A high-order closed-form solution for interfacial stresses in externally sandwich FGM plated RC beams

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Abstract.   In this paper, an improved theoretical solution for interfacial stress analysis is presented for simply supported concrete beam bonded with a sandwich FGM plate. Interfacial stress analysis is presented for simply supported concrete beam bonded with a sandwich plate. This improved solution is intended for application to beams made of all kinds of materials bonded with a thin plate, while all existing solutions have been developed focusing on the strengthening of reinforced concrete beams, which allowed the omission of certain terms. It is shown that both the normal and shear stresses at the interface are influenced by the material and geometry parameters of the composite beam. A numerical parametric study was performed for different simulated cases to assess the effect of several parameters. Numerical comparisons between the existing solutions and the present new solution enable a clear appreciation of the effects of various parameters. The results of this study indicated that the FGM sandwich panel strengthening systems are effective in enhancing flexural behavior of the strengthened RC beams.

Keywords: RC beam; sandwich plate; interfacial stresses; strengthening; functionally graded material

1. Introduction

Few decades, polymer composites were introduced to the construction industry as new alternative structural materials and strengthening system for existing structures. One of the applications that attracted the attention of civil engineers was the external strengthening of existing structures constructed from different conventional materials. The concept of sandwich structure has been known for decades for its superiority to resist both out-of-plane as well as in-plane loads. However, most of the applications were focused in utilizing sandwich panels as stand-alone members, especially in aerospace industry and recently in bridge applications. The majority of the FRP bridge deck applications in USA are in the form of sandwich construction. One of the main aspects of the bonded strengthening technology is the stress analysis of the reinforced structure. In particular, reliable evaluation of the adhesive shear stress and of the stress in the CFRP plates is mandatory in order to predict the beam's failure load. Recently, the authors conducted a numerical

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study on the static behaviour of RC beams strengthened with composites in different directions (Rabahi *et al.* 2015a, Yeghnem *et al.* 2017, Alam and Al Riyami 2018, Keskin *et al.* 2017, Hassaine Daouadji *et al.* 2016c, Smith and Teng 2001, Hadi and Yuan 2017, Yang and Wu 2007, Bouakaz *et al.* 2014, Mahi *et al.* 2014, Ge *et al.* 2018, Guenaneche and Tounsi 2014, Krour *et al.* 2014, Huang *et al.* 2018, Touati *et al.* 2015, Zidani *et al.* 2015, Adim *et al.* 2016a, b, Benferhat *et al.* 2016a and Yang *et al.* 2010). Numerical examples and a parametric study are presented to illustrate the governing parameters that control the stress concentrations at the edge of the FRP strip. Finally, the results of these investigations show that the interface bond-stresses are non-uniformly distributed along the reinforced boundaries. It is believed that the present results will be of interest to civil and structural engineers and researchers.

The simple approximate closed-form solutions discussed in this paper provide a useful but simple tool for understanding the interfacial behaviour and for exploitation in developing a design rule. The present work concerns the shear and normal stresses concentrations at the ends of the sandwich FGM plate. In this paper, the details of the interfacial shear and normal stress are analyzed by the improved theoretical solutions. The effects of the material and geometry parameters on the interface stresses are considered and compared with that resulting from literature.

2. Method of solution

2.1 Assumptions

The present analysis takes into consideration the transverse shear stress and strain in the beam and the plate but ignores the transverse normal stress in them. One of the analytical approach proposed by Hassaine Daouadji *et al.* 2016c for concrete beam strengthened with a bonded sandwich FGM plate (Fig. 1) was used in order to compare it with a finite element analysis. The analytical approach (Hassaine Daouadji *et al.* 2016c) is based on the following assumptions:

- Elastic stress strain relationship for concrete, sandwich plate and adhesive;
- There is a perfect bond between the FGM plate and the beam;
- The adhesive is assumed to only play a role in transferring the stresses from the concrete to the composite plate reinforcement;
- The stresses in the adhesive layer do not change through the direction of the thickness.

Since the functionally graded materials is an orthotropic material. In analytical study (Hassaine Daouadji *et al.* 2016a, b, the classical plate theory is used to determine the stress and strain behaviours of the externally bonded composite plate in order to investigate the whole mechanical performance of the composite – strengthened structure.

2.2 Properties of the FGM constituent materials

The FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function or exponential function to describe the volume fractions. However, only a few studies used sigmoid function to describe the volume fractions. Consider an elastic FGM plate of uniform thickness *h*, which is made of a ceramic and metal, is considered in this study. The material properties, young's modulus and the poisson's ratio, on the upper and lower surfaces are
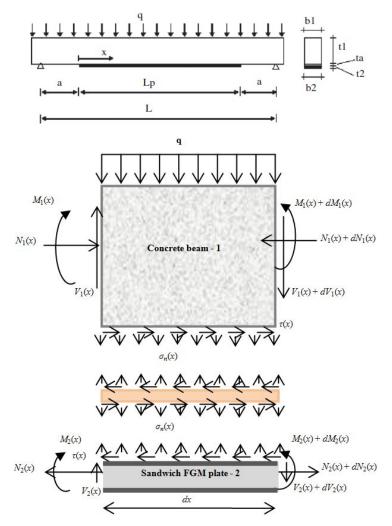
Fig. 1 Simply supported RC beam strengthened with sandwich FGM plate

different but are preassigned according to the performance demands (Abdelhak *et al*. 2016, Meziane *et al*. 2015, Bellifa *et al*. 2015, Benferhat *et al*. 2016b, Bensattalah *et al*. 2016, Hadji *et al*. 2016, Mantari and Soares 2014 and Bourada *et al*. 2015). However, the Young's modulus and the Poisson's ratio of the beams vary continuously only in the thickness direction (*z*-axis) i.e., $E = E(z), v = v(z)$. Hassaine Daouadji (2013) indicated that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus. Thus, Poisson's ratio of the plate is assumed to be constant. However, the Young's modules in the thickness direction of the FGM plates vary with power-law functions (P-FGM). The material properties of P-FGM plates are assumed to vary continuously through the thickness. Three homogenization methods are deployable for the computation of the Young's modulus $E(z)$ namely

$$E(z) = E_2 + (E_1 - E_2)(\frac{z}{h} + \frac{1}{2})^p \qquad (1)$$

where E_2 is the Young's modulus of the homogeneous plate; E_2 denote Young's modulus of the bottom (as metal) and top E_1 (as ceramic) surfaces of the FGM plate, respectively; E_2 is Young's modulus of the homogeneous plate; and p is a parameter that indicates the material variation through the plate thickness. For the power law distribution P-FGM, the Young's modulus is given as Hassaine Daouadji (2013).

The linear constitutive relations of a FG plate can be written as

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}
\tag{2}
$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively.

The computation of the elastic constants Q_{ij} in the plane stress reduced elastic constants, defined as

$$
Q_{11} = Q_{22} = \frac{E(z)}{1 - v^2} \qquad Q_{12} = \frac{v\,E(z)}{1 - v^2} \qquad Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1 + v)}
\tag{3}
$$

where A_{ij}, D_{ij} are the plate stiffness, defined by

$$
A_{ij} = \int_{-h/2}^{h/2} Q_{ij} dz \qquad\qquad D_{ij} = \int_{-h/2}^{h/2} Q_{ij} z^2 dz
\tag{4}
$$

where A'_{11}, D'_{11} are defined as

$$
A'_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} \qquad\qquad D'_{11} = \frac{D_{22}}{D_{11}D_{22} - D_{12}^2}
\tag{5}
$$

2.3 Shear stress distribution along the sandwich FGM plate – concrete interface

The governing differential equation for the interfacial shear stress (Hassaine Daouadji *et al.* 2016c) is expressed as

$$
\frac{d^2\tau(x)}{dx^2} - K_1 \left(A'_{11} + \frac{b_2}{E_1 A_1} + \frac{(y_1 + t_2/2)(y_1 + t_a + t_2/2)}{E_1 I_1 D'_{11} + b_2} b_2 D'_{11} \right) \tau(x)
$$
$$
+ K_1 \left(\frac{(y_1 + t_2/2)}{E_1 I_1 D'_{11} + b_2} D'_{11} \right) V_T(x) = 0
\tag{6}
$$

Where

$$K_1 = \frac{1}{\left(\dfrac{t_a}{G_a} + \dfrac{t_1}{4G_1} \right)} \tag{7}$$

For simplicity, the general solutions presented below are limited to loading which is either concentrated or uniformly distributed over part or the whole span of the beam, or both. For such loading, $d^2 V_T(x)/dx^2 = 0$, and the general solution to Eq. (6) is given by

$$\tau(x) = B_1 \cosh(\lambda x) + B_2 \sinh(\lambda x) + m_1 V_T(x) \tag{8}$$

Where

$$\lambda^2 = K_1 \left(A'_{11} + \frac{b_2}{E_1 A_1} + \frac{(y_1 + t_2/2)(y_1 + t_a + t_2/2)}{E_1 I_1 D'_{11} + b_2} b_2 D'_{11} \right) \tag{9}$$

$$m_1 = \frac{K_1}{\lambda^2} \left(\frac{(y_1 + t_2/2)}{E_1 I_1 D'_{11} + b_2} D'_{11} \right) \tag{10}$$

And B_1 and B_2 are constant coefficients determined from the boundary conditions. In the present study, a simply supported beam has been investigated which is subjected to a uniformly distributed load (Fig. 1). The interfacial shear stress for this uniformly distributed load at any point is written as (Hassaine Daouadji *et al*. 2016c)

$$\tau(x) = \left[\frac{m_2 a}{2}(L-a) - m_1 \right] \frac{q e^{-\lambda x}}{\lambda} + m_1 q \left(\frac{L}{2} - a - x \right) \qquad 0 \le x \le L_P \tag{11}$$

Where q is the uniformly distributed load and x; a; L and L_p are defined in Fig. 1.

2.4 Normal stress distribution along the sandwich FGM plate – concrete interface

The following governing differential equation for the interfacial normal stress (Hassaine Daouadji *et al*. 2016c)

$$\frac{d^4 \sigma_n(x)}{dx^4} + K_n \left(D'_{11} + \frac{b_2}{E_1 I_1} \right) \sigma_n(x) - K_n \left(D'_{11} \frac{t_2}{2} - \frac{y_1 b_2}{E_1 I_1} \right) \frac{d\tau(x)}{dx} + \frac{q K_n}{E_1 I_1} = 0 \tag{12}$$

The general solution to this fourth–order differential equation is

$$\sigma_n(x) = e^{-\beta x} \left[C_1 \cos(\beta x) + C_2 \sin(\beta x) \right] + e^{\beta x} \left[C_3 \cos(\beta x) + C_4 \sin(\beta x) \right] - n_1 \frac{d\tau(x)}{dx} - n_2 q \tag{13}$$

For large values of x it is assumed that the normal stress approaches zero and, as a result, $C_3 = C_4 = 0$. The general solution therefore becomes

$$\sigma_n(x) = e^{-\beta x}\left[C_1 \cos(\beta x) + C_2 \sin(\beta x)\right] - n_1 \frac{d\tau(x)}{dx} - n_2 q \tag{14}$$

Where

$$\beta = \sqrt[4]{\frac{K_n}{4}\left(D_{11}^{'} + \frac{b_2}{E_1 I_1}\right)} \tag{15}$$

$$n_1 = \left(\frac{y_1 b_2 - D_{11}^{'} E_1 I_1 t_2 / 2}{D_{11}^{'} E_1 I_1 + b_2}\right) \qquad\qquad n_2 = \frac{1}{D_{11}^{'} E_1 I_1 + b_2} \tag{16}$$

As is described by Hassaine Daouadji *et al*. 2016c, the constants C_1 and C_2 in Eq. (13) are determined using the appropriate boundary conditions and they are written as follows

$$C_1 = \frac{K_n}{2\beta^3 E_1 I_1}\left[V_T(0) + \beta M_T(0)\right] - \frac{n_3}{2\beta^3}\tau(0) + \frac{n_1}{2\beta^3}\left(\frac{d^4\tau(0)}{dx^4} + \beta \frac{d^3\tau(0)}{dx^3}\right) \tag{17}$$

$$C_2 = -\frac{K_n}{2\beta^2 E_1 I_1} M_T(0) - \frac{n_1}{2\beta^2}\frac{d^3\tau(0)}{dx^3} \tag{18}$$

$$n_3 = b_2 K_n\left(\frac{y_1}{E_1 I_1} - \frac{D_{11}^{'} t_2}{2b_2}\right) \tag{19}$$

The above expressions for the constants C_1 and C_2 has been left in terms of the bending moment $M_T(0)$ and shear force $V_T(0)$ at the end of the soffit plate. With the constants C_1 and C_2 determined, the interfacial normal stress can then be found using Eq. (13).

3. Numerical verification and discussions

3.1 Comparison with approximate solutions

The present simple solution is compared, in this section, with some approximate solutions available in the literature. These include Tounsi *et al*. (2008) and Hassaine Daouadji *et al*. (2016c) solutions uniformly distributed loads. A comparison of the interfacial shear and normal stresses from the different existing closed – form solutions and the present solution is undertaken in this section. An undamaged beams bonded with CFRP, FGM and sandwich FGM plate soffit plate is considered. The beam is simply supported and subjected to a uniformly distributed load. A summary of the geometric and material properties is given in Table 1. The results of the peak interfacial shear and normal stresses are given in table 2 for the beams strengthened by bonding CFRP, FGM and sandwich FGM plate. As it can be seen from the results, the peak interfacial stresses assessed by the present theory are smaller compared to those given by Tounsi *et al*. (2008) and Hassaine Daouadji *et al*. (2016c) solutions. This implies that adherend shear deformation

Table 1 Geometric and material properties

Component	Width (mm)	Depth (mm)	Young's modulus (MPa)	Poisson's ratio	Shear modulus (MPa)
RC beam	$b_1 = 200$	$t_1 = 300$	$E_1 = 30\ 000$	0.18	-
Adhesive layer	$b_a = 200$	$t_a = 4$	$E_a = 3000$	0.35	-
CFRP plate	$b_2 = 200$	$t_2 = 4$	$E_2 = 140\ 000$	0.28	$G_{12} = 5000$
FGM (ZrO_2) plate	$b_2 = 200$	$t_2 = 4$	$E_2 = 200\ 000$	0.3	$G_{12} = 5000$

is an important factor influencing the adhesive interfacial stresses distribution. Figs. 2(a) and (b) plots the interfacial shear and normal stresses near the plate end for the example RC beam bonded with a sandwich FGM plate for the uniformly distributed load case. Overall, the predictions of the different solutions agree closely with each other. The interfacial normal stress is seen to change sign at a short distance away from the plate end. The present analysis gives lower maximum interfacial shear and normal stresses than those predicted by Tounsi *et al*. (2008) and Hassaine Daouadji *et al*. (2016c), indicating that the inclusion of adherend shear de formation effect in the beam and soffit plate leads to lower values of σ_{max} and τ_{max}. However, the maximum interfacial shear and normal stresses given by Tounsi *et al*. (2008) and Hassaine Daouadji *et al*. (2016c) methods are lower than the results computed by the present solution. This difference is due to the assumption used in the present theory which is in agreement with the beam theory. Hence, it is apparent that the adherend shear deformation reduces the interfacial stresses concentration and thus renders the adhesive shear distribution more uniform. The interfacial normal stress is seen to change sign at a short distance away from the plate end.

The results of the peak interfacial shear and normal stresses are given in Table 2 for the RC beam with a CFRP, FGM and sandwich soffit plate. Table 2 shows that, for the UDL case, the present solution gives results which generally agree better with those from Tounsi's (Tounsi *et al*. 2008) and Hassaine Daouadji's (Hassaine Daouadji *et al*. 2016c) solutions. The latter two again give similar results. In short, it may be concluded that all solutions are satisfactory for RC beams bonded with a thin plate as the rigidity of the soffit plate is small in comparison with that of the RC beam. Those solutions which consider the additional bending and shear deformations in the soffit plate due to the interfacial shear stresses give more accurate results. The present solution is the only solution which covers the uniformly distributed loads and considers this effect and the effects of other parameters.

Table 2 Comparison of peak interfacial shear and normal stresses (MPa)

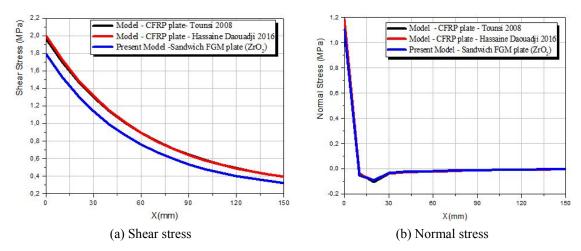| Model	Beam - Uniformly Distributed Load	Shear	Normal
Tounsi *et al*. (2008)	RC beam with CFRP plate	1.96203	1.1694
Hassaine Daouadji *et al*. (2016c)	RC beam with CFRP plate	1.9982	1.1887
Present model	RC beam with P-FGM plate ($P = 2$)	1.7801	1.0945
Present model	RC beam with sandwich plate Homogeneous face sheet and FGM core ($P = 2$)	1.7895	1.1046

(a) Shear stress   (b) Normal stress

Fig. 2 Comparison of interfacial shear and normal stress for sandwich FGM plate RC beam with the analytical results

Table 3 Effect of plate stiffness on interfacial stresses in FGM strengthened RC beam

	FGM plate - ZrO_2						
	Ceramic $P = 0$	$P = 0,5$	$P = 2$	$P = 5$	$P = 10$	$P = 100$	Metal: $P = = \infty$
Shear stress	2.29383	2.06355	1.78010	1.60980	1.52388	1.42472	1.41202
Normal stress	1.26050	1.20763	1.09456	1.01908	.99022	.98252	.98556
	Sandwich plate : Homogeneous face sheet and FGM core - ZrO_2						
	Ceramic $P = 0$	$P = 0,5$	$P = 2$	$P = 5$	$P = 10$	$P = 100$	Metal: $P = = \infty$
Shear stress	2.19097	2.00645	1.78955	1.66507	1.60407	1.53532	1.52664
Normal stress	1.28005	1.21124	1.10466	1.04003	1.01188	0.98795	0.98592

3.2 Parametric studies

To better understand the behavior of bonded beam repairs, which will help engineers in optimizing their design parameters, the effects of several parameters were investigated. The material used for the present studies is an RC beam bonded with a FGM plate. The beams are simply supported and subjected to a uniformly distributed load. A summary of the geometric and material properties is given in Table 1. The span of the RC beam is 3000 mm, the distance from the support to the end of the plate is 300 mm and the uniformly distributed load is 50 kN/m.

The Table 3 give interfacial normal and shear stresses for the RC beam bonded with a sandwich and FGM plate, which demonstrates the effect of plate material properties on interfacial stresses. The results show that, as the plate material becomes softer (from FGM plate then sandwich FGM plate), the interfacial stresses become smaller, as expected. This is because, under the same load, the tensile force developed in the plate is smaller, which leads to reduced interfacial stresses. The position of the peak interfacial shear stress moves closer to the free edge as the plate becomes less stiff. Effect of plate stiffness on interfacial stress Fig. 3 and Table 3 gives interfacial normal and shear stresses for the RC beam bonded with a FGM plate, which demonstrates the effect of plate
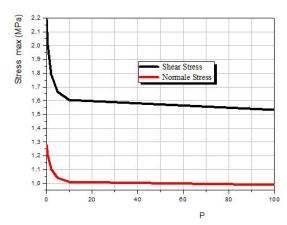
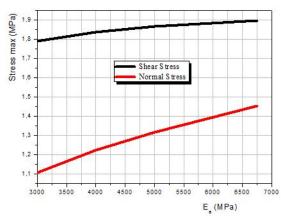Fig. 3 Effect of degree of homogeneity on inter-facial stresses in sandwich FGM plate strengthened RC beam

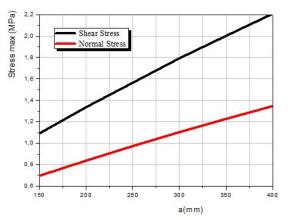Fig. 4 Influence of length of unstrengthened region on edge stresses for RC beam with a bonded sandwich FGM plate

Fig. 5 Effect of adhesive moduli on interfacial stresses in sandwich FGM plate streng-thened RC beam
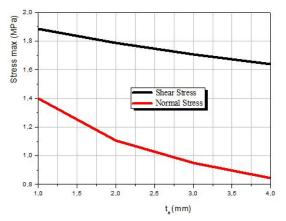

Fig. 6 Effect of adhesive layer thickness on edge shear stresses in sandwich FGM plate strengthened RC beam

material properties on interfacial stresses. The results show that, as the plate material becomes softer (Sandwich with homogeneous face sheet and FGM core), the interfacial stresses become smaller, as expected. This is because, under the same load, the tensile force developed in the plate is smaller, which leads to reduced interfacial stresses. The position of the peak interfacial shear stress moves closer to the free edge as the plate becomes less stiff. The influence of the length of the ordinary-beam region (the region between the support and the end of the composite strip on the edge stresses) appears in Fig. 4. It is seen that, as the plate terminates further away from the supports, the interfacial stresses increase significantly. This result reveals that in any case of strengthening, including cases where retrofitting is required in a limited zone of maximum bending moments at midspan, it is recommended to extend the strengthening strip as possible to the lines. The adhesive layer is a relatively soft, isotropic material and has a smaller stiffness. The four sets of Young's moduli are considered here, which are 3, 4, 5 and 6.7 GPa. The Poisson's ratio of the

adhesive is kept constant. The numerical results in Fig. 5 show that the property of the adhesive hardly influences the level of the interfacial stresses, whether normal or shear stress, but the stress concentrations at the end of the plate increase as the Young's modulus of the adhesive increases. Then Fig. 6 show the effects of the thickness of the adhesive layer on the interfacial stresses. Increasing the thickness of the adhesive layer leads to a significant reduction in the peak interfacial stresses. Thus using thick adhesive layer, especially in the vicinity of the edge, is recommended. In addition, it can be shown that these stresses decrease during time, until they become almost constant after a very long time.

4. Conclusions

The interfacial stresses in the sandwich FGM–RC hybrid beam were investigated by an improved theoretical solution. The adherend shear deformations have been included in the theoretical analyses by assuming linear shear stress distributions through the thickness of the adherends. The obtained solution could serve as a basis for establishing simplified FGM theories or as a benchmark result to assess other approximate methodologies. The present solution is thus recommended as a widely applicable and accurate solution with due simplicity for application to beams bonded with a soffit plate, particularly when the plate is relatively stiff.

Acknowledgments

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