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Damage identification in structural elements through curvature mode shapes and nonlinear energy operator

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Abstract. Damage detection in structural elements using curvature mode shape technique has become a research focus of increasing interest during the last few years. A noticeable deficiency of curvature mode shape, however, is its susceptibility to measurement noise, easily impairing its advantage of sensitivity to damage. To overcome this drawback, a nonlinear operator called Teager Energy Operator (TEO) is incorporated. The efficacy of TEO is analytically verified through modal curvatures in a steel reinforced concrete cantilever beam with an induced damage (stiffness loss) along its length. The applicability of the proposed curvature mode shape technique is experimentally validated for detecting simulated damage (mass attached) in an aluminum plate from mode shapes acquired by a non-contact Scanning Laser Doppler Vibrometer (SLDV). Normal responses (out of plane flexure response) are measured with SLDV. The excitation is given with the help of a data-physics exciter based on the frequencies obtained from the free vibration tests using a roving hammer attached with a forced transducer. The proposed algorithm is to compute a robust nonlinear operator - TEO using the attained curvature, dynamic rotation and displacement mode shapes. This algorithm has been successfully implemented and tested for detecting damages on beam and plate elements, using finite element simulations as well as laboratory experiments. It is anticipated that the suggested approach facilitates damage localization in a comparatively fast and accurate manner.

Keywords: damage detection; health monitoring; curvature mode shape; energy operator; scanning laser vibrometer; beam/plate elements

1. Introduction

Structural Health Monitoring (SHM) is an essential component of maintenance for engineering systems (Worden and Dulieu-Barton 2004). The research community has contributed vastly to this field over the past three decades, aiming to improve the efficiency and accuracy of detection and localization of damages in engineered systems (Farrar and Lieven 2007).

The importance of having robust health monitoring system that can detect and locate progressive deterioration in structures or abrupt damage induced by extreme loading events is well recognized in civil engineering field. A wide variety of different damage identification methods is available. Vibration-based damage detection method is one among the important technique and it

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has been studied extensively in the past by many researchers (Pandey *et al.* 1991, Ndambi *et al.* 2002, Kim *et al.* 2003, Wang *et al.* 2009, Lu *et al.* 2017, Yin *et al.* 2018). The other papers in this field are described subsequently and by no-means the list is exhaustive. Study by Pandey *et al.* (1991) and Ratcliffe (1997) show curvature mode shape as parameter to detect damage in structure by using the mode shape data from undamaged and damaged structures, and they are found it can be better indicators for damage identification compared to natural frequency. The curvature mode shape appears sensitive to structural discontinuity and therefore it is well suited for identification of damages in various structural elements. Despite the prevalence of their use, curvature mode shapes have a noticeable drawback of susceptibility to noise, caused by the second order differentiation of mode shapes. This differentiation can amplify slight noise present in a mode shape, usually producing a noise-dominated curvature mode shape with obscured damage signature (Cao and Qiao 2009). Several researchers have attempted to tackle this problem from the perspective of optimal sampling interval or signal processing, but no satisfactory solutions have been obtained (Ciambella and Vestroni 2015, Dessi and Camerlengo 2015).

Damage detection systems utilize the illustrious fact that material discontinuities affect elastic waves propagating in solids. Wave frequencies that are most sensitive to damage depend on the type of structure, the material, and the form of damage (Gopalakrishnan and Doyle 1994). Monitoring structural elements using wave propagation mechanics (Doyle 1997) is an alternative method and is of paramount importance in civil engineering field. In structures with local discontinuities, the dynamics of wave propagation and its mechanics have been extensively studied in the past. The main issue in modelling of the wave propagation phenomena by continuous models is that the use of such models is limited to structures of simply geometrics and specific boundary conditions. In inhomogeneous materials, the energy reflections are instigated by the mismatch of the impedance properties amongst various regions. The mismatch can be created by introducing local changes in the material properties or in the geometry of the structure (Kumar *et al.* 2017, Saravanan *et al.* 2018). New damage detection methods are based on analyzing incongruities in elastic wave propagation in structures. Elastic waves are generated and sensed by an array of transducers either embedded in, or bonded to, the surface of a structure. There are many classical work done to damage detection in structural elements and other composite structures (Lakshmanan *et al.* 2010a, Lakshmanan *et al.* 2010b, Raghuprasad *et al.* 2013, Saravanan *et al.* 2017, Lei *et al.* 2017, Zheng *et al.* 2018, Bagherahmadi *et al.* 2018). The Scanning Laser Doppler Vibrometer (SLDV) is a scientific instrument which is used for steady state analysis as well for wave propagation analysis. SHM using SLDV through lamb wave motion and other wave propagation sensing is studied extensively for damage detection. In order to increase the depth of understanding on SHM using SLDV technology, suitable references are provided as classical literatures (Brehmer and Sinapius 2004, Mallet *et al.* 2004, Staszewski *et al.* 2004, Leong *et al.* 2005, Ostachowicz *et al.* 2006, Żak *et al.* 2012, Saravanan *et al.* 2015).

The pioneering work and the connected papers, which inspire the present work, include that of Sung *et al.* (2013), Radzieński *et al.* (2013), Cao *et al.* (2014), Janeliukstis *et al.* (2017) and Mohammadzadeh *et al.* (2018). It will be important and advantageous to develop a regime that can overcome the drawback of curvature mode shape to provide a reliable damage identification method. To that end, this study explores a regime to improve, based on the synergy of a non-linear Teager energy operator (TEO) (Teager and Teager 1983, Kaiser 1990, 1993) in characterizing damage location. The present research paper aims at solving emerging problems in the structural engineering field for SHM of typical structural beam and plate elements, by providing novel methodology and practical understanding of curvature mode shape and nonlinear energy operator.

The analytical study based on finite element modeling (FEM) using ABAQUS dynamic explicit analysis (Hibbitt *et al*. 2011) is investigated for damage detection (loss in stiffness along its length) in steel reinforced concrete (SRC) beam element for verifying the proposed algorithm. The research paper illustrates an experimental investigation for localizing damage in a structural element (aluminum plate with added mass as induced damage) using low-frequency elastic waves by a scanning technique based on a non-contact SLDV through appropriate signal processing technique on measured signal. TEO is calculated from obtained responses namely, curvature, dynamic rotation and displacement mode shapes. Also, an experimental investigation using non-contact SLDV is examined to detect bolt loosening using modal curvature technique.

2. Background theory

2.1 Curvature mode shape

Any curve can be considered as an equivalent circle and the reciprocal of the radius of the equivalent circle gives the curvature (Bucciarelli 2009). Curvature mode shape can be related to the flexural stiffness of beam element cross-section (c/s).

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \tag{1}$$

where v is the deflection, $\frac{d^2v}{dx^2}$ is the curvature at a section, M is the bending moment at a section, E is the modulus of elasticity and I is the second moment of c/s area. The moment-curvature relationship to produce a differential equation for the transverse displacement, $v(x)$ of the structural element at every point along the neutral axis (as shown in Fig. 1) is given as follows,

$$\frac{d\varphi}{ds} = \frac{\frac{d^2v}{dx^2}}{\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{3/2}} \tag{2}$$

where s is the independent variable, the distance along the curved, deformed neutral, x axis; $\frac{dv}{dx}$ is the rotation of the c/s. The moment curvature equations for a plate are analogous to the beam moment-deflection equation as in Eq. (1). It is given as,

$$\begin{aligned} M_x &= D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \\ M_z &= -D(1-v)\left(\frac{\partial^2 w}{\partial x \partial y}\right) \end{aligned} \tag{3}$$

where, $D = \frac{Eh^3}{12(1-v^2)}$; h is the height of the plate. The factor D is called the plate stiffness or flexural rigidity and plays the same role in the plate theory as does the flexural rigidity term EI in the beam theory.

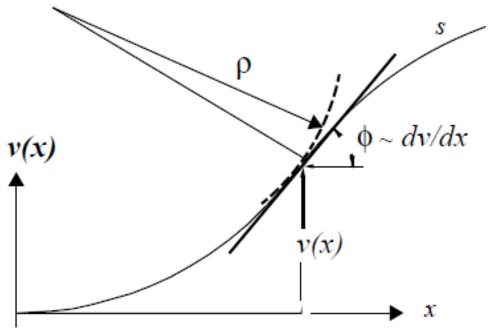


Fig. 1 Curvature of the deformed neutral axis (Bucciarelli 2009)

The curvature can further be defined as the rate of change of slope. The curvature variation is calculated by central difference method which is the derivative of dynamic rotation (Pandey *et al.* 1991). The damage gets magnify for higher order curvature mode shapes.

$$\frac{M}{EI} = \frac{d^2 v}{dx^2} = \frac{v_{n+1} - 2v_n + v_{n-1}}{\Delta h^2} = \frac{\theta_{n+1} - \theta_n}{\Delta h} \tag{4}$$

2.2 Teager Energy Operator (TEO)

TEO is defined in both the continuous and discrete domains and are very useful tools for analyzing single component signals from an energy point of view (Teager and Teager 1983, Kaiser 1990, 1993). TEO possess simplicity, efficiency and good time-resolution. The discrete version of the TEO is proposed with the aim of calculating the instantaneous energy of a temporal signal. The second order differential equation defines an object with mass m suspended by a string with constant stiffness k as,

$$\frac{d^2 x}{dt^2} + \frac{k}{m} x = 0 \tag{5}$$

This is a simple model of a mechanical-acoustical system, where the object may oscillate, thus creating pressure waves in the surrounding medium. The solution to equation is a periodic oscillation which is given by,

$$x(t) = Acos(\omega t + \varphi) \tag{6}$$

where $x(t)$ is the position of the object at time t, A is the amplitude of the oscillation, $\omega = \sqrt{k/m}$ is the frequency of the oscillation, and φ - is the initial phase. If $\varphi \neq 0$, then the object is not initially in equilibrium. The total energy of the object in Newtonian physics is given as the sum of the potential energy of the spring and the kinetic energy of the object,

$$E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 \tag{7}$$

By substituting, $v = \frac{dx}{dt}$ and $x = Acos(\omega t + \varphi)$ in Eq. (7), the energy which is implicitly function of time and proportional to A and ω.

$$E = \frac{1}{2} m\omega^2 A^2 \tag{8}$$

In continuous time, the TEO is defined as,

$$\psi\big(x(t)\big) = \dot{x}^2(t) - x(t)\ddot{x}(t) \tag{9}$$

where ψ is the TEO; $\dot{x}$ and $\ddot{x}$ are the first and second derivative of x. Substituting Eq. (6), into Eq. (9) yields,

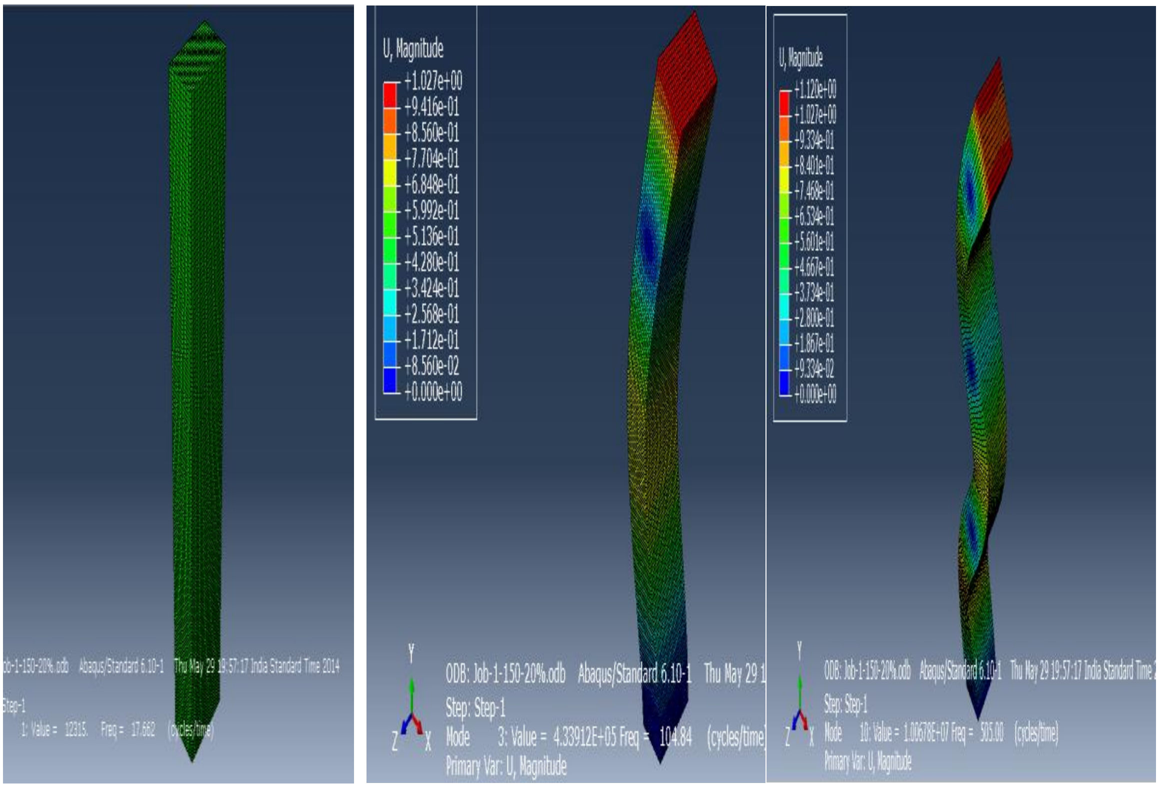
$$\begin{aligned}\psi\big(x(t)\big) &= (-A\omega \sin(\omega t))^2 - A\cos(\omega t)\ (-\omega^2 A\cos(\omega t)) \\ &= A^2\omega^2\big(sin^2(\omega t) + cos^2(\omega t)\big)\psi\big(x(t)\big) = A^2\omega^2\end{aligned} \tag{10}$$

which is the amplitude and frequency product squared. The energy operator output for sinusoidal sources produces the normalized energy.

3. Analytical investigation

In order to investigate the damage location using the curvature mode shapes and TEO, an analytical investigation is carried out using the ABAQUS software – dynamic explicit analysis. If a damage is introduced in a structural element, it reduces the flexural rigidity (*EI*) of the structure at the damaged region, which increases the magnitude of curvature at that section of the structure. The changes in the curvature are local in nature and hence can be used to detect and locate a damage in the structure. A cantilever beam made of SRC is modelled in FEM using ABAQUS with following dimensions, 300 mm x 500 mm x 3000 mm as shown in Fig 2 (a). The properties used for modelling is same as the real condition of SRC elements with Poisson's ratio as 0.15 and M30 grade concrete and Fe415 steel for reinforcement. Modulus of elasticity for concrete is calculated using, $E = 5000\sqrt{f_{ck}}$ from codal provision, IS 456 (BIS 2000) and f_{ck} is the characteristic compressive strength of concrete. In SRC beam model, the damage is induced at one third of the span (i.e.) at 1000 mm by reducing 20% of Young's modulus of concrete (providing only 80% *EI*). The mesh is of eight node hexahedron having 24 degrees of freedom with linear brick element and the size of each mesh is provided as 20 mm uniformly in FEM. The obtained second and fourth mode shape results are presented in Fig. 2 (b-c).

A modal analysis is done for the above said beam; a single line is taken up from the beam for computing the dynamic rotation and the curvature from the displacements obtained. The dynamic rotation is nothing but the first order derivative of the displacements and the curvature is the second order derivative of the displacements. The dynamic rotation and curvature mode shapes are obtained for the second and fourth mode as shown in Fig. 3 respectively. From the dynamic rotation plots, it is clearly evident that there is a kink at section where the damage is induced. But, the curvature mode shapes are more smoothened than dynamic rotation and the kink is clearly visible. Comparing to the lower modes, the damage can be clearly localized only in the fourth mode. For better damage localization and visualization, a robust non-linear energy operator, TEO is utilized. As discussed earlier, from the obtained, curvature dynamic rotation and displacement mode shapes, TEO is computed and illustrated in Fig. 4 for second and fourth mode respectively. It can be clearly seen that the energy operator helps in shoot up the damage alone and retain other places approximately constant value. The concept behind TEO is, damage gets magnified for



a) FEM model b) second mode shape c) fourth mode shape

Fig. 2 SRC cantilever beam model

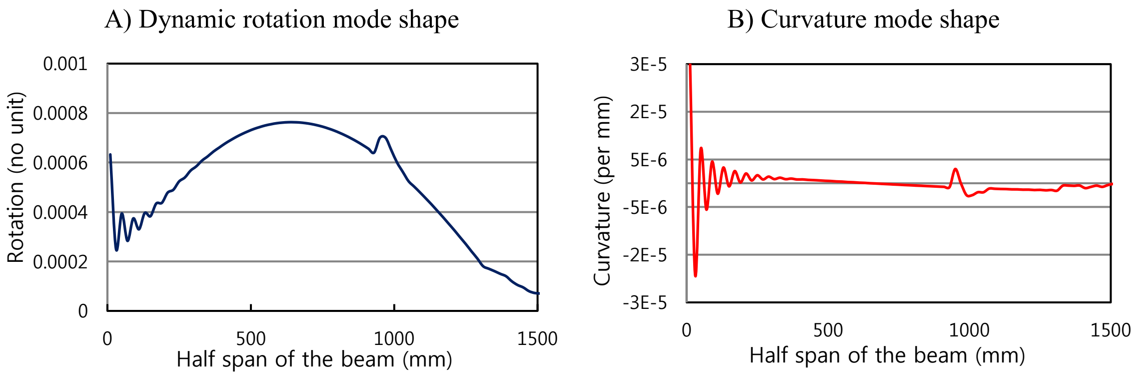
higher order curvature mode shapes but when the damage is located at the zero crossing of a mode shape then it is not identified. This can be explained from Fig. 4, that the damage can be identified accurately from fourth mode comparing to other lower modes. Thus, the proposed methodology using TEO helps in precisely identifying the damage location.

4. Experimental investigation

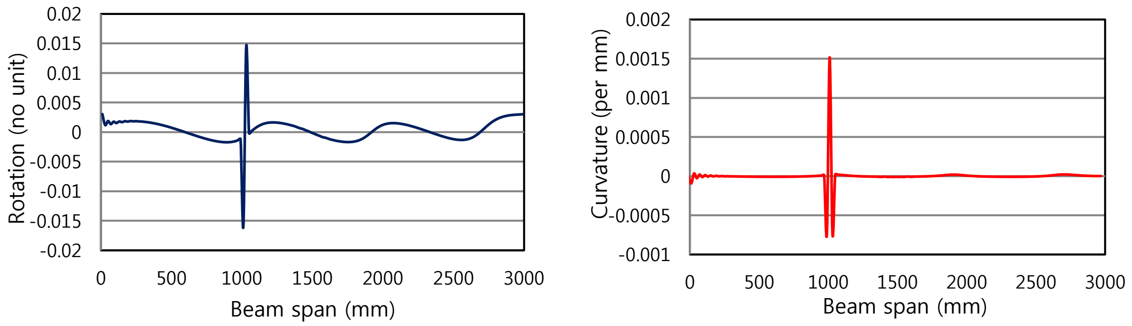
The experimental investigation is carried out in laboratory condition for detecting damage location in aluminum plate specimen using the modal curvature and TEO. This section describes the test specimen, equipment and experimental procedure used in this study. The two different sets of experimental measurements are carried out, namely free vibration test and the laser vibrometry test.

4.1 Free vibration test setup

The free vibration test has been conducted on the aluminum plate to find the frequencies of the mode shapes. The dimensions of an aluminum plate are, 0.760 m x 0.420 m x 0.004 m, with

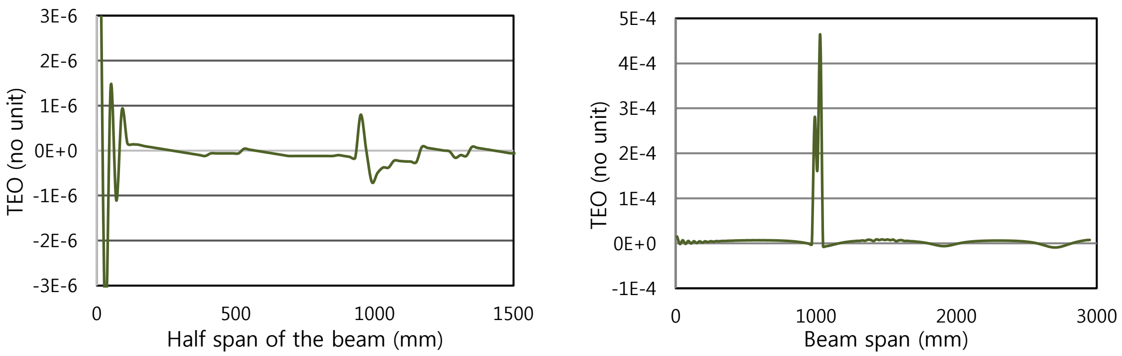


(a) for second mode



(b) for fourth mode

Fig. 3 Analytical results for SRC cantilever beam with induced damage



(a) for second mode (b) for fourth mode

Fig. 4 Teager energy plot for cantilever beam with induced damage

restrained displacements at the boundaries (Fig. 5). The plate is restrained at both the ends using 6 bolts on either side. The accelerometer pickup is attached and the roving hammer with force transducer is used for applying impact load on the aluminum plate as shown in Fig. 5(a). The oscilloscope and condition amplifier used for the vibration test are shown in Fig. 5(b). Two types of tips are used in the roving hammer for finding out the symmetric and anti-symmetric modes, i.e. a plastic tip and a rubber tip. The rubber tip is used to find out the lower mode frequencies, as the contact duration of this tip is quite higher, (i.e. higher time lower frequency). Similarly, the plastic

a) Accelerometer pickup and roving hammer b) Oscilloscope and condition amplifier

Fig. 5 Free vibration test setup: aluminum plate

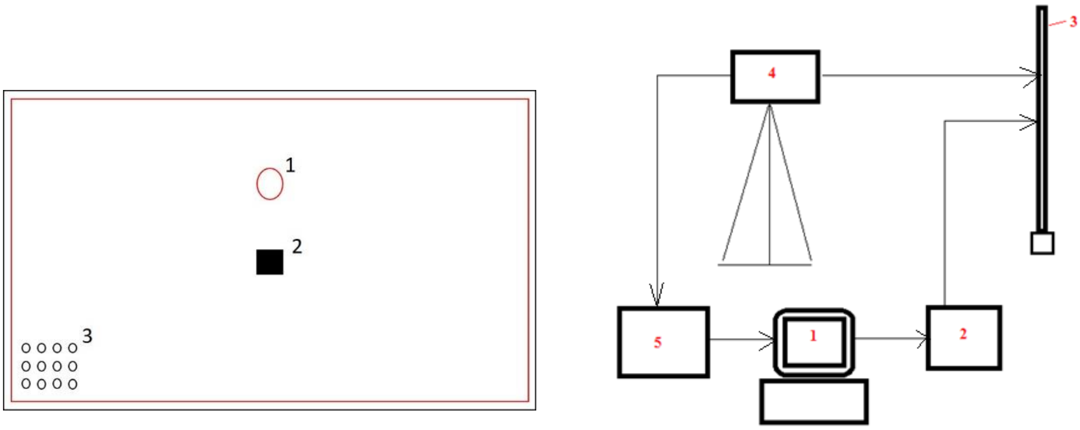
Table 1 Experimental result: Frequencies for various modes

Modes	1	2	3	4	5	6
Frequency (Hz)	26.65	67.50	126.25	185.00	272.50	382.50

tip is used to find out the higher mode frequencies, as the contact duration is lower, (i.e. lower time higher frequency). The number of trials are carried out by changing the position of the pickup and the place of hit to get the correct frequencies of the respective mode shapes and the obtained modal frequency values are tabulated in Table 1. The symmetric and anti-symmetric modes are (1,3,5) and (2,4,6) respectively.

4.2 SLDV test

The aim of the experiment is to detect the simulated damage in the prepared specimen through non-contact method as illustrated in Fig. 6 and 7. The experiment is conducted on same aluminum plate of 0.760 m x 0.420 m x 0.003 m dimensions as shown in Fig. 5. The excitation is given with the help of a data-physics exciter based on the frequencies obtained from the free vibration tests using a roving hammer attached with a forced transducer (Fig. 7 (a)). Damage has been simulated by attaching a small additional mass of 20 grams fixed on the plate surface [square metal] having dimension of 0.025 m x 0.025 m x 0.004 m and it is attached by cyano acrylate adhesive (Fig 7 (b)). All measurements are recorded using a PSV–500 Scanning Laser Doppler Vibrometer (SLDV). The Vibrometer consist of optics (scanning head), electronics (front end), and control (Data Management system). In order to verify the simulated damage, experimental measurements are taken using the application of laser vibrometry. Fig. 6 (a-b) show the schematic picture of analyzed specimen and test set-up. It also shows the localization of measurement points defined for SLDV and the location of excitation. Fig. 7 shows the experimental set-up used for gathering responses from the examined structural element. During the experiment the specimen has been isolated from the influence of external vibration source and propagating waves have been measured.



a) Specimen: 1- excitation point; 2- additional mass; 3- some measurement points

b) Setup: 1-Computer; 2-Exciter; 3- Specimen; 4-SLDV; 5- DAQ

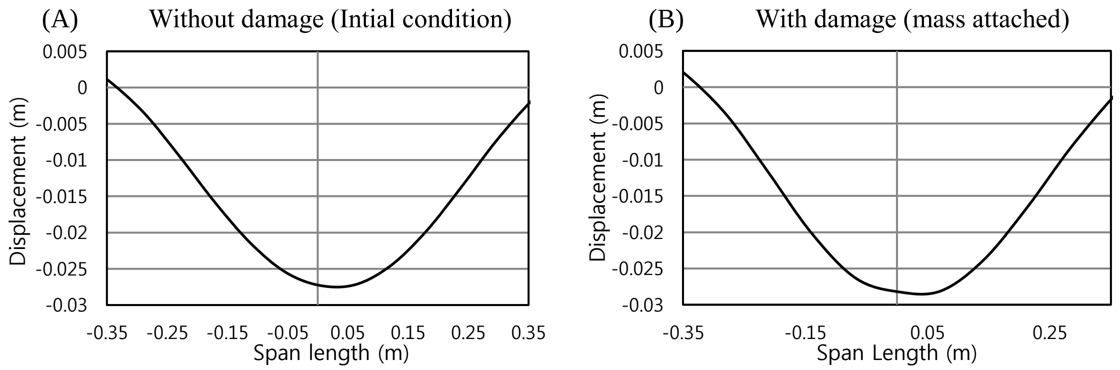
Fig. 6 Schematic view of measurement setup

a) Measured plate specimen

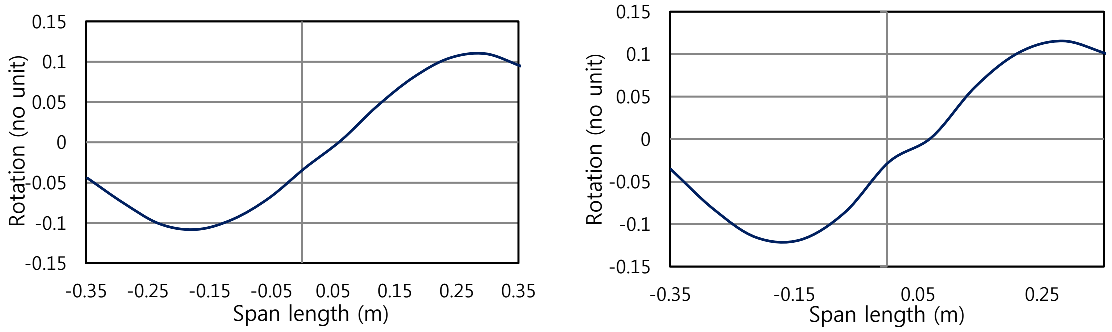
b) Top view

Fig. 7 Experimental setup for damage detection using SLVD

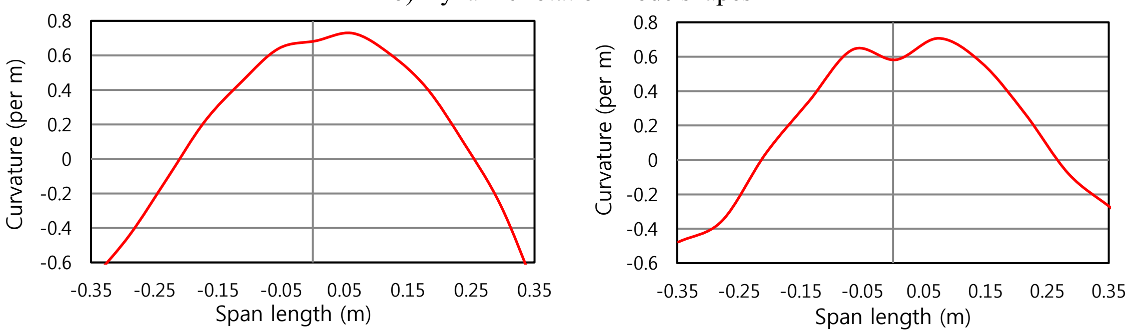
Normal responses (out of plane flexure response) are measured with SLDV. The velocity and displacement acquisition are carried out using a modified Mach – Zehnder interferometer. The benefit of using SLDV is to perform fast, accurate, high sensitivity and non-contact measurement capabilities during which entire surface can be speedily skimmed and robotically investigated with elastic and interactively shaped restrained grids. Hence, the influence of unwanted effects which could alter the results is removed. The laser vibrometer unit robotically moves to each point on the scan grid, measures the response and validates each measured points by checking the signal-to-noise ratio. A small amount of laser beam reflected from the measurement surface to sensor head due to dispersion would result in low signal-to-noise ratio. The response obtained from the SLDV is further processed to get the displacement, dynamic rotation, curvature mode shapes and non-linear operator TEO. The experiment is conducted for two cases, namely without damage (initial condition with no mass attached) and with damage (mass attached) conditions. The obtained responses are processed and plotted for symmetric (first mode - 26.25 Hz) and anti-symmetric (sixth mode - 382.5 Hz) modes in Fig. 8 and 9 respectively. The modal curvature plots (Figs. 8 (c)



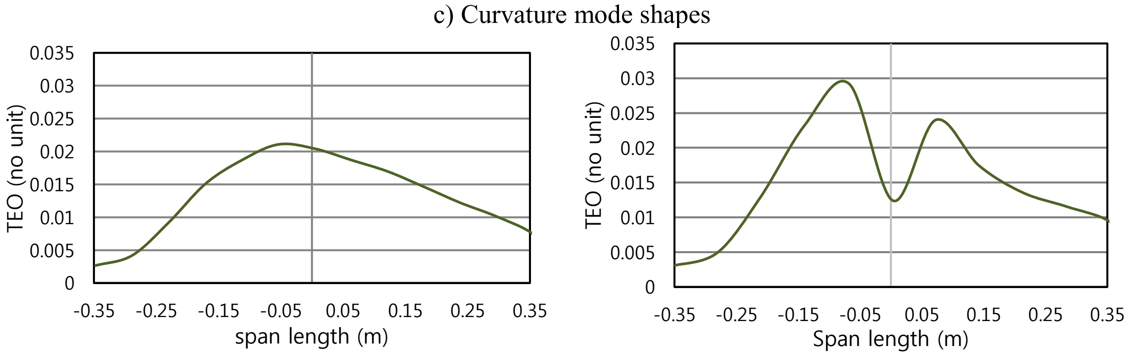
a) Displacement mode shapes



b) Dynamic rotation mode shapes

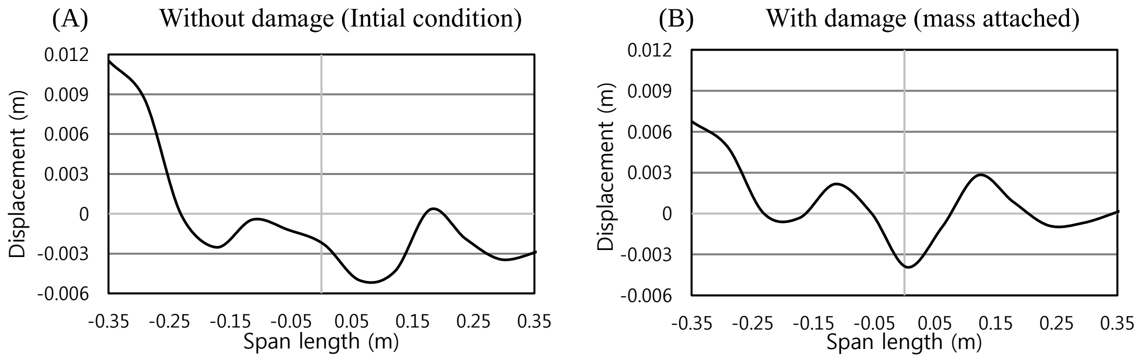


c) Curvature mode shapes

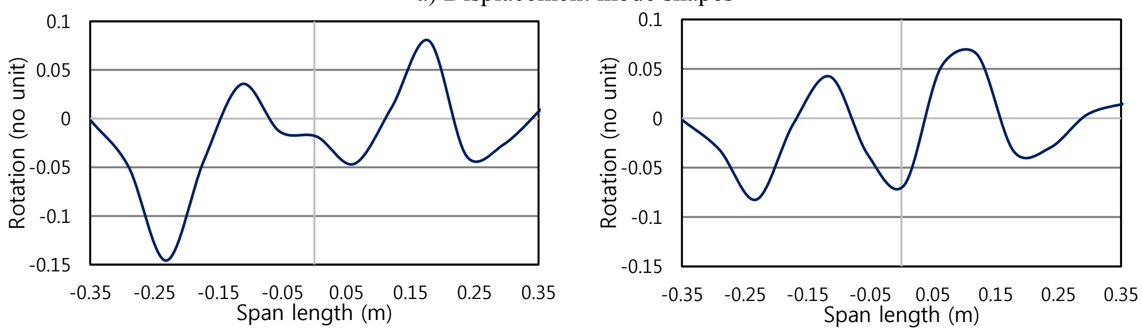


d) Teager energy plot

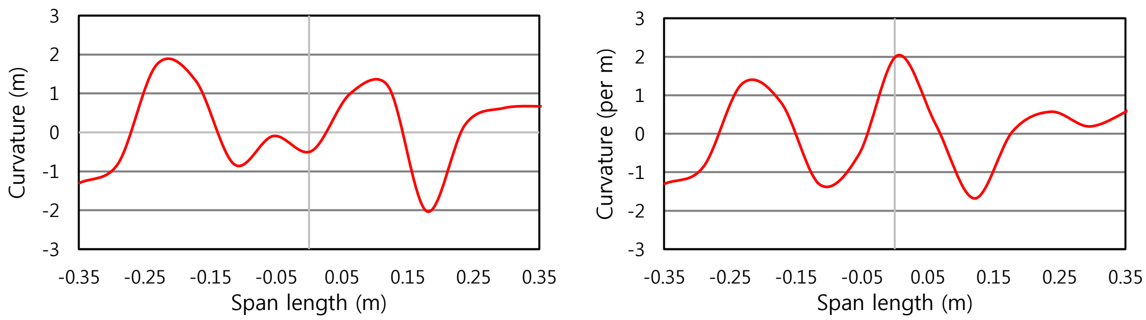
Fig. 8 Obtained from SLDV for first natural frequency of the plate (26.25 Hz - symmetric mode)



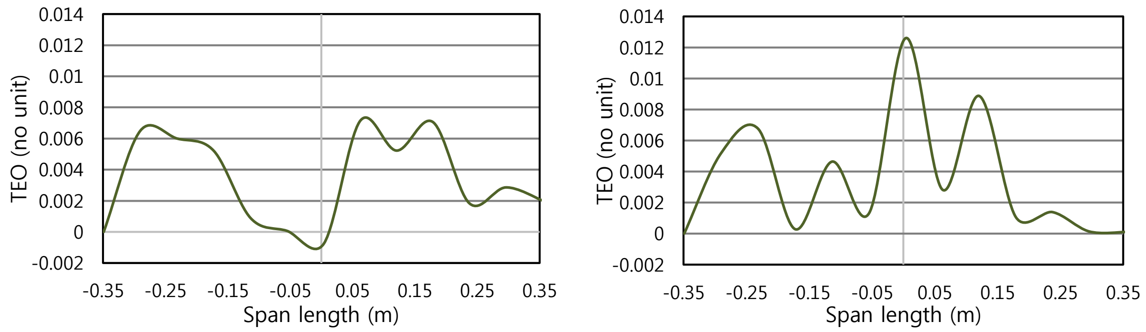
a) Displacement mode shapes



b) Dynamic rotation mode shapes



c) Curvature mode shapes



d) Teager energy plot

Fig. 9 Obtained from SLDV for sixth natural frequency of the plate (382.5 Hz - antisymmetric mode)

a) Plate with PZT patch - excited with 1 kHz b) loosened bolt (side view)

Fig. 10 Experiment set up for the bolt loosening

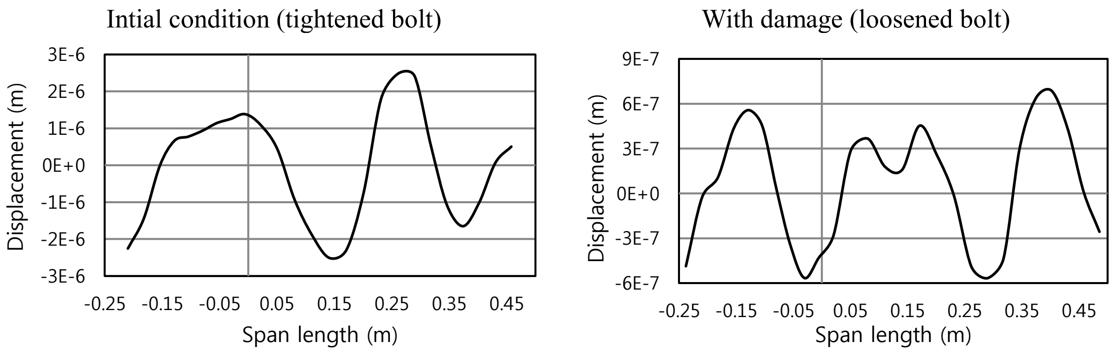
and 9 (c)) clearly show that there is a kink at the place where the damage is induced and further by using TEO, it helps to magnify and locate the damage precisely (Figs. 8 (d) and 9 (d)). As discussed earlier, for some cases, occurrence of zero crossing at particular frequencies, could not able to detect the damage location.

4.3 SLDV test for identifying bolt loosening problem

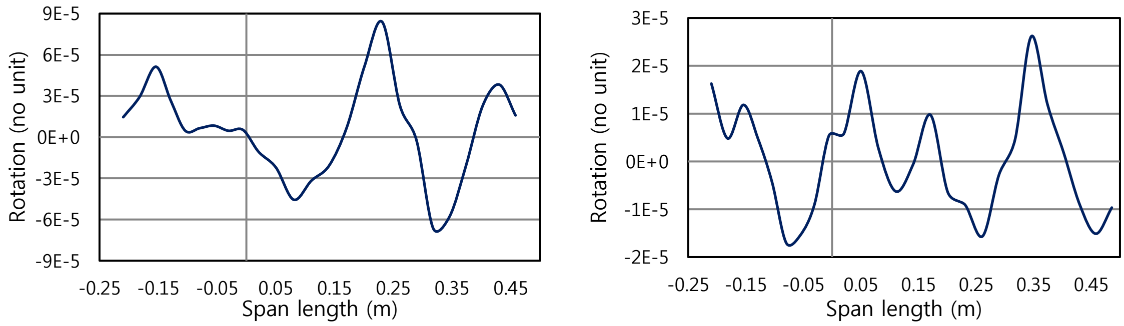
The experiment is repeated on the same aluminum plate of 0.760 m x 0.420 m x 0.004 m dimensions, devoid of any additional mass, bolt loosening (second, third and fourth from top on one edge) are treated as damages for the measured specimen as shown in Fig. 10. The input excitation of 1 kHz is given using internal function generator using PZT sensor patch [PI Ceramic 151], which is placed on sub-surface location at middle of the aluminum plate. The obtained mode shape results along the plate length and width is shown in the Fig. 11 and 12 respectively. The results are shown both for the tightened bolt condition (reference data) and the loosened bolt condition (damage induced) respectively. Though we can obtain the damage location from the proposed energy operator, further studies are required for obtaining the accurate results. In future, we would like to extend it.

5. Conclusions

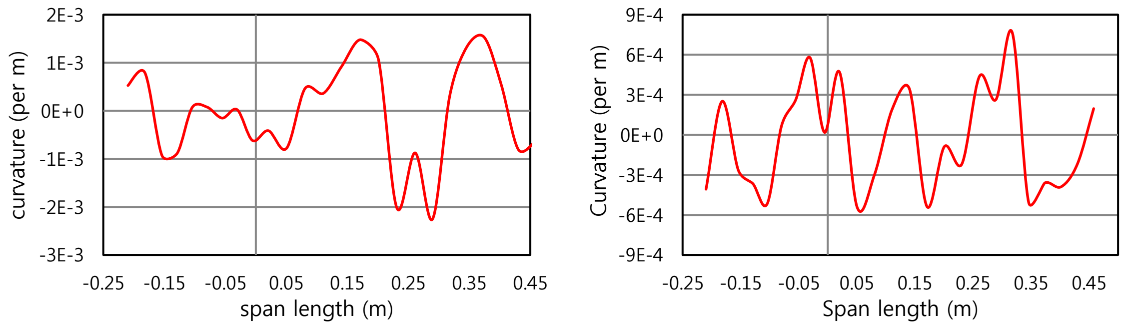
The damage detection in structural elements from the curvature modes is comparatively better than the displacement and dynamic rotational modes. In this research work both analytical and experimental investigations are carried out for a damage localization method using a robust special non-linear energy operator-TEO, computed using curvature, dynamic rotation and displacement mode shapes. The proposed method aids in shoot up and locating damage precisely. In the



a) Displacement mode shapes



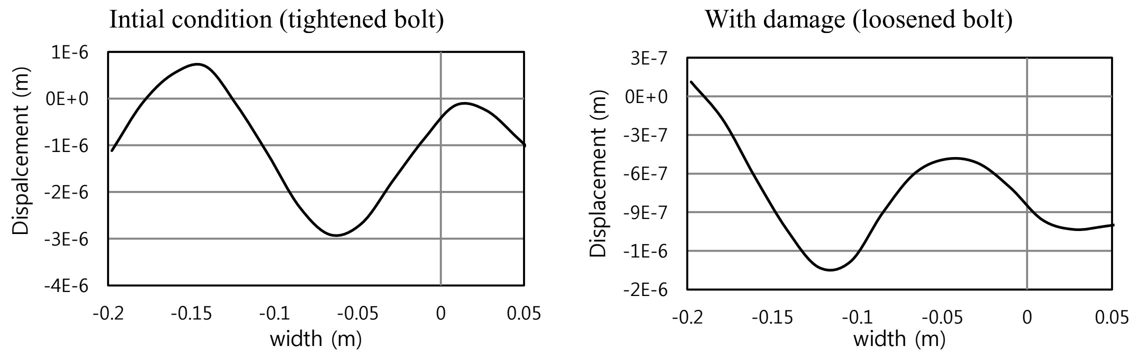
b) Dynamic rotation mode shapes



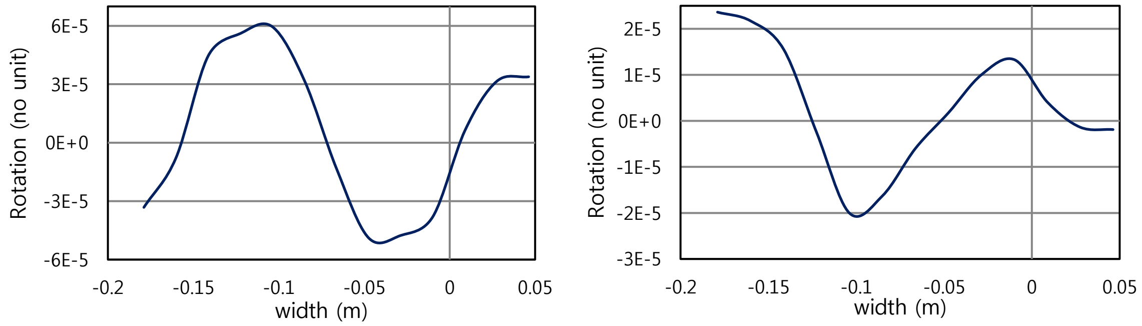
c) Curvature mode shapes

Fig. 11 Experimental results along plate span length for bolt loosening problem

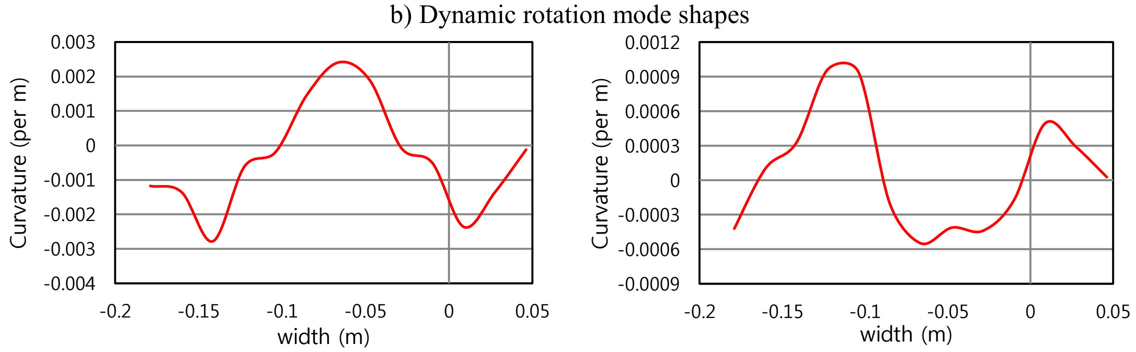
analytical study, FEM of cantilever beam with stiffness loss at particular section is examined and damage location is predicted accurately. The change in curvature increases with reduction in the value of *EI*, and therefore, the amount of damage can be obtained from the magnitude of change in curvature. Based on this verification study, an experimental investigation on plate element is carried out using SLDV and derived curvature mode shapes and TEO plots illustrated the damage localization precisely. An experimental investigation using non-contact SLDV is examined to detect bolt loosening using modal curvature technique. In future, sensitivity studies need to be accomplished concerning the non-linearity rising out of a severe damage and during multi-damage scenario. Also, the present study has to be extended for actual damage detection within more complex structures. In summary, the method shows great potential for rapid damage localization of structural elements and further work is in progress in this direction for other large structures.



a) Displacement mode shapes



b) Dynamic rotation mode shapes



c) Curvature mode shapes

Fig. 12 Experimental results along plate width for bolt loosening problem

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