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Rayleigh waves in orthotropic magneto-thermoelastic media under three GN-theories

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Abstract. The present work is considered to study the two-dimensional problem in an orthotropic magneto-thermoelastic media and examined the effect of thermal phase-lags and GN-theories on Rayleigh waves in the light of fractional order theory with combined effect of rotation and hall current. The boundary conditions are used to derive the secular equations of Rayleigh waves. The wave properties such as phase velocity, attenuation coefficient are computed numerically. The numerical simulated results are presented graphically to show the effect of phase-lags and GN-theories on the Rayleigh wave phase velocity, attenuation coefficient, stress components and temperature change. Some particular cases are also discussed in the present investigation.

Keywords: attenuation coefficient; fractional order; hall current; orthotropic; phase lags; phase velocity; Rayleigh wave propagation; rotation

1. Introduction

The wave theory is one of the important branches in the solid mechanics and continuum dynamics. The study of elastic surface waves in different media along with the different interfaces has great importance in thermoelastic wave theory. The interface waves require at least one of the two mediums is solid while the other medium may be a vacuum, air, a liquid or a solid. In addition to longitudinal and transverse waves there are various types of surface waves. The surface waves which can transmit along the stress free surface of a solid are called Rayleigh waves. These elastic waves are helpful to examine the nature of earthquakes, damage of large buildings as these are much responsible than the other seismic waves. These waves are generally non-dispersive in nature. The magneto-thermoelastic theories are concerned with the interacting effects of an externally applied magnetic field on elastic and thermo-elastic deformations in the solids. When heat is supplied to the body then mechanical waves are produced with thermal expansion. It was observed that the interactions between the thermal and mechanical fields occurred through the Lorentz force, Ohm's law and the electric field created by the velocity of a material particle moving in a magnetic field. When the magnetic field is strong, the influence of hall current and rotation cannot be ignored. The

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interaction between the externally applied magnetic field and thermoelastic deformations give rise to the coupled field of magnetothermoelasticity. Moreover, the fractional order theory of generalized thermoelasticity is well known part of solid mechanics. Most of the practical problems which contain differential equations of fractional order are solved with the help of fractional order theory. The applications of Fractional calculus are used in diverse fields like in quantum mechanics, nuclear physics, biology, fluid mechanics, control theory etc. The most important advantage of using differential equation of fractional order is due to their non-localization property. It was Abel, who first introduced the fractional derivatives in the formulation of tautochrone problem. Oldham and Spainer (1974) gave some alternate definitions of fractional thermoelasticity. The Propagation of Rayleigh waves along with isothermal and insulated boundaries was discussed by Chadwick and Windle (1964). Rayleigh (1885) was the one who introduced first these waves to find the solution of a free vibration problem of an elastic half space. Marin (1999) obtained the existence and uniqueness solutions for the mixed initial-boundary value problems in thermoelasticity of dipolar bodies. Abbas (2011) studied the two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation. Ahmed and Abo-Dahab (2012) studied the influence of initial stress and gravity field on propagation of Rayleigh and Stoneley waves in a thermoelastic orthotropic granular medium. Abd-Alla *et al*. (2012) investigated the effect of rotation on propagation of Rayleigh waves in orthotropic elastic media under initial stress and gravity. Zakaria (2014) examined the effect of hall current in a micropolar magneto-thermoelastic solid due to ramp type heat. Abbas (2014) studied the interactions in a thermoelastic fibre reinforced anisotropic plate with fractional order theory of type GN-II. Deswal and Kalkal (2014) studied the plane wave problem in a micropolar magneto-thermoelastic half-space with fractional order. Mahmoud (2014) studied the effect of non-homogeneity, magnetic field and gravity field on Rayleigh waves in an initially stressed elastic half-space of orthotropic material subject to rotation. Das and Kanoria (2014) studied the finite thermal waves in a magneto-thermoelastic rotating medium. Abbas (2015) studied the vibrations in a thermoelastic hollow sphere in the context of generalized thermoelastic theory with one relaxation time with the help of eigen value approach. Abbas (2015) studied the effect of fractional parameter in an infinite isotropic thermoelastic body with spherical cavity. Marin *et al*. (2015) extended the domain of influence theorem for generalized thermoelasticity of anisotropic material with voids. Marin *et al*. (2015) studied the double porosity structure for micropolar bodies. Xiong and Tian (2017) investigated the transient thermo-piezo-elastic responses of a functionally graded piezoelectric plate due to thermal shock. Kumar *et al.* (2017) studied the Rayleigh wave propagation problem in anisotropic magnetothermoelastic medium. Abd-Alla *et al.* (2017) studied the rotational effect on thermoelastic Stoneley, Love and Rayleigh waves in fibre-reinforced anisotropic general viscoelastic media of higher order. Abbas and Marin (2017) examined the analytical solution of thermoelastic interaction in a half space due to pulsed laser heating. Hobiny and Abbas (2018) studied the thermal damages in skin tissues induced by moving heat source and examined the effects of heat source velocity on the temperature of skin tissue and thermal damages. Shahsavari *et al.* (2018) studied a higher-order gradient model for wave propagation of porous FG nanoplates. Shaw *et al.* (2018) studied the characteristics of Rayleigh wave propagation in orthotropic medium. Hobiny and Abbas (2018) studied the analytical solutions of photo-thermo-elastic waves in a non-homogenous semiconducting material. Biswas and Mukhopadhyay (2018) studied the Rayleigh wave propagation problem in Orthotropic medium with three phase lags by using Eigen function expansion method. Biswas and Abo-Dahab (2018) studied the effect of phase lags on Rayleigh wave propagation in initially stressed magneto- thermoelastic orthotropic medium. Singh and Verma (2019) studied the Rayleigh wave propagation in isotropic medium by using five different theories of thermoelasticity. Othman *et al.* (2019) studied the effect of gravity field on the fibre-reinforced thermoelastic medium with two temperature and three phase-lag model of heat transfer by using GN-II and GN-III theory. Shaw and Othman (2019) studied the Rayleigh wave propagation problem in orthotropic medium with elastic half space and two temperature theory of generalized thermoelasticity. Horrigue and Abbas (2020) studied the propagation of plane waves in a fiber-reinforced anisotropic thermoelastic half space under the effect of magnetic field. Lata and Himanshi (2020, 2021a, 2021b) studied the various orthotropic thermoelastic problems of generalized thermoelasticity with fractional order heat transfer. Abouelregal *et al.* (2020) derived the fundamental equations in generalized thermoelastic diffusion with four lags and higher-order time-fractional derivatives. Ezzat (2020) studied the fractional thermo-viscoelastic response of biological tissue with variable thermal material properties. Lata and Kaur (2021) examined the interactions in a homogeneous isotropic modified couple stress thermoelastic solid with multi-dual-phase-lag heat transfer and two-temperature. Lata and Singh (2021) studied the Stoneley wave propagation problem in non-local isotropic magneto-thermoelastic solid with multi-dual-phase-lag heat transfer. Draiche *et al*. (2021) investigated the flexural response of laminated composite plates using a simple quasi-3D HSDT. Mudhaffar *et al.* (2021) studied the Hygro-thermo-mechanical bending behaviour of advanced functionally graded ceramic metal plate resting on a viscoelastic foundation. Bouafia *et al*. (2021) analyzed the bending and

free vibration characteristics of various compositions of FG-plates on elastic foundation via quasi 3D HSDT model. Djilali *et al*. (2021) studied the original four-variable quasi-3D shear deformation theory for the static and free vibration analysis of new type of sandwich plates with both FG face sheets and FGM hard core. Zaitoun *et al*. (2022) studied the influence of the visco-Pasternak foundation parameters on the buckling behavior of a sandwich functional graded ceramic–metal plate in a hygrothermal environment. Hebali *et al*. (2022) studied the effect of the variable visco-Pasternak foundations on the bending and dynamic behaviors of FG plates using integral HSDT model. Tahir *et al*. (2022) studied the effect of three-variable viscoelastic foundation on the wave propagation in functionally graded sandwich plates via a simple quasi-3D HSDT. Vinh and Tounsi studied the (2022) studied the free vibration analysis of functionally graded doubly curved nanoshells using nonlocal first-order shear deformation theory with variable nonlocal parameters. Rachid *et al*. (2022) studied the mechanical behaviour and free vibration analysis of of FG doubly curved shells on elastic foundation by using new modified displacements field model of 2D and quasi-3D HSDTs. Including all the above work, we conclude that the Rayleigh wave problem in an orthotropic magneto-thermoelastic media with fractional order heat transfer with combined effect of rotation, hall current in generalized thermoelasticity has not been considered yet. So in the present study, we investigated the effect of phase-lags and GN-theories on Rayleigh wave phase velocity, attenuation coefficient, displacement components, stress components and thermodynamical temperature. The numerically produced results are depicted graphically to show the effect of phase-lags and GN-theories.

2. Basic equations

Following Lata and Himanshi (2021a), the equation of motion for an orthotropic thermoelastic medium rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega\vec{n}$, where $\vec{n}$ is unit vector representing the direction of axis of rotation and taking into account Lorentz force is given as

$$\sigma_{ij,j} + F_i = \rho\left[\ddot{u}_i + (\mathbf{\Omega} \times (\mathbf{\Omega} \times \vec{u}))_i + (2\,\mathbf{\Omega} \times \dot{\vec{u}})_i\right], \tag{1}$$

Lorentz force component is given by

$$F_i = \mu_0 \ (\vec{J} \times \vec{H_0})_i \,, \tag{2}$$

Where, $\vec{H} = (0, H_0, 0)$ is the magnetic field strength, $\vec{J}$ is the current density vector, μ_0 is magnetic permeability.

The additional terms $\mathbf{\Omega} \times (\mathbf{\Omega} \times \vec{u})$ and $2\, \mathbf{\Omega} \times \dot{\vec{u}}$ on the right side of above Eq. (2) are centripetal acceleration and Coriolis acceleration respectively.

Following Lata and Zakhmi (2020), heat equation in anisotropic medium with three-phase-lags and fractional order heat transfer is given by

$$K_{ij} \left(1 + \frac{\tau_t^{\alpha}}{\alpha!} \frac{\partial}{\partial t^{\alpha}}\right) \dot{T}_{,ji} + K_{ij}^{*} \left(1 + \frac{\tau_v^{\alpha}}{\alpha!} \frac{\partial^{\alpha}}{\partial t^{\alpha}}\right) T_{,ji} = \left[1 + \frac{\tau_q^{\alpha}}{\alpha!} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\right] \left[\rho\, C_E \ddot{T} + \beta_{ij} T_0 \ddot{e}_{ij}\right], \tag{3}$$

Where,

$$\beta_{ij} = c_{ijkl}\alpha_{ij}, \beta_{ij} = \beta_i \delta_{ij,} \, K_{ij} = K_i \, \delta_{ij} \,, K_{ij}^{*} = K_i^{*} \delta_{ij},$$

i is not summed $(i, j = 1,2,3)$ and δ_{ij} is Kronecker delta.

Following Kumar *et al*. (2016), the above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the hall current effect.

$$\vec{J} = \frac{\sigma_0}{1+m^2} \left[\vec{E} + \mu_0 \ \left(\dot{\vec{u}} \times \vec{H} - \frac{1}{en_e} \vec{J} \times \vec{H_0}\right)\right], \tag{4}$$

Also the strain displacement relations are

$$e_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i}\right), \ \ i, j = 1,2,3. \tag{5}$$

Here, dot (.) represents the partial derivative w.r.t time and (,) denote the partial derivative w.r.t spatial coordinate. $c_{ijkl} = (c_{klij} = c_{jikl} = c_{ijlk})$ is the tensor of elastic constant, ρ is the density, T_0 is the reference temperature such that $|\frac{T}{T_0}| \ll 1$, u_i are the components of displacement vector $\vec{u}$, C_E is the specific heat at constant strain, , $\sigma_{ij} = (\sigma_{ji})$ are the components of stress tensor. T is the temperature change, α_{ij} is the coefficient of linear thermal expansion, β_{ij} is the tensor of thermal moduli, $\mathbf{\Omega}$ is the angular velocity of the solid, K_{ij} are the components of thermal conductivity and K_{ij}^{*} are the materialistic constants respectively. τ_q , τ_t and τ_v are respectively, the phase lag of the heat flux, the phase lag of the temperature gradient and the phase lag of the thermal displacement, H is the magnetic strength, $\vec{J}$ is the current density vector, $\vec{E}$ is the intensity vector of electric field, m is the hall parameter given by $m = \omega_e t_e = \frac{\sigma_0 \mu_0 H_0}{en_e}$, where t_e is the electron collision time. Where, $\omega_e = \frac{e\mu_0 H_0}{m_e}$ is the electron frequency, $\sigma_0 = \frac{e^2 t_e n_e}{m_e}$ is the electrical conductivity, e is the charge on electron, m_e is the mass of electron and n_e is the no of density of electrons.

3. Formulation of the problem

We consider a perfectly conducting homogeneous orthotropic magneto-thermoelastic medium rotating with an angular velocity $\mathbf{\Omega} = \Omega\vec{n},$ initially at uniform temperature T_0 in the context of three-phase-lag fractional order model of thermoelasticity with an initial magnetic field $\vec{H} = (0, H_0, 0)$, towards y-axis. The origin of the coordinate system (x, y, z) is taken on $(z = 0)$.We choose x-axis in the direction of wave propagation in such a way that all the particles on a line parallel to the y-axis are equally displaced, so that $v = 0$ and u, w, T are independent of y. The surface of half-space is subjected to thermomechanical sources. For the 2D problem in xz-plane, we take

$$u = u(x, z, t), v = 0, w = w\,(x, z, t), T = T(\,x, z, t), \tag{6}$$

Let us assume that

$$\mathbf{E} = 0,\ \ \boldsymbol{\Omega} = (\,0, \Omega, 0), \tag{7}$$

From the generalized ohm's law

$$J_2\ = 0, \tag{8}$$

Also the current density components by using Eq. (4) are given by

$$J_1\ = \frac{\sigma_0 \mu_0 H_0}{1+m^2}\left(m\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right), \tag{9}$$

$$J_3\ = \frac{\sigma_0 \mu_0 H_0}{1+m^2}\left(\frac{\partial u}{\partial t} + m\ \frac{\partial w}{\partial t}\right), \tag{10}$$

Following Kumar and Chawla (2014), the stress-strain relations in an orthotropic medium is given by

$$\sigma_{xx} = \ C_{11}\, e_{xx} + C_{13}\, e_{zz} - \beta_1 T, \tag{11}$$

$$\sigma_{zz} = \ C_{13}\, e_{xx} + \ C_{33}\, e_{zz} - \beta_3\, T, \tag{12}$$

$$\sigma_{xz} = 2C_{55}\, e_{xz}, \tag{13}$$

where,

$$e_{11} = \ \frac{\partial u}{\partial x}\ , e_{33} = \ \ \frac{\partial w}{\partial z}\ , e_{13} = \ \ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \ \frac{\partial w}{\partial x}\right),$$
$$\beta_1 = C_{11}\, \alpha_1 + \ C_{13}\, \alpha_3\ , \beta_3 = \ C_{13}\, \alpha_1 + \ C_{33}\, \alpha_3. \tag{14}$$

Eqs. (1) and (3) with the aid of (2), (5), (6)-(14) reduce to the form

$$C_{11}\frac{\partial^2 u}{\partial x^2} + \ C_{55}\frac{\partial^2 u}{\partial z^2} + \ (C_{13} + \ C_{55}\ )\ \frac{\partial^2 w}{\partial x \partial z} - \beta_1\frac{\partial T}{\partial x} - \mu_0\, j_3 H_0 = \ \rho\ \left(\frac{\partial^2 u}{\partial t^2} - \Omega^2\, u + 2\Omega\frac{\partial w}{\partial t}\right), \tag{15}$$

$$(C_{13}\ + C_{55})\frac{\partial^2 u}{\partial x \partial z}\ + C_{55}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial T}{\partial z} + \ \mu_0\, j_1 H_0 = \ \rho\ (\frac{\partial^2 w}{\partial t^2} - \Omega^2\, w - 2\Omega\frac{\partial u}{\partial t}), \tag{16}$$

$$K_1\left(1 + \frac{\tau_t^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\dot{T}_{,xx} + K_3\left(1 + \ \frac{\tau_t^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\ \dot{T}_{,zz} + \ K_1^*\left(1 + \ \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,xx} + \ K_3^*\ \left(1 + \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\ T_{,zz}\ = \left[1 + \ \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right]\left[\,\rho\, C_E \ddot{T}\ +\ T_0\ \frac{\partial^2}{\partial t^2}\ \left(\beta_1\frac{\partial u}{\partial x}\ +\ \beta_3\ \frac{\partial w}{\partial z}\right)\right]. \tag{17}$$

In the above equations, we use the contracting subscript notations

$(\,11 \to 1,\ 22 \to 2,\ 33 \to 3, 23 \to 4,\ 13 \to 5,\ 12 \to 6)$ to relate C_{ijkl} to C_{mn}

Where $i, j, k, l = 1,2,3$ and $m, n = 1,2,3,4,5,6$

We assume that the medium is initially is at rest. Then the undisturbed state is maintained at reference temperature. Then the initial and regularity conditions are

$$u(x, z, 0) = 0 = \ \dot{u}(x, z, 0),$$
$$w(x, z, 0) = 0 = \ \ \dot{w}\ (x, z, 0),$$
$$T(x, z, 0) = 0 = \ \dot{T}(x, z, 0),\ \text{For } z \geq 0\ , -\infty < x < \infty;$$

$$u(x,z,t) = w\,(x,z,t) = T(x,z,t) = 0, \qquad \text{For } t > 0 \text{ when } z \rightarrow \infty$$

To facilitate the solution the following dimensionless quantities are used

$$x' = \frac{x}{L}\,, z' = \frac{z}{L}, u' = \frac{\rho c_1^2}{LT_0\beta_1}u\,, w' = \frac{\rho c_1^2}{LT_0\beta_1}w\,, t' = \frac{C_1}{L}t,$$

$$\sigma'_{zz} = \frac{\sigma_{zz}}{T_0\beta_1}, \sigma'_{xz} = \frac{\sigma_{xz}}{T_0\beta_1}, T' = \frac{T}{T_0}, \Omega' = \frac{L}{C_1}\Omega,$$

where

$$c_1^2 = \frac{c_{11}}{\rho}. \tag{18}$$

Using dimensionless quantities given by (18) in Eqs. (15)-(17) and suppressing the primes for convenience yield

$$\left(\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial^2 u}{\partial z^2} + \delta_2\frac{\partial^2 w}{\partial x\partial z}\right) - M\left(\frac{\partial u}{\partial t} + m\frac{\partial w}{\partial t}\right) - \frac{\partial T}{\partial x} = \left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right), \tag{19}$$

$$\left(\delta_3\frac{\partial^2 w}{\partial z^2} + \delta_1\frac{\partial^2 w}{\partial x^2} + \delta_2\frac{\partial^2 u}{\partial x\partial z}\right) + M\left(m\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) - \varepsilon\frac{\partial T}{\partial z} = \left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right), \tag{20}$$

$$\epsilon_1\tau_t^1\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial x^2}\right) + \epsilon_2\tau_t^1\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right) + \epsilon_3\tau_v^1\left(\frac{\partial^2 T}{\partial x^2}\right) + \epsilon_4\tau_v^1\left(\frac{\partial^2 T}{\partial z^2}\right) = \tau_q^1\left[\frac{\partial^2 T}{\partial z^2} + \epsilon_5\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \varepsilon\frac{\partial w}{\partial z}\right)\right], \tag{21}$$

Where,

$$\delta_1 = \frac{c_{55}}{c_{11}}, \delta_2 = \frac{c_{13}+c_{15}}{c_{11}}, \delta_3 = \frac{c_{33}}{c_{11}}, \epsilon_1 = \frac{K_1}{\rho\,L\,C_1C_E}, \epsilon_2 = \frac{K_3}{\rho\,LC_1C_E}, \epsilon_3 = \frac{K_1^*}{\rho c_1^2 C_E}, \epsilon_4 = \frac{K_3^*}{\rho c_1^2 C_E},$$

$$\epsilon_5 = \frac{\beta_1^2 T_0}{\rho^2 c_1^2 C_E}, \varepsilon = \frac{\beta_3}{\beta_1}, \quad \tau_t^1 = \left(1 + \frac{\tau_t^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right),$$

$$\tau_q^1 = \left(1 + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right),$$

$$\tau_v^1 = \left(1 + \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right), M = \frac{\sigma_0\mu_0^2 H_0^2 L}{\rho C_1(1+m^2)}.$$

4. Solution of the problem

Following Biswas (2021), we take Rayleigh wave solution of the form

$$(u,w,T) = (u^*,w^*,T^*)(z)e^{i\xi(x-ct)}, \tag{22}$$

Here $c = \omega/\xi$ is the phase velocity of the wave. ξ is a wave number and ω is the angular frequency of the wave. By using (22) in Eqs. (19)-(21), we obtain a system of homogeneous equations in (u^*,w^*,T^*) i.e.

$$u^*[\,p_1 + \delta_1 D^2] + w^*[p_2 + i\xi\delta_2 D] - [i\xi]T^* = 0, \tag{23}$$

$$u^*[p_3 + i\xi\delta_2 D] + w^*[p_4 + \delta_3 D^2] - [\varepsilon D]T^* = 0, \tag{24}$$

$$u^*\left[p_5\varepsilon_5\,\tau'_q\right] + w^*\left[p_6\varepsilon\varepsilon_5\tau'_q D\right] + \left[\,p_7\varepsilon_1\tau'_t + p_8\varepsilon_2\tau'_t D^2 - \xi^2\varepsilon_3\tau'_v + \varepsilon_4\tau'_v D^2 + p_6\tau'_q\right]T^* = 0, \tag{25}$$

Where,

$$D = \frac{d}{dz},$$

$$p_1 = \Omega^2 + \xi^2(c^2 - 1) + Mi\xi c\,,$$

$$p_2 = i\xi c\,(mM + 2\,\Omega),$$
$$p_3 = -\,p_2,$$
$$p_4 = \Omega^2 + \xi^2(c^2 - \delta_1) + Mi\xi c,$$
$$p_5 = i\xi^3 c^2,$$
$$p_6 = \xi^2 c^2,$$
$$p_7 = i\xi^3 c,$$
$$p_8 = -i\xi c,$$
$$\tau_t' = 1 + \frac{\tau_t^\alpha}{\alpha!}\,(-i\xi c)^\alpha,$$
$$\tau_v' = 1 + \frac{\tau_v^\alpha}{\alpha!}\,(-i\xi c)^\alpha,$$
$$\tau_q' = 1 + \frac{\tau_q^\alpha}{\alpha!}\,(-i\xi c)^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!}\,(-i\xi c)^{2\alpha}.$$

The above resulting equations have non-trivial solution if the determinant of the coefficients (u^*, w^*, T^*) vanishes and we obtain the following characteristic equation.

$$(PD^6 + QD^4 + RD^2 + S\,)\,(u^*, w^*, T^*) = 0, \tag{26}$$

Where,

$$D = \frac{d}{dz},$$
$$P = \tau_v'[\delta_3\delta_1\varepsilon_4] + \tau_t'\,[\varepsilon_2\delta_1\,\delta_3 p_8],$$
$$Q = \tau_t'\left[p_1p_8\varepsilon_2\delta_3 + p_4p_8\delta_1\varepsilon_2 + \varepsilon_1\delta_1\delta_3p_7 + \xi^2{\delta_2}^2p_8\varepsilon_2\right] + \tau_v'\,[\,p_1\delta_3\varepsilon_4 + p_4\varepsilon_4\delta_1 - \xi^2\delta_1\,\delta_3\varepsilon_3 + \varepsilon_4\xi^2\delta_2^2\,] + \tau_q'\,[\,p_6\,\delta_1\,\delta_3 + p_6\varepsilon_5\delta_1\varepsilon^2\,],$$
$$R = \tau_t'[\,p_1p_4\,p_8\varepsilon_2 + \varepsilon_1\delta_3p_1\,p_7 + \varepsilon_1\delta_1p_4\,p_7 - p_2p_3\,p_8\varepsilon_2 + \varepsilon_1\,p_7\,\xi^2\delta_2^2] + \tau_v'[\,p_1p_4\varepsilon_4 - \xi^2p_1\,\delta_3\varepsilon_3 - \xi^2p_4\,\delta_1\varepsilon_3 - p_2p_3\,\varepsilon_4 - \delta_2^2\,\xi^4\varepsilon_3] + \tau_q'\,[\,p_1\,p_6\,\delta_3 + p_1p_6\,\varepsilon_5\varepsilon^2 + \delta_1\,p_4p_6 + p_6\xi^2\delta_2^2 - i\xi\delta_2\varepsilon\varepsilon_5\,p_5 + \xi^2\delta_2\,\varepsilon\varepsilon_5\,p_6 + i\xi\delta_3\,\varepsilon_5\,p_5\,],$$
$$S = \tau_t'\,[\,p_1p_4\,p_7\varepsilon_1 - p_2p_3\,p_7\varepsilon_1] + \tau_v'\,[\,-p_1\,p_4\,\xi^2\varepsilon_3 + p_2p_3\xi^2\varepsilon_3] + \tau_q'\,[\,-p_2p_3\,p_6 + p_1p_4\,p_6 + i\xi\varepsilon_5\,p_4\,p_5].$$

The Eq. (26) is cubic in λ_j^2 ; $j = 1,2,3$. Therefore the solution which satisfy the radiation conditions $u, w, T \to 0$ as $z \to \infty$ is given by

$$u^* = \sum_{j=1}^{3} A_j\,e^{-\lambda_j z}, \tag{27}$$

$$w^* = \sum_{j=1}^{3} d_jA_je^{-\lambda_j z}, \tag{28}$$

$$T^* = \sum_{j=1}^{3} l_jA_je^{-\lambda_j z}, \tag{29}$$

Here $\pm\lambda_j$ are the roots of Eq. (26) and A_j are arbitrary constants and the coupling constants d_j and l_j ; ($j = 1,2,3$)are given by

$$d_j = \frac{\lambda_j^4A^* + \lambda_j^2B^* + C^*}{\lambda_j^4A' + \lambda_j^2B' + C'}, \quad \text{j} = 1, 2, 3.$$
$$l_j = \frac{\lambda_j^4P^* + \lambda_j^2Q^* + R^*}{\lambda_j^4A' + \lambda_j^2B' + C'}, \ \text{j} = 1, 2, 3. \tag{30}$$

where,

$$A^* = \tau_t'\,[\delta_1\,p_8\varepsilon_2] + \tau_v'[\delta_1\,\varepsilon_4\,],$$
$$B^* = \tau_t'[\,p_1p_8\varepsilon_2 + \delta_1\,\varepsilon_1p_7\,] + \tau_v'\,[p_1\varepsilon_4 - \delta_1\xi^2\varepsilon_3\,],$$

$$C^* = \tau_t' \, [\, p_1 p_7 \, \varepsilon_1] + \tau_v'[-p_1 \xi^2 \varepsilon_3 \,] \, \tau_q' \, [\, p_1 p_6 \;\; + i \, \xi \varepsilon_5 \, p_5 \; \xi^2],$$
$$A' = \tau_t' \, [p_8 \, \varepsilon_2 \delta_3 \,] + \tau_v' [\, \varepsilon_4 \delta_3 \,],$$
$$B' = \tau_t' \, [\;\; p_4 p_8 \, \varepsilon_2 + \; p_7 \, \varepsilon_1 \delta_3 \,] + \tau_v' [\, p_4 \, \varepsilon_4 - \delta_3 \varepsilon_3 \xi^2] + \tau_q' [\, \delta_3 p_6 \;\; + \varepsilon^2 \varepsilon_5 p_6 \;],$$
$$C' = \tau_t' [\, p_7 p_4 \, \varepsilon_1] + \tau_v' [\, -\epsilon_3 \xi^2 p_4 \,] + \tau_q' [\, p_4 p_6],$$
$$P^* = [\delta_1 \delta_3],$$
$$Q^* = [\, p_1 \delta_3 + \; p_4 \delta_1 + \; \xi^2 {\delta_2}^2],$$
$$R^* = \;\; [\, p_1 p_4 - \;\; p_2 p_3 \,].$$

4. Boundary conditions

Following Kumar *et al*. (2017), we take the following boundary conditions at the interface z=0

$$(1) \;\; \sigma_{zz} = 0, \tag{31}$$

$$(2) \;\; \sigma_{xz} = 0, \tag{32}$$

$$(3) \, \frac{\partial T}{\partial z} = \; 0 \;\;\; at \;\; z = 0. \tag{33}$$

5. Derivations of the secular equations

Making the use of (11)-(14), (18) and (27)-(29) in (30)-(32), we get three linear equations as

$$\sum_{j=1}^{3} \eta_{qj} A_j = 0 \, , q = 1,2,3 \tag{34}$$

Where, $\eta_{1j} = i\xi \frac{c_{13}}{\rho c_1^2} - \frac{c_{33}}{\rho c_1^2} d_j mj - \varepsilon \, l_j,$

$$\eta_{2j} = -\frac{c_{55}}{\rho c_1^2} mj + \frac{c_{55}}{\rho c_1^2} d_j i\xi, \;\; \eta_{3j} = \; l_j, \;\; \mathrm{j} = 1, 2, 3 \tag{35}$$

The system of Eq. (33) has a non-trivial solution if the determinant of unknowns A_j; j=1, 2, 3 vanishes i.e.

$$\left|\eta_{ij}\right|_{3\times3} = 0, \tag{36}$$

These resulting secular equations have all the information about the phase velocity, attenuation coefficient depth and specific loss for a Rayleigh waves in an orthotropic magneto-thermoelastic medium with combined effect of rotation and hall current in the context of fractional order theory of generalized thermoelasticity.

5.1 Phase velocity

Phase velocity is defined as the speed at which waves propagating at a particular frequency and it depend on the real component of the wave number. The phase velocities is given by

$$C = \frac{\omega}{Re\,(\xi)}\,, \tag{37}$$

Where, ξ is a wave number and ω is a angular frequency of the wave.

5.2 Attenuation coefficient

The attenuation coefficient is the gradual loss of flux intensity through a medium, and it depends on the imaginary component of the wavenumber. The attenuation coefficient is defined as

$$Q = img\ (\xi), \tag{38}$$

where, ξ is a wave number.

6. Particular cases

1. If we put $K_1 = K_3 = 0$ in Eq. (17), the problem reduces for the case Rayleigh wave propagation in orthotropic magneto-thermoelastic rotating medium without energy dissipation (GN-II type) with three-phase-lag fractional order model.

2. If $C_{11} = C_{33,}\ 2C_{55} = C_{11} - C_{33}$, we get the expressions for Rayleigh wave propagation in transversely isotropic magneto-thermoelastic medium with combined effect of hall current and rotation with GN-III type fractional order model with three-phase-lags.

3. If $C_{11} = C_{33} = \lambda + 2\mu, C_{13} = \lambda, C_{55} = \mu,\ \beta_1 = \beta_3 = \beta, K_1 = K_3 = K, K_1^* = K_3^* = K^*,$ we get the expressions for Rayleigh wave propagation for isotropic solid with three-phase-lag fractional order theory of generalized thermoelasticity.

4. If we put $\tau_t = \tau_v = \tau_q = 0$, and $K_1^* = K_3^* = 0,$ in Eq. (17) then the resulting equation represents heat equation for coupled theory of thermoelasticity.

5. If we put $K_1^* = K_3^* = 0$, in Eq. (17), then the problem reduces for the case GN-I type fractional order model in generalized thermoelasticity.

6. If we put $\tau_t = \tau_v = \tau_q = 0$, in Eq. (17), then the resulting equation reduces for the case GN-III type model of thermoelasticity.

7. Numerical results and discussion

Following Lata and Himanshi (2021a), cobalt material has been taken for the purpose of numerical computations with L=1,

Quantity	Value	Unit
c_{11}	3.071×10^{11}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{13}	1.650×10^{11}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{33}	3.581×10^{11}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{55}	1.510×10^{11}	$\mathrm{Kgm^{-1}s^{-2}}$
c_E	4.27×10^{2}	$\mathrm{JKg^{-1}K^{-1}}$
β_1	7.04×10^{6}	$\mathrm{Nm^{2}K^{-1}}$
β_3	6.90×10^{6}	$\mathrm{Nm^{2}K^{-1}}$
T_0	298	K
K_1	6.90×10^{2}	$\mathrm{Wm^{-1}K^{-1}}$
K_3	7.01×10^{2}	$\mathrm{Wm^{-1}K^{-1}}$
K_1^*	1.313×10^{2}	$\mathrm{Ws^{-1}}$

K_3^*	1.54×10^2	Ws^{-1}
ρ	8.836×10^3	Kgm^{-3}
τ_t	1.5×10^{-7}	S
τ_v	1.0×10^{-7}	S
τ_q	2.0×10^{-7}	S
μ_0	1.2571×10^{-6}	Hm^{-1}
H_0	1	$Jm^{-1}nb^{-1}$
σ_0	9.36×10^5	$col^2/\ cm.sec$

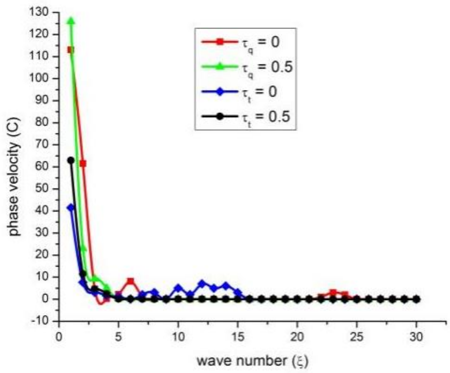

Fig**.** 1 Variation of phase velocity C with wave number ξ

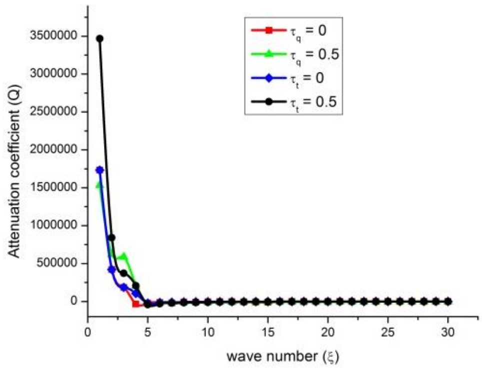

Fig**.** 2 Variation of attenuation Q coefficient with wave number ξ

With the help of the above parametric values, the numerical results are obtained with the help of octave programming language and presented graphically to show the impact of three theories (GN-I, GN-II and GN-III) and phase-lags on the Rayleigh wave phase velocity, attenuation coefficient, stress components and temperature change with respect to wave number ξ corresponding to fix value of rotation, hall current and fractional parameter.

8.1 Effect of phase-lags with $\alpha = 0.8, \Omega = 0.3, m = 0.5$

1. The red solid line with centre symbol (□) relates to $\tau_q = 0$
2. The green solid line with centre symbol (Δ) relates to $\tau_q = 1.5$
3. The blue solid line with centre symbol (◊) relates to $\tau_t = 0$
4. The black solid line with centre symbol (○) relates to $\tau_t = 1.5$

Figs. 1-2 give the variation of phase velocity and attenuation coefficient w.r.t wave number $'\xi'$ with and without phase-lag effects corresponding to two phase lags (phase lag of temperature gradient and phase-lag of heat flux) with two different values of each. From the graphs, it is clear that in the initial range near the boundary value of the phase velocity declines sharply for all the cases then oscillates little when there is no phase-lag effect $(\tau_q = 0, \tau_t = 0)$ after that exhibits steady state behaviour for $(\tau_q = 1.5, \tau_t = 1.5)$ in the remaining range with increasing value of wave number. The value of attenuation coefficient also decreases in the starting range $0 < \xi < 5$, then remains constant throughout for both the cases. The variation of normal and tangential stress has been shown in Figs. 3-4. We see that in the range $0 < \xi < 7.5$, the value of both the components

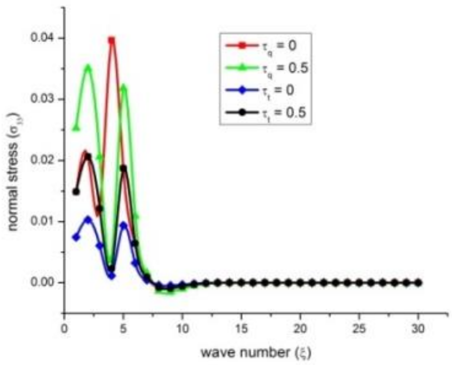

Fig. 3 Variation of normal stress σ_{zz} with wave number ξ

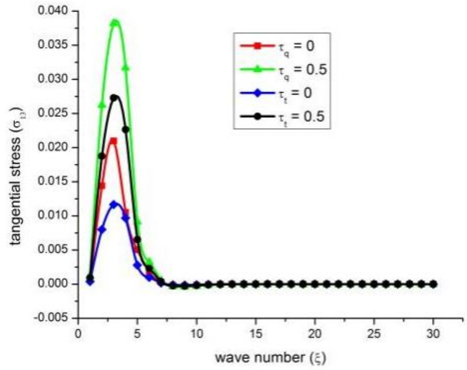

Fig. 4 Variation of tangential stress σ_{xz} with wave number ξ

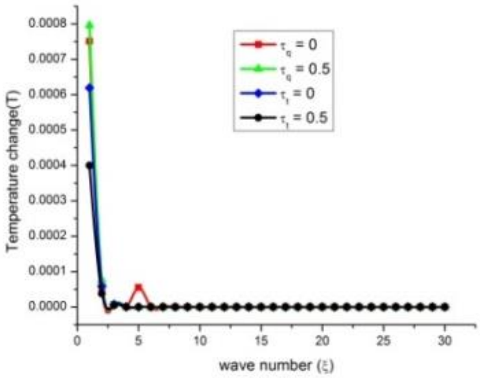

Fig. 5 Variation of temperature change with wave number ξ

oscillates with different amplitudes corresponding to two phase-lags (phase lag of heat and temperature gradient) afterwards they follow a similar behaviour i.e., remains constant in the rest of the range with increasing value of wave number. Normal stress attains its highest peak at $\xi = 5$, when there is no effect of phase-lag of heat $(\tau_q = 0)$, and tangential stress attains its maximum value at $\xi = 2.5$, respectively. Fig. 5 illustrates the nature of temperature change w.r.t wave number corresponding to two different values of phase lags respectively. We observed that the value of temperature change decreases sharply in the range $0 < \xi < 2.5$, then remains constant when wave number approaches to its maximum value.

8.2 Effect of GN-theories with $\alpha = 0.8, \Omega = 0.3, m = 0.5$

1. The red solid line with centre symbol (Δ) with $K_{ij}^* = 0$, corresponds to GN-I theory.
2. The green solid line with centre symbol $(\lozenge)$ with $K_{ij} = 0$ corresponds to GN-II theory.
3. The blue solid line with centre symbol $(\circ)$ corresponds to GN-III theory.

Here in this case, we discussed the effect of GN-theories on the Rayleigh wave characteristics with fix value of rotation, fractional parameter and hall current. Fig. 6 gives the variation of phase velocity w.r.t $'\xi'$ for GN-I, GN-II and GN-III types of theories respectively. We noticed that initially

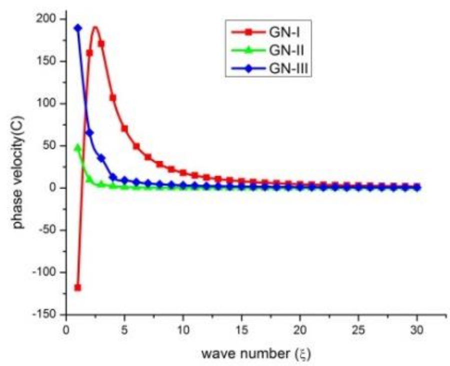

Fig. 6 Variation of phase velocity C with wave number ξ

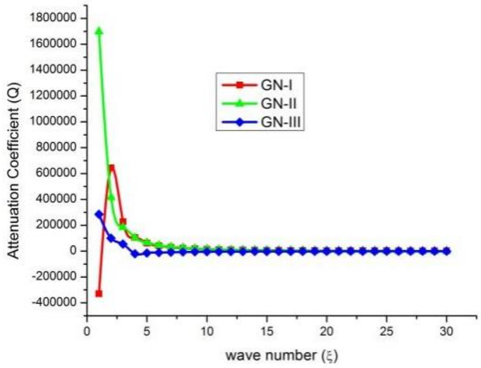

Fig. 7 Variation of attenuation coefficient Q with wave number ξ

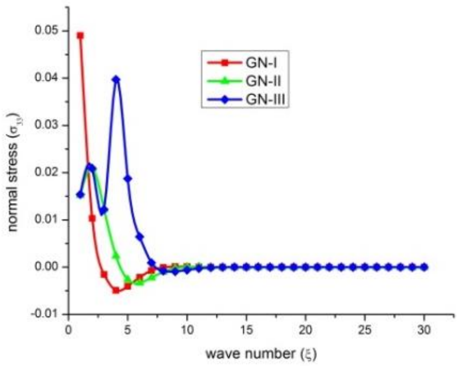

Fig. 8 Variation of normal stress with wave number ξ

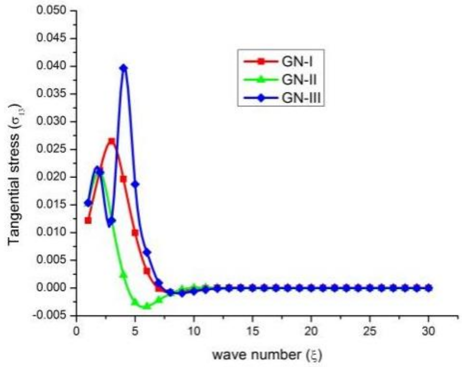

Fig. 9 Variation of tangential stress with wave number ξ

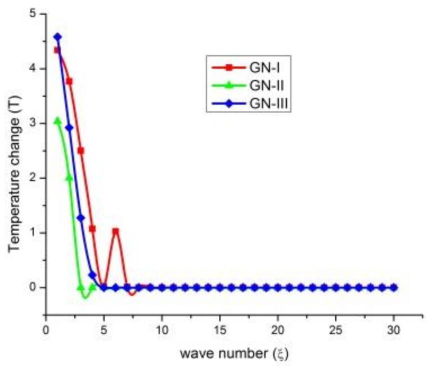

Fig. 10 Variation of temperature change with wave number ξ

the value of the phase velocity increases for the case GN-I and attains a peak value at $\xi = 2.5$ then decreases. While for the case GN-II and GN-III it declines in the range $0 < \xi < 5$ and all the curves intersect each other with similar value in the rest of the range. Fig. 7 gives the change in the value of attenuation coefficient corresponding to three different theories. The trends are similar but for the case GN-I the amplitude of the attenuation coefficient curve is less oscillatory as compared to phase-velocity. The nature of normal stress is described in Fig. 8. We observed that in the range $0 < \xi < 7.5$ for GN-I there is a sharp decrease in the beginning is followed by smooth increase afterwards follow steady state behaviour. For GN-II and GN-III the value of normal stress increases in the starting then decreases and all the curves meet each other with constant value in the remaining range. Fig. 9 describes the variation in the tangential stress w.r.t $'\xi'$ for GN-I, GN-II and GN-III types of theories respectively. It is clear from the graphs that the trends are same as in case of normal stress with different magnitudes. The value of temperature change has been shown in Fig. 10 respectively. Here, we noticed that for all the three cases in the range $0 < \xi < 5$ its value declines sharply then remains same throughout.

9. Conclusions

From the above investigation, the following conclusions are made.

• We conclude that the phase-lags and GN-theories has a significant impact on the Rayleigh waves in a two-dimensional magneto-thermoelastic orthotropic rotating media in the context of hall current and fractional order heat transfer. The properties of Rayleigh waves like phase velocity, attenuation coefficient, stress components and temperature change shows the variation under the effect of phase-lags and GN-theories.

• From the graphs, we see that in both the cases (phase-lag and GN-theories effect) for small value of non-dimensional wave number in the initial range the amount of large variations (oscillatory) has been noticed in the value of phase velocity, attenuation coefficient, stress components and temperature change respectively.

• However, with increasing value of wave number all the components follow steady state behaviour in the whole range.

• The problem is theoretical but the results produced in this paper are helpful for those researchers which are working in the field of dynamics, magneto-thermoelasticity, earthquakes, seismology and geo-physics etc.

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