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Modeling free vibration analysis of osteon as bone unite

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Abstract. This paper investigated vibrational behavior of the osteon as bone unit in the different situations. This study can lead to increase our knowledge of our body. In this paper free vibration of the osteon with considering it as composite material has been studied. The effect of numbers of lamellae and radius of those on natural frequency of osteon are subtle; while thickness of lamellae have decreasing trend on natural frequency of osteon. The presence of nerve and blood in haversian canal change trend of natural frequency, absolutely. Using the nonlocal strain gradient theory (NSGT) leads to effectiveness of scale parameter on equations of motion and the obtained results. The governing equations are derived by Hamilton's principles. A parametric study is presented to examine the effect of various parameters on vibrational behaviour of osteon. The results can also be regarded as a benchmark in vibration analysis behavior of osteon as bone unite.

Keywords: bone; lamellae; osteon; haversian sys; free vibration

1. Introduction

Bone as a biological structure is the solid compound of skeleton of creatures and the part of framework of body, among duties of this member is, making strength in the body, protection of some tissues. Bones are the place to produce white blood cells and red blood cells. They are a source of minerals, in particular calcium; bones transmit minerals whenever the body needs them. Most bones are composed of two parts: a) spongy bone, b) cortical bone. The outer part (cortical bone) of the rigid bone is made of collagen and calciumy structure as hydroxyapatite and so on. However, there are other tissues such as blood vessel and nerve in it. This section of bones are made of units with regular arrangement are known as the Haversian system. Haversian system consists of a central hole (Havers's duct) that contains the nerves and vessels and bony centered cylinders that are called lamellae which surrounds the central hole. Every 5-6 lamellae make the osteon unit (Cowin 2001).

Investigation of mechanical properties of bone have been studied, widely; but there are few analysis about the constituent units or Micro/Nano structure of bone like Haversian system, osteon and lamellae.

As importance of bone in structure of body, investigation of vibration behavior of components of this vital member was investigated. Those components of cortical bone include of osteons, lamellae and so on. In this paper, the single lamellae's free vibration characteristic has been studied

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for different set of parameters. The component of stiffness matrix was presented in Eq. (1). In this case density ρ was considered $49850\ kg/m^3$.

Rho *et al*. (1998) surveyed published papers which are about mechanical properties of bone and finally with some questions delivered the work to others. Liu *et al*. (1999) by experimental approach, measured the flexural modulus and fatigue life of the lamellae. Weiner *et al*. (1999) studied structure of bone with SEM and TEM and obtained young modules, flexural modules and fatigue strength of lamellae. Hoffler *et al*. (2000) achieved to mechanical properties of bone such hardness and elastic modulus by nanoindentation test. Sun *et al*. (2016) studied the mechanical properties of lamellae in various plane. Hassenkam *et al*. (2004) performed tests with AFM to extracted mechanical properties of bone with focus approach on the nanostructure of bone. Currey *et al*. (2006) extracted coefficients of variation of mechanical properties of bone; and then currey (9) investigated hierarchal structure of bone. Faingold *et al*. (2013) with nanoindentation and SEM evaluated properties of a single lamellae in various plates. Ren *et al*. (2015) studied various aspects of mechanical properties of bone such as hierarchical structure of bone and fluid flow in bone and hydrostatic pressure. Also, they studied components of bone with AFM-IR test and Fourier-Transform Infrared (FTIR) spectroscopy (2015), Mitchell *et al*. (2015) for better understanding of lamellae, investigated the arrangement of fibers, thickness, orientation and compositions of lamellae. Xie *et al*. (2017) studied time-dependent properties of bone and dependence of those to volume fraction of compounds of it.

Meanwhile a pin-moment model of flexoelectric actuators was presented by Wang *et al*. (2018) and an electro-hydrostatic actuator for hybrid active-passive vibration isolation by Henderson *et al*. (2018). Also Active vibration compensator on moving vessel by hydraulic parallel mechanism examined by Tanaka (2018).

Unal *et al*. (2018) investigated hygro- electrical properties of bone. Hamed *et al*. (2010) modeled cortical bone as composite material and hierarchical levels and extracted elastic modulus and stiffness matrix of cortical bone; then they did same work with continuum approach and finite element approach and achieved to good agreement with experimental results (Hamed *et al*. 2012). Vercher *et al*. (2013) estimated elastic constants by using Halpin-Tsai equation and used it for finite element simulation. Ranglin *et al*. (2009) By their innovation method, measured the mechanical properties of bone. Korsa and Mares (19) with mathematical analysis achieved the coefficients(mechanical constants) of lamellae of bone's osteon. Korsa *et al*. (2012) studied the mechanical properties of osteon with micro mechanical model and experimental test. Dullemeijer *et al*. (1981) investigated relation between fibers' angle of lamellae and shear modulus of osteon by experimental tests. Ebrahimi *et al*. (2019) Modeled wave dispersion of single lamellae and found physical relations in their model for lamellae.

In this paper modeled osteon as cylindrical shell; in this way Zine *et al*. (2018) surveyed a novel method for analysis of bending and vibration of plates and shells. In view of solving method karami *et al*. (2018) used the nonlocal strain gradient theory for problem of wave propagation of doubly curved nanoshell. Civalek (2017) investigated free vibration of carbon nano tube reinforced cylindrical shell and plate. Mercan and Civalek (2017) was analyzed buckling of shell by HDQ method; material of that paper was silicon carbide nanotube. Bakhadda *et al.* (2018) and others (Zaoui *et al*. 2019, Bouhadra *et al*. 2018, Belabed *et al*. 2018, Cherif *et al*. 2018, Civalek *et al*. 2007, Mohammadimehr and Shahedi 2017, Demir and Civalek 2017) worked on vibration of nano beams and plates.

2. Mechanical model

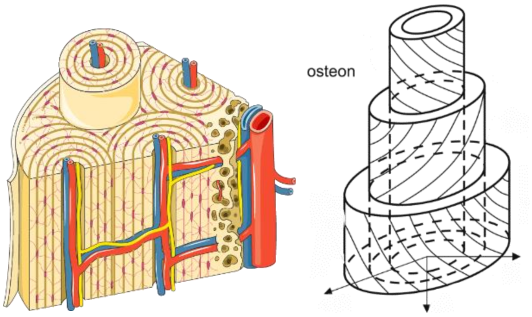

Fig. 1 A single lamella with showing angular coordinate system element

The single osteon can be modeled as laminar composite cylindrical shell which conveying fluid (Fig. 1). In the present paper the nonlocal strain gradient theory (NSGT) is used for modeling osteon (Lim *et al*. 2015). The stiffness matrix of a single lamellae (a layer of osteon) which is obtained experimentally can be expressed as follows (Lim *et al*. 2015)

$$C(GPa) = \begin{bmatrix} 16.08 & 6.21 & 0 \\ 6.22 & 16.07 & 0 \\ 0 & 0 & 9.88 \end{bmatrix} \tag{1}$$

The osteon displacement fields are denoted by U, V, and W in x, θ, and z coordinates. The values of these displacements based on the love thin shell theory can express as follows (Mokhtari and Beni 2016)

$$\begin{aligned} U &= u_0 - z w_{,x} \\ V &= v_0 - (z/R)(w_{,\theta} - v_0) \\ W &= w_0 \end{aligned} \tag{2}$$

where u_0, v_0, w_0 are axial, circumferential, and radial displacements, respectively. In these equations, has been called radius by R, thickness by h and mass density by ρ. We use x, θ and z as angular coordinate elements, perpendicular to the lamellae axis. The strain components ε_x, ε_θ, $\gamma_{x\Theta}$ at an arbitrary point of the shell, are as (Mokhtari and Beni 2016)

$$\varepsilon_x = u_{,x} - z w_{,xx} \tag{3}$$

$$\varepsilon_\theta = \frac{v_{,\theta} + w}{R} - \frac{z}{R^2}\frac{\partial^2 w}{\partial \theta^2} \tag{4}$$

$$\gamma_{x\theta} = \frac{1}{2}\left[\frac{u_{,\theta}}{R} + v_{,x} - 2z\frac{w_{,x\theta}}{R}\right] \tag{5}$$

The stress-strain relationships based on NSGT for a single layer shell can be written as

$$\bar{\sigma}_{ij} = (1 - (l_s)^2 \nabla^2) C_{ijkl} \bar{\varepsilon}_{kl} \tag{6}$$

where l_s ls is nonlocal coefficient and C is stiffness matrix; By solving problem for a osteon which compose of n shell layers we will have:

$$\sigma_{ij} = (1 - (l_s)^2 \nabla^2) Q_{ijkl} \varepsilon_{kl}$$

$$Q = [T][C][T]^T$$

$$T = \begin{bmatrix} \mathrm{Cos}^2\alpha & \mathrm{Sin}^2\alpha & -2\,\mathrm{Sin}\,\alpha\,\mathrm{Cos}\,\alpha \\ \mathrm{Sin}^2\alpha & \mathrm{Cos}^2\alpha & 2\,\mathrm{Sin}\,\alpha\,\mathrm{Cos}\,\alpha \\ \mathrm{Sin}\,\alpha\,\mathrm{Cos}\,\alpha & -\mathrm{Sin}\,\alpha\,\mathrm{Cos}\,\alpha & \mathrm{Cos}^2\alpha - \mathrm{Sin}^2\alpha \end{bmatrix}$$

And for using Hamilton's principle we need to obtain strain energy Π, kinetic energy *K* and work of external forces *W*

$$\int_0^t \delta(\Pi - T + W)\,dt = 0 \tag{7}$$

Strain energy Π of lamellae is expressed as follows

$$\Pi = \frac{1}{2}\int_{\forall} \sigma_{ij}\,\varepsilon_{kl}\,d\forall \tag{8}$$

And for kinetic energy *K* we have

$$T = \frac{1}{2}\rho\int_V (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\,dV \tag{9}$$

And blood flow in haversian canal as external forces

$$\delta W = \int (\Omega^2(\rho_f/\rho_b)(R/h)(K_rR)^{-1}(\frac{J_n(K_rR)}{J_n{}'(K_rR)})w\delta w)\,dA \tag{10}$$

In which

$$\Omega = \omega R\sqrt{\frac{\rho_s}{E_2}}\,(K_rR)^2 = \Omega^2(\frac{C_s}{C_f})^2 - (K_mR)^2$$

By substituting Eqs. (8), (9), (10) into (7) and integrating by parts, equations of motion can be written as follows

$$\begin{aligned}
\delta u&: \left\{A_{11}\frac{\partial^2 u}{\partial x^2} + A_{12}\left(\frac{\partial^2 v}{R\partial x\partial\theta} + \frac{1}{R}\frac{\partial w}{\partial x}\right)\right\} + \frac{1}{R}\left\{A_{33}(\frac{1}{R}\frac{\partial^2 u}{\partial^2\theta} + \frac{\partial^2 v}{\partial x\partial\theta})\right\} - \\
&\frac{1}{2R^2}\left\{-A_{44}l^2\left[\frac{1}{R}\left(\frac{1}{R}\frac{\partial^2 u}{\partial\ \ \theta^2} - \frac{\partial^2 v}{\partial x\partial\theta}\right) + \frac{1}{R}\frac{\partial^3 w}{\partial x\partial\theta^2}\right]\right\} + \frac{1}{2R^2}\left\{-A_{44}l^2\left[-\frac{1}{R^2}\frac{\partial^2 u}{\partial\theta^2}\right]\right\} \\
&+\frac{1}{2R}\left\{-\frac{A_{44}l^2}{2}\left[-\frac{\partial^4 v}{\partial x^3\partial\theta} - \frac{1}{R^2}\frac{\partial^2 v}{\partial\theta\partial x} + \frac{1}{R}\frac{\partial^4 u}{\partial x^2\partial\theta^2} + \frac{1}{R^2}\frac{\partial^3 w}{\partial x\partial\theta^2}\right]\right\} + \\
&\frac{1}{2R^2}\frac{\partial^2}{\partial\theta^2}\left\{-\frac{A_{44}l^2}{2}\left[\frac{1}{R^2}\frac{\partial^4 u}{\partial\theta^4} - \frac{1}{R}\frac{\partial^4 v}{\partial x\partial\theta^3} - \frac{1}{R}\frac{\partial^3 w}{\partial x\partial\theta^2}\right]\right\} = I_0\frac{\partial^2 u}{\partial t^2}
\end{aligned} \tag{16}$$

$$\begin{aligned}
\delta v&: \left\{A_{33}\left(\frac{1}{R}\frac{\partial^2 u}{\partial x\partial\theta} + \frac{\partial^2 v}{\partial x^2}\right)\right\} + \frac{1}{R}\left\{A_{11}\left(\frac{1}{R}\frac{\partial w}{\partial\theta} + \frac{1}{R}\frac{\partial^2 v}{\partial\theta^2}\right) + A_{12}\frac{\partial^2 u}{\partial\theta\partial x}\right\} \\
&-\frac{1}{2R}\left\{-A_{44}l^2\left(\frac{1}{R}\frac{\partial^2 v}{\partial x^2} - \frac{1}{R}\frac{\partial^3 w}{\partial x^2\partial\theta}\right)\right\} + \frac{1}{2R}\frac{\partial}{\partial x}\left\{-A_{44}l^2\left[\frac{1}{R}\left(\frac{1}{R}\frac{\partial^2 u}{\partial x\partial\theta} - \frac{\partial^2 v}{\partial x^2}\right) + \frac{1}{R}\frac{\partial^3 w}{\partial x^2\partial\theta}\right]\right\}
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{R^2}\left\{-\frac{A_{44}l^2}{4}\left(\frac{1}{R^2}\frac{\partial^2 v}{\partial^2\theta}+\frac{\partial^3 w}{\partial x^2\partial\theta}-\frac{1}{R^2}\frac{\partial^3 w}{\partial\theta^3}\right)\right\}\\
&-\left\{-\frac{A_{44}l^2}{4}\left[-\frac{\partial^4 v}{\partial x^4}-\frac{1}{R^2}\frac{\partial^2 v}{\partial x^2}+\frac{1}{R}\frac{\partial^4 u}{\partial x^3\partial\theta}+\frac{1}{R^2}\frac{\partial^3 w}{\partial x^2\partial\theta}\right]\right\}\\
&-\frac{1}{R^2}\left\{-\frac{A_{44}l^2}{4}\left[-\frac{\partial^2 v}{\partial x^2}-\frac{v}{R^2}+\frac{1}{R}\frac{\partial^2 u}{\partial x\partial\theta}+\frac{1}{R^2}\frac{\partial w}{\partial\theta}\right]\right\}\\
&-\frac{1}{R}\left\{-\frac{A_{44}l^2}{4}\left[\frac{1}{R^2}\frac{\partial^4 u}{\partial\theta^3\partial x}-\frac{1}{R}\frac{\partial^4 v}{\partial x^2\partial\theta^2}-\frac{1}{R}\frac{\partial^3 w}{\partial x^2\partial\theta}\right]\right\}=I_0\frac{\partial^2 v}{\partial t^2}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\delta w:&-\frac{1}{R}\left\{A_{11}\left(\frac{w}{R}+\frac{1}{R}\frac{\partial v}{\partial\theta}\right)+A_{12}\frac{\partial u}{\partial x}\right\}-\frac{1}{2R^2}\left\{-\frac{A_{44}l^2}{2}\left(\frac{1}{R^2}\frac{\partial^3 v}{\partial\theta^3}+\frac{\partial^4 w}{\partial x^2\partial\theta^2}-\frac{1}{R^2}\frac{\partial^4 w}{\partial\theta^4}\right)\right\}\\
&-\frac{1}{2R^2}\left\{-\frac{A_{44}l^2}{2}\left[-\frac{\partial^3 v}{\partial x^2\partial\theta}-\frac{1}{R^2}\frac{\partial v}{\partial\theta}+\frac{1}{R}\frac{\partial^3 u}{\partial x\partial\theta^2}+\frac{1}{R^2}\frac{\partial^2 w}{\partial\theta^2}\right]\right\}\\
&+\frac{1}{2R}\left\{-\frac{A_{44}l^2}{2}\left[\frac{1}{R^2}\frac{\partial^3 u}{\partial x\partial\theta^2}-\frac{1}{R}\frac{\partial^3 v}{\partial x^2\partial\theta}-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right]\right\}\\
&+\frac{1}{2}\left\{-\frac{A_{44}l^2}{2}\left(\frac{1}{R^2}\frac{\partial^3 v}{\partial x^2\partial\theta}+\frac{\partial^4 w}{\partial x^4}-\frac{1}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right)\right\}-\frac{1}{2R}\left\{-A_{44}l^2\left(\frac{1}{R}\frac{\partial^3 v}{\partial x^2\partial\theta}-\frac{1}{R}\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right)\right\}\\
&+\frac{1}{2R}\left\{-A_{44}l^2\left[\frac{1}{R}\left(\frac{1}{R}\frac{\partial^3 u}{\partial\theta^2\partial x}-\frac{\partial^3 v}{\partial\theta\partial x^2}\right)+\frac{1}{R}\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right]\right\}\\
&-\Omega^2(\rho_f/\rho_b)(R/h)(K_rR)^{-1}\left(\frac{J_n(K_rR)}{J_n{}'(K_rR)}\right)=I_0\left(\frac{\partial^2 w}{\partial t^2}\right)
\end{aligned}
\tag{18}
$$

In which:

$$
\begin{aligned}
&\{A_{11}\}=\int_{-\square/2}^{\square/2}c_{11}\,dz,\{A_{12}\}=\int_{-\square/2}^{\square/2}c_{12}\,dz,\\
&\{A_{33}\}=\int_{-\square/2}^{\square/2}c_{66}\,dz,\{A_{44}\}=\int_{-\square/2}^{\square/2}\mu\,dz,\\
&\mu=(e_0a/l)^2\\
&I_0=\int hdz
\end{aligned}
$$

3. Solution method

For solving the vibration problem of osteon, the following equation can be extracted from Eqs. (16), (17) and (18)

$$
\begin{bmatrix}F_{11} & F_{12} & F_{13}\\ F_{21} & F_{22} & F_{23}\\ F_{31} & F_{32} & F_{33}\end{bmatrix}\cdot\begin{bmatrix}U\\ V\\ W\end{bmatrix}=0 \tag{20}
$$

In which, F_{ij} are functions of k_x, c and n. since matrix $\begin{bmatrix}U\\ V\\ W\end{bmatrix}$ can't be equal to zero, then

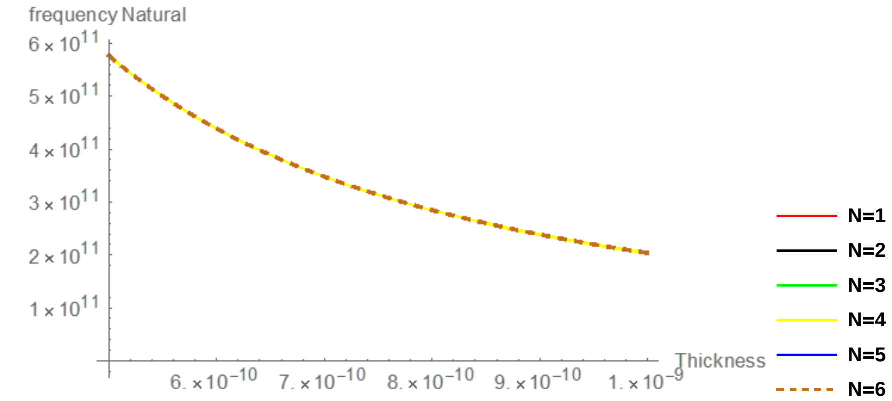

Fig. 2 Plot of variation in natural frequency versus variation in thickness for osteon with various number of layers

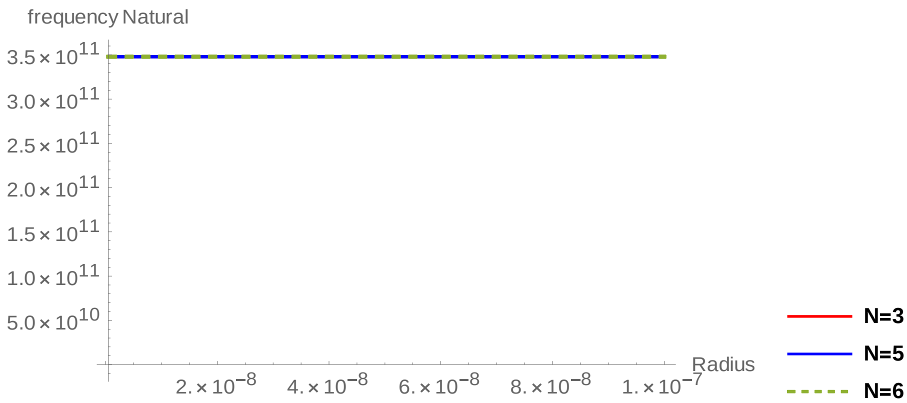

Fig. 3 Variation in natural frequency versus variation in mean radius of osteon for osteon with 3, 5 and 6 layers

determinant of the following matrix should be equal to zero

$$det\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} = 0 \tag{21}$$

4. Result and discussion

Investigation of mechanical properties of bone have been studied, widely; but there are few analysis about the constituent units or Micro/Nano structure of bone like Haversian system, osteon and lamellae. As importance of bone in structure of body, investigation of vibration behavior of components of this vital member was investigated. Those components of cortical bone include of osteons, lamellae and so on. In this paper, the single lamellae's free vibration characteristic have been studied for different set of parameters. The component of stiffness matrix was presented in (1). In this case density ρ was considered 49850.

First, we examine the variation of natural frequency in the osteons with different number of

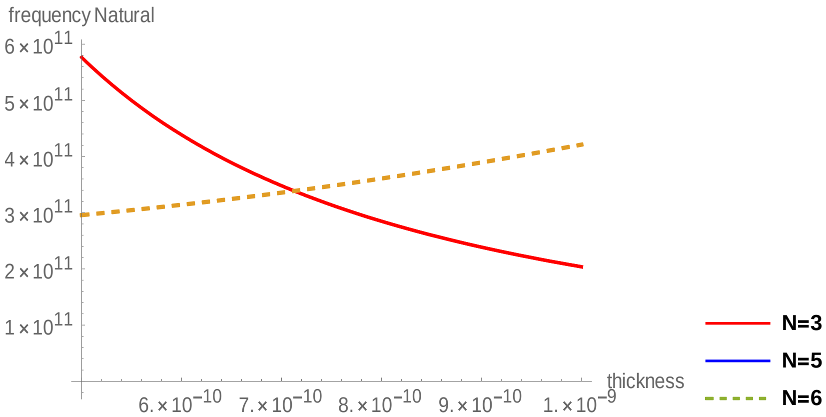

Fig. 4 Plot of variation of natural frequency of osteon with and without fluid flow effect

layers. In this part was considered mean value of radius: 7×10^{9}. It can be seen as shown in the Fig. 2 that plots of every 6 conditions are the same and have a decreasing trends. We are witness that there are those likelihood with 0.00001 precision. Every plot is monitor of variation of natural frequency versus variation of thickness H.

In Fig. 3 has been shown that variation of radius of lamellae there aren't any effect on natural frequency of whole osteon with different number of layers. In this figure was considered thickness of osteon 7×10^{-10}. In this plot was considered natural frequency for 3, 5 and 6 lamellae.

Fig. 4, compared the trends of variation of natural frequency versus thickness with and without considering effect of fluid flow in Haversian canal. It has been shown that dry osteon was exhausted entirely different behavior in comparison with the osteon which conveying capillaries and nerves. In the first one there is a decremental trend against the second one which has an incremental trend and those have a cross point in thickness H 7×10^{-10} approximately.

5. Conclusions

In this article, is explored free vibration of osteon with considering effect of blood flow and nerves within haversian canal. Finally, with using some parametric study, the parameters such as radius R, thickness H, layers number in osteon and fluid flow is investigated. It is found that the variation in radius of lamellae hasn't effected on natural frequency of osteon. In addition, observed the effect of thickness H is decremental. Figures are showing the numbers of lamellae in one osteon have no effect on natural frequency of it. Comparison between osteon with and without blood within it show that osteon behavior in those situations are completely different but in H=7×10^{-10} m both of them have same natural frequency.

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CC

Abbreviations

NSGT	nonlocal strain gradient theory	SEM	scanning electron microscopy
TEM	Transmission electron microscopy	AFM	Atomic force microscopy
FTIR	Fourier-Transform Infrared	U, V, W	Displacement fields
ρ	density	x, θ, z	coordinates
R	Radius	εx, $\varepsilon\theta$, $\gamma_{x\Theta}$	Strain components
h	Thickness	σ_{ij}	Stress components
l_s	Size parameter	Π	Strain energy
T	Kinetic energy	W	work