Advances in Materials Research, *Vol. 9, No. 1 (2020) 33-48*
DOI: https://doi.org/10.12989/amr.2020.9.1.033

A review of effects of partial dynamic loading on dynamic response of nonlocal functionally graded material beams

Ridha A. Ahmed, Raad M. Fenjan, Luay Badr Hamad and Nadhim M. Faleh*

Al-Mustansiriah University, Engineering Collage P.O. Box 46049, Bab-Muadum, Baghdad 10001, Iraq

(*Received December 15, 2019, Revised February 14, 2020, Accepted March 1, 2020*)

Abstract. With the use of differential quadrature method (DQM), forced vibrations and resonance frequency analysis of functionally graded (FG) nano-size beams rested on elastic substrate have been studied utilizing a shear deformation refined beam theory which contains shear deformations influence needless of any correction coefficient. The nano-size beam is exposed to uniformly-type dynamical loads having partial length. The two parameters elastic substrate is consist of linear springs as well as shear coefficient. Gradation of each material property for nano-size beam has been defined in the context of Mori-Tanaka scheme. Governing equations for embedded refined FG nano-size beams exposed to dynamical load have been achieved by utilizing Eringen's nonlocal differential law and Hamilton's rule. Derived equations have solved via DQM based on simply supported-simply supported edge condition. It will be shown that forced vibrations properties and resonance frequency of embedded FG nano-size beam are prominently affected by material gradation, nonlocal field, substrate coefficients and load factors.

Keywords: forced vibrations; DQM; FG nanobeam; dynamic load; elastic substrate; nonlocal elasticity theory

1. Introduction

In a FG material, all material properties may change from one side to another side by means of a prescribed distribution. These two sides may be ceramic or metal. Mechanical characteristics of a FG material can be described based on the percentages of ceramic and metal phases. The material distribution in FG materials may be characterized via a power-law function. FG materials are not always perfect because of porosity production in them. Existence of porosities in the FG materials may significantly change their mechanical characteristics. For example, the elastic moduli of porous FG material is smaller than that of perfect FG material. Up to now, many authors focused on wave propagation, vibration and buckling analyzes of FG structures having porosities (Jabbari *et al*. 2008, Chikh *et al*. 2016, Sobhy 2016, Lal *et al*. 2017, Bensaid and Kerboua 2019, Bekhadda *et al*. 2019). Also, there are several investigations concerning with the analysis of FG structures in thermal environments (Bouderba *et al*. 2016, El-Hassar *et al*. 2016).

Recently, this kind of materials have found their applications in nano-scale structures. Vibration behavior of a nano-scale plate is not the same as a macro-scale plate (Lee *et al*. 2006, Zalesak *et al*. 2016). This is because small-size effects are not present at macro scale. So, mathematical

*Corresponding author, Professor, E-mail: dr.nadhim@uomustansiriyah.edu.iq; drnadhim@gmail.com

http://www.techno-press.org/?journal=amr&subpage=5 ISSN: 2234-0912 (Print), 2234-179X (Online)

modeling of a nanoplate can be done with the use of nonlocal elasticity (Eringen 1983) incorporating only one scale parameter (Berrabah *et al*. 2013, Zenkour and Abouelregal 2014a, b, Aissani *et al*. 2015, Besseghier *et al*. 2015, 2017, Elmerabet *et al*. 2017, Bouadi *et al*. 2018, Yazid *et al*. 2018). Due to the ignorance of strain gradient effect in nonlocal elasticity theory, a more general theory will be required (Natarajan *et al*. 2012, Daneshmehr and Rajabpoor 2014, Belkorissat *et al*. 2015, Ebrahimi and Barati 2016, Sobhy and Radwan 2017, Larbi Chaht *et al*. 2015, Belmahi *et al*. 2019, Alassadi *et al*. 2019). Strain gradients at nano-scale are observed by many researchers (Lam *et al*. 2003, Lim *et al*. 2015, Mirsalehi *et al*. 2017). Thus, nonlocal-strain gradient theory was introduced as a general theory which contains an additional strain gradient parameter together with nonlocal parameter (Li *et al*. 2015, 2018, Li and Hu 2015, 2016, 2017, Barati and Zenkour 2017, Fenjan *et al*. 2019). The scale parameters used in nonlocal strain gradient theory can be obtained by fitting obtained theoretical results with available experimental data and even molecular dynamic (MD) simulations.

This paper uses a higher order shear deformation beam formulation having three variables without using of shear correction factor. Based upon differential quadrature (DQ) approach and nonlocal elasticity formulation, forced vibrational analysis of shear deformable functionally graded (FG) nanobeam on elastic medium under partial dynamical load has been performed. The presented formulation incorporates a scale factor for examining vibrational behaviors of nano-dimension beams. The material properties for FG beam are defined employing a power-law form. It is supposed that the nano-sized beam is exposed to transverse dynamic load for excitation frequency. The governing equations achieved by Hamilton's principle are solved implementing DQM. Presented results indicate the prominence of material gradient index, nonlocal coefficient, material gradient coefficient, load location and substrate factors on vibrational properties of FG nano-size beam.

2. Theories and formulations

2.1 Effective properties for FGMs based upon neutral axis location

FG materials have variable properties in transverse direction of the beam affected by the location of neutral axis (Tang *et al*. 2020). For incorporating exact location of neutral axis, the z_{ms}, z_{ns} have been measured from the middle and neutral axes, respectively. This leads to below relation

$$\int_{-h/2}^{h/2} E(z_{ms})(z_{ms} - h_0)dz_{ms} = 0 \tag{1}$$

so that the location ($z = h_0$) may be determined as

$$h_0 = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms}\, dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms})\, dz_{ms}} \tag{2}$$

Based upon Mori-Tanaka scheme, the effective local bulk modulus, K_e, and shear modulus μ_e may be defined by (Ebrahimi *et al*. 2016)

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m(K_c - K_m)/(K_m + 4\mu_m/3)} \tag{3}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m(\mu_c - \mu_m)/[(\mu_m + \mu_m(9K_m + 8\mu_m)/(6(K_m + 2\mu_m))]} \tag{4}$$

so that subscripts m and c are corresponding to metallic and ceramic constituents, respectively. Also, the below relation exists for volume fractions of the two constituents

$$V_c + V_m = 1 \tag{5}$$

in such a way that ceramic constituent has below volume fraction as a function of material exponent (p)

$$V_c(z_{ns}) = \left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^p \tag{6}$$

Next, the effective Young's modulus (E), Poisson's ratio (ν) and mass density ρ may be expressed by

$$E(z_{ns}) = \frac{9K_e\mu_e}{3K_e + \mu_e} \tag{7}$$

$$\nu(z_{ns}) = \frac{3K_e - 2\mu_e}{6K_e + 2\mu_e} \tag{8}$$

$$\rho(z_{ns}) = \rho_c V_c + \rho_m V_m \tag{9}$$

2.2 Kinematic relations

Shear deformation are shown to have great influence on mechanical characteristics of nano-size beams (Tang *et al*. 2019a, b). By defining exact location of neutral axis, the displacement components based on axial u, bending w_b and shear w_s displacements may be introduced as (Besseghier *et al*. 2017, Fenjan *et al*. 2019)

$$u_x(x, z_{ns}) = u(x) - z_{ns}\frac{\partial w_b}{\partial x} - f(z_{ns})\frac{\partial w_s}{\partial x} \tag{10a}$$

$$u_z(x, z_{ns}) = w_b(x) + w_s(x) \tag{10b}$$

In this study, the shear strain function $f(z_{ns})$ is defined by

$$f(z_{ns}) = z_{ns} + h_0 - sin(\xi(z_{ns} + h_0))/\xi \tag{11}$$

where $\xi = \pi/h$. Finally, the strains based on the three-unknown beam model have been obtained as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z_{ns}\frac{\partial^2 w_b}{\partial x^2} - f(z_{ns})\frac{\partial^2 w_s}{\partial x^2} \tag{12a}$$

$$\gamma_{xz} = g(z_{ns})\frac{\partial w_s}{\partial x} \tag{12b}$$

where $g(z_{ns}) = 1 - df(z_{ns})/dz_{ns}$. Next, one might express the Hamilton's rule as follows based on strain energy (U) and kinetic energy (T)

$$\int_0^t \delta(U + V - K)dt = 0 \tag{13}$$

and $V$is the work of non-conservative loads. Based on above relation we have

$$\delta U = \int_v \sigma_{ij}\delta\varepsilon_{ij}dV = \int_v (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz})dV \tag{14}$$

Placing Eqs. (12a)-(12b) into Eq. (14) leads to

$$\delta U = \int_0^L \left(N\frac{\partial\delta u}{\partial x} - M_b\frac{\partial^2\delta w_b}{\partial x^2} - M_s\frac{\partial^2\delta w_s}{\partial x^2} + Q\frac{\partial\delta w_s}{\partial x}\right)dx \tag{15}$$

where

$$(N, M_b, M_s) = \int_{-h/2-h_0}^{h/2-h_0}(1, z_{ns}, f)\sigma_{xx}\,dz_{ns}, Q = \int_{-h/2-h_0}^{h/2-h_0} g\sigma_{xz}\,dA \tag{16}$$

The variation for the work of non-conservative force is expressed by

$$\delta V = \int_0^L ((q + q_{dynamic})\delta(w_b + w_s))dx \tag{17}$$

The external force q due to Winkler-Pasternak substrate may be defined as

$$q = -k_w(w_b + w_s) + k_p\frac{\partial^2(w_b + w_s)}{\partial x^2} \tag{18}$$

in such a way that k_w and k_p define Winkler and Pasternak factors of substrate, respectively. Also, the kinetic energy variation is obtained as

$$\begin{aligned}\delta K = \int_0^L \Bigg(& I_0\left[\frac{\partial u}{\partial t}\frac{\partial\delta u}{\partial t} + \left(\frac{\partial w_b}{\partial t} + \frac{\partial w_s}{\partial t}\right)\left(\frac{\partial\delta w_b}{\partial t} + \frac{\partial\delta w_s}{\partial t}\right)\right] - I_1\left(\frac{\partial u}{\partial t}\frac{\partial^2\delta w_b}{\partial x\partial t} + \frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial\delta u}{\partial t}\right)\\ &+I_2\left(\frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial^2\delta w_b}{\partial x\partial t}\right) - J_1\left(\frac{\partial u}{\partial t}\frac{\partial^2\delta w_s}{\partial x\partial t} + \frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial\delta u}{\partial t}\right) + K_2\left(\frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial^2\delta w_s}{\partial x\partial t}\right)\\ &+J_2\left(\frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial^2\delta w_s}{\partial x\partial t} + \frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial^2\delta w_b}{\partial x\partial t}\right)\Bigg)dx\end{aligned} \tag{19}$$

so that

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_{-h/2-h_0}^{h/2-h_0} \rho(z_{ns})(1, z_{ns}, f, z_{ns}^2, z_{ns}f, f^2)dz_{ns} \tag{20}$$

Substituting Eqs. (15)-(19) into Eq. (13) then collecting the coefficients for field variables results in three equations of motion

$$\frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} - J_1 \frac{\partial^3 w_s}{\partial x \partial t^2} \tag{21}$$

$$\begin{aligned} \frac{\partial^2 M_b}{\partial x^2} = q_{dynamic} + I_0 \left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2} \right) + I_1 \frac{\partial^3 u}{\partial x \partial t^2} - I_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} \\ -J_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} + k_w(w_b + w_s) - k_p \frac{\partial^2 (w_b + w_s)}{\partial x^2} \end{aligned} \tag{22}$$

$$\begin{aligned} \frac{\partial^2 M_s}{\partial x^2} + \frac{\partial Q}{\partial x} = q_{dynamic} + I_0 \left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2} \right) + J_1 \frac{\partial^3 u}{\partial x \partial t^2} - J_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} \\ -K_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} + k_w(w_b + w_s) - k_p \frac{\partial^2 (w_b + w_s)}{\partial x^2} \end{aligned} \tag{23}$$

2.3 The nonlocal elasticity model for refined FGM nanobeams

In the context of nonlocal elastic field theory, the stress situation of every points within a structure may be defined as a function of strain of all neighboring points. Thus, a constitutive scheme has been employed for expressing the nonlocal stress field σ_{ij} at point *x* based on below relation

$$\sigma_{ij} = \int_V \lambda(|x' - x|, \tau)\, C_{ijkl}\varepsilon_{kl}(x')dV(x') \tag{24}$$

where C_{ijkl} and ε_{kl} are the elastic material properties and strain field, and the nonlocal kernel $\lambda(|x' - x|, \tau)$ contains the effects of the strains of point x'on the stresses of point *x* within the structure and $|x' - x|$ defines Euclidean distances. In differential form, the nonlocal stress-strain relations may be expressed by

$$(1 - (e_0 a)^2 \nabla^2)\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \tag{25}$$

where $\nabla^2$is used as Laplacian operator. Accordingly, the constitutive relations based on nonlocal refined FG nano-size beam may be introduced as

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z_{ns})\varepsilon_{xx} \tag{26}$$

$$\sigma_{xz} - \mu \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G(z_{ns})\gamma_{xz} \tag{27}$$

where $\mu = (e_0 a)^2$. Integration of Eqs. (26) and (27) about the beam thickness results in the below forces and moments

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A \frac{\partial u}{\partial x} - B \frac{\partial^2 w_b}{\partial x^2} - B_s \frac{\partial^2 w_s}{\partial x^2} \tag{28}$$

$$M_b - \mu \frac{\partial^2 M_b}{\partial x^2} = B \frac{\partial u}{\partial x} - D \frac{\partial^2 w_b}{\partial x^2} - D_s \frac{\partial^2 w_s}{\partial x^2} \tag{29}$$

$$M_s - \mu \frac{\partial^2 M_s}{\partial x^2} = B_s \frac{\partial u}{\partial x} - D_s \frac{\partial^2 w_b}{\partial x^2} - H_s \frac{\partial^2 w_s}{\partial x^2} \tag{30}$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = A_s \frac{\partial w_s}{\partial x} \tag{31}$$

where the cross-sectional rigidities are calculated as follows

$$(A, B, B_s, D, D_s, H_s) = \int_{-h/2-h_0}^{h/2-h_0} E(z_{ns})(1, z_{ns}, f, {z_{ns}}^2, z_{ns}f, f^2)\, dz_{ns} \tag{32}$$

$$A_s = \int_{-h/2-h_0}^{h/2-h_0} g^2 G(z_{ns})\, dz_{ns} \tag{33}$$

Three governing equations for presented beam model exposed to uniformly-type dynamical loads in terms of displacements have been stablished via placing Eqs. (28)-(31) into Eqs. (21)-(23) as follows

$$\begin{aligned} &A\left(\frac{\partial^2 u}{\partial x^2} + g \frac{\partial^3 u}{\partial t \partial x^2}\right) - B\left(\frac{\partial^3 w_b}{\partial x^3} + g \frac{\partial^4 w_b}{\partial t \partial x^3}\right) - B_s\left(\frac{\partial^3 w_s}{\partial x^3} + g \frac{\partial^4 w_s}{\partial t \partial x^3}\right) - I_0 \frac{\partial^2 u}{\partial t^2} \\ &+ I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} + J_1 \frac{\partial^3 w_s}{\partial x \partial t^2} + \mu\left(I_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} - I_1 \frac{\partial^5 w_b}{\partial x^3 \partial t^2} - J_1 \frac{\partial^5 w_s}{\partial x^3 \partial t^2}\right) = 0 \end{aligned} \tag{34}$$

$$\begin{aligned} &B\left(\frac{\partial^3 u}{\partial x^3}\right) - D\left(\frac{\partial^4 w_b}{\partial x^4}\right) - D_s\left(\frac{\partial^4 w_s}{\partial x^4}\right) - I_0\left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}\right) - I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} \\ &+ J_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} - k_w(w_b + w_s) + k_p \frac{\partial^2 (w_b + w_s)}{\partial x^2} + \mu\left(+I_0\left(\frac{\partial^4 w_b}{\partial x^2 \partial t^2} + \frac{\partial^4 w_s}{\partial x^2 \partial t^2}\right)\right. \\ &\left.+ I_1 \frac{\partial^5 u}{\partial x^3 \partial t^2} - I_2 \frac{\partial^6 w_b}{\partial x^4 \partial t^2} - J_2 \frac{\partial^6 w_s}{\partial x^4 \partial t^2} + k_w \frac{\partial^2 (w_b + w_s)}{\partial x^2} - k_p \frac{\partial^4 (w_b + w_s)}{\partial x^4}\right) \\ &= q_{dynamic} - \mu \frac{\partial^2 q_{dynamic}}{\partial x^2} \end{aligned} \tag{35}$$

$$\begin{aligned} &B_s\left(\frac{\partial^3 u}{\partial x^3}\right) - D_s\left(\frac{\partial^4 w_b}{\partial x^4}\right) - H_s\left(\frac{\partial^4 w_s}{\partial x^4}\right) + A_s\left(\frac{\partial^2 w_s}{\partial x^2}\right) - I_0\left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}\right) \\ &- J_1 \frac{\partial^3 u}{\partial x \partial t^2} + J_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} + K_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} - k_w(w_b + w_s) + k_p \frac{\partial^2 (w_b + w_s)}{\partial x^2} \end{aligned} \tag{36}$$

$$+\mu\left(+I_0\left(\frac{\partial^4 w_b}{\partial x^2\partial t^2}+\frac{\partial^4 w_s}{\partial x^2\partial t^2}\right)+J_1\frac{\partial^5 u}{\partial x^3\partial t^2}-J_2\frac{\partial^6 w_b}{\partial x^4\partial t^2}-K_2\frac{\partial^6 w_s}{\partial x^4\partial t^2}\right.$$
$$\left.+k_w\frac{\partial^2(w_b+w_s)}{\partial x^2}-k_p\frac{\partial^4(w_b+w_s)}{\partial x^4}\right)=q_{dynamic}-\mu\frac{\partial^2 q_{dynamic}}{\partial x^2} \tag{36}$$

3. Solution procedure

In the present chapter, differential quadrature method (DQM) has been utilized for solving the governing equations for FG nanobeam. According to DQM, at an assumed grid point (x_i, y_j) the derivatives for function F are supposed as weighted linear summation of all functional values within the computation domains as

$$\frac{d^nF}{dx^n}\Big|_{x=x_i}=\sum_{j=1}^{N}c_{ij}^{(n)}F(x_j) \tag{37}$$

where

$$C_{ij}^{(1)}=\frac{\pi(x_i)}{(x_i-x_j)\,\pi(x_j)}\qquad i,j=1,2,\dots,N,\qquad i\neq j \tag{38}$$

in which $\pi(x_i)$ is defined by

$$\pi(x_i)=\prod_{j=1}^{N}(x_i-x_j),\quad i\neq j \tag{39}$$

And when $i=j$

$$C_{ij}^{(1)}=c_{ii}^{(1)}=-\sum_{k=1}^{N}C_{ik}^{(1)},\quad i=1,2,\dots,N,\quad i\neq k,\qquad i=j \tag{40}$$

Then, weighting coefficients for high orders derivatives may be expressed by

$$\begin{aligned}
C_{ij}^{(2)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(1)}\\
C_{ij}^{(3)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(2)}=\sum_{k=1}^{N}C_{ik}^{(2)}C_{kj}^{(1)}\\
C_{ij}^{(4)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(3)}=\sum_{k=1}^{N}C_{ik}^{(3)}C_{kj}^{(1)}\qquad i,j=1,2,\dots,N.\\
C_{ij}^{(5)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(4)}=\sum_{k=1}^{N}C_{ik}^{(4)}C_{kj}^{(1)}
\end{aligned} \tag{41}$$

$$C_{ij}^{(6)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(5)} = \sum_{k=1}^{N} C_{ik}^{(5)} C_{kj}^{(1)} \tag{41}$$

According to presented approach, the dispersions of grid points based upon Gauss-Chebyshev-Lobatto assumption are expressed as

$$x_i = \frac{a}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right] \quad i = 1, 2, \ldots, N, \tag{42}$$

Next, the displacement components may be determined by

$$u(x,t) = U(x)e^{i\omega t} \tag{43}$$

$$\{w_b, w_s\}(x,t) = \{W_b, W_s\}(x)e^{i\omega t} \tag{44}$$

where W_b and W_s denote vibration amplitudes and ω defines the vibrational frequency. Then, it is possible to express obtained boundary conditions as

$$w_b = w_s = 0, \quad \frac{\partial^2 w_b}{\partial x^2} = \frac{\partial^2 w_s}{\partial x^2} = 0 \quad \text{for} \quad \text{S-S} \tag{45}$$

Now, one can express the modified weighting coefficients for all edges simply-supported as

$$\begin{aligned} \bar{C}_{1,j}^{(2)} = \bar{C}_{N,j}^{(2)} = 0, \quad i = 1, 2, \ldots, M, \\ \bar{C}_{i,1}^{(2)} = \bar{C}_{1,M}^{(2)} = 0, \quad i = 1, 2, \ldots, N. \end{aligned} \tag{46}$$

and

$$\bar{C}_{ij}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(1)} \bar{C}_{kj}^{(2)} \qquad \bar{C}_{ij}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(1)} \bar{C}_{kj}^{(3)} \tag{47}$$

Inserting Eqs. (43)-(44) into Eqs. (34)-(36) gives

$$\{[K] + [M]\omega^2\} \begin{Bmatrix} U \\ W_b \\ W_s \end{Bmatrix} = 0 \tag{48}$$

where ω defines vibration frequencies; [K] and [M] define the stiffness and mass matrices for FGM nano-size beam, respectively. Next, the dimensionless frequency and foundation factors have been selected by

$$\widetilde{\omega}_n = \omega_n L^2 \sqrt{\frac{\rho_c A}{E_c I}}, \quad K_u = k_u \frac{L^4}{E_c I}, \quad K_l = k_l \frac{L^4}{E_c I} K_s = k_s \frac{L^2}{E_c I}, \tag{49}$$

In this paper, forced vibration of the nanobeam is due to applied partial dynamic load with sinusoidal variation as defined below

$$q_{dynamic} = \sum_{n=1}^{\infty} Q_n s\,in\left[\frac{n\pi}{L}x\right] sin\,\omega\,t \tag{50}$$

$$Q_n = \frac{2}{L}\int_{x_0-c}^{x_0+c} \sin\left[\frac{n\pi}{L}x\right] q(x)dx = \frac{4q_0}{n\pi} sin\left[\frac{n\pi}{L}x_0\right] sin\left[\frac{n\pi}{L}c\right] \tag{51}$$

so that Q_n defines the Fourier coefficients and $q(x) = q_0$ indicates the uniform load magnitude and x_0 is load central location.

In order to perform forced vibrational study, placing the displacement fields and the dynamical force expressed in Eq. (50) into Eq. (48) results in below system

$$\{[K] + \omega_{ex}^2[M]\}\begin{Bmatrix} U \\ W_b \\ W_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_{dynamic} - \mu\dfrac{\partial^2 q_{dynamic}}{\partial x^2} \\ q_{dynamic} - \mu\dfrac{\partial^2 q_{dynamic}}{\partial x^2} \end{Bmatrix} \tag{52}$$

in which ω_{ex} is the excitation frequency. Solving Eq. (52) results in amplitude-frequency curves which are discussed in following section. The dimensionless excitation frequency and forced vibration amplitude have been selected as

$$\Omega = \omega_{ex}L^2\sqrt{\frac{\rho_c A}{E_c I}}, \bar{W}_{uniform} = W\frac{E_c I}{L^4 q_0} \tag{53}$$

4. Discussions on results

Thorough the present section, results are provided for forced vibration investigation of scale-dependent FGM nano-scale beams formulated by a three-unknown refined beam theory and

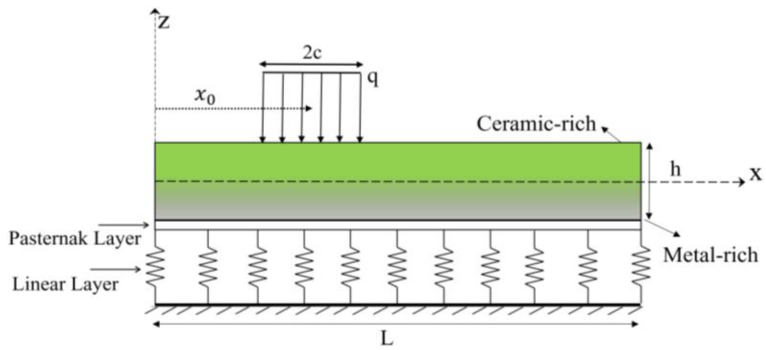

Fig. 1 Configuration of embedded FGM nano-size beam under partial dynamical load

Table 1 Material properties for FGM constituents

Property	Steel	Alumina (Al_2O_3)
E	210 (GPa)	390 (GPa)
ρ	7800 (kg/m^3)	3960 (kg/m^3)
ν	0.3	0.24

Table 2 Comparison of the dimensionless frequency for nonlocal FG nanobeams (L/h = 20)

B.C.	β	p = 0.1		p = 5		p = 1	
		CBT (Eltaher et al. 2012	Present HOBT	CBT (Eltaher et al. 2012	Present HOBT	CBT (Eltaher et al. 2012	Present HOBT
S-S	0	9.2129	9.16130	7.8061	7.71504	7.0904	6.96751
	1	8.7879	8.74014	7.4458	7.36037	6.7631	6.6472
	2	8.4166	8.37218	7.1312	7.05050	6.4774	6.36736
	3	8.0887	8.04711	6.8533	6.77674	6.2251	6.12012

Table 3 Comparison of dynamic deflection of the nanobeams based on refined shear defamation and Euler-Bernoulli beam theories (L/h = 10)

μ	Ω = 7		Ω = 7.5	
	CBT	Present HOBT	CBT	Present HOBT
0	0.0521	0.0591	0.0604	0.0631
1	0.0607	0.0679	0.0701	0.0753
2	0.1123	0.1246	0.3145	0.3226

nonlocal elasticity. The nano-size beam under a periodic dynamical loading has been depicted in Fig. 1. Table 1 presents material coefficients for the FG material. Accordingly, the present formulation and DQ solution is capable of giving accurate results of nanobeams. Also, Table 3 present a comparison between obtained dynamic deflections of the nanobeam based on refined and classic beam (CBT) theories at different nonlocal parameters. According to this table, obtained dynamic deflections based on refined beam theory are greater than those of classic beam theory.

In Fig. 2, the variations of normalized deflections of a FG nano-dimension beam versus excitation frequency of mechanical loading are represented for several nonlocality (μ) coefficients when L/h = 10. By selecting μ = 0, the deflections and vibrational frequencies based upon classic beam assumption will be derived. Actually, selecting μ = 0 gives the deflections in the context of classic elasticity theory and discarding nonlocal impacts. Exerting higher values of excitation frequency leads to larger deflections and finally resonance of the beam. It can be understand from Fig. 2 that normalized deflection of system will rise with nonlocality coefficient. This observation is valid for excitation frequencies before resonance. So, forced vibration behavior of the nanobeam system is dependent on scale effects.

Figs. 3 and 4 respectively indicate the influences of Winkler and Pasternak factors on dynamical deflections of FGM nano-size beams versus external frequency (Ω) assuming L/h = 10 and p = 1. Also, load location and configuration have been selected as x_0/L = 0.5 and c/L = 0.5.

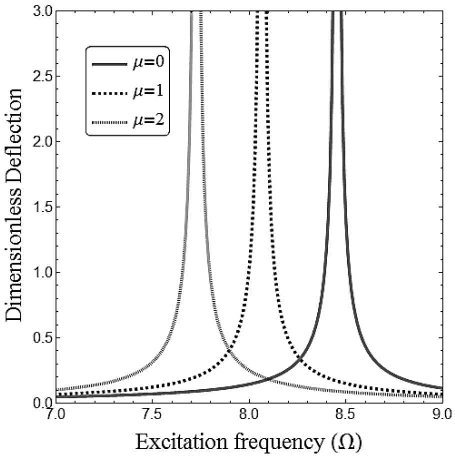

Fig. 2 Deflection-frequency results for FGM nano-size beam with varying external frequency and various nonlocal factors (L/h = 10, $K_w = 0$, $K_p = 0$, p = 0.2, $x_0/L = 0.5$, c/L = 0.5)

Different magnitudes of Winkler factor ($K_w = 0, 25, 50, 75$) and Pasternak factor ($K_p = 0, 5, 10, 15$) have been selected. One can observe that a rise in the magnitude of Winkler and Pasternak factors leads to reduction in vibration amplitudes of FGM nano-size beams. Actually, the nano-size beams become more rigid via increasing in foundation factors which leads to postponement of resonance frequency. An important finding is that Pasternak factor indicates more significant impact on deferment of resonance frequency. This is because Pasternak factor is corresponding to continuous interactions with the nano-size beam but Winkler factor leads to discontinuous interactions with the nano-size beam. Accordingly, the forced vibrations of FGM nano-size beams have been significantly influenced by elastic substrate.

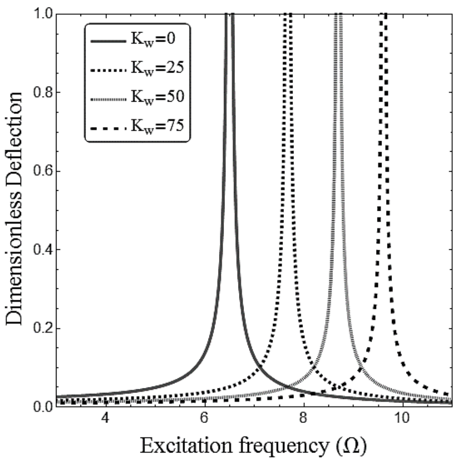

Fig. 3 Deflection-frequency results for FGM nano-size beam with varying external frequency and various Winkler factors (L/h = 10, $K_p = 0$, p = 1, $x_0/L = 0.5$, c/L = 0.5)

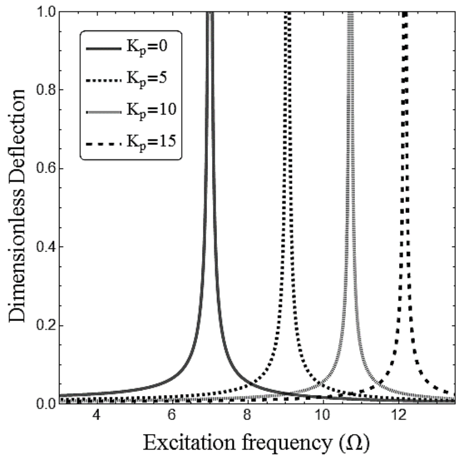

Fig. 4 Deflection-frequency results for FGM nano-size beam with varying external frequency and various Pasternak factors (L/h = 10, K_w = 10, p = 1)

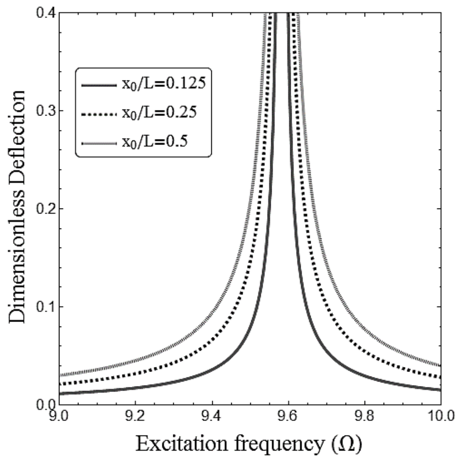

Fig. 5 Deflection-frequency results for FGM nano-size beam with varying external frequency and different locations of dynamical load (L/h = 10, K_w = 25, K_p = 5, p = 1, μ = 1, c/L = 0.125)

Study of the influence of dynamical force position (x_0/L) on normalized deflections of FGM nano-size beams versus external to natural frequency ratios (Ω/ω_n) has been carried out in Fig. 5. For the figure, other factors are selected as K_w = 25, K_p = 5, p = 1, μ = 1 and c/L = 0.125. One may observe that as the dynamical force moves away from the beam edges, the dynamical deflections increase. It means that the region of frequency– deflection curves for FGM nano-size beams become wider and the maximum amplitudes tend to take place at a higher external frequency.

Impacts of material FG exponent (p = 0, 0.2, 1, 5) on dynamical deflections of FGM nano-size beams exposed to partial dynamical force with respect to the ratio of external to natural frequency (Ω/ω_n) have been illustrated in Fig. 6 assuming L/h = 10, K_w = 25, K_p = 5 and μ = 1 nm^2. One may see that at $\Omega/\omega_n = 1$, the resonance frequency takes place. Furthermore, at $\Omega/\omega_n < 1$ and $\Omega/\omega_n > 1$ the dynamical bending of FGM nano-size beam is remarkably influenced by FGM gradation. It is

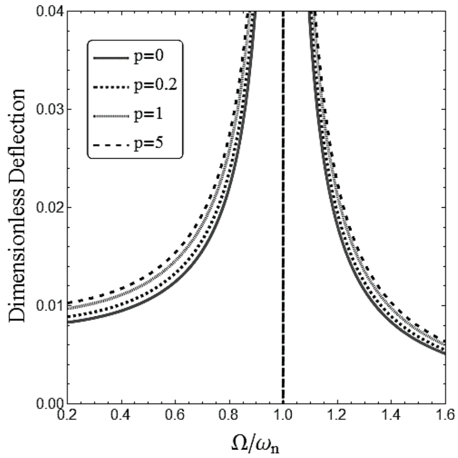

Fig. 6 Deflection-frequency results for FGM nano-size beam with varying external frequency and for various FG exponents (L/h = 10, K_w = 25, K_p = 5, μ = 1)

found that the magnitudes of dynamical deflections reduce via increase of material exponent (p). This is owning to higher portions of metallic constituent via increase of material exponent. Thus, choosing reliable values for material exponent is crucial for reasonable design of FG nano-size structures when they are exposed to dynamical excitation.

5. Conclusions

The presented article employed a higher order shear deformation beam formulation having three variables without using of shear correction factor. Based upon differential quadrature (DQ) approach and nonlocal elasticity formulation, forced vibrational analysis of shear deformable functionally graded (FG) nanobeam on elastic medium under partial dynamical load was performed. The presented formulation incorporated a scale factor for examining vibrational behaviors of nano-dimension beams. The material properties for FG beam were defined employing a power-law form. The governing equations achieved by Hamilton’s principle were solved implementing DQM. Presented results indicated the prominence of material gradient index, nonlocal coefficient, material gradient coefficient, load location and substrate factors on vibrational properties of FG nano-size beam. Especially, it was found that as the dynamical force moves away from the beam edges, the dynamical deflections increase. Aslo, it was observed that nonlocal factor increment results in smaller values for resonance frequency of FGM nano-size beam.

Acknowledgments

The authors would like to thank Mustansiriyah university (www.uomustansiriyah.edu.iq) Baghdad-Iraq, for their support in the present work.

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