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Bending and stability analysis of size-dependent compositionally graded Timoshenko nanobeams with porosities

Ismail Bensaid*[1] and Ahmed Guenanou[2]

[1]IS2M Laboratory, Faculty of Technology, Mechanical engineering Department, University of Tlemcen, Tlemcen, Algeria
[2]Faculty of Technology, Mechanical engineering Department, University of Tlemcen, Tlemcen, Algeria

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Abstract. In this article, static deflection and buckling of functionally graded (FG) nanoscale beams made of porous material are carried out based on the nonlocal Timoshenko beam model which captures the small scale influences. The exact position of neutral axis is fixed, to eliminate the stretching and bending coupling due to the unsymmetrical material change along the FG nanobeams thickness. The material properties of FG beam are graded through the thickness on the basis of the power-law form, which is modified to approximate the material properties with two models of porosity phases. By employing Hamilton's principle, the nonlocal governing equations of FG nanobeams are obtained and solved analytically for simply-supported boundary conditions via the Navier-type procedure. Numerical results for deflection and buckling of FG nanoscale beams are presented and validated with those existing in the literature. The influences of small scale parameter, power law index, porosity distribution and slenderness ratio on the static and stability responses of the FG nanobeams are all explored.

Keywords: nonlocal elasticity theory; bending; buckling; FG nanobeam; porosities

1. Introduction

Today, the use of miniaturized structures such as nanorods, carbon nanotubes, nanorings, etc., are known as high technology in various engineering fields of nanotechnology (Wang *et al*. 2007, Dresselhaus *et al*. 2004, Hong and Myung 2007). It is demonstrated that size effect gains importance on mechanical properties by using both experimental and atomistic simulation, when the dimensions of these structures become very small at order micro and nanoscale. It is well-know that traditional continuum mechanics (local theories) which assume that the stress at a point depends only on the strain at the same point, fail to take the size effect showed in nanostructures due to the need of additional length scale parameter. To fill this gap, the nonlocal elasticity theory pioneered by Eringen (1972, 1983), which is one of size-dependent continuum theories, assumes that the stress at a point depends on strains at all points in the continuum (Reddy and Pang 2008,

∗Corresponding author, Ph.D., E-mail: bensaidismail@yahoo.fr

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Heireche *et al*. 2008a, b, Benzair *et al*. 2008, Amara *et al*. 2010, Thai 2012a, Tounsi *et al*. 2013a, b, c, Adda Bedia *et al*. 2015, Besseghier *et al*. 2015, Hadj Elmerabet *et al*. 2017, Ebrahimi and Daman 2017). The use of nonlocal elasticity of Eringen to study various types of structures at micro and nanoscale has received much attention nowadays, for example, in the case of static investigation of nanostructures (Peddieson *et al*. 2003, Reddy 2007, Tounsi *et al*. 2013d, Aissani *et al*. 2015, Kheroubi and Tounsi 2016), buckling (Adali 2008, Murmu and Pradhan 2009, Murmu and Adhikari 2011, Pradhan and Reddy 2011, Ansari *et al*. 2011, Berrabah *et al*. 2013, Ould Youcef *et al*. 2015, Krenich *et al*. 2017), and dynamic (Aydogdu 2009, Aghbabaei and Reddy 2009, Şimşek 2010, Şimşek 2011, Şimşek 2012, Thai *et al*. 2012b, Benguediab *et al*. 2014, Bessaim *et al*. 2015).

Functionally graded materials (FGMs) are a new sorts of advanced composite materials in which the volume fractions of two or more material constituents such as a combination of ceramic-metal vary smoothly and continuously as a function of spatial arrangement through the thickness directions Koizumi (1997), which will lead to a reduction in thermal stresses and stress concentration at intersection with free surfaces found in traditional composite materials. Also FG materials present some particular characteristics such as ability to control deformation, multi functionality, corrosion resistance, dynamic reaction, minimization or eliminate stress concentrations, smoothing the transition of thermal stress and resistance to oxidation. In view of these superiority, FGMs have attracted wide attention recently in various engineering applications, in which many number of investigations were reported in the scientific literatures (El Meiche *et al*. 2011, Bourada *et al*. 2012, Tounsi *et al*. 2013e, Bouderba *et al*. 2013, Yaghoobi and Torabi 2013, Ould Larbi *et al*. 2013, Belabed *et al*. 2014, Chakraverty and Pradhan 2014, Liang *et al*. 2014, Zidi *et al*. 2014, Khalfi *et al*. 2014, Fekrar *et al*. 2014, Ait Amar Meziane *et al*. 2014, Hebali *et al*. 2014, Hamidi *et al*. 2015, Ait Yahia *et al*. 2015, Ait Atmane *et al*. 2015, Ziane *et al*. 2015, Bouchafa *et al*. 2015, Bourada *et al*. 2015, Tounsi *et al*. 2016, Houari *et al*. 2016, Benbakhti *et al*. 2016, Hebali *et al*. 2016, Chikh *et al*. 2016, Bensaid *et al*. 2017).

With the rapid progress of technology, functionally graded (FG) beams and plates are now greatly exploited in micro/nanoelectromechanical systems (MEMS/NEMS), i.e., the components in the form of shape memory alloy, sensors, thin films with a global thickness in micro- or nano-scale, etc. and which have many advantages over the isotropic nanoscale structures, like cited earlier. To applying accurately this kinds of new materials in micro/nano structures, their behaviors (static, buckling, vibration) have attracted the attention of several researcher in the literature. To this end, Eltaher *et al*. (2012) employed the finite element method (FEM) to study free vibration of thin FG nanobeam based on the nonlocal Euler–Bernoulli beam hypothesis. Simsek and Yurtcu (2013) developed an analytical solution to explore the static and stability of simply-supported Timoshenko FG nanobeams by using the nonlocal elasticity theory. The bending and stability behavior of FG size-dependent nanobeams were studied by Larbi Chaht *et al*. (2015), and taking in the consideration the thickness stretching effect. Zemri *et al.* (2015) contributed to a refined nonlocal shear deformation theory beam theory for investigating mechanical behavior of FG nanobeams. The free vibration properties of nanoplate resting on elastic foundation were studied by Belkorissat *et al*. (2015), by developing a new nonlocal hyperbolic refined plate model needless of any shear correction factors (SCFs). More recently, Bounouara *et al*. (2016) extended the zeroth-order shear deformation theory developed by Shimpi for the first time, to study the vibration behaviors of FG nanoplates embedded in an elastic medium based on nonlocal elasticity theory of Eringen. Ebrahimi and Salari (2015a) have studied by developing an analytical solution the thermal buckling and free vibration of FG Timoshenko nanobeam based on the nonlocal elasticity theory. Free vibration analysis of FG thick nanobeam in hygro-thermal surrounding has been investigated by Ebrahimi and Barati (2016), using a unified formulation. Ahouel *et al*. (2016) contributaed to the mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. More recently Bouafia *et al*. (2017) developed a novel nonlocal quasi-3D theory for bending and free flexural vibration characteristics of functionally graded nanobeams. Thermo-mechanical vibration of rotating axially non-homogeneous nonlocal Timoshenko beam has been presented by Azimi *et al*. (2017a), using differential quadrature method (DQM). Again, Azimi *et al*. (2017b), studied the vibration of rotating functionally graded Timoshenko nanobeams subjected to a nonlinear thermal distribution. In a recent study by Shafiei *et al*. (2016), the nonlinear thermal buckling of axially functionally graded micro and nanobeams has been studied based on the nonlocal elasticity and the modified couple stress theories, respectively, and employing generalized differential quadrature method (GDQM).

In FGM manufacture, micro voids or porosities can happen regularly within the materials during the process of sintering, due to high difference in solidification between the material constituents Zhu *et al*. (2001), the porosity is in contrast to the harmful high performance composite material. The impact of this failure has been the subject of much attention, as evidenced by the large number of studies on this subject. To this end, it is important to take into consideration the porosity impact when designing and analyzing FGM structures. Wattanasakulpong and Chaikittiratana (2015) used the collocation method to examine the porosity effect on vibration of FGM beams. The effects of thickness stretching and porosity on mechanical behavior of FGM beams resting on elastic foundations were examined by Ait Atmane *et al*. (2015). Ebrahimi *et al*. (2016) explored analytically the thermal vibration response of FGM beams by taking into account the impact of porosities. Mouaici *et al*. (2016) studied the influence of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory. Recently Ebrahimi and Jafari (2017) provided a four-variable refined shear-deformation beam theory for thermo mechanical vibration analysis of temperature-dependent FGM beams with porosities.

So, based on the above literature survey, there is no reported work examined the effect of different porosity model on static and stability of FG nanobeams. The problem of imperfection in nano FG structures is not well-investigated up to now and there is a requirement for further investigations.

This paper investigates the static deflection and buckling of porous FG nanobeam based on the nonlocal Timoshenko beam theory, which can capture the small scale effect. Two types of porosity distributions, namely, even and uneven along the thickness directions are considered. The material properties of the FG nanobeam are assumed to be graded continuously in the thickness direction according to the modified power-law model. The exact position of physical neutral surface is determined, in order to eliminate stretching-bending coupling due to the unsymmetrical material variation along the thickness. The governing equations of motion are derived by Hamilton's principle and solved analytically by Navier-type procedure for simply-supported boundary conditions. Numerical examples are provided to prove the impacts of nonlocal parameter, power law-index, porosity coefficient, and thickness to length ratios on the bending and buckling of FG nanobeams.

2. Theory and formulation

2.1 Power-law functionally graded beams having porosities

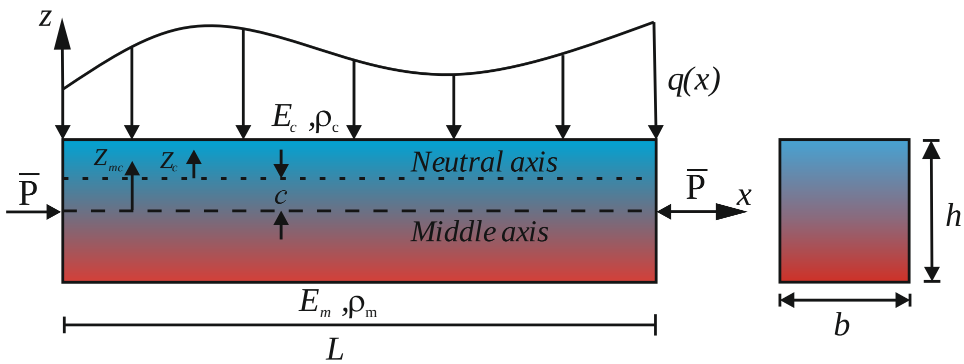

Fig. 1 Geometry, position of middle surface and neutral surface of functionally graded nanobeam

Consider a uniform FG nanobeam of thickness *h*, length *L*, and width *b* made by mixing of two distinct materials (metal and ceramic) is studied here (Fig. 1). The coordinate *x* is along the longitudinal direction and *z* is along the thickness direction. For such beams, the neutral surface may not coincide with its geometric mid-surface (Ould Larbi *et al*. 2013, Yaghoobi and Feraidoon 2010) which leads to bending-extension coupling due to the unsymmetrical material variation along the thickness. By considering the exact position of neutral axis, this coupling can be eliminated.

To capture exact position of neutral axis, two different datum planes are considered for the measurement of *z*, namely, z_{ms} and z_{ns} measured from the middle surface, and the neutral surface of the beam, respectively (Fig. 1). Satisfying the first moment with respect to Young's modulus being zero, the exact position of neutral axis is determined as follows (Ould Larbi *et al*. 2013, Bousahla *et al*. 2014, Fekrar *et al*. 2014, Bourada *et al*. 2015)

$$\int_{-h/2}^{h/2} E(z_{ms})(z_{ms}-c)\,dz_{ms}=0 \tag{1}$$

In which, the position of neutral surface can be obtained as

$$c=\frac{\int_{-h/2}^{h/2} E(z_{ms})\,z_{ms}\,dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms})\,z_{ms}\,dz_{ms}} \tag{2}$$

The volume-fraction of ceramic V_c is expressed based on z_{ms} and z_c coordinates (Fig. 1) as

$$V_c=\left(\frac{z_{ms}}{h}+\frac{1}{2}\right)^k=\left(\frac{z_c+c}{h}+\frac{1}{2}\right)^k \tag{3}$$

where *k* represents the material distribution parameter which takes the value greater or equal to zero and *c* is the distance of neutral surface from the mid-surface (Fig. 1). Material non homogeneous properties of a functionally graded material beam may be obtained by means of the Voigt rule of mixture (Eltaher *et al*. 2012, Bourada *et al*. 2012, Larbi Chaht *et al*. 2015, Tounsi *et*

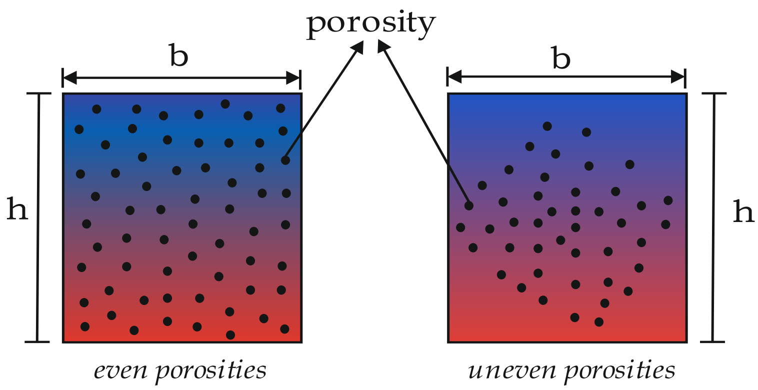

Fig. 2 through thickness FGM beam with evenly and unevenly distributed porosities

al. 2013a, Bouderba *et al*. 2013, Hebali *et al*. 2014, Zidi *et al*. 2014, Bakora and Tounsi 2015, Hamidi *et al*. 2015, Mahi *et al*. 2015, Akbaş 2015, Bennoun *et al*. 2016, Salima *et al*. 2016). Thus, using Eq. (3), the material inhomogeneous properties of FG nanobeam P, such as Young's modulus (E), Poisson's ratio (v), the shear modulus (G), and the mass density (ρ), can be described by

$$P(z_c) = (P_t - P_b)\left(\frac{z_c + c}{h} + \frac{1}{2}\right)^k + P_b \tag{4}$$

Here P_t and P_b are the corresponding material property at the top and bottom surfaces of the FG nanobeam.

Consider an imperfect FG beam with two kinds of porosities that distributed identical in two phases of ceramic and metal due to defect during production. The modified rule of mixture proposed by Wattanasakulpong and Ungbhakorn (2014) is used as

$$P = P_b\left(V_m - \frac{\alpha}{2}\right) + P_t\left(V_c - \frac{\alpha}{2}\right) \tag{5}$$

where α denotes the volume fraction of porosities $\alpha \prec\prec 1$, for perfect FGM α is set to zero, V_c (Previously defined) and V_m are the volume fraction of ceramic and metal, respectively, and the compositions represent in relation to

$$V_c + V_m = 1 \tag{6}$$

Combining Eqs. (5)-(6), the effective material properties of FG beam with even porosities (FGM-I) can be expressed in the following form

$$P(z_c) = (P_t - P_b)\left(\frac{z_c + c}{h} + \frac{1}{2}\right)^k + P_b - \frac{\alpha}{2}(P_t + P_b) \tag{7}$$

It is noted that the FGM-I has porosity phases with even repartition of volume fraction through the cross section. While, the FGM-II has porosity phases spreading frequently around the middle zone of the cross-section and the amount of porosity seems to be linearly decreases to zero at the top and bottom of the cross-section. Fig. 2 exhibits an examples of cross section areas of FGM-I and-II with porosities phases. For second type, uneven distribution of porosities (defined as FGM-II), the effective material properties are replaced by following form

$$P(z_c) = (P_t - P_b)\left(\frac{z_c + c}{h} + \frac{1}{2}\right)^k + P_b - \frac{\alpha}{2}(P_t + P_b)\left(1 - \frac{2|z_c + c|}{h}\right) \tag{8}$$

2.2 Kinematics

Based on the Timoshenko beam theory, the displacement field at any point of the beam can be written as (Azimi *et al*. 2017a, b)

$$u_x(x,z,t) = u(x,t) + z\varphi(x,t) \tag{9}$$

$$u_z(x,z,t) = w(x,t) \tag{10}$$

where t is time, u and w are the axial and the transverse displacement of any point on the neutral axis; φ is the total bending rotation of the cross-sections at any point on the neutral axis. The nonzero strains of the Timoshenko beam theory are obtained as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \varphi}{\partial x} \tag{11}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \varphi \tag{12}$$

where ε_{xx} and γ_{xz} are the normal strain and shear strain, respectively.

The governing equations and the boundary conditions will be derived by using the Hamilton's principle Reddy (2000)

$$\delta\int_0^T (U + V)\,dt = 0 \tag{13}$$

where δU is the variation of the strain energy; δV represents the potential energy; and the variation of the kinetic energy is given by δK. The variation of the strain energy of the beam can expressed by the following form

$$\begin{aligned} \delta U &= \int_0^L \int_A (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz})\,dA\,dx \\ &= \int_0^L \left(N\frac{d\delta u}{dx} - M\frac{d\delta\varphi}{dx} + Q\left(\frac{d\delta w}{dx} - \frac{d\delta\varphi}{dx}\right)\right)dx \end{aligned} \tag{14}$$

Where (N), (M) and (Q) are the stress resultants, defined as

$$(N, M) = \int_A (1, z)\sigma_{xx}\,dA \quad and \quad Q = \int_A k_s \tau_{xz}\,dA \tag{15}$$

where k_s is the shear correction factor.

The variation of the potential energy by the applied loads can be written as

$$\delta V = \int_0^L q\delta w\, dx + \overline{P}\frac{dw}{dx}\frac{d\delta w}{dx}dx \tag{16}$$

where (q) and ($\overline{P}$) are the transverse and axial loads, respectively.

Substituting the expressions for (δU) and (δV) from Eqs. (14) and (16) into Eq. (13) and integrating by parts, and collecting the coefficients of (δu), (δw), and ($\delta\varphi$), the following equations of motion of the proposed beam theory are obtained

$$\delta u : \frac{dN}{dx} = 0 \tag{17}$$

$$\delta w : \frac{dQ}{dx} + q - \overline{P}\frac{d^2 w}{dx^2} = 0 \tag{18}$$

$$\delta\varphi : \frac{d^2 M}{dx^2} - \frac{dQ}{dx} = 0 \tag{19}$$

2.3 The nonlocal elasticity model

Contrary to the classical (local) theory, in the nonlocal elasticity theory of Eringen (1972, 1983), the stress at a reference point x is considered to be a functional of the strain field at every point in the body. For example, in the non-local elasticity, the uniaxial constitutive law is expressed as elasticity Eringen (1972, 1983).

$$\sigma_{xx} - \mu\frac{d^2\sigma_{xx}}{dx^2} = E(z_c)\varepsilon_{xx} \tag{20}$$

$$\tau_{xz} - \mu\frac{d^2\tau_{xz}}{dx^2} = G(z_c)\gamma_{xz} \tag{21}$$

$\mu = (e_0 a)^2$ is a nonlocal parameter revealing the nanoscale effect on the response of nanobeams, e_0 is a constant appropriate to each material and a is an internal characteristic length. In general, a conservative estimate of the nonlocal parameter is $e_0 a < 2.0$ nm for a single wall carbon nanotube (Wang 2005, Heireche *et al*. 2008a, b, Tounsi *et al*. 2013b, c)

Using Eqs. (20), (21), (11), (12) and (15), the force-strain and the moment–strain relations of the nonlocal FG Timoshenko beam theory can be obtained as

$$N - \mu\frac{d^2 N}{dx^2} = A_{xx}\frac{du}{dx} - B_{xx}\frac{d\varphi}{dx} \tag{22}$$

$$M - \mu\frac{d^2 M}{dx^2} = B_{xx}\frac{du}{dx} - D_{xx}\frac{d\varphi}{dx} \tag{23}$$

$$Q-\mu\frac{d^2Q}{dx^2}=k_sA_{xz}\left(\frac{dw}{dx}-\varphi\right) \tag{24}$$

In which the cross-sectional rigidities are defined as follows

$$\left(A_{xx},B_{xx},D_{xx}\right)=\int_A E\left(z_c\right)\left(1,z_c,z_c{}^2\right)dA \tag{25}$$

$$A_{xz}=k_s\int_A G\left(z_c\right)dA \tag{26}$$

By substituting Eqs. (22), (24), (25) into Eqs. (17), (18), (19), the nonlocal equations of motion can be expressed in terms of displacements (u, w, φ) as follows

$$A_{xx}\frac{d^2u}{dx^2}-B_{xx}\frac{d^2\varphi}{dx^2}=0 \tag{27}$$

$$A_{xz}\left(\frac{d^2w}{dx^2}+\frac{d\varphi}{dx}\right)+q-\mu\frac{d^2q}{dx^2}-\overline{P}\left(\frac{d^2w}{dx^2}-\mu\frac{d^4w}{dx^4}\right)=0 \tag{28}$$

$$B_{xx}\frac{d^2u}{dx^2}+D_{xx}\frac{d^2\varphi}{dx^2}-A_{xz}\left(\frac{dw}{dx}-\varphi\right)=0 \tag{29}$$

The equations of motion of local beam theory can be obtained from Eqs above by setting the nonlocal parameter μ equal to zero.

3. Closed-form solution of simply supported FG nanobeam

Here, the above nonlocal governing equations of motion are solved analytically for bending and buckling problems. The Navier solution procedure is used to determine the analytical solutions for a simply supported FG nanobeam. The solution is assumed to be of the form

$$\begin{Bmatrix} u \\ w \\ \varphi \end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix} U_n\cos(\alpha x)e^{i\omega t} \\ W_n\sin(\alpha x)e^{i\omega t} \\ \varphi_n\sin(\alpha x)e^{i\omega t} \end{Bmatrix} \tag{30}$$

where U_n, W_n, and φ_n are arbitrary parameters to be determined, ω is the eigenfrequency associated with nth eigenmode, and $\alpha=n\pi/L$. The transverse load q is also expanded in the Fourier sine series as

$$q(x)=\sum_{n=1}^{\infty}Q_n\sin(\alpha x),\quad Q_n=\frac{2}{L}\int_0^L q(x)\sin(\alpha x)dx \tag{31}$$

Table 1 Dimensionless transverse deflections ($\overline{w}$) of the perfect FG nanobeam under uniform load

		Nonlocal parameter, e_0a (nm)									
L/h	*k*	0		0.5		1		1.5		2	
		REF[(a)]	Present	REF[(a)]	Present	REF[(a)]	Present	REF[(a)]	Present	REF[(a)]	Present
10	0	5.3383	5.3383	5.4659	5.4659	5.8487	5.8486	6.8467	6.4864	7.3798	7.3794
	0.3	3.2169	3.2155	3.2938	3.2923	3.5245	3.5229	3.9090	3.9071	4.4472	5.6438
	1	2.4194	2.4194	2.4772	2.4772	2.6508	2.6508	2.9401	2.9400	3.3451	3.3449
	3	1.9249	1.9249	1.9710	1.9710	2.1091	2.1090	2.3393	2.3392	2.6615	2.6614
	10	1.5799	1.5799	1.6176	1.6176	1.7310	1.7309	1.9190	1.9198	2.1843	2.1842
30	0	5.2227	5.2227	5.2366	5.2366	5.2784	5.2784	5.3480	5.3480	5.4455	5.4455
	0.3	3.1486	3.1472	3.1570	3.1556	3.1822	3.1807	3.2241	3.2227	3.2829	3.2814
	1	2.3732	2.3732	2.3795	2.3795	2.3985	2.3985	2.4301	2.4301	2.4744	2.4744
	3	1.8894	1.8894	1.8944	1.8944	1.9094	1.9095	1.9347	1.9347	1.9700	1.9700
	10	1.5489	1.5489	1.5530	1.5530	1.5654	1.5654	1.5860	1.5860	1.6149	1.6149

[(a)]Şimşek and Yurtçu (2013)

The Fourier coefficients Q_n related with some typical loads are given as follows

$$Q_n = q_0 , \quad n = 1 \text{ for sinusoidal load} \tag{32a}$$

$$Q_n = \frac{4q_0}{n\pi} , \quad n = 1,3,5.... \text{for uniform load} \tag{32b}$$

$$Q_n = \frac{2q_0}{L}\sin\frac{n\pi}{2}, \; n = 1,2,3.... \text{ for point load } \boldsymbol{Q_0} \text{ at the midspan} \tag{32c}$$

Substituting the expansions of u, w, φ, and q from Eqs. (30) and (31) into Eqs. (27), (28), (29), the analytical solutions can be obtained from the following equations

$$\begin{bmatrix} S_{11} & 0 & S_{13} \\ 0 & S_{22}-\xi & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{Bmatrix} U_n \\ W_n \\ \varphi_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ \lambda Q_n \\ 0 \end{Bmatrix}, \tag{33}$$

Where

$$S_{11} = A_{xx}\alpha^2 , \; S_{13} = -B_{xx}\alpha^2 , \; S_{23} = -A_{xz}\alpha , \; S_{22} = A_{xz}\alpha^2 , S_{33} = D_{xx}\alpha^2 + A_{xz} , \quad \xi = \lambda\overline{P}\alpha^2 , \quad \lambda = 1 + \mu\alpha^2 \tag{34}$$

4. Numerical results and discussion

Through this section, the size-dependent static and stability responses of porous FG nanobeams

Table 2 Dimensionless critical buckling load (N_{cr}) of the perfect FG nanobeam

		Nonlocal parameter, e_0a (nm)									
L/h	k	0		0.5		1		1.5		2	
		REF[(a)]	Present	REF[(a)]	Present	REF[(a)]	Present	REF[(a)]	Present	REF[(a)]	Present
10	0	2.4056	2.4056	2.3477	2.3477	2.1895	2.1895	1.9686	1.9685	1.7247	1.7247
	0.3	3.9921	3.9938	3.8959	3.8977	3.6365	3.6351	3.2667	3.2681	2.8621	2.8634
	1	5.3084	5.3084	5.1805	5.1805	4.8315	4.8315	4.3437	4.3437	3.8059	3.8059
	3	6.6720	6.6720	6.5113	6.5113	6.0727	6.0727	5.4596	5.4596	4.7835	4.7835
	10	8.1289	8.1289	7.9379	7.9332	7.3987	7.3987	6.6518	6.6518	5.8281	5.8281
30	0	2.4603	2.4603	2.4536	2.4536	2.4336	2.4336	2.4011	2.4011	2.3570	2.3569
	0.3	4.0811	4.0829	4.0699	4.0718	4.0368	4.0386	3.9828	3.9846	3.9096	3.9113
	1	5.4146	5.4146	5.3998	5.3998	5.3559	5.3559	5.2843	5.2843	5.1871	5.1871
	3	6.8011	6.8011	6.7825	6.8725	6.7273	6.7273	6.6373	6.6373	6.5153	6.5153
	10	8.2962	8.2962	8.2735	8.2735	8.2062	8.2062	8.0964	8.0964	7.9476	7.9476

[(a)] Şimşek and Yurtçu (2013)

Table 3 the influence of porosity volume fraction, porosity distribution, power law index on the dimensionless transverse deflections ($\overline{w}$) of a FG porous nanobeam. (L/h=10, μ=1)

FGM type	α	k=0	k=0.3	k=1	k=3	k=10
FGM (I)	0	5.8486	3.5229	2.6508	2.1090	1.7309
	0.1	7.7981	4.2290	3.0406	2.3323	1.8710
	0.2	11.69	5.3448	3.5829	2.6118	2.0362
FGM (II)	0	5.4886	5.5229	2.6508	2.1090	1.7309
	0.1	6.2492	3.6930	2.7495	2.1645	1.7650
	0.2	6.7108	3.8862	2.8582	2.2234	1.8005

Table 4 the influence of porosity volume fraction, porosity distribution, power law index on the dimensionless critical buckling load (N_{cr}) of a FG porous nanobeam. (L/h=10, μ=1)

FGM type	α	k=0	k=0.3	k=1	k=3	k=10
FGM (I)	0	2.1895	3.6151	4.8315	6.0727	7.3987
	0.1	1.6421	3.0281	4.2122	5.4914	6.8448
	0.2	1.0947	2.3961	3.5748	4.9039	6.2898
FGM (II)	0	2.1895	3.2334	4.0864	5.2076	6.8309
	0.1	2.0490	3.0968	3.9437	5.0443	6.6707
	0.2	1.9079	2.9637	3.8052	4.8806	6.5089

are explored based nonlocal Timoshenko beam model. Computations have been implemented for the following material and beam properties: E_1=1 TPa, E_2=0.25 TPa, v_1=v_2=0.3. The shear correction factor is taken as k_s=5/6 for Timoshenko beam theory (Larbi Chaht *et al*. 2014). For convenience, the following dimensionless amounts are used in presenting the numerical results in graphical and tabular forms

$$\overline{w} = 100w\frac{E_1 I}{q_0 L^4} \quad \text{for uniform load} \tag{35}$$

$$P_{cr} = \frac{\overline{P}L^2}{E_1 I} \tag{36}$$

The correctness of presented deflection and buckling results are checked with those of first order beam theory obtained by (Şimşek and Yurtçu 2013) for a perfect FG beam and the obtained results are tabulated in Tables 1 and 2 in the following discussions.

In Table 1 the nondimensional maximum deflections $\overline{w}$ of the simply supported FG nanobeams are examined for various values of the gradient index (k=0, 0.3, 1, 10), nonlocal parameters (μ=0, 0.5, 1, 1.5, 2 (nm)^2) and two different values of slenderness ratios (L/h=10, 30) based on analytical Navier solution method. It is mentioned that when e_0a vanish corresponds to local beam theory. It can be concluded that the results of the present beam theory based on physical neutral surface position are in excellent agreement with those predicted by TBT (Şimşek and Yurtçu 2013) for all values of thickness ratio L/h, power law index k and nonlocal parameter e_0a and thus validates the proposed model. A variation of the material distribution parameter k leads to a significant change in the maximum deflection. One can also notice that an increase in the nonlocal parameter gives rise to an increase in the maximum deflection, which highlights the significance of the nonlocal effect.

Variations of the nondimensional critical buckling loads for various values of thickness ratio L/h, gradient indexes k and nonlocal parameter e_0a are presented in Table 2 for perfect FG nanobeam. As can be noted also, that the obtained results are in good concordance with the results provided in the literature those of Şimşek and Yurtçu (2013) again. The critical buckling load decreases as the nonlocal parameter rises. This emphasizes the significance of the nonlocal effect on the buckling response of beams, because the nonlocal parameter softens the nanobeam. By varying the material distribution parameter k leads to a reduction in the buckling load, because decreasing in ceramics phase constituent, and hence, stiffness of the beam.

In the following, the impact of porosity model named even and uneven, referred to (FGPM-I and FGPM-II), porosity volume fraction, power-law exponent the non-dimensional deflection and buckling of the porous FG nanobeam will be investigated.

Variations of the nondimensional maximum deflections ($\overline{w}$) of the simply supported porous FG nanobeams based on neutral surface position, for different porosity parameters (α=0, 0.1, 0.2), power law index (k=0, 0.3, 1, 3, 10), and constant values of (L/h=20, μ=1) are explored in Table 3. Two models of porosity distributions are chosen (even and uneven) FGM-I and FGM-II respectively. It should be noted that e_0a vanishes corresponds to local beam theory. It can be observed from this table that by growing the power law index k leads to a significant change in the maximum deflection($\overline{w}$), because an increase in the power law index leads to the increment in the flexibility of the FG nanobeams, rise in the metal phase. Also from this table it is mentioned that increasing porosity parameter increases maximum deflections($\overline{w}$), due to the increase in internal pores in the FG nanobeams. In the last, it is showed that FGM-I model estimates higher values for deflections of porous FG nanobeam compared to FGP-II model.

Table 3 displays the variations of critical buckling load (N_{cr}) of the porous Timoshenko FG nanobeams based on neutral surface position, for various porosity parameters (α=0, 0.1, 0.2),

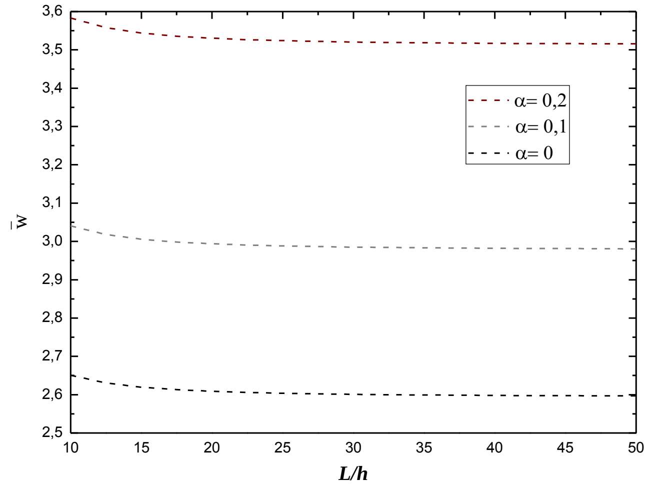

Fig. 3 Influence of the aspect ratio on dimensionless static deflection of FG (I) nanobeam for various porosity parameter (k=1, μ=1)

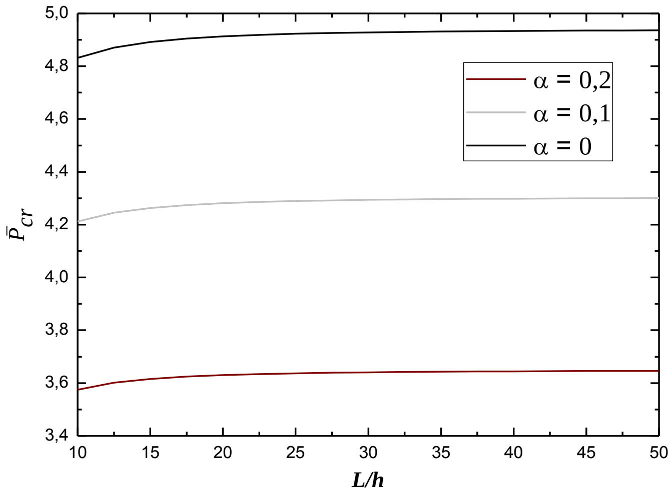

Fig. 4 Influence of the aspect ratio on dimensionless buckling load of FG (I) nanobeam for various porosity parameter (k=1, μ=1)

power law index (k=0, 0.3, 1, 3, 10), and constant values of (L/h=20, μ=1), depending on two models of porosity distributions (even and uneven) FGM-I and FGM-II respectively. It can be observed from this table that, an increase of the material distribution parameter k leads to a significant change in the buckling load; this is due to the flexibility impact in the FG nanobeam. The porosity parameter has a significant influence on the dimensionless critical buckling load (N_{cr}), due to the beneficial effect of internal pores in the FG nanobeams. It is showed that FGM-I model has more significant effect on buckling load (N_{cr}) compared to FGP-II model.

Figs. 3 and 4 display the variations of static and buckling responses of FG porous nanobeam based on the neutral surface position versus aspect ratio for several fraction of porosity volume and fixed values of (k=1, μ=1). We can see from these curves that for different values of porosity

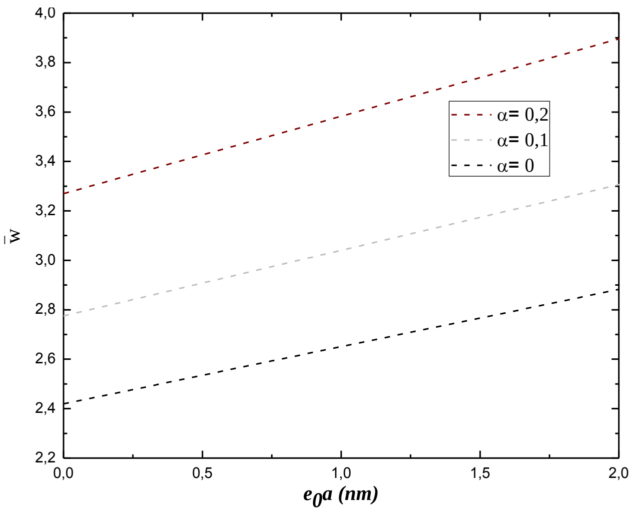

Fig. 5 Influence of the nonlocal parameter on dimensionless deflection of FG (I) nanobeam for various porosity parameter (k=1, L/h=10)

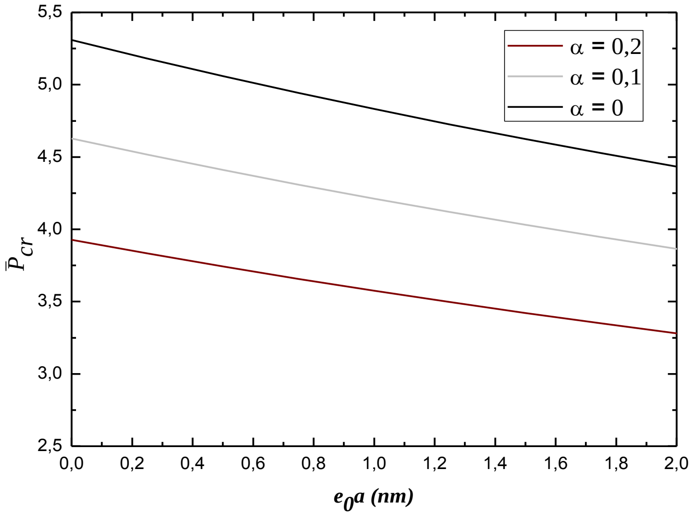

Fig. 6 Influence of the nonlocal parameter on dimensionless buckling load of FG (I) nanobeam for various porosity parameter (k=1, L/h=10)

coefficient, as slenderness ratios increase, the deflection decreases and the non-dimensional buckling load increase, these influences are more significant for lower values of thickness ratio (L/h), and this effect is too small for long FG nanobeams.

Influence of the nonlocal scale parameter on non-dimensional static deflection and buckling responses of porous FG nanobeam type FGM-I based on the neutral surface position for various values of porosity coefficient α is illustrated in Figs. 5 and 6, respectively at L/h=10, k=1. It is deduced that a rise in nonlocal parameter leads to an increment in transverse displacement and a decrement in the critical buckling load; the responses vary linearly in function of the nonlocal scale parameter.

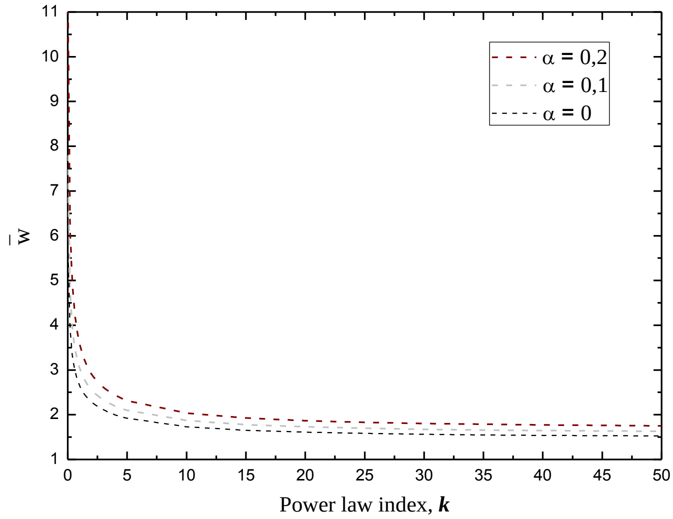

Fig. 7 Effect of the power law index on dimensionless deflection of FG (I) nanobeam for various porosity parameter (L/h=10, μ=1)

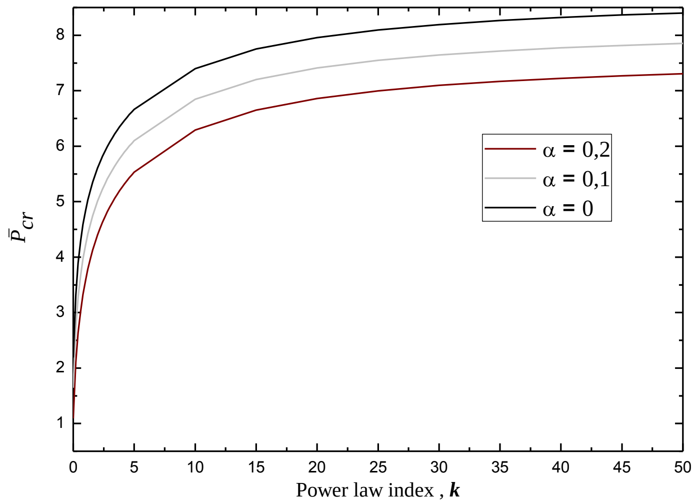

Fig. 8 Effect of the power law index on dimensionless buckling load of FG (I) nanobeam for various porosity parameter (L/h=10, μ=1)

Furthermore, the porosity parameter has a significant impact on the non-dimensional static deflection and buckling load of the porous FG nanobeam.

The effect of the material distribution parameter (k) on the static deflection and buckling responses of porous FG nanobeam type (FGM-I) for different values of porosity coefficient is demonstrated in Figs. 7 and 8, respectively. It can be seen from these figures that both the dimensionless deflection ($\overline{w}$) decreases whereas the dimensionless buckling load rises as the material distribution parameter increases. The raison is that an increase in the material distribution parameter yields to a degradation in the stiffness of the porous FG nanobeam. Besides, the porosity coefficient has significant influences on the both static deflection and critical buckling load.

5. Conclusions

In this paper, Static bending and buckling of the FG porous nanoscale beam are investigated based on the nonlocal Timoshenko beam theory including neutral surface position. The governing equations of motions are derived by using the Hamilton's principle, and solved analytically by Navier-type solution for simply-supported boundary conditions. The material properties of FG plate were supposed to be graded across the thickness direction according to the modified power-law model. The simple formulation of the problem is developed based on the new reference physical surface. Accuracy of the results obtained is checked with those available in the literature and good agreements were observed. According to the numerical and graphical results, it is found that the dimensionless deflection and buckling responses for FGM porous beams are considerably affected by side-to-thickness ratios, nonlocal parameter, power law index and porosity parameter. It is also shown that even porosity distribution (FGM-I) gives higher deflection and lower buckling load compared with uneven porosity distribution (FGM-II). Therefore, the neutral surface position and porosity impacts should be considered in the analysis behavior of FG nanostructures. Finally, the formulation lends itself particularly well for behavioral analysis of FG nanostructures that contain two scale parameters for modeling the size-dependent more accurately (Li *et al*. 2016, Şimşek 2016).

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