ is specified} \\
&\partial M_x^b/\partial x+2\partial M_{xy}^b/\partial y=0 && \text{or} && \text{w is specified} \\
&M_x^b=0 && \text{or} && \partial w/\partial x \text{ is specified} \\
&M_x^s=0 && \text{or} && \phi \text{ is specified} \\
&M_{xy}^s=0 && \text{or} && \psi \text{ is specified}
\end{aligned} \tag{16}$$

and along y=0 and y=b edges, the boundary conditions are as follows

$$\begin{aligned}
&N_y=0 && \text{or} && v_0 \text{ is specified} \\
&N_{xy}=0 && \text{or} && u_0 \text{ is specified} \\
&\partial M_y^b/\partial y+2\partial M_{xy}^b/\partial x=0 && \text{or} && w \text{ is specified} \\
&M_y^b=0 && \text{or} && \partial w/\partial y \text{ is specified} \\
&M_y^s=0 && \text{or} && \psi \text{ is specified} \\
&M_{xy}^s=0 && \text{or} && \phi \text{ is specified}
\end{aligned} \tag{17}$$

Using expressions of force and moment resultants from Eqs. (12) and (13) into the Eq. (15), the governing equations of the present theory in terms of five unknowns (u_0, v_0, w_0, ϕ and ψ) can be written as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x\partial y}+\frac{A_{11}}{R}\frac{\partial w_0}{\partial x}-B_{11}\frac{\partial^3 w_0}{\partial x^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x\partial y^2}$$
$$+As_{11}\frac{\partial^2 \phi}{\partial x^2}+As_{66}\frac{\partial^2 \phi}{\partial y^2}+\left(As_{12}+As_{66}\right)\frac{\partial^2 \psi}{\partial x\partial y}=0 \tag{18}$$

$$\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}+A_{22}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}-B_{22}\frac{\partial^3 w_0}{\partial y^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2\partial y}+\frac{A_{12}}{R}\frac{\partial w_0}{\partial y}$$
$$+\left(As_{12}+As_{66}\right)\frac{\partial^2 \phi}{\partial x\partial y}+As_{22}\frac{\partial^2 \psi}{\partial y^2}+As_{66}\frac{\partial^2 \psi}{\partial x^2}=0 \tag{19}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}-\frac{A_{11}}{R}\frac{\partial u_0}{\partial x}+B_{22}\frac{\partial^3 v_0}{\partial y^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2\partial y}-\frac{A_{12}}{R}\frac{\partial v_0}{\partial y}+\frac{2B_{11}}{R}\frac{\partial^2 w_0}{\partial x^2}$$
$$-D_{11}\frac{\partial^4 w_0}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0}{\partial y^4}+\frac{2B_{12}}{R}\frac{\partial^2 w_0}{\partial y^2}-\frac{A_{11}}{R^2}w_0+Bs_{11}\frac{\partial^3 \phi}{\partial x^3}$$
$$+\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 \phi}{\partial x\partial y^2}-\frac{As_{11}}{R}\frac{\partial \phi}{\partial x}+Bs_{22}\frac{\partial^3 \psi}{\partial y^3}+\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 \psi}{\partial x^2\partial y}-\frac{As_{12}}{R}\frac{\partial \psi}{\partial y}-q=0 \tag{20}$$

$$As_{11}\frac{\partial^2 u_0}{\partial x^2}+As_{66}\frac{\partial^2 u_0}{\partial y^2}+\left(As_{12}+As_{66}\right)\frac{\partial^2 v_0}{\partial x\partial y}-Bs_{11}\frac{\partial^3 w_0}{\partial x^3}-\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 w_0}{\partial x\partial y^2}+\frac{As_{11}}{R}\frac{\partial w_0}{\partial x}$$
$$+Ass_{11}\frac{\partial^2 \phi}{\partial x^2}+Ass_{66}\frac{\partial^2 \phi}{\partial y^2}-Acc_{55}\phi+\left(Ass_{12}+Ass_{66}\right)\frac{\partial^2 \psi}{\partial x\partial y}=0 \tag{21}$$

$$\left(As_{12}+As_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}+As_{66}\frac{\partial^2 v_0}{\partial x^2}+As_{22}\frac{\partial^2 v_0}{\partial y^2}-Bs_{22}\frac{\partial^3 w_0}{\partial y^3}-\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 w_0}{\partial x^2\partial y}+\frac{As_{12}}{R}\frac{\partial w_0}{\partial y}$$
$$+\left(Ass_{12}+Ass_{66}\right)\frac{\partial^2 \phi}{\partial x\partial y}+Ass_{22}\frac{\partial^2 \psi}{\partial y^2}+Ass_{66}\frac{\partial^2 \psi}{\partial x^2}-Acc_{55}\psi=0 \tag{22}$$

3. Closed-formed analytical solutions for cylindrical shells

Closed-formed analytical solutions for the simply supported laminated composite and sandwich cylindrical shells are obtained using Navier's technique. Following are the simply supported boundary conditions.

$$\begin{gathered}
N_x\left(x=0,x=a\right)=0;\quad N_y\left(y=0,y=b\right)=0\\
v_0\left(x=0,x=a\right)=0;\quad u_0\left(y=0,y=b\right)=0\\
w\left(x=0,x=a,y=0,y=b\right)=0\\
\psi\left(x=0,x=a\right)=0;\quad \phi\left(y=0,y=b\right)=0\\
M_x^b\left(x=0,x=a\right)=0;\quad M_y^b\left(y=0,y=b\right)=0\\
M_x^s\left(x=0,x=a\right)=0;\quad M_y^s\left(y=0,y=b\right)=0
\end{gathered} \tag{23}$$

The uniform transverse load acting on the cylindrical shell is presented in the following form.

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{16q_0}{mn\pi^2}\sin\alpha_1 x\sin\alpha_2 y \tag{24}$$

where, $\alpha_1=m\pi/a$, $\alpha_2=n\pi/a$; q_0 represents the intensity of the load and (m, n) are odd integers. To satisfy simply supported boundary conditions stated in Eq. (23), the unknown variables of the present generalized shell theory are presented in the following Fourier series.

$$\begin{aligned}
\{u_0 \quad \phi\} &= \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\{u_{mn} \quad \phi_{mn}\}\cos\alpha_1 x\sin\alpha_2 y\\
\{v_0 \quad \psi\} &= \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\{v_{mn} \quad \psi_{mn}\}\sin\alpha_1 x\cos\alpha_2 y\\
\{w_0\} &= \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty} w_{mn}\sin\alpha_1 x\sin\alpha_2 y
\end{aligned} \tag{25}$$

where u_{mn}, v_{mn}, w_{mn}, ϕ_{mn}, ψ_{mn} are the unknown coefficients of the respective Fourier series. Using transverse load from Eq. (24), assumed displacement variables from Eq. (25) and governing Eqs. (18)-(22), one can get the following systems of equations.

$$[K]\{\Delta\}=\{F\} \tag{26}$$

Displacements and stresses in laminated composite and sandwich shells are determined using solution of Eq. (26). Elements of stiffness matrix [K] are given below.

$$\begin{aligned}
&K_{11}=A_{11}\alpha_1^2+A_{66}\alpha_2^2,\quad K_{12}=\left(A_{12}+A_{66}\right)\alpha_1\alpha_2,\\
&K_{13}=-\left[\frac{A_{11}}{R}\alpha_1+B_{11}\alpha_1^3-\left(B_{12}+2B_{66}\right)\alpha_1\alpha_2^2\right],\\
&K_{14}=As_{11}\alpha_1^2+As_{66}\alpha_2^2,\quad K_{15}=\left(As_{12}+As_{66}\right)\alpha_1\alpha_2,\quad K_{22}=A_{66}\alpha_1^2+A_{22}\alpha_2^2,\\
&K_{23}=-\left[\frac{A_{12}}{R}\alpha_2+B_{22}\alpha_2^3-\left(B_{12}+2B_{66}\right)\alpha_1^2\alpha_2\right], K_{24}=\left(As_{12}+As_{66}\right)\alpha_1\alpha_2,\\
&K_{25}=As_{66}\alpha_1^2+As_{22}\alpha_2^2,\\
&K_{33}=D_{11}\alpha_1^4+2\left(D_{12}+2D_{66}\right)\alpha_1^2\alpha_2^2+D_{22}\alpha_2^4+\frac{2B_{11}}{R}\alpha_1^2+\frac{2B_{12}}{R}\alpha_2^2+\frac{A_{11}}{R^2},\\
&K_{34}=-\left[Bs_{11}\alpha_1^3+\left(Bs_{12}+2Bs_{66}\right)\alpha_1\alpha_2^2+\frac{As_{11}}{R}\alpha_1\right],\\
&K_{35}=-\left[Bs_{22}\alpha_2^3+\left(Bs_{12}+2Bs_{66}\right)\alpha_1^2\alpha_2+\frac{As_{12}}{R}\alpha_2\right],\\
&K_{44}=Ass_{11}\alpha_1^2+Ass_{66}\alpha_2^2+Acc_{55},\quad K_{45}=\left(Ass_{12}+Ass_{66}\right)\alpha_1\alpha_2,\\
&K_{55}=Ass_{22}\alpha_2^2+Ass_{66}\alpha_1^2+Acc_{55}
\end{aligned} \tag{27}$$

Since the stiffness matrix [K] is a symmetric matrix, $K_{21}=K_{12}$, $K_{13}=K_{31}$, $K_{23}=K_{32}$, $K_{14}=K_{41}$, $K_{24}=K_{42}$, $K_{34}=K_{43}$, $K_{15}=K_{51}$, $K_{25}=K_{52}$, $K_{35}=K_{53}$, $K_{45}=K_{54}$. Elements of displacement and force vectors are as follows.

$$\begin{aligned}
\{\Delta\}&=\{u_{mn} \quad v_{mn} \quad w_{mn} \quad \phi_{mn} \quad \psi_{mn}\}^T\\
\{F\}&=\frac{16}{mn\pi^2}\{0 \quad 0 \quad q_0 \quad 0 \quad 0\}^T
\end{aligned} \tag{28}$$

4. Numerical results and discussions

In this section, displacements and stresses of isotropic, laminated composite and sandwich cylindrical shells under uniform loading is presented. Numerical results are presented in Tables 1 through 4 and graphically in Figs. 2 through 13. The following material properties are used to obtained numerical results:

Material 1

$$E_1 = E_2 = E_3 = 210\ GPa, G_{13} = G_{23} = G_{12} = G = \frac{E}{2(1+\mu)}, \mu_{12} = \mu_{32} = \mu_{31} = \mu = 0.3 \tag{29}$$

Material 2

$$E_1 = E_2 = 0.04,\ E_3 = 0.5,\ G_{13} = G_{23} = 0.06,\ G_{12} = 0.016, \mu_{12} = \mu_{32} = \mu_{31} = 0.25 \tag{30}$$

Material 3

$$\frac{E_1}{E_2} = 25,\ \frac{E_3}{E_2} = 1,\ \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5,\ \frac{G_{23}}{E_2} = 0.2, \mu_{12} = \mu_{13} = \mu_{23} = 0.25 \tag{31}$$

The following non-dimensional forms are used to present the displacements and stresses.

$$\bar{w}\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{100 E_3 h^3}{q_0 a^4} w,\quad (\bar{\sigma}_x, \bar{\sigma}_y)\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{h^2}{q_0 a^2}(\sigma_x, \sigma_y),$$
$$\bar{\tau}_{xy}\left(0,0,\frac{z}{h}\right) = \frac{h^2}{q_0 a^2}\tau_{xy},\quad (\bar{\tau}_{zx})\left(0,\frac{b}{2},\frac{z}{h}\right) = \frac{h}{q_0 a}\tau_{zx},\quad \bar{\tau}_{yz}\left(\frac{a}{2},0,\frac{z}{h}\right) = \frac{h}{q_0 a}\tau_{yz}. \tag{32}$$

where E_3 is modulus of elasticity of middle layer in z-direction. Following three types of cylindrical shells are analyzed in the present study.

1. Isotropic cylindrical shell
2. Laminated cylindrical shell (0°/90° and 0°/90°/0°)
3. Sandwich cylindrical shell (0°/core/0°)

All graphical results are plotted for R/a=5 and a/h=4.

4.1 Recovery of transverse shear stresses

In multilayered laminated composite structures, if interlaminar transverse shear stresses are obtained using the constitutive relations it leads to the discontinuity at the layer interface and thus violates the equilibrium conditions. Therefore, in the present study, the transverse shear stresses are evaluated by direct integration of three-dimensional stress equilibrium equations of theory of elasticity neglecting the body forces.

$$\tau_{xz}^{(k)} = -\sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left(\frac{\partial \sigma_x^{(k)}}{\partial x} + \frac{\partial \tau_{xy}^{(k)}}{\partial y}\right) dz + C_1$$
$$\tau_{yz}^{(k)} = -\sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left(\frac{\partial \sigma_y^{(k)}}{\partial y} + \frac{\partial \tau_{xy}^{(k)}}{\partial x}\right) dz + C_2 \tag{33}$$

Table 1 Non-dimensional transverse displacements (10 $\overline{w}$) for laminated cylindrical shells under uniform load with (a=10h)

R/a	Model	Theory	Isotropic	0°/90°	0°/90°/0°	0°/*core*/0°
0.5	Model 1	PSDST	11.022	5.6951	4.6324	9.6018
	Model 2	TSDST	11.022	5.6947	4.6474	9.6175
	Model 3	HSDST	11.022	5.6947	4.6310	9.5996
	Model 4	ESDST	11.022	5.6927	4.6637	9.6175
	Model 5	FSDST	11.024	5.0992	4.4113	7.6574
	Asadi *et al*. (2012)	FSDTQ	12.108	5.9687	4.4754	---
	Asadi *et al*. (2012)	3D-FEM	12.242	5.9629	4.7425	---
	Tornabene *et al*. (2012)	FSDT-GDQ	---	5.9627	4.4752	---
1	Model 1	PSDST	26.325	12.164	8.1755	13.486
	Model 2	TSDST	26.324	12.157	8.2240	13.519
	Model 3	HSDST	26.325	12.150	8.1710	13.482
	Model 4	ESDST	26.328	12.072	8.2679	13.519
	Model 5	FSDST	26.222	12.394	7.4570	9.8544
	Asadi *et al*. (2012)	FSDTQ	28.333	12.738	7.8589	---
	Asadi *et al*. (2012)	3D-FEM	28.415	12.679	8.6399	---
	Tornabene *et al*. (2012)	FSDT-GDQ	---	12.736	7.8587	---
2	Model 1	PSDST	39.188	16.771	10.064	14.966
	Model 2	TSDST	39.185	16.755	10.138	15.007
	Model 3	HSDST	39.188	16.741	10.057	14.961
	Model 4	ESDST	39.194	16.571	10.203	15.007
	Model 5	FSDST	38.926	17.432	8.9857	10.594
	Asadi *et al*. (2012)	FSDTQ	40.956	17.141	9.5159	---
	Asadi *et al*. (2012)	3D-FEM	40.875	17.009	10.661	---
	Tornabene *et al*. (2012)	FSDT-GDQ	---	17.139	9.5157	---

The transverse shear stresses of k^{th} lamina can be obtained by integrating of Eq. (33) with respect to the thickness coordinate i.e., z. The layerwise integration of Eq. (33) is need to be performed. The integration constants (C_1 and C_2) are determined by imposing the continuity conditions at layer interface as well as shear stress boundary conditions at top and bottom surfaces of the shell.

4.2 Discussion on numerical results

A comparison of non-dimensional transverse displacement for isotropic, laminated composite and sandwich cylindrical shells subjected to uniform load is shown in Table 1. The present results are compared with those presented by Asadi *et al*. (2012) and Tornabene *et al*. (2012). The numerical results are obtained for different *R/a* ratios and *a/h*=0. It can be seen from Table 1 that the present results are in good agreement with those available in the literature for isotropic and laminated cylindrical shells. Numerical results for sandwich cylindrical shells are not presented by Asadi *et al*. (2012) and Tornabene *et al*. (2012).

Values of non-dimensional displacements and stresses for the isotropic cylindrical shell subjected to uniform load are presented in Table 2. Numerical results are obtained using six displacement

Table 2 Non-dimensional displacements and stresses of isotropic cylindrical shell under uniform load with (a=10h)

R/a	Model	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Model 1	PSDST	4.5284	0.2578	0.2571	0.2182	0.4752	0.4747
	Model 2	TSDST	4.5281	0.2580	0.2572	0.2185	0.4742	0.4737
	Model 3	HSDST	4.5284	0.2578	0.2570	0.2182	0.4753	0.4741
	Model 4	ESDST	4.5292	0.2584	0.2576	0.2193	0.4803	0.4789
	Model 5	FSDST	4.4926	0.2562	0.2571	0.2214	0.4841	0.4836
10	Model 1	PSDST	4.6309	0.2756	0.2752	0.2103	0.4834	0.4832
	Model 2	TSDST	4.6305	0.2757	0.2753	0.2105	0.4824	0.4823
	Model 3	HSDST	4.6309	0.2756	0.2751	0.2103	0.4835	0.4827
	Model 4	ESDST	4.6317	0.2761	0.2757	0.2114	0.4885	0.4870
	Model 5	FSDST	4.5933	0.2738	0.2752	0.2135	0.4922	0.4921
20	Model 1	PSDST	4.6572	0.2830	0.2828	0.2052	0.4855	0.4854
	Model 2	TSDST	4.6569	0.2831	0.2829	0.2054	0.4845	0.4845
	Model 3	HSDST	4.6572	0.2830	0.2827	0.2052	0.4856	0.4849
	Model 4	ESDST	4.6580	0.2835	0.2833	0.2063	0.4906	0.4891
	Model 5	FSDST	4.6192	0.2811	0.2828	0.2085	0.4943	0.4943
50	Model 1	PSDST	4.6647	0.2869	0.2868	0.2018	0.4861	0.4860
	Model 2	TSDST	4.6643	0.2870	0.2869	0.2020	0.4851	0.4851
	Model 3	HSDST	4.6647	0.2869	0.2867	0.2018	0.4862	0.4855
	Model 4	ESDST	4.6655	0.2875	0.2874	0.2029	0.4912	0.4899
	Model 5	FSDST	4.6265	0.2850	0.2868	0.2051	0.4949	0.4949
100	Model 1	PSDST	4.6657	0.2881	0.2881	0.2006	0.4862	0.4861
	Model 2	TSDST	4.6654	0.2882	0.2882	0.2008	0.4852	0.4852
	Model 3	HSDST	4.6657	0.2881	0.2881	0.2006	0.4863	0.4863
	Model 4	ESDST	4.6665	0.2887	0.2886	0.2017	0.4913	0.4911
	Model 5	FSDST	4.6275	0.2862	0.2881	0.2039	0.4950	0.4950
∞ (Plate)	Model 1	PSDST	4.6661	0.2893	0.2893	0.1994	0.4862	0.4862
	Model 2	TSDST	4.6657	0.2894	0.2894	0.1996	0.4852	0.4852
	Model 3	HSDST	4.6661	0.2893	0.2893	0.1993	0.4863	0.4863
	Model 4	ESDST	4.6669	0.2898	0.2898	0.2005	0.4913	0.4913
	Model 5	FSDST	4.6279	0.2893	0.2893	0.2027	0.4950	0.4950
	Pagano (1970)	Elasticity	4.4381	0.2873	0.2873	--	0.4949	0.4949
	Sayyad and Ghugal (2014a)	SSNPT	4.4238	0.3049	0.3049	0.1941	0.4944	0.4944

models recovered from the present generalized shell theory. All results are presented for a/h=10 and R/a=5, 10, 20, 50, 100. Material properties defined by Eq. (29) are used. Numerical results show that the non-dimensional displacements and stresses predicted by various shear deformation theories approach each other as the R/a ratio increases. Results obtained by PSDT, TSDT and HSDT are in excellent agreement with each other whereas ESDT slightly overestimates the values. Displacements and stresses obtained for isotropic plate ($R/a\to\infty$) by using all higher-order theories are in excellent agreement with exact elasticity solutions presented by Pagano (1970).

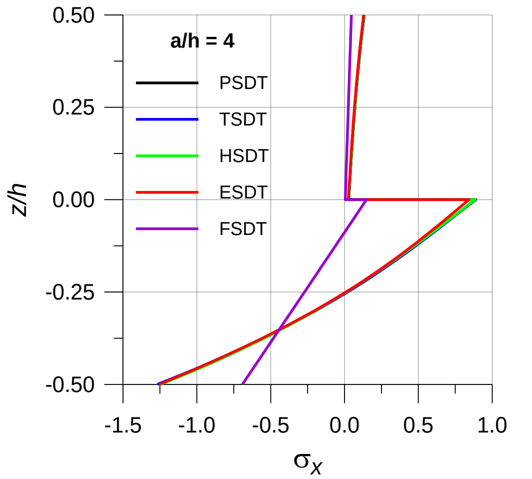

Fig. 2 Through-the-thickness distribution of in-plane normal stress (σ_x) for (0°/90°) laminated composite cylindrical shells subjected to uniform load

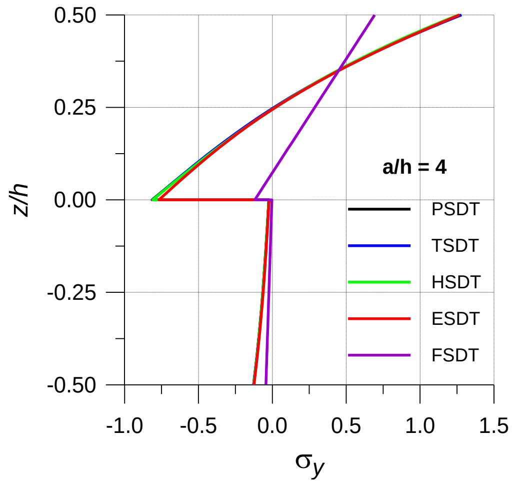

Fig. 3 Through-the-thickness distribution of in-plane normal stress (σ_y) for (0°/90°) laminated composite cylindrical shells subjected to uniform load

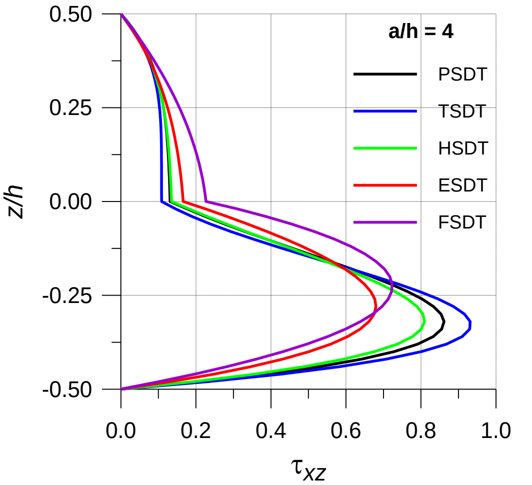

Fig. 4 Through-the-thickness distribution of transverse shear stress (τ_{xz}) for (0°/90°) laminated composite cylindrical shells subjected to uniform load

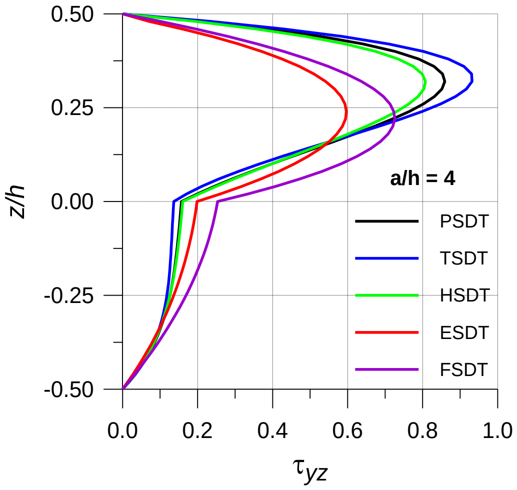

Fig. 5 Through-the-thickness distribution of transverse shear stress (τ_{yz}) for (0°/90°) laminated composite cylindrical shells subjected to uniform load

The present generalized shell theory is applied for the stress analysis of (0°/90°) anti-symmetric and (0°/90°/0°) symmteric laminated composite cylindrical shells. Displacements and stresses of the two-layer (0°/90°) anti-symmetric laminated cylindrical shell under uniform load are presented in Table 3. Both the layers are of equal thickness i.e., $h/2$. Material properties defined by Eq. (31) and non-dimensional form defined by Eq. (32) are used to represent the numerical results. It is observed that the transverse displacement obtained by using PSDT, TSDT and HSDT are in excellent agreement with each other. ESDT underestimates the transverse displacement for all R/a ratios. It is observed that the non-dimensional transverse displacement and stresses are increased with respect to increase in *R/a* ratios. Figs. 2 and 3 show through-the-thickness distributions of in-plane normal stresses for (0°/90°) anti-symmetric laminated cylindrical shell under uniform load. Maximum normal stresses are observed at extreme fibres of thickness. In-plane normal stress (σ_x) is maximum at z=-h/2 and zero at z=-0.25h whereas in-plane normal stress (σ_y) is maximum at z=h/2 and zero at z=0.25h. To ascertain the continuity of transverse shear stress at layer interface, they are obtained using Eq. (33). Figs. 4 and 5 show through-the-thickness distributions of transverse shear stresses for ($0^0/90^0$) anti-symmetric laminated cylindrical shell under uniform load. Transverse shear stresses are zero at top and bottom surfaces of the shell whereas maximum at z=±0.25h.

Table 3 Non-dimensional displacements and stresses of two-layer (0°/90°) laminated composite cylindrical shell under uniform load (a=10h)

R/a	Model	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Model 1	PSDST	1.8740	1.0530	0.1220	0.1157	0.1885	0.2348
	Model 2	TSDST	1.8720	1.0551	0.1221	0.1160	0.1830	0.2301
	Model 3	HSDST	1.8702	1.0518	0.1219	0.1151	0.1885	0.2348
	Model 4	ESDST	1.8482	1.0542	0.1216	0.1140	0.1940	0.2420
	Model 5	FSDST	1.8621	1.0169	0.1222	0.1121	0.2181	0.2648
10	Model 1	PSDST	1.9059	1.0851	0.1254	0.1078	0.2024	0.2261
	Model 2	TSDST	1.9038	1.0873	0.1255	0.1081	0.1969	0.2213
	Model 3	HSDST	1.9019	1.0837	0.1253	0.1073	0.2024	0.2261
	Model 4	ESDST	1.8791	1.0860	0.1249	0.1063	0.2077	0.2340
	Model 5	FSDST	1.8938	1.0486	0.1256	0.1049	0.2322	0.2560
20	Model 1	PSDST	1.9141	1.0965	0.1266	0.1034	0.2088	0.2207
	Model 2	TSDST	1.9120	1.0987	0.1266	0.1037	0.2033	0.2159
	Model 3	HSDST	1.9101	1.0951	0.1265	0.1029	0.2088	0.2207
	Model 4	ESDST	1.8870	1.0974	0.1260	0.1020	0.2140	0.2292
	Model 5	FSDST	1.9019	1.0597	0.1268	0.1005	0.2386	0.2506
50	Model 1	PSDST	1.9164	1.1018	0.1271	0.1006	0.2124	0.2171
	Model 2	TSDST	1.9143	1.1041	0.1271	0.1009	0.2069	0.2124
	Model 3	HSDST	1.9124	1.1004	0.1270	0.1001	0.2124	0.2171
	Model 4	ESDST	1.8893	1.1027	0.1265	0.0993	0.2175	0.2260
	Model 5	FSDST	1.9042	1.0647	0.1273	0.0977	0.2422	0.2470
100	Model 1	PSDST	1.9167	1.1033	0.1272	0.0997	0.2135	0.2159
	Model 2	TSDST	1.9146	1.1056	0.1272	0.1000	0.2080	0.2111
	Model 3	HSDST	1.9127	1.1019	0.1271	0.0992	0.2135	0.2159
	Model 4	ESDST	1.8896	1.1042	0.1267	0.0983	0.2187	0.2249
	Model 5	FSDST	1.9045	1.0661	0.1275	0.0967	0.2434	0.2458
∞ (Plate)	Model 1	PSDST	1.9169	1.1047	0.1274	0.0987	0.2147	0.2147
	Model 2	TSDST	1.9147	1.1070	0.1274	0.0990	0.2100	0.2100
	Model 3	HSDST	1.9128	1.1033	0.1273	0.0982	0.2147	0.2147
	Model 4	ESDST	1.8897	1.1056	0.1268	0.0974	0.2198	0.2238
	Model 5	FSDST	1.9046	1.0674	0.1276	0.0958	0.2445	0.2445
	Pagano (1970)	Elasticity	1.9320	1.0860	0.1300	---	0.2460	0.2480
	Sayyad and Ghugal (2014a)	SSNPT	1.9070	1.1057	0.1307	0.0978	0.2100	0.2100

Table 4 Non-dimensional displacements and stresses of three-layer ($0^0/90^0/0^0$) laminated composite cylindrical shells under uniform load with (a=10h)

R/a	Model	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Model 1	PSDST	1.0757	0.8172	0.0463	0.0710	0.6186	0.3810
	Model 2	TSDST	1.0842	0.8223	0.0467	0.0716	0.6137	0.3774
	Model 3	HSDST	1.0749	0.8167	0.0462	0.0710	0.6189	0.3818
	Model 4	ESDST	1.0916	0.8415	0.0472	0.0719	0.6217	0.3913
	Model 5	FSDST	0.9531	0.7590	0.0407	0.0607	0.7017	0.4126
10	Model 1	PSDST	1.0864	0.8311	0.0473	0.0654	0.6241	0.3847
	Model 2	TSDST	1.0950	0.8364	0.0477	0.0660	0.6192	0.3810
	Model 3	HSDST	1.0856	0.8306	0.0473	0.0653	0.6243	0.3854
	Model 4	ESDST	1.1026	0.8560	0.0483	0.0662	0.6270	0.3944
	Model 5	FSDST	0.9614	0.7705	0.0415	0.0557	0.7067	0.4158
20	Model 1	PSDST	1.0891	0.8360	0.0477	0.0624	0.6254	0.3856
	Model 2	TSDST	1.0977	0.8414	0.0481	0.0630	0.6206	0.3819
	Model 3	HSDST	1.0883	0.8355	0.0477	0.0624	0.6257	0.3863
	Model 4	ESDST	1.1054	0.8611	0.0487	0.0632	0.6283	0.3948
	Model 5	FSDST	0.9635	0.7746	0.0419	0.0530	0.7079	0.4166
50	Model 1	PSDST	1.0898	0.8383	0.0479	0.0606	0.6258	0.3858
	Model 2	TSDST	1.0985	0.8437	0.0484	0.0611	0.6210	0.3822
	Model 3	HSDST	1.0890	0.8378	0.0479	0.0605	0.6261	0.3865
	Model 4	ESDST	1.1062	0.8634	0.0489	0.0614	0.6287	0.3954
	Model 5	FSDST	0.9641	0.7765	0.0421	0.0514	0.7083	0.4168
100	Model 1	PSDST	1.0899	0.8389	0.0480	0.0600	0.6259	0.3859
	Model 2	TSDST	1.0986	0.8443	0.0484	0.0605	0.6210	0.3822
	Model 3	HSDST	1.0891	0.8384	0.0480	0.0599	0.6262	0.3866
	Model 4	ESDST	1.1063	0.8641	0.0490	0.0608	0.6288	0.3951
	Model 5	FSDST	0.9642	0.7771	0.0421	0.0508	0.7083	0.4169
∞ (Plate)	Model 1	PSDST	1.0900	0.8395	0.0481	0.0593	0.6259	0.3859
	Model 2	TSDST	1.0986	0.8449	0.0485	0.0599	0.6210	0.3822
	Model 3	HSDST	1.0892	0.8390	0.0480	0.0593	0.6262	0.3866
	Model 4	ESDST	1.1063	0.8647	0.0490	0.0601	0.6273	0.3952
	Model 5	FSDST	0.9642	0.7776	0.0422	0.0503	0.7083	0.4169
	Pagano (1970)	Elasticity	1.1539	0.8708	0.0529	---	0.6279	0.4009
	Sayyad and Ghugal (2014a)	SSNPT	1.0954	0.8436	0.0510	0.0594	0.6139	0.3553

Table 4 shows displacements and stresses in (0°/90°/0°) symmteric laminated composite cylindrical shells under uniform load. All the layers are of equal thickness i.e., h/3 and made up of material properties defined by Eq. (31). It is observed that the distributions of these stresses according to PSDT, TSDT and HSDT are in excellent agreement with each other whereas ESDT and FSDT underestimate the same. Figs. 6 and 7 show through-the-thickness distributions of in-plane normal stresses for (0°/90°/0°) symmteric laminated cylindrical shell under uniform load. In symmetric lamination scheme, maximum normal stresses are observed at extreme fibres of thickness and zero at z=0 i.e., mid-plane. Figs. 8 and 9 show through-the-thickness distributions of transverse

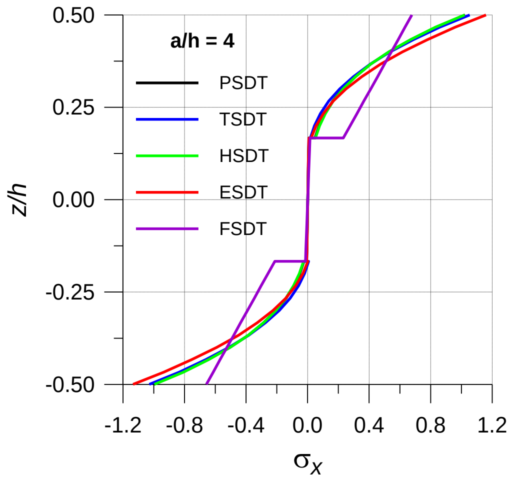

Fig. 6 Through-the-thickness distribution of in-plane normal stress (σ_x) for (0°/90°/0°) laminated composite cylindrical shells subjected to uniform load

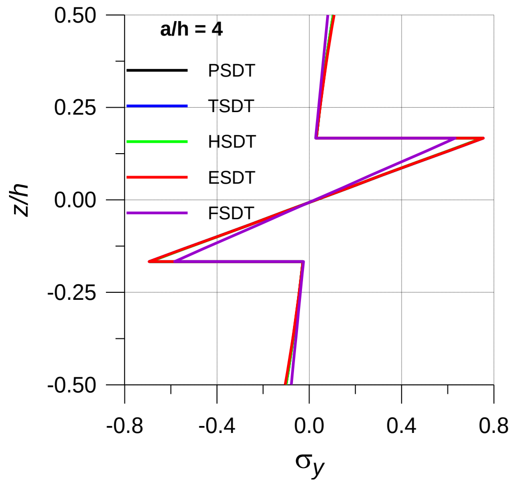

Fig. 7 Through-the-thickness distribution of in-plane normal stress (σ_y) for (0°/90°/0°) laminated composite cylindrical shells subjected to uniform load

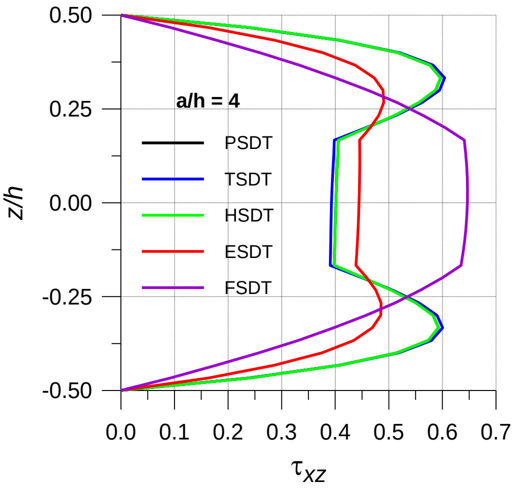

Fig. 8 Through-the-thickness distribution of transverse shear stress (τ_{xz}) for ($0^0/90^0/0^0$) laminated composite cylindrical shells subjected to uniform load

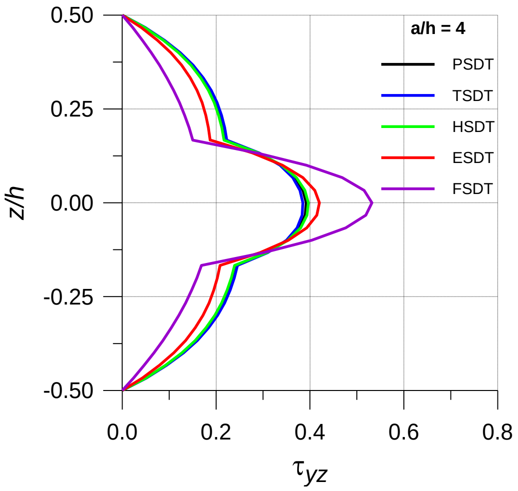

Fig. 9 Through-the-thickness distribution of transverse shear stress (τ_{yz}) for ($0^0/90^0/0^0$) laminated composite cylindrical shells subjected to uniform load

shear stresses for (0°/90°/0°) symmteric laminated cylindrical shell under uniform load.

In the last problem, the present generalized shell theory is applied for the stress analysis of symmetric sandwich (0°/*core*/0°) cylindrical shells under uniform load. Top and bottom layers of the shell i.e., face sheets are of thickness 0.1h whereas middle layer i.e., core is of thickness 0.8h, where h is the overall thickness of the shell. The material properties of the core material are given by Eq. (30) whereas those of the face sheets are given by Eq. (31). Non-dimensional displacements and stresses for this problem are presented in Table 5. It is pointed out from Table 5 that all the

Table 5 Non-dimensional displacements and stresses of three-layer (0°/*core*/0°) sandwich cylindrical shell under uniform load (a=10h)

R/a	Model	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Model 1	PSDST	1.5436	1.6349	0.0999	0.1780	0.5388	0.1672
	Model 2	TSDST	1.5480	1.6395	0.1004	0.1786	0.5382	0.1672
	Model 3	HSDST	1.5430	1.6345	0.0998	0.1779	0.5390	0.1673
	Model 4	ESDST	1.5481	1.6530	0.1003	0.1784	0.5398	0.1672
	Model 5	FSDST	1.0819	1.5467	0.0707	0.1358	0.5678	0.1543
10	Model 1	PSDST	1.5505	1.6486	0.1102	0.1642	0.5408	0.1682
	Model 2	TSDST	1.5550	1.6533	0.1107	0.1648	0.5402	0.1682
	Model 3	HSDST	1.5499	1.6482	0.1101	0.1641	0.5410	0.1683
	Model 4	ESDST	1.5550	1.6651	0.1105	0.1648	0.5414	0.1682
	Model 5	FSDST	1.0852	1.5551	0.0777	0.1257	0.5691	0.1550
20	Model 1	PSDST	1.5523	1.6536	0.1153	0.1570	0.5413	0.1685
	Model 2	TSDST	1.5567	1.6583	0.1158	0.1576	0.5407	0.1684
	Model 3	HSDST	1.5517	1.6532	0.1152	0.1570	0.5415	0.1685
	Model 4	ESDST	1.5568	1.6694	0.1156	0.1577	0.5419	0.1684
	Model 5	FSDST	1.0860	1.5580	0.0812	0.1205	0.5695	0.1552
50	Model 1	PSDST	1.5528	1.6559	0.1183	0.1527	0.5415	0.1686
	Model 2	TSDST	1.5572	1.6606	0.1189	0.1533	0.5408	0.1685
	Model 3	HSDST	1.5522	1.6555	0.1183	0.1526	0.5416	0.1686
	Model 4	ESDST	1.5573	1.6715	0.1186	0.1535	0.5420	0.1685
	Model 5	FSDST	1.0863	1.5594	0.0833	0.1173	0.5696	0.1552
100	Model 1	PSDST	1.5528	1.6566	0.1193	0.1512	0.5415	0.1686
	Model 2	TSDST	1.5573	1.6613	0.1199	0.1518	0.5408	0.1685
	Model 3	HSDST	1.5522	1.6562	0.1193	0.1512	0.5416	0.1686
	Model 4	ESDST	1.5573	1.6720	0.1196	0.1520	0.5421	0.1685
	Model 5	FSDST	1.0863	1.5598	0.0840	0.1162	0.5696	0.1552
∞ (Plate)	Model 1	PSDST	1.5528	1.6572	0.1203	0.1497	0.5426	0.1686
	Model 2	TSDST	1.5573	1.6620	0.1209	0.1503	0.5423	0.1685
	Model 3	HSDST	1.5523	1.6568	0.1203	0.1497	0.5427	0.1686
	Model 4	ESDST	1.5574	1.6969	0.1206	0.1506	0.5423	0.1685
	Model 5	FSDST	1.0863	1.5601	0.0847	0.1152	0.5696	0.1552
	Pagano (1970)	Elasticity	1.7537	1.8098	0.1717	0.1336	0.5453	0.1021
	Sayyad and Ghugal (2014a)	SSNPT	1.5500	1.6579	0.1285	0.1479	0.5384	0.1692

results predicted by the present theories for sandwich cylindrical shell for all R/a ratios are in excellent agreement with each other. Figs. 10 and 11 show that the maximum in-plane normal stresses are occurred in face sheets due to stiff fibrous composite material whereas in-plane normal stresses in middle core are very small due to low Young's and shear moduli of soft-core material. Figs. 12 and 13 show through-the-thickness distributions of transverse shear stresses for (0°/*core*/0°) sandwich cylindrical shells subjected to uniform load.

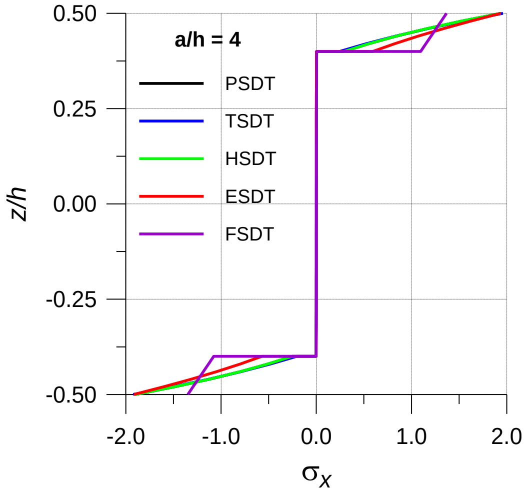

Fig. 10 Through-the-thickness distribution of in-plane normal stress (σ_x) for (0°/*core*/0°) sandwich cylindrical shells subjected to uniform load

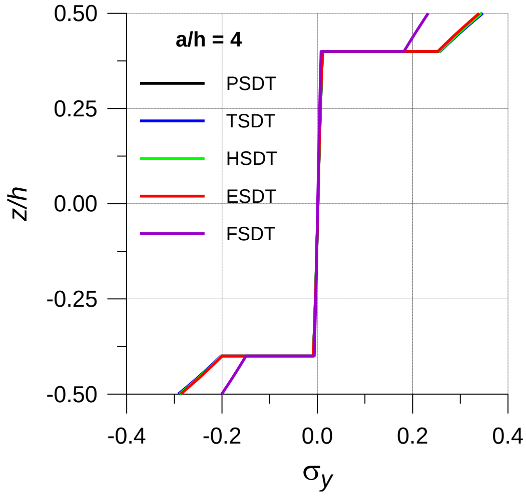

Fig. 11 Through-the-thickness distribution of in-plane normal stress (σ_y) for (0°/*core*/0°) sandwich cylindrical shells subjected to uniform load

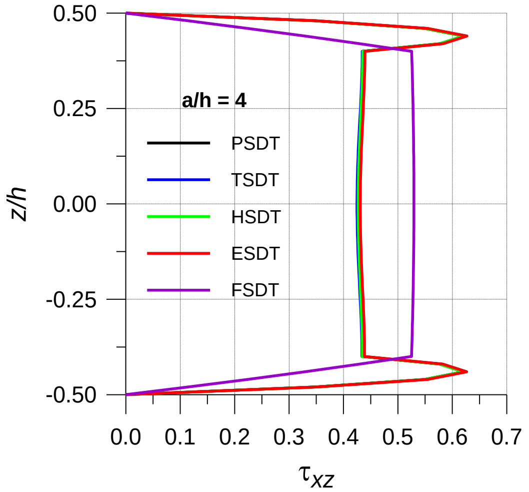

Fig. 12 Through-the-thickness distribution of transverse shear stress (τ_{xz}) for (0°/*core*/0°) sandwich cylindrical shells subjected to uniform load

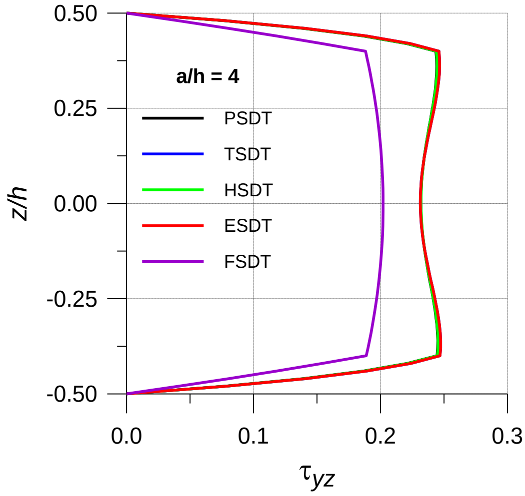

Fig. 13 Through-the-thickness distribution of transverse shear stress (τ_{yz}) for (0°/*core*/0°) sandwich cylindrical shells subjected to uniform load

5. Conclusions

In this study, a generalized shell theory is presented and applied for the stress analysis of laminated composite and sandwich cylindrical shells. The present generalized shell theory is developed with the inclusion of different shape functions in-terms of thickness coordinate to account for the effect of transverse shear deformation. The generalized governing equations are derived by using the principle of virtual work. Closed-formed solutions for the stress analysis of simply supported cylindrical shells are obtained using Navier's solution technique. Numerical results obtained using all the models are in excellent agreement with each other. Transverse shear stresses are recovered from 3-D stress equilibrium equations of elasticity for laminated composite and sandwich cylindrical shells. Therefore, it is recommended that the present theory can be extended for the static analysis of laminated composite and sandwich shells of double curvature.

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