

The effects of thermal relaxation times in living tissues under the TPL bio-heat model with experimental study

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Abstract. In the present article, the effects of three thermal relaxation times in living tissue under the three-phase-lag (TPL) bioheat model are introduced. Using the Laplace transforms, the analytical solution of the temperature and the resulting thermal damages in living tissues are obtained. The experimental data are used to validate the analytical solutions. By the formulations of Arrhenius, the thermal damage of tissue is estimated. Numerical outcomes for the temperature and the resulting of thermal damages are presented graphically. The effects of parameters, such as thermal relaxation times, blood perfusion rate on tissue temperature are also discussed in detail.

Keywords: bio-heat transfer; laplace transforms; living tissues; three-phase lag model

1. Introduction

Thermic therapeutics include all treatments of pernicious diseases (Tumors and cancer cells) depending on the transfer of thermal energy into or out of the body in different shapes (Radiofrequency, Ultrasound Electromagnetic radiation, etc.). It is applied as a low invasive substitutional to classic eradication in treating cancerous cells and benign diseases. In a clinical frame, the main objective of thermal therapy is to obtain an efficient treatment of cancerous cells without harming surrounding sanitary tissue. Counting on the rise in temperature and duration of treatment, thermic therapeutics is illustrated by Habash *et al.* (2006). In artificial hyperthermia-therapeutic, the heating up at the place of the tumor cell results in the changed physiology of unhealthy cells then results in mortification or apoptosis-inducing. The degree of initial tissue mortification count on the heat implemented on the tissue. Mortification is characterized by negative diseased cells damage and the inflaming response produced from the nearby tissues whereas apoptosis represents a hereditarily restrained cell death. Hyperthermia-therapeutic is an auxiliary treatment that is applied in addition to proper therapy, for instance, radiotherapeutic or chemotherapeutic. Accordingly, hyperthermia-therapeutic enhances the effect of orthodox medical treatment in the frame of multimodal therapy concepts. For achieving the favorable outcome of hyperthermia-therapeutic, always needed control and accurate predictions of temperature (Gupta *et al.* 2010, Dombrovsky *et al.* 2012, Das *et al.* 2013, Padra and Salva 2013, Di Michele *et al.* 2015).

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Their heterogeneous internal structure complicates the heat transfer analysis in living tissues. Convection between tissue and blood, metabolic heat generation, evaporation, heat conduction in blood vessels and solid tissue matrix, and perfusion through capillary tubes within the tissues are all examples of heat transfer in living tissues.

The heat transfer in living biological tissue is modeled by Pennes' (1948) bioheat transfer model, which is based on Fourier's law of heat conduction

$$q(\mathbf{r}, t) = -k\nabla T(\mathbf{r}, t). \quad (1)$$

In which $q(\mathbf{r}, t)$ is denoted the local heat flux, $\nabla T(\mathbf{r}, t)$ is the temperature gradient k is the thermal conductivity. Because of its simplicity, Pennes' bioheat transfer model was widely used in heat transfer in tissue. Nevertheless, there are some drawbacks to Pennes' model, considering it supposes that the spread speed of thermal disturbance is unlimited. Indeed, heat is regularly seen spreading at a slow rate in living tissues that seem to have a very diverse internal structure (Kumar *et al.* 2016). Therefore, other methods for resolving the dilemma in Pennes' model have been offered. Cattaneo (Cattaneo 1958) and Vernotte (Vernotte 1958) concurrently adjusted Fourier law to know C–V fundamental relation, which is given as

$$q(\mathbf{r}, t + \tau_q) = -k\nabla T(\mathbf{r}, t). \quad (2)$$

Pennes' bioheat transfer model was widely used in heat transfer in tissue because of its simplicity. However, there are some disadvantages for Pennes' bioheat transfer model as it assumes that the propagation speed of thermal disturbance is infinite. In fact, heat is always found to propagate with a finite speed within living biological tissues as they have highly non-homogeneous inner structure (Kumar *et al.* 2015). In this consecutive order, other models have been proposed for solving the paradox, which is occurred in Pennes' bioheat transfer model. Cattaneo (1958) and Vernotte (1958) simultaneously modified Fourier law to know C–V constitutive relation which is given as: bioheat transfer model, which is based on the Fourier's law of heat conduction bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in non-homogeneous tissue is a very complex process. It demonstrates with multiple phenomenological mechanisms including conduction in tissue, convection between blood and tissues, blood perfusion or advection and diffusion through micro-vascular cells, and metabolic heat generation (Afrin *et al.* 2011). The generalization theory of bioheat transfer in biological tissues has been reviewed as follows: The heat transfer in living biological tissue is modeled by Pennes (1948) bioheat transfer model, which is based on the Fourier's law of heat conduction The heat transfer in non-homogeneous tissue is a very complex process. It demonstrates with multiple phenomenological mechanisms including conduction in tissue, convection between blood and tissues, blood perfusion or advection and diffusion through micro-vascular cells, and metabolic heat generation (Afrin *et al.* 2011). The generalization theory of bioheat transfer in biological tissues has been reviewed as follows: The heat transfer in living biological tissue is modeled by

Pennes (1948) bioheat transfer.

In which τ_q is relaxation time because of heat transfer. C–V model assumes that temperature gradient and heat flow occur at the same position at various times of instant. The delay time between the temperature gradient and heat flux is known as relaxation time and thermalization time. Due to the imperfect thermal model for some circumstances, Tzou (2014) stated a dual-phase-lag (DPL) constitutive relation, i.e.

$$q(\mathbf{r}, t + \tau_q) = -k\nabla T(\mathbf{r}, t + \tau_T), \quad (3)$$

where temperature gradient and heat flux at the same point are at different times of instant. The lagging time τ_T interpreted as that caused by micro-structural interaction and is called phase-lag in a temperature gradient. Another lagging time τ_q is relaxation time in view of the ephemeral effect of heat inertia which is named phase-lag of heat flow (Antaki 2005). Kumar *et al.* (2016) studied and compared the results of DPL, monocular-phase-lag, and traditional Fourier bioheat transfer models in living tissues. The non-linear DPL bioheat transfer model in spherical biological tissues was studied by Saeed and Abbas (2020). Hobiny and Abbas (2021) have been studied the Fractional-order bioheat transfer model. Mondal *et al.* (2020) studied the heat transport equation for this problem involving the memory-dependent derivative on a slipping interval in the context of the Lord-Shulman (LS) model. The three-phase-lag for thermoelastic media has been studied (Kumar and Chawla 2011, Kumar and Chawla 2013, Akbarzadeh *et al.* 2014). Quintanilla and Racke (2008) have discussed the stability in TPL heat conduction equation and the relations among the three parameters. Fahmy (2019) has studied the novel boundary element model for describing thermo-mechanical interactions in anisotropic laser-induced tissue hyperthermia. Hassan *et al.* (2018) studied the exploration of convective thermal transfers and flow characteristics synthesis. Marin (1996) studied the generalized solutions in the elasticity of micropolar materials with voids. Dahab and Abbas (2011) studied the generalized magneto-thermoelasticity in an infinitely long annular cylinder with variable thermal conductivity under LS model. Abbas *et al.* (2022) studied the nonlocal heat conduction technique in biological tissues generated by laser irradiation. Hobiny and Abbas (2018) investigated the theoretical analysis of thermal damage in skin tissues due to moving heat source. Kabiri and Talaei (2022) studied the effect of non-Fourier bioheat transfer on bone drilling temperature in orthopedic surgery. Abdalla *et al.* (2022) presented the analytical and numerical solutions to study the effects of fractional derivative of bioheat model in living tissue. Kumari and Singh (2022) studied the effects of space-fractional three-phase-lag bioheat transfer model during thermal therapy. The authors (Marin *et al.* 2015, Abbas and Alzahrani 2016, Abbas and Kumar 2016, Alzahrani and Abbas 2016, Abbas and Marin 2017, Eftekhari 2018, Kahya and Turan 2018, Lata 2018, Kaur and Lata 2020, Saeed *et al.* 2020, Alzahrani and Abbas 2021, Lata and Kaur 2021) presented the solutions of several problems under generalized thermoelastic theories

The present paper proposes the three-phase lag (TPL) bioheat transfer model to investigate the temperature distribution in living spherical tissue for a more realistic physical phenomenon. The validation of (TPL) model of bioheat transfer with experimental data (Stolwijk and Hardy 1966) is presented in the situation of fever. Laplace transform technique is used to solve the model for analyzing the temperature distribution in living tissue. We analyzed the significant effect of lag times due to heat flow, temperature gradient, and thermal displacement gradient on tissue temperature. The effect of variability of other parameters on temperature profile in living skin tissue is discussed in detail.

2. Mathematical model

Under (Xu *et al.* 2008, Ahmadikia *et al.* 2012, Kumar and Rai 2020), the TPL bioheat equation in spherical living tissues is defined as

$$\begin{aligned} & \left[k^* + (k + k^* \tau_v) \frac{\partial}{\partial t} + k \tau_\theta \frac{\partial^2}{\partial t^2} \right] \left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right] \\ & = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} + \frac{\tau_q^2}{2} \frac{\partial^3}{\partial t^3} \right) \left(\rho c \frac{\partial T}{\partial t} - Q_b - Q_m - Q_{ext} \right), \end{aligned} \quad (4)$$

where t , τ_θ , τ_q , and τ_v are defined respectively by the time, the phase lag of the temperature gradient, the phase lag of the heat flux density, and the phase lag for the gradient of thermal displacement, k^* is the constant material characteristic of the model, T is the tissue temperature, ρ is the density of the mass of tissues, k is the tissue thermal conductivity, c is the tissues specific heat and the heat source of blood perfusion is denoted by Q_b which is expressed as

$$Q_b = \omega_b \rho_b c_b (T_b - T), \quad (5)$$

where T_b is the blood temperature, ρ_b is the density of the mass of blood, c_b is the specific heat of blood, the rate of blood perfusion is denoted by ω_b , and Q_m is denote the heat generated by metabolic mechanisms because of different physiological operations that occur in the rest of the body. Due to Mitchell *et al.* (1970), the metabolically generated heat can define as

$$Q_m = Q_{mo} \times 2^\beta \left(\frac{T - T_o}{10} \right), \quad (6)$$

where β is the constant associated metabolic, Q_{mo} is the reference metabolic heating resource, and T_o denoted to the initial temperature of local tissue, The dependences on metabolic heating generation can be generally estimated as linear functions of the temperature of local tissue as below

$$Q_m = Q_{mo} \left(1 + \beta \left(\frac{T - T_o}{10} \right) \right). \quad (7)$$

where Q_{ext} represent the heat generated from the heating source per unit volume of tissue, which is submitted by (Gardner *et al.* 1996) as a laser heating source and determined as

$$Q_{ext}(r, t) = I_o \mu_a [u(t) - u(t - \tau_p)] \left[C_1 e^{-\frac{k_1}{\Delta} r} - C_2 e^{-\frac{k_2}{\Delta} r}, \right] \quad (8)$$

where k_1 , C_1 , k_2 , and C_2 are the diffuse reflectance function of R_d which is given in (Gardner *et al.* 1996), $u(t)$ is the unit step functions, I_o is the density of laser, τ_p the time of tissue exposure to the laser, Δ is permeation depth, and μ_a is the absorption coefficients. The permeation depth was clarified by (Gardner *et al.* 1996) as

$$\Delta = \frac{1}{\sqrt{3\mu_a(\mu_a + \mu_s(1 - g))}}, \quad (9)$$

where g is the anisotropy factor and μ_s the scattering coefficient. Currently, initial conditions and boundary conditions can be described by

$$T(r, 0) = T_b, \quad \frac{\partial T(r, 0)}{\partial t} = 0, \quad (10)$$

$$\begin{aligned}
& - \left[k^* + (k + k^* \tau_v) \frac{\partial}{\partial t} + k \tau_\theta \frac{\partial^2}{\partial t^2} \right] \frac{\partial T(r_L, t)}{\partial r} = 0, \\
& - \left[k^* + (k + k^* \tau_v) \frac{\partial}{\partial t} + k \tau_\theta \frac{\partial^2}{\partial t^2} \right] \frac{\partial T(r_o, t)}{\partial r} = 0.
\end{aligned} \tag{11}$$

For appropriateness, the non-dimensional parameters are expressed as

$$\begin{aligned}
T' &= \frac{T - T_o}{T_o}, T'_b = \frac{T_b - T_o}{T_o}, (t', \tau'_\theta, \tau'_q, \tau'_v, \tau'_p) = \frac{k}{\rho c L^2} (t, \tau_\theta, \tau_q, \tau_v, \tau_p), r' = \frac{r}{L} \\
f &= \frac{L^2}{k T_o}, (k'_1, k'_2) = \frac{L}{\delta} (k^1, k^2), (Q'_b, Q'_m, Q'_{ext}) = f(Q_b, Q_m, Q_{ext}), f_1 = f k^*.
\end{aligned} \tag{12}$$

In the context of these non-dimensional parameters in (12), government equation the (4) can be written as

$$\begin{aligned}
& \left[f_1 + (1 + f_1 \tau_v) \frac{\partial}{\partial t} + \tau_\theta \frac{\partial^2}{\partial t^2} \right] \left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right] \\
& = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} + \frac{\tau_q^2}{2} \frac{\partial^3}{\partial t^3} \right) \left(\frac{\partial T}{\partial t} - Q_b - Q_m - Q_{ext} \right),
\end{aligned} \tag{13}$$

$$T(r, 0) = T_b, \frac{\partial T(r, 0)}{\partial t} = 0, \tag{14}$$

$$\left[f_1 + (1 + f_1 \tau_v) \frac{\partial}{\partial t} + \tau_\theta \frac{\partial^2}{\partial t^2} \right] \frac{\partial \bar{T}(r_o, t)}{\partial r} = 0, \left[f_1 + (1 + f_1 \tau_v) \frac{\partial}{\partial t} + \tau_\theta \frac{\partial^2}{\partial t^2} \right] \frac{\partial \bar{T}(r_L, t)}{\partial r} = 0. \tag{15}$$

3. Solution of the problem

For a function $\Psi(r, t)$, its Laplace transform is written as

$$\bar{\Psi}(r, s) = L[\Psi(r, t)] = \int_0^\infty \Psi(r, t) e^{-st} dt, \tag{16}$$

where s the Laplace transform parameter. Then, by utilizing (16), the Eqs. (13), and (15) can be replaced by

$$\frac{d^2 \bar{T}}{dr^2} + \frac{2}{r} \frac{d \bar{T}}{dr} - n^2 \bar{T} = -n_1 e^{-k_1 r} - n_2 e^{-k_2 r}, \tag{17}$$

$$\frac{\partial \bar{T}(r_o, s)}{\partial r} = 0, \frac{\partial \bar{T}(r_L, s)}{\partial r} = 0, \tag{18}$$

where

$$n^2 = \frac{\left(s + s^2\tau_q + s^3\frac{\tau^2 q}{2}\right)\left(s + \frac{L^2\omega_b\rho_b c_b}{k} - \frac{L^2 Q_{mo}\beta}{10k}\right)}{\frac{f_1 + (1 + f_1\tau_v)s + s^2\tau_\theta}{L^2 I_o \mu_a} \frac{C_1(1 - e^{-s\tau_p})}{kT_o}}, n_1$$

$$= \frac{L^2 I_o \mu_a}{kT_o} \frac{C_1(1 - e^{-s\tau_p})}{f_1 + (1 + f_1\tau_v)s + s^2\tau_\theta},$$

$$n_2 = -\frac{L^2 I_o \mu_a}{kT_o} \frac{C_2(1 - e^{-s\tau_p})}{f_1 + (1 + f_1\tau_v)s + s^2\tau_\theta}.$$

Then

$$\bar{T}(r, s) = \frac{A_1}{r} e^{nr} + \frac{A_2}{r} e^{-nr} - \left(\frac{2k_1 n_1 e^{-k_1 r}}{(k_1^2 - n^2)^2 r} + \frac{2k_2 n_2 e^{-k_2 r}}{(k_2^2 - n^2)^2 r} \right) - \left(\frac{n_1 e^{-k_1 r}}{(k_1^2 - n^2)} + \frac{n_2 e^{-k_2 r}}{(k_2^2 - n^2)} \right), \quad (19)$$

where

$$A_1 = ((M_1 S_4 - M_1 M_2 S_4 k_1 - M_1 S_4 k_1 p) e^{-pk_1} + (M_3 S_4 - M_3 M_4 S_4 k_2 - M_3 S_4 k_2 p) e^{-pk_2} + (-M_1 S_2 + LM_1 S_2 k_1 + M_1 M_2 S_2 k_1) e^{-Lk_1} + (-M_3 S_2 + LM_3 S_2 k_2 + M_3 M_4 S_2 k_2) e^{-Lk_2}) / (S_1 S_4 - S_2 S_3),$$

$$A_2 = -((M_1 S_3 - M_1 M_2 S_3 k_1 - M_1 S_3 k_1 p) e^{-pk_1} + (M_3 S_3 - M_3 M_4 S_3 k_2 - M_3 S_3 k_2 p) e^{-pk_2} + (-M_1 S_1 + LM_1 S_1 k_1 + M_1 M_2 S_1 k_1) e^{-Lk_1} + (-M_3 S_1 + LM_3 S_1 k_2 + M_3 M_4 S_1 k_2) e^{-Lk_2}) / (S_1 S_4 - S_2 S_3),$$

$$M_1 = \frac{n_1}{(k_1^2 - n^2)}, M_2 = \frac{2k_1}{(k_1^2 - n^2)}, M_3 = \frac{n_2}{(k_2^2 - n^2)}, M_4 = \frac{2k_2}{(k_2^2 - n^2)},$$

$$S_1 = \frac{(np-1)e^{pn}}{p^2}, S_2 = -\frac{(np+1)e^{-pn}}{p^2}, S_3 = \frac{(nL-1)e^{Ln}}{L^2}, S_3 = -\frac{(nL+1)e^{-Ln}}{L^2}, r_o = p \cong 0, r_L = L.$$

A numerically inverse approach adopted which is based on the approximate value approach of the Riemann sum, is used to obtain the distributions of temperature. based on this process, in the domain of Laplace, any function can be transformed into the time domain as in (Jou 1998)

$$\Psi(r, t) = \left(R_e \sum_{n=0}^{\infty} (-1)^n \bar{\Psi} \left(r, m + \frac{in\pi}{t} \right) + \frac{1}{2} R_e [\bar{\Psi}(r, m)] \right) \frac{e^{mt}}{t}, \quad (20)$$

where $i = \sqrt{-1}$ and R_e is known as the real part.

4. Results and discussion

In this study, the variation of temperature in living tissues has examined under the hyperbolic bioheat model. and has been chosen idealistic values of heat characteristics in living tissues for the numerical computations (Askarizadeh and Ahmadikia 2014)

$$\rho_b = 1060(\text{kg})(\text{m}^{-3}), c_b = 3860(\text{J})(\text{kg}^{-1})(\text{k}^{-1}), \omega_b = 1.87 \times 10^{-3}(\text{s}^{-1}), T_b = 36.5^\circ\text{C},$$

$$I_o = 3 \times 10^5(\text{W})(\text{m}^{-2}), \tau_p = 15(\text{s}), L = 0.03(\text{m}), Q_m = 1.19 \times 10^3(\text{W})(\text{m}^{-3}), g = 0.9,$$

$$C_1 = 3.09 + 5.44R_d - 2.12e^{-21.5R_d}, K_2 = 1.63e^{3.4R_d}, C_2 = 2.09 - 1.47R_d - 2.12e^{-21.5R_d},$$

$$K_1 = 1 - \left(1 - \frac{1}{\sqrt{3}}\right) e^{-20.1R_d}, R_d = 0.05, \mu_s = 12000(\text{m}^{-1}), \mu_a = 40(\text{m}^{-1}), T_o = 36.5^\circ\text{C},$$

$$c = 4187(\text{J})(\text{kg}^{-1})(\text{k}^{-1}), \rho = 1000(\text{kg})(\text{m}^{-3}), k = 0.628(\text{W})(\text{m}^{-1})(\text{k}^{-1}).$$

In the biological applied sciences in living tissue, the rigorous diagnosis of burn is one of the main features, It is a fundamental part of thermal therapy. Moritz and Henriques (Henriques Jr.

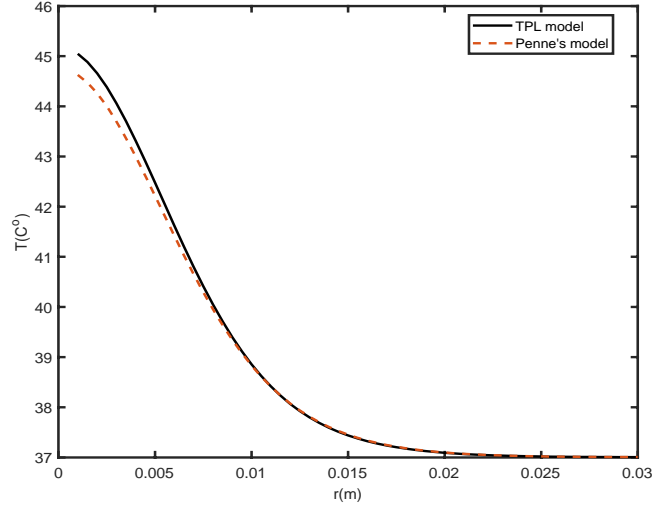


Fig. 1 The distribution of temperature according to the distance

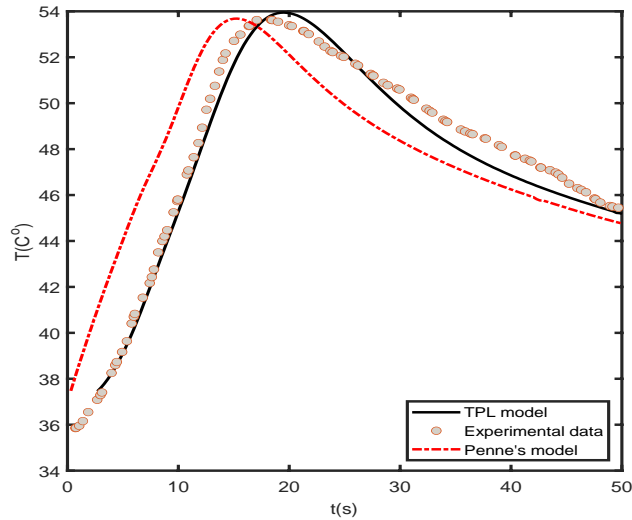


Fig. 2 Thermal behavior overtime at $r \cong 0$

and Moritz 1947, Moritz and Henriques 1947) developed a method that is used to measure thermal damages. The non-dimensional quantity of thermal damages Ω can be determined by

$$\Omega = \int_0^t B \exp\left(-\frac{E_a}{RT}\right) dt \quad (21)$$

where $B = 3.1 \times 10^{98} (\text{s}^{-1})$ is the frequency factor, $R = 8.314 \text{ J}/(\text{mal k})$ is the constant of universal gas, and $E_a = 6.27 \times 10^5 \text{ J}/\text{mal}$ the activation energy. There are great common features between pigskin and human skin, particularly in vascular organizations, some experimental studies indicated that. The experimental data have been examined the heating of pigskin with a laser by

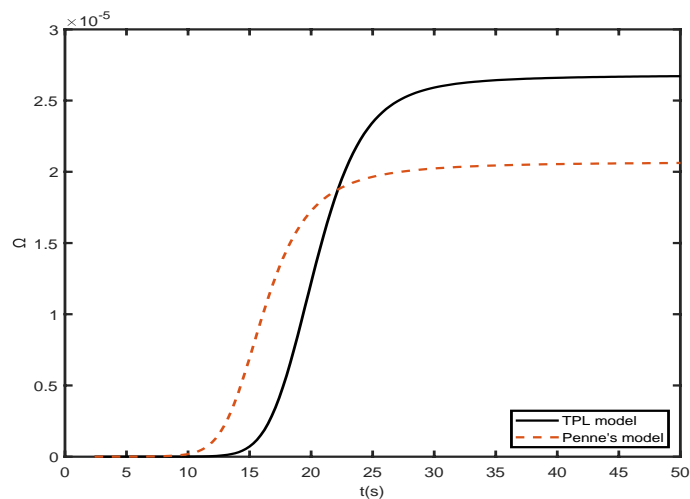
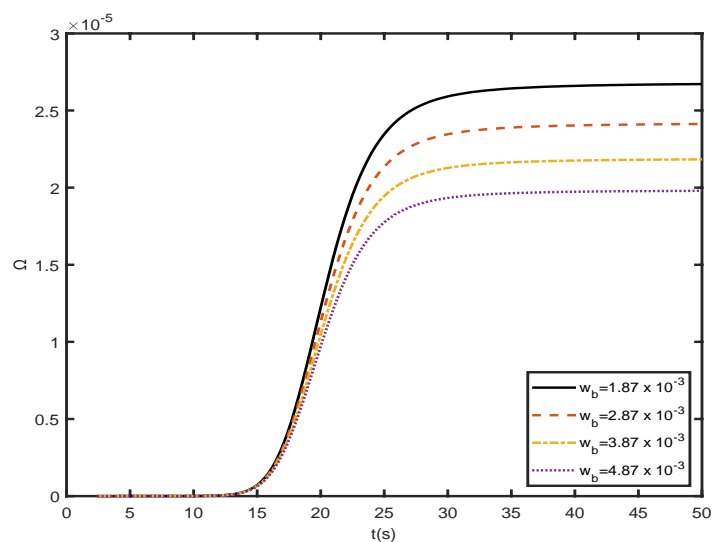
Fig. 3 Thermal damage along with time at $r \cong 0$ 

Fig. 4 The effect of the rate of blood perfusion on thermal damage

Museux *et al.* (2012). the analytical solution results are worthy of attention compared with that collected from the experimental survey. The results here are predicated on the skin tissues with the aforesaid attributes, In addition to the blood parameters and laser attributes. MATLAB (R2020a) software has been utilized in making the computations and the results are given graphically in Figs. 1-5. In these figures, the calculation was performed whereas $T_o = T_b = 37^\circ\text{C}$

Fig. 1 illustrates the temperature varies according to the radial distance r at $t = 50$ s for the three-phase lag (TPL) and penne's bioheat transfer models when the laser is applied steadily $\tau_p = 15$ s. The graph shows clearly that the temperature begins with the maximum value and then begins to decay until it reaches the initial value) $T_b = 37^\circ\text{C}$. Fig. 2 displays the time histories of surface temperature in the present experimental data. The figure also shows the good compatibility

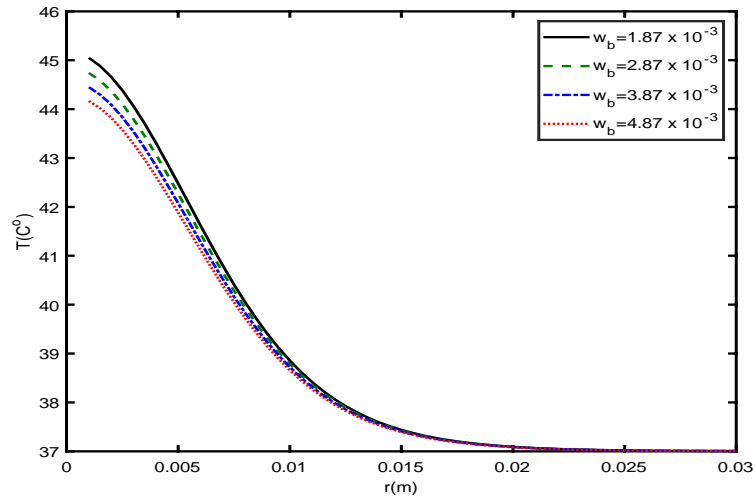


Fig. 5 The effect of the rate of blood perfusion on thermal damage

betwixt the outcomes of the mathematical solution with delay times and the outcomes received from the experimental study. Fig. 3 exhibits the estimated thermal damage occurring to the skin's surface over time. The effect of the rate of the blood perfusion on thermal damage to the surface of the skin, as well as on temperature distributions, is clearly shown in Figs. 4 and 5. In conclusion, parametric analyses are devoted to identifying appropriate procedures for choosing serious design variables to reach thermal effectiveness in hyperthermia treatments.

5. Conclusions

This work presents the temperature variations and the resulting thermal damages in spherical biological tissues under three phase lag model. The non-dimensional formulations are solved by employing the semi-analytical method based on the Laplace transform. The comparisons between the existing experimental studies and our numerical solution indicate that a three-phase lag model is an effective tool for estimating the bioheat transfer in living tissues.

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