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A nonlocal integral Timoshenko beam model for free vibration analysis of SWCNTs under thermal environment

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Abstract. In this paper, the nonlocal integral Timoshenko beam model is employed to study the free vibration characteristics of singled walled carbon nanotubes (*SWCNTs*) including the thermal effect. Based on the nonlocal continuum theory, the governing equations of motion are formulated by considering thermal effect. The influences of small scale parameter, the chirality of *SWCNTs*, the vibrational mode number, the aspect ratio of *SWCNTs* and temperature changes on the thermal vibration properties of single-walled nanotubes are examined and discussed. Results indicate significant dependence of natural frequencies on the nonlocal parameter, the temperature change, the aspect ratio and the chirality of *SWCNTs*. This work should be useful reference for the application and the design of nanoelectronics and nanoelectromechanical devices that make use of the thermal vibration properties of *SWCNTs*.

Keywords: nonlocal continuum theory; nonlocal integral Timoshenko beam model; singled walled carbon nanotubes (SWCNTs); small-scale effect; thermal effect; vibration characteristics

1. Introduction

The discovery of carbon nanotubes "*CNTs*" was by Iijima (1991), since this time, they have attracted worldwide attention. Recently, the analysis of *CNTs* has been of large interest to several scientific researchers because of their exceptional mechanical, electronic, electrochemical, physical and thermal properties (Robertson 2004). These properties support CNTs to be the suitable element for nanoelectronics, nanodevices, nanosensors and nanocomposites (Dai *et al*. 1996, Dharap *et al*. 2004). There are certain experimental investigations of the effectives properties and behaviors of "*CNTs*" as the Young's modulus "E", shear modulus "*G*", buckling behavior and vibration responses. For examples, experimental investigations by Treacy *et al*. (1996), who used the technique of transmission electron microscopy (TEM) to measure the Young's modulus of MWCNTs, reported a mean value of 1.8 Tpa with a variation from 0.40 to

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4.15 Tpa. Tombler *et al*. (2000) and Salvetat *et al*. (1999) have used the technique of atomic force microscope (AFM) and they found the average Young's modulus of MWCNTs to be 1.2 TPa and 0.81 TPa.

In the experiment, however, it is very difficult to measure the mechanical properties of CNTs, directly due to their very small size. Computer simulation has been regarded as a powerful tool for modeling the properties of CNTs. Generally, two major types of computational approaches to the simulation of mechanical properties of CNTs modelling. These are (*a*) atomistic approach such as Ab-initio (Ye *et al*. 2001), molecular dynamics "MD" (Frankland 2003) and tight binding "TBMD" (Hernandez *et al*. 1998) and (*b*) continuum mechanics approaches (Wang 2005). Among the available modeling techniques, molecular dynamics simulation numerically solves Newton's equations of motion, thus allowing structural fluctuations to be remarked with respect to time (Tersoff and Ruoff 1994). However, MD simulation is limited to systems with a maximum atom number of about 10^9 by the scale and cost of computation (Tadmor *et al*. 1999). So only single-walled nanotubes with small deflection can be simulated using the MD method. Hernandez *et al*. (1998) studies of the mechanical properties of SWCNTs using MD method which tight-binding approach (TBMD). SWCNTs have the highest Young's modulus and the Poisson's ratio, which were found to be 1.24 TPa and 0.262, respectively. Bao *et al*. (2004) used MD approach and reported the Young's modulus average values were reported 935.805 ± 0.618 GPa, 935.287 ± 2.887 GPa and 918.309 ± 10.392 GPa for Armchair, Zigzag and Chiral SWCNTs, respectively.

It is well known that classical continuum mechanics theory is size-independent, because it cannot incorporate the small-scale effect in nano-scale structure. The local continuum mechanics theory assumes that the stress at a point depends only on the strain at the same point, whereas in the nonlocal continuum mechanics theory originated by Eringen (1972 and 1983), the stress state at a given point as a function of the strain states of all points in the continuum mechanics. There are many works in the literature that have used this theory. Wang *et al*. (2006) investigated the small-scale effect on elastic buckling of CNTs with nonlocal continuum models. The Dynamic instability on CNTs is investigated by Sedighi and Yaghootian (2016) based on nonlocal continuum elasticity. Timesli (2020) presented an explicit analytical formula to examine the stability of DWCNTs using Donnell shells continuum approach. The small-scale and thermal effect on dynamic response of an embedded Armchair SWCNTs is investigated by Hamidi *et al*. (2018) by employing the non-local Timoshenko beam model. Bensattalah *et al*. (2018) investigated on mechanical stability of CNT reposed on foundation type Kerr by employing the nonlocal continuum theory and EBT formulations. Based on Energy equivalent model, Eltaher *et al*. (2018) have analyzed the vibrational behaviors of Material Size-Dependent CNTS. Bensattalah *et al*. (2019a) proposed a novel nonlocal Timoshenko beam theory to examine the free vibrational response of the chiral single-walled CNTs. Ahmed *et al*. (2019) examined the post-buckling response of imperfect FG nanobeams based on higher order nonlinear refined beam theory. Based on Eringen's differential law and Timoshenko beam theory, Jalaei and Civalek (2019) studied the dynamic instability of imperfect (FG) nanobeam subjected to axially oscillating loads. Also, Gafour *et al*. (2020) investigated the dynamic behavior of porous FG nanobeam by employing non-local higher order shear deformation theory. Abdulrazzaq *et al*. (2020) analyzed the thermal stability of clamped E-FG nano-size plate rested on an elastic substrate using a nonlocal refined theory. In similar works, the non-classical theory was used by several researchers to examine the static and dynamic response of the CNTs and others materials and structures (Attia and Rahman 2018, Karami *et al*. 2018a, b, Al-Maliki *et al*. 2019, Karami and Janghorban 2019a, b, Belmahi *et al*. 2019, Hamad *et al*. 2019, Eltaher *et al*. 2019a, b, c, Ebrahimi and Barati 2019, Attia *et al*. 2019, Shanab *et al*. 2020, Bensattalah *et al*. 2019b, 2020, Fenjan *et al*. 2020, Ghandourah and Abdraboh 2020, Tahouneh *et al*. 2020, Arefi and Żur 2020, Yuan *et al*. 2020, Mirjavadi *et al*. 2020a, Asrari *et al*. 2020, Ahmed *et al*. 2020a, Eltaher *et al*. 2020,Thanh *et al*. 2020, Civalek *et al*. 2021).

Therefore, the aim of this study is to analyze the free vibration of single-walled carbon nanotubes SWCNTs using a nonlocal integral Timoshenko elastic beam theory. Young's modulus of SWCNTs is predicted using MD simulation carried out by Bao *et al*. (2004). The effects of both small scale parameter, the chirality of SWCNTs, the vibrational mode number, the aspect ratio of SWCNTs and temperature changes on the frequency of SWCNTs are studied and discussed. This work should be useful guidance for the study and design of the next generation of nanodevices that make use of the thermal vibration properties of SWCNTs.

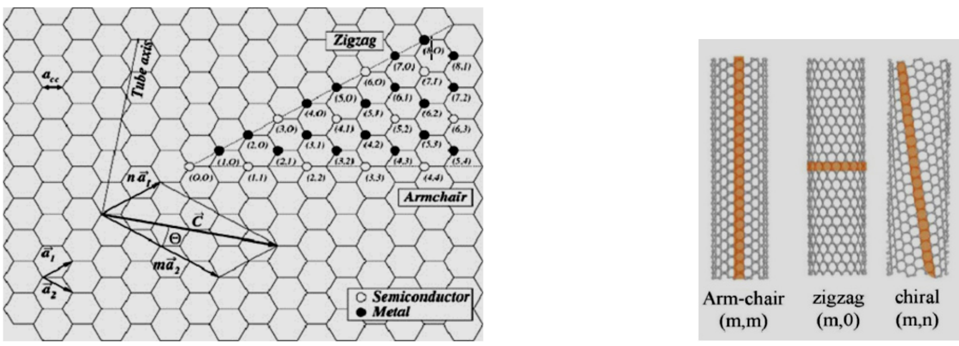
2. Structure of carbon nanotube

The SWCNTs are generated by rolling up a graphene sheet into a seamless cylinder. This structures may be described by the tube chirality, defined or represented through a pair of indices (n, m) referred to as the chiral vector $\vec{C}_h$, and the chiral angle θ (see Fig. 1(a)). The chiral vector $\vec{C}_h$ is defined as shown in the equation below

$$\vec{C}_h = n\vec{a}_1 + m\vec{a}_2 \tag{1}$$

Where: $\vec{a}_1$ and $\vec{a}_2$ are the unit cell vectors of the two-dimensional lattice formed by the grapheme sheets, *n* and *m* are two integers. The direction of the CNT axis is perpendicular to this chiral vector. The chiral angle "θ_c" expression is given in the function of the integers "n, m" as (Dresselhaus *et al*. 2003)

$$\theta_c = cos^{-1}\left(\frac{2n+m}{2\sqrt{n^2+nm+m^2}}\right);\ \theta_c = sin^{-1}\left(\frac{\sqrt{3}m}{2\sqrt{n^2+nm+m^2}}\right);\ \theta_c = tan^{-1}\left(\frac{\sqrt{3}m}{2n+m}\right) \tag{2}$$



(a) (b)

Fig. 1 (a) The 2D graphene sheet diagram showing a vector structure classification used to define CNT structure (Dresselhaus *et al*. 1996); (b) CNTs structure with different chiralities: armchair, zigzag and chiral

Depending on the chiral indices (n, m) and the chiral angles, CNTs can be classified to zigzag and armchair structures as shown in Fig. 1(b). For armchair CNTs, the chiral indices n and m are equal (n = m) and $\theta_c = 30°$ while for zigzag CNTs, m = 0 and $\theta_c = 0°$. For other values of indices (n $\neq$ m), CNTs are known as chiral and $0° < \theta_c < 30°$. The length of the unit vector a is defined as $a = \sqrt{3}a_{C-C}$ with the equilibrium carbon–carbon (C–C) covalent bond length a_{C-C} usually taken to be 0.1421 nm (Wilder *et al.* 1998). The nanotube radius "r" expression is given as

$$r = a\frac{\sqrt{n^2 + nm + m^2}}{2\pi} \tag{3}$$

3. Nonlocal Timoshenko elastic beam models of SWCNTs

In this article the Eringen model of nonlocal elasticity is adopted. In the theory of nonlocal elasticity (Eringen 1972 and 1983), the stress "σ" at a reference point "x' is supposed to be a functional of the strain accordance with atomic theory of lattice dynamics and experimental observations on phonon dispersion. In the limit when the effects of strains at points other than x are neglected, one obtains classical or local theory of elasticity. The basic equations for linear, homogeneous, isotropic, nonlocal elastic solid with zero body force are given by Eringen (1983)

$$\begin{aligned} &\sigma_{ij,j} = 0 \\ &\sigma_{ij}(x) = \int T(|x - x^{'}|, h)C_{ijkl}(x^{'})dV(x^{'}), \quad \forall x \in V \\ &\varepsilon_{ij} = \frac{1}{2}(u_{ij} + u_{ji}) \end{aligned} \tag{4}$$

where C_{ijkl} is the elastic modulus tensor of classical isotropic elasticity; σ_{ij} and ε_{ij} are stress and strain tensors respectively, and u_i is displacement vector. $T(|x - x'|, h)$ is the nonlocal modulus or attenuation function incorporating into the constitutive equations to characterize the nonlocal effects at the reference point x produced by local strain at the source $x^{'}$. $|x - x^{'}|$ is the Euclidean distance, and $h = \frac{e_0 a}{L}$ is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wavelength), where e_0 is a constant appropriate to each material, a is an internal characteristic length, e.g. length of C—C bond, lattice parameter, granular distance, and L is an external characteristic length. Nonlocal constitutive relations for present nanotubes /nanobeams can be written as

$$\sigma_x - (e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} = E\varepsilon_x \quad \Rightarrow \quad (1 - (e_0 a)^2 \nabla^2)\sigma_x = E\varepsilon_x \tag{5a}$$

$$\tau_{xz} - (e_0 a)^2 \frac{\partial^2 \tau_{xz}}{\partial x^2} = G\gamma_{xz} \quad \Rightarrow \quad (1 - (e_0 a)^2 \nabla^2)\tau_{xz} = G\gamma_{xz} \tag{5b}$$

where E is the Young's modulus *and* G is the shear modulus of the material, ∇^2 is Laplace operator. γ is the shear strain and σ_x is the axial stress. The parameter $e_0 a$ is the parameter leading to small scale effect on the response of structures in nanosize.

Based on the integral Timoshenko beam theory, the displacement field is given as

$$
\begin{aligned}
u(x,z,t) &= -zk\int \theta(x,t)dx \\
w(x,z,t) &= w_0(x,t)
\end{aligned}
\tag{6}
$$

Where θ and w_0 are the two unknowns' variables. The indeterminate integral appears in the above displacement field shall be resolved by the type of solutions used. k is the coefficient depends of the geometry.

The expressions of the axial and shear strains ε_x and γ_{xz} associated with the displacement field of Eq. (6) is obtained as

$$
\begin{aligned}
\varepsilon_x &= -zk\theta \\
\gamma_{xz} &= -kA'\frac{\partial\theta}{\partial x}+\frac{\partial w_0}{\partial x}
\end{aligned}
\tag{7}
$$

Where A' is coefficient defined via adopted solutions type. In this case $A' = -1/\lambda^2$.

The expressions of the shear force and bending moment M are given as

$$
\begin{aligned}
M &= \int z\sigma dA \\
V &= \int \tau dA
\end{aligned}
\tag{8}
$$

Using Eqs. (5), (7) and (8). The nonlocal shear force and bending moment are obtained as

$$M = (e_0a)^2\frac{\partial^2 M}{\partial x^2} - EIk\theta, \tag{9a}$$

$$V = (e_0a)^2\frac{\partial^2 V}{\partial x^2} + \beta GA\left(\frac{\partial w}{\partial x} - kA'\frac{\partial\theta}{\partial x}\right), \tag{9b}$$

Where A is the cross-section area of the beam. β is the form factor of shear depending on the shape of the cross section and $I = \int_A z^2\, dA$ is the inertia moment. The recommended value of β, the adjustment coefficient, is 0,9 for a circular shape of the cross area (Timoshenko 1921).

Based on the Hamilton's principle (Ebrahimi and Barati 2017, Fenjan *et al*. 2019a, b, Dehghan *et al*. 2019, Hamed *et al*. 2020, Attia and Mohamed 2020, Barati and Shahverdi 2020, Dehsaraji *et al*. 2020, She *et al*. 2020, Shariati *et al*. 2020, Mirjavadi *et al*. 2020b, Kachapi 2020, Khazaei and Mohammadimehr 2020, Bahaadini *et al*. 2020, Mohamed *et al*. 2020, Karami *et al*. 2021). The equations of motion of an integral Timoshenko beam theory under thermal effect are obtained as follow

$$-\frac{\partial V}{\partial x} + N_t\frac{\partial^2 w}{\partial x^2} = -\frac{\partial^2 w}{\partial t^2}\rho A \tag{10a}$$

$$-Mk + \frac{\partial V}{\partial x}kA' = \rho I k^2 A'^2\frac{\partial^4\theta}{\partial x^2\partial t^2} \tag{10b}$$

Where ρ is the mass density of the material, N_t denotes an additional axial force and is dependent on temperature θ_T and thermal coefficient α of the nanotube; the force can be expressed as

$$N_t = -\alpha EA\theta_T \tag{11}$$

Substituting Eq. (10) into Eq. (9) leads to

$$M = -EIk\theta + (e_0a)^2\left[\rho Ik^2A'^2\frac{\partial^6\theta}{\partial x^4\partial t^2} - \rho AkA'\frac{\partial^4 w}{\partial x^2\partial t^2} - N_t kA'\frac{\partial^4 w}{\partial x^4}\right] \tag{12a}$$

$$V = \beta GA\left(\frac{\partial w}{\partial x} - kA'\frac{\partial\theta}{\partial x}\right) + (e_0a)^2\left[\rho A\frac{\partial^3 w}{\partial x^2\partial t} + N_t\frac{\partial^3 w}{\partial x^3}\right] \tag{12b}$$

Replacing Eq. (12) into Eq. (10), the nonlocal equations of motion in function of displacement terms θ and w_0 is obtained as

$$EIk^2\theta + \beta GAkA'\left(\frac{\partial^2 w}{\partial x^2} - kA'\frac{\partial^2\theta}{\partial x^2}\right) = \rho Ik^2A'^2\left[\frac{\partial^4\theta}{\partial x^2\partial t^2} - (e_0a)^2\frac{\partial^6\theta}{\partial x^4\partial t^2}\right] \tag{13a}$$

$$\beta GA\left(-kA'\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\right) + N_t\left[\frac{\partial^2 w}{\partial x^2} - (e_0a)^2\frac{\partial^4 w}{\partial x^4}\right] = \rho A\left[-(e_0a)^2\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^2 w}{\partial t^2}\right] \tag{13b}$$

In the present work, the SWCNT is supposed to be simply supported in the end edges; the solutions of the above equation of motion (Eq. (13)) are resolved via Navier method (Zouatnia and Hadji 2019, Safa *et al*. 2019, Mohammadimehr *et al*. 2020, Yaghoobi and Taheri 2020). The displacement coefficient is expressed as

$$w(x,t) = \bar{W}e^{i\omega t}\,sin(\lambda_n x) \tag{14a}$$

$$\theta(x,t) = \bar{\theta}e^{i\omega t}\,sin(\lambda_n x), \tag{14b}$$

Where $\overline{W}$ and $\bar{\theta}$ arbitrary parameters to be determined, ω are is the eigenfrequency associated with N^{th} eigenmode. i is the imaginary unit and λ_n is the wave number with $\lambda_n = \frac{N\pi}{L}$.

Substituting the Navier Solutions of Eq. (14) into nonlocal equations of motion of Eq. (13), the correspondent frequency relation can be obtained as

$$(\omega_{NT})^2 = \frac{1}{2}\left(\alpha_n \pm \sqrt{\alpha_n^2 - 4\beta_n}\right) \tag{15}$$

Where

$$\alpha_n = \frac{N_x{\lambda_n}^2}{\rho A} + \frac{\beta IG{\lambda_n}^4A'^2 + \beta AG{\lambda_n}^2A'^2 + EI}{\rho IA'^2{\lambda_n}^2\left[1 + (e_0a)^2{\lambda_n}^2\right]} \tag{16a}$$

$$\beta_n = \frac{E\beta G}{\rho^2 A'^2\left[1+(e_0a)^2{\lambda_n}^2\right]^2} + N_t \frac{\left(EI+\beta AGA'^2{\lambda_n}^2\right)}{\rho^2 IA'^2{\lambda_n}^2\left[1+{\lambda_n}^2(e_0a)^2\right]} \tag{16b}$$

The Timoshenko beam model can be reduced to Euler – beam model by eliminating the effect of rotary inertia and the shear force. to get the local model just put $e_0 = 0$.

4. Results and discussions

Based on the formulations obtained above with the nonlocal integral Timoshenko beam model, the effect of small scale parameter, the chirality of SWCNTs, the vibrational mode number, the aspect ratio of SWCNTs and temperature changes on the thermal vibration properties of single-walled nanotubes are discussed here. The ratios of the results with nonlocal parameter and temperature change to those without nonlocal parameter or temperature change are, respectively, given by

$$\chi_N = \frac{\omega_{NT}}{\omega_{LT}}; \qquad \chi_{th} = \frac{\omega_{NT}}{\omega_{NT}^0} \tag{17}$$

where ω_{LT} and ω_{NT} are the frequency based on the local and nonlocal integral Timoshenko beam model including thermal effect and ω_{NT}^0 is the frequency based on the nonlocal integral Timoshenko beam model without thermal effect ($\theta_T = 0$). As previously mentioned Jiang *et al.* (2004) found that the coefficients of thermal expansion for CNTs are negative at low or room temperature and become positive at high temperature. Consequently, the value of the ratio χ was herein calculated for both cases of low and high temperatures. For the case of room or low temperature, we suppose $\alpha = -1.6 \times 10^{-6} K^{-1}$ (Yao and Han 2006) and for the case of high temperature, we suppose $\alpha = 1.1 \times 10^{-6} K^{-1}$ (Yao and Han 2006). The parameters used in calculations for the SWCNT are given as follows: the effective thickness of CNTs taken to be $0.34\,nm$ (Bao *et al.* 2004), the mass density $\rho = 2.3\,gcm^{-3}$, Poisson ratio is $\nu = 0.25$ and the shear module can be determined from the relation $G = \frac{0.5E}{(1+\nu)}$.

The Young's moduli used in this study of Armchair, Zigzag and Chiral SWCNTs are calculated by Bao *et al.* (2004) using MD approach. It was reported that the Young's modulus average values were 935.805 ± 0.618 GPa, 935.287 ± 2.887 GPa and 918.309 ± 10.392 GPa for Armchair, Zigzag and Chiral SWCNTs, respectively (see Table 1) and weakly affected by the tube chirality and radius. The results given by MD approach are in good agreement with the available experimental results (Salvetat *et al.* 1999, Frankland 2003).

On the other hand, parameter e_0 was given as 0.39 by Eringen (1983). A conservative estimate of the scaling effect parameter $0\,nm \le e_0a \le 2\,nm$ for SWCNTs is proposed by Wang *et al.* (2006) with nonlocal continuum models. The product e_0a instead of e_0 alone was evaluated by Wang *et al.* (2007) by using the asymptotic value of frequency of the DWCNTs $\omega = \frac{1}{e_0a}\sqrt{\frac{E}{\rho}}$ derived in the analysis of wave propagation via a nonlocal Timoshenko beam model, where ρ is the mass density of carbon nanotubes. On the basis of the frequency equation, Wang found the scale coefficient $e_0a < 2.1\,nm$ for a CNT by substituting the available experimental vibration frequency 0.1 THz (Krishnan *et al.* 1998) or $e_0a > 2.1\,nm$ supposing that the measured

Table 1 Values of Young's modulus for different tube radius and different chiralities using MD simulation by Bao *et al*. (2004)

(n, m)	Radius (nm)	Young's modulus E (GPa)
	Armchair	
(8, 8)	0.542	934.960
(10, 10)	0.678	935.470
(12, 12)	0.814	935.462
(14, 14)	0.949	935.454
(16, 16)	1.085	939.515
(18, 18)	1.220	934.727
(20, 20)	1.356	935.048
	Zigzag	
(14, 0)	0.548	939.032
(17, 0)	0.665	938.553
(21, 0)	0.822	936.936
(24, 0)	0.939	934.201
(28, 0)	1.096	932.626
(31, 0)	1.213	932.598
(35, 0)	1.370	933.061
	Chiral	
(12, 6)	0.525	927.671
(14, 6)	0.696	921.616
(16, 8)	0.828	928.013
(18, 9)	0.932	927.113
(20, 12)	1.096	904.353
(24, 11)	1.213	910.605
(30, 8)	1.358	908.792

frequency value for the CNT was greater than 10 THz according to the observation of Yoon *et al*. (2004). It is clear that a large range of values for the scale coefficient e_0a is possible due to different vibration frequencies. Therefore, in this study, the nonlocal parameters are values $0\ nm \leq e_0a \leq 5\ nm$ to investigate nonlocal effects on the responses of SWCNTs.

Fig. 2 shows the variation of the frequency ratio χ_N (i.e., ratio of the frequency ω_{NT} of the nonlocal integral Timoshenko beam to the corresponding local integral Timoshenko beam ω_{LT}) versus the length–to-diameter ratio $\frac{L}{d}$ for six vibration mode numbers (N) with scale parameter e_0a is $1\ nm$ without consideration of thermal effect ($\theta_T = 0$). It can be seen from Fig. 2 that the frequency ratios χ_N of armchair (8,8) carbon nanotube are smaller than unity for all modes at different length-to-diameter ratios. This means that the frequencies obtained by the nonlocal integral Timoshenko beam are smaller than those given by local integral Timoshenko beam, indicating that the inclusion of effects of small scale lead to a reduction in the vibration frequencies. This under prediction of frequency values is amplified for higher vibration modes and

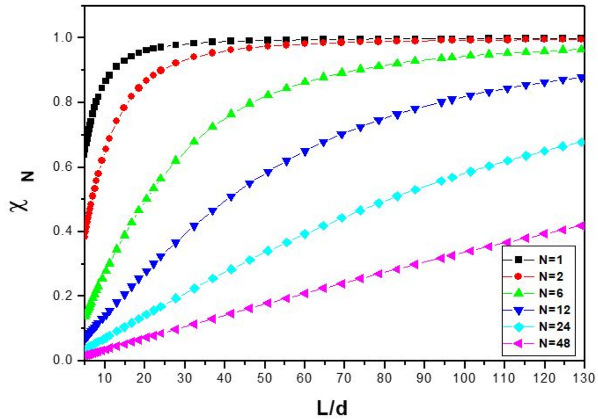


Fig. 2 The values of ratio χ_N of armchair (8,8) carbon nanotube with respect to length–to-diameter ratio $\frac{L}{d}$ for different mode numbers (N) in the case of without thermal effect ($\theta_T = 0$) with $e_0a = 1\ nm$

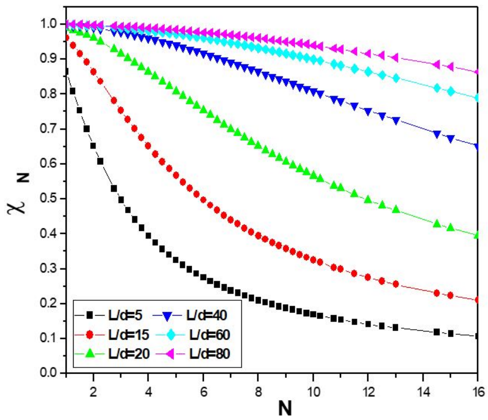


Fig. 3 Relation between the values of ratio χ_N and the mode numbers (N) for different aspect ratios $\frac{L}{d}$ of armchair (8,8) carbon nanotube in the case of without thermal effect ($\theta = 0$) With $e_0a = 1\ nm$

for small length-to-diameter ratios. This fact is clearly shown in Fig. 2. For the fundamental mode $N = 1$, the frequency ratio approaches unity (i.e., the nonlocal integral Timoshenko and local integral Timoshenko results become close to each other) for long tubes. For frequencies corresponding to higher modes, say $N = 48$, the frequency ratios are significantly smaller than unity, especially at small values of $\frac{L}{d}$.

For example, the frequency ratios at $\frac{L}{d} = 5$ are 0.6521, 0.1434 and 0.0362 for the N = 2, N = 12 and N = 48, respectively. With the increase of $\frac{L}{d}$ (i.e., $\frac{L}{d} > 80$), the effects of the small scale on the vibration frequencies become negligible as can be observed from the curves associated with the N = 1, N = 2 and N = 6 in Fig. 2. However, it is worth noting that even for slender *CNTs* with $\frac{L}{d}$ as large as 130, the small scale still have an appreciable effect on the frequency associated with high vibration modes (see results for the N = 24 and N = 48 in Fig. 2).

Fig. 3 illustrate relationship between the frequency ratios χ_N and vibration mode numbers (N) of armchair (8,8) carbon nanotube at different values length–to-diameter ratio $\frac{L}{d}$ in the case of without thermal effect ($\theta_T = 0$). It can be seen in Fig. 3 that the frequency ratios χ_N decreases with increasing the value N. In addition, we observe that increasing the ratio $\frac{L}{d}$ results in increasing in the ratio χ_N. It is also observed from Fig. 3 that the frequency ratios χ_N are less than unity. However, for the length-to-diameter ratios $L/d > 80$, the frequency ratio is seen to virtually approach unit. For short CNTs, say $L/d = 5$, the frequency ratios are smaller than unity especially at higher modes N. This result indicates that the effects of small scale lead to a reduction of the vibration frequencies and the reduction is amplified at higher vibration modes and for small length-to-diameter ratios. So, when the length-to-diameter ratios are small and when considering high vibration modes, nonlocal integral Timoshenko beam should be employed for a better prediction of the frequencies instead of local integral Timoshenko beam model which neglects the effects of small scale.

The relation between the values of ratio χ_N and the small-scale effect e_0a for different values of $\frac{L}{d}$ of zigzag (21,0) carbon nanotube with $N = 2$ and $\theta_T = 0$ is shown in Fig. 4. It is clear that the small scale effect is significant for short CNTs ($\frac{L}{d} \leq 15$). Conversely, for a slender CNT of L/d = 90, the frequency for the nonlocal integral Timoshenko beam is close to that furnished by the local beam model, indicating the negligible effect of small scale in long CNTs.

The variation in the ratios χ_N with the nonlocal parameter e_0a for various aspect ratios $\frac{L}{d}$is shown in Fig. 5. The obtained results show that the frequency ratios χ_N exhibit a dependence on the nonlocal parameter. For example, the frequency ratios at $\frac{L}{d} = 4$ are 0,9610; 0,8669; 0,6562 and 0,5710 for the $e_0a = 1$, $e_0a = 2$, $e_0a = 4$ and $e_0a = 5$, respectively. It is found from Fig. 5 that the vibrational characteristics for the SWCNT are related to the nonlocal parametere_0a. On the other hand, the effect of small scale parameter on the frequency ratio is seen to be very small and is negligible for $\frac{L}{d} > 50$ In addition, it is clearly seen from Figure that the frequency ratios χ_N are less than unity for different small-scale parameter. This means that the application of the

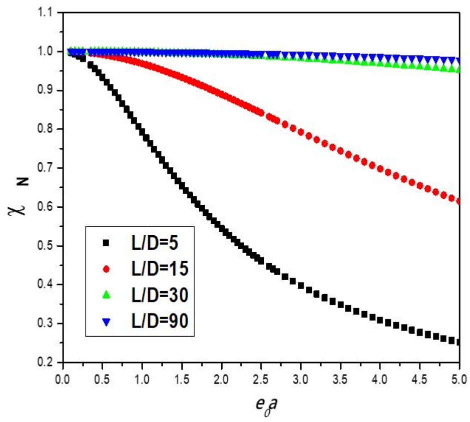


Fig. 4 The values of ratio χ_N of armchair (8,8) carbon nanotube with respect to length–to-diameter ratio $\frac{L}{d}$ for different mode numbers(N) in the case of without thermal effect ($\theta = 0$); with $e_0a = 1\ nm$

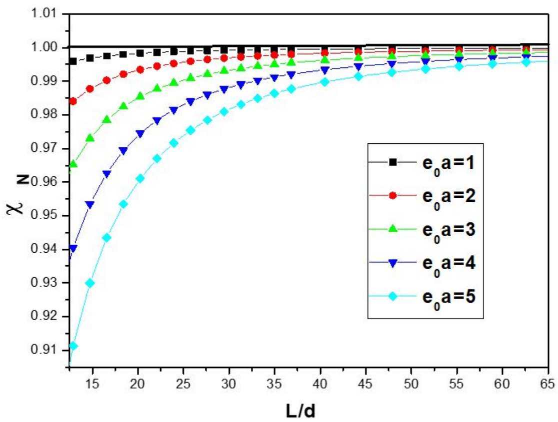


Fig. 5 Relation between the values of ratio χ_N and the aspect ratios $\frac{L}{d}$ for different small-scale effect e_0a of chiral (30,8) carbon nanotube in the case of without thermal effect ($\theta_T = 0$) with N = 1

local integral Timoshenko beam model for CNT analysis would lead to an overprediction of the frequency if the small length scale effect between the individual carbon atoms in CNTs is neglected.

The values of ratio χ_N of armchair (8,8) SWCNTs with different mode numbers N for $\frac{L}{d} = 20$ based on the non-local integral Timoshenko beam model are listed in Table 2. From the table, it can be noted that the ratio χ_N is in inverse relation with the mode numbers N and scale

Table 2 Values of frequency ratios χ_N for different mode numbers N, nonlocal parameter e_0a and aspect ratio $\frac{L}{d}$ of armchair (8,8) SWCNT

N	$\frac{L}{d} = 20$			$\frac{L}{d} = 40$		
	$e_0a = 1\,nm$	$e_0a = 1.5\,nm$	$e_0a = 2\,nm$	$e_0a = 1\,nm$	$e_0a = 1.5\,nm$	$e_0a = 2\,nm$
1	0,98957	0,97698	0,96014	0,99736	0,99409	0,98957
2	0,96014	0,91643	0,86418	0,98957	0,97698	0,96014
3	0,91643	0,83651	0,75318	0,97698	0,95033	0,91643
4	0,86418	0,75318	0,65148	0,96014	0,91643	0,86418
5	0,80853	0,67545	0,56624	0,93971	0,87774	0,80853
6	0,75318	0,60675	0,49683	0,91643	0,83651	0,75318
7	0,70044	0,5475	0,44052	0,89102	0,79455	0,70044
8	0,65148	0,49683	0,39453	0,86418	0,75318	0,65148
9	0,60675	0,45353	0,35657	0,83651	0,7133	0,60675
10	0,56624	0,4164	0,32486	0,80853	0,67545	0,56624
11	0,52971	0,38437	0,29807	0,78064	0,6399	0,52971
12	0,49683	0,35657	0,27519	0,75318	0,60675	0,49683
16	0,39453	0,27519	0,2099	0,65148	0,49683	0,39453

parameters e_0a. It can be concluded also that the biggest values of the ratio χ_N are obtained for slender armchair (8,8) SWCNT.

Figs. 6 and 7 are presented to show variations of the frequency ratio χ_N against the ratio $\frac{L}{d}$ at different chirality and different armchair of SWCNTs in the case ($\theta_T = 0$) with the nonlocal parameter $e_0a = 1nm$ and $N = 2$. However, for CNTs with larger values of diameter, this dependence becomes very weak. The reason for this phenomenon is that a carbon nanotube with smaller diameter d has a larger curvature, which results in a more significant distortion of $C - C$ bonds.

The relationship between the frequency ratios χ_N and the vibrational mode number N for different chirality of SWCNTs with $\frac{L}{d} = 10$, $e_0a = 1\,nm$ and $\theta_T = 0\,K$ are shown in Figs. 8 and 9. it is clearly seen in Fig. 9 that the distributions of ratio χ_N for different chiralities of

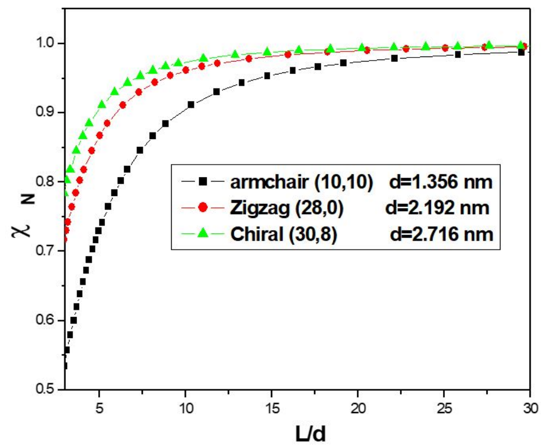


Fig. 6 The effect of aspect ratios $\frac{L}{d}$ on the frequency ratio χ_N for different chirality of SWCNTs with $N = 2$ and $e_0a = 1\,nm$ in the case ($\theta_T = 0$)

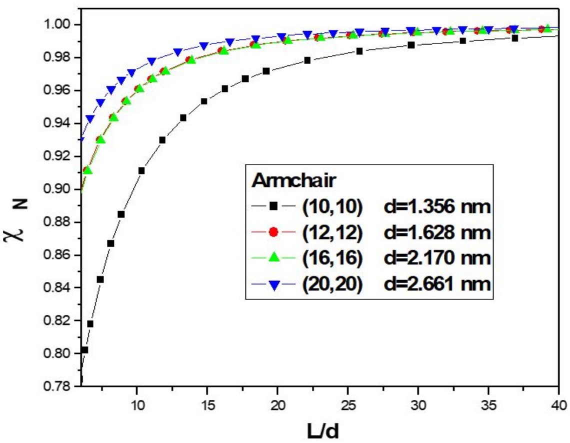


Fig. 7 The effect of aspect ratios $\frac{L}{d}$ on the frequency ratio χ_N for different armchair of SWCNTs with $N = 2$ and $e_0a = 1\,nm$ in the case ($\theta_T = 0$)

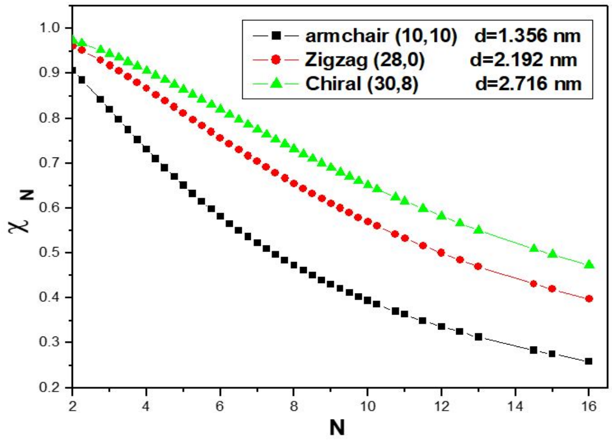


Fig. 8 The effect of vibrational mode number *N* on the frequency ratio χ_N for different chirality of SWCNTs with $N = 2$ and $e_0a = 1\ nm$ in the case ($\theta_T = 0$)

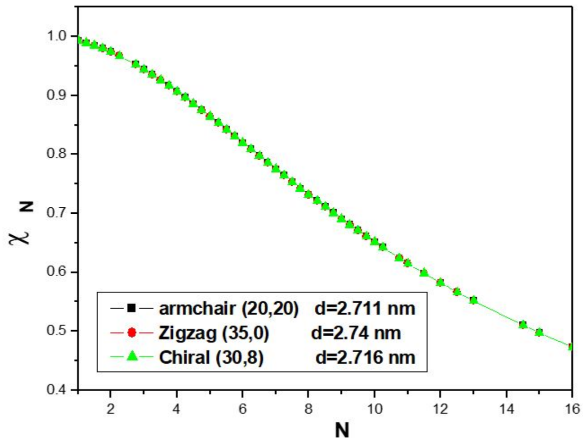


Fig. 9 The effect of vibrational mode number *N* on the frequency ratio χ_N for different chirality of SWCNTs with the same radius are identical, $N = 2$ and $e_0a = 1\ nm$ in the case ($\theta_T = 0$)

SWCNT with the same radius are identical. Comparing Figs. 8 and 9, the distributions of ratio χ_N are independent of the chirality vector of CNTs, but are dependent on the radius of CNTs.

The relationship among the ratios χ_{th} (i.e., ratio of the frequency ω_{th} of the nonlocal integral Timoshenko beam including thermal effect to the corresponding nonlocal integral Timoshenko beam without thermal effect ω_{th}^0), the vibrational mode number *N*, and the temperature change θ_T on the vibration frequencies of chiral (30.8) carbon nanotube for both cases of low and high temperatures are shown in Figs. 10 and 11. The relationship among the ratio χ_{th}, the aspect ratio $\frac{L}{d}$, and the temperature change θ are shown in Figs. 12 and 13.

In the case of room or low temperature, it is clearly seen from Figs. 10 and 12 that the frequency ratios χ_{th} are more than unity. This means that the values of frequencies ω_{NT} obtained by the nonlocal integral Timoshenko beam considering the thermal effect are larger than

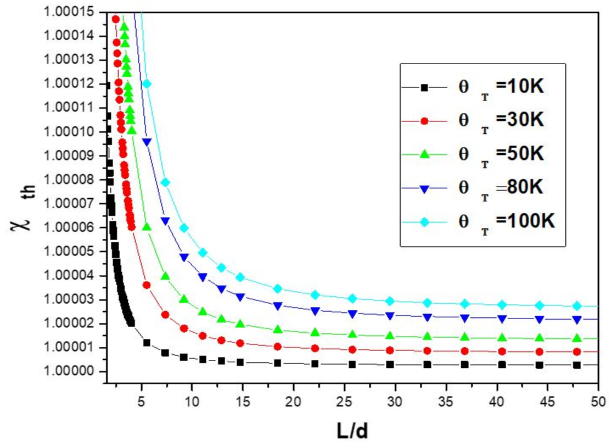


Fig. 10 Relation between the values of ratio χ_{th} and the with the aspect ratio $\frac{L}{d}$ for different various temperature θ_T of chiral (30.8) carbon nanotube in the case of low or room temperature. The value of $e_0 a = 3\,nm$ and $N = 3$

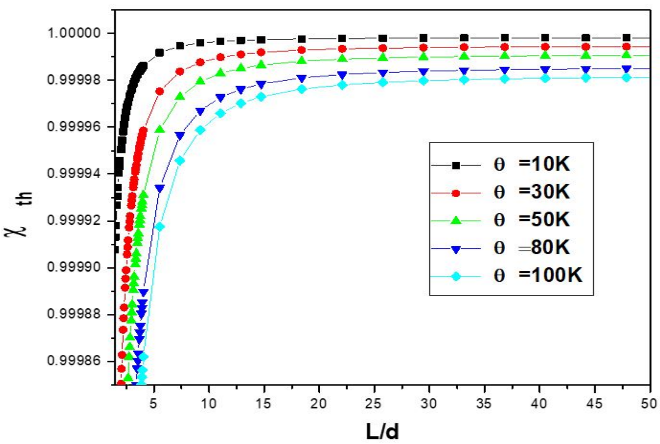


Fig. 11 Relation between the values of ratio χ_{th} and the with the aspect ratio $\frac{L}{d}$ for different various temperature θ_T of chiral (30.8) carbon nanotube in the case of high temperature. The value of $e_0 a = 3\,nm$ and $N = 3$

those given by local integral Timoshenko beam ignoring the influence of temperature change. Conversely in the case of high temperature, it can be seen from Figs. 11 and 13 that the frequency ratios χ_{th} are less than unity. This means that the values of frequencies ω_{NT} obtained by thenonlocal integral Timoshenko beam considering the thermal effect are smaller than those excluding the influence of temperature change. In addition, we observe that in the case of room or low temperature, the thermal effect on the frequency diminishes with increasing the aspect ratio $\frac{L}{d}$ and becomes more significant with the increase of the number *N* and temperature change θ_T. Conversely to the case of high temperature, the thermal effect on the frequency becomes less significant with the increase of the vibrational mode number N and increases with increasing the

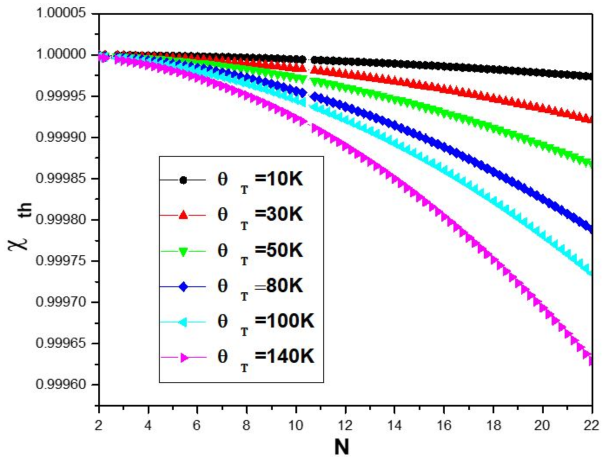


Fig. 12 Relation between the values of ratio χ_{th} and the with the aspect ratio N for different various temperature θ_T of chiral (30.8) carbon nanotube in the case of low or room temperature. With $e_0a = 3\ nm$ and $\frac{L}{d} = 20$

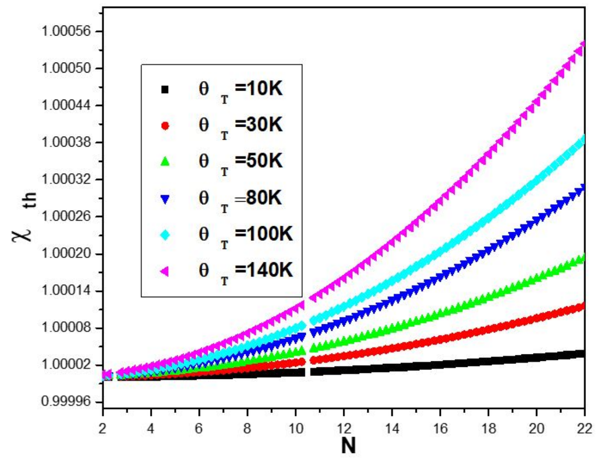


Fig. 13 Relation between the values of ratio χ_{th} and the with the aspect ratio N for different various temperature θ_T of chiral (30.8) carbon nanotube in the case of high temperature. With $e_0a = 3\ nm$ and $\frac{L}{d} = 20$

aspect ratio $\frac{L}{d}$ and temperature change θ_T.

In the case of room or low temperature, the relationship between the values of ratio χ_{th} and the temperature θ_T of chiral (30.8) carbon nanotube for different mode numbers (N) with $e_0a = 3nm$ and $\frac{L}{d} = 5$ is shown in Fig. 14, for different small-scale effect e_0a with $N = 3$ and $\frac{L}{d} = 5$ is presented in Fig. 15. From these figures, it can be clearly observed that frequency ratios χ_{th} vary linearly with the temperature change. It can be seen that the ratio χ_{th} increases monotonically as the temperature θ_T increases, indicating that the effect of temperature change leads to an increase of the fundamental frequency. It is clearly seen from these figures that the

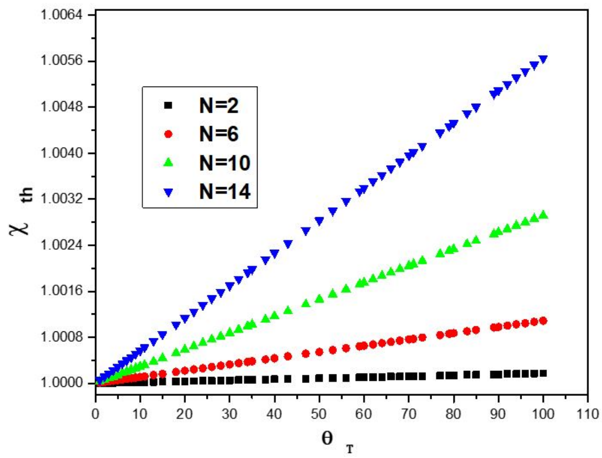


Fig. 14 Relationship between the values of ratio χ_{th} and the temperature θ_T for different mode numbers (N) of chiral (30.8) carbon nanotube in the case of low or room temperature. The value of $e_0a = 3\ nm$ and $\frac{L}{d} = 5$

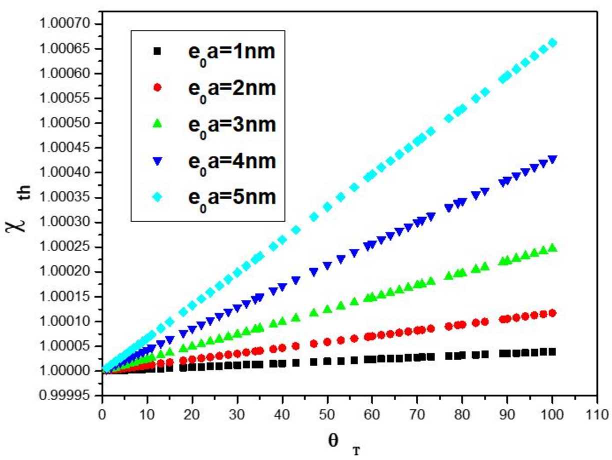


Fig. 15 Relationship between the values of ratio χ_{th} and the temperature θ_T for different small-scale effect e_0a of chiral (30.8) carbon nanotube in the case of low or room temperature. The value of $N = 3$ and $\frac{L}{d} = 5$

frequency ratios χ_{th} are greater than unity. Therefore, we observe that the frequency ratios χ_{th} becomes more significant with increasing the vibrational mode number *N* and the nonlocal parameter e_0a.

5. Conclusions

This work investigates the thermal vibration response of SWCNTs based on Eringen's nonlocal elasticity theory and the integral Timoshenko beam theory. Theoretical formulations include the shear deformation and nonlocal parameter effects. The nonlocal governing equations with appropriate boundary conditions for SWCNTs were derived and solved via Hamilton's principle and Navier's procedure. According to the study, the results showed the dependence of the frequency ratios on the chirality of carbon nanotube, the vibrational mode number, the aspect ratio, nonlocal parameter effect and the temperature change. From the results presented herein, it can be clearly seen that the small scale effect reduces of the vibrational characteristic and the reduction is amplified at higher values of vibration modes and for small length-to-diameter ratios. The influence of temperature change on the vibration frequencies of the single walled carbon nanotubes for both cases of low and high temperatures is also discussed. It was concluded that at low or room temperature the frequency ratios χ_{th} are more than unity, whereas the frequency ratios χ_{th} are less than unity for the case of high temperature. It can be detected that at low or room temperature, the frequency ratios for the SWCNT increase as the temperature change increases. Conversely to the case of high temperature, it can be noted that the thermal effect on the frequency becomes less significant with the increase temperature change. It is also shown that the values of frequency accounting for the thermal effect are larger than those ignoring the influence of temperature change for the case of room or low temperature, whereas the values of frequency with the thermal effect are smaller than those excluding the thermal effect for the case of high temperature. This work is expected to be useful in the design and analyze the thermal vibration properties of nano scale physical devices. The developed formulations can be extend to examine the response of the others type of materials (Kar *et al*. 2016, Katariya *et al*. 2017, Al-Maliki *et al*. 2020, Benferhat *et al*. 2020, Tayeb and Daouadji 2020, Ahmed *et al*. 2020b, Hadji and Bernard 2020, Rostami and Mohammadimehr 2020, Mahesh and Harursampath 2020, Civalek and Avcar 2020, Tayeb *et al*. 2020, Abed and Majeed 2020, Benferhat *et al*. 2021, Hashim and Sadiq 2021, Pourmoayed *et al*. 2021).

References

Abed, Z.A.K. and Majeed, W.I. (2020), "Effect of boundary conditions on harmonic response of laminated plates", *Compos. Mater. Eng., Int. J.*, **2**(2), 125-140. http://doi.org/10.12989/cme.2020.2.2.125

Abdulrazzaq, M.A., Fenjan, R.M., Ahmed, R.A. and Faleh, N.M. (2020), "Thermal buckling of nonlocal clamped exponentially graded plate according to a secant function based refined theory", *Steel Compos. Struct., Int. J.*, **35**(1), 147-157. https://doi.org/10.12989/scs.2020.35.1.147

Ahmed, R.A., Fenjan, R.M. and Faleh, N.M. (2019), "Analyzing post-buckling behavior of continuously graded FG nanobeams with geometrical imperfections", *Geomech. Eng., Int. J.*, **17**(2), 175-180. https://doi.org/10.12989/gae.2019.17.2.175

Ahmed, R.A., Moustafa, N.M., Faleh, N.M. and Fenjan, R.M. (2020a), "Nonlocal nonlinear stability of higher-order porous beams via Chebyshev-Ritz method", *Struct. Eng. Mech., Int. J.*, **76**(3), 413-420. https://doi.org/10.12989/sem.2020.76.3.413

Ahmed, R.A., Fenjan, R.M., Hamad, L.B. and Faleh, N.M. (2020b), "A review of effects of partial dynamic loading on dynamic response of nonlocal functionally graded material beams", *Adv. Mater. Res., Int. J.*, **9**(1), 33-48. https://doi.org/10.12989/amr.2020.9.1.033

Al-Maliki, A.F., Faleh, N.M. and Alasadi, A.A. (2019), "Finite element formulation and vibration of nonlocal refined metal foam beams with symmetric and non-symmetric porosities", *Struct. Monit. Maint., Int. J.*, **6**(2), 147-159. https://doi.org/10.12989/smm.2019.6.2.147

Al-Maliki, A.F.H., Ahmed, R.A., Moustafa, N.M. and Faleh, N.M. (2020), "Finite element based modeling and thermal dynamic analysis of functionally graded graphene reinforced beams", *Adv. Computat. Des., Int. J.*, **5**(2), 177-193. https://doi.org/10.12989/acd.2020.5.2.177

Arefi, M. and Żur, K.K. (2020), "Free vibration analysis of functionally graded cylindrical nanoshells resting on Pasternak foundation based on two-dimensional analysis", *Steel Compos. Struct., Int. J.*, **34**(4), 615-623. https://doi.org/10.12989/scs.2020.34.4.615

Asrari, R., Ebrahimi, F. and Kheirikhah, M.M. (2020), "On post-buckling characteristics of functionally graded smart magneto-electro-elastic nanoscale shells", *Adv. Nano Res., Int. J.*, **9**(1), 33-45. https://doi.org/10.12989/anr.2020.9.1.033

Attia, M.A. and Rahman, A.A.A. (2018), "On vibrations of functionally graded viscoelastic nanobeams with surface effects", *Int. J. Eng. Sci.*, **127**, 1-32. https://doi.org/10.1016/j.ijengsci.2018.02.005

Attia, M.A. and Mohamed, S.A. (2020), "Nonlinear thermal buckling and postbuckling analysis of bidirectional functionally graded tapered microbeams based on Reddy beam theory", *Eng. Comput.* https://doi.org/10.1007/s00366-020-01080-1

Attia, M.A., Shanab, R.A., Mohamed, S.A. and Mohamed, N.A. (2019), "Surface energy effects on the nonlinear free vibration of functionally graded Timoshenko nanobeams based on modified couple stress theory", *Int. J. Struct. Stabil. Dyn.*, **19**(11), 1950127. https://doi.org/10.1142/s021945541950127x

Bahaadini, R., Hosseini, M. and Amiri, M. (2020), "Dynamic stability of viscoelastic nanotubes conveying pulsating magnetic nanoflow under magnetic field", *Eng. Comput.*, **37**(4), 2877-2889. https://doi.org/10.1007/s00366-020-00980-6

Bao, W.X., Zhu, C.C. and Cui, W.Z. (2004), "Simulation of Young's modulus of single-walled carbon nanotubes by molecular dynamics", *Physica B: Condensed Matter*, **352**(1-4), 156-163. https://doi.org/10.1016/j.physb.2004.07.005

Barati, M.R. and Shahverdi, H. (2020), "Finite element forced vibration analysis of refined shear deformable nanocomposite graphene platelet-reinforced beams", *J. Braz. Soc. Mech. Sci. Eng.*, **42**, 33. https://doi.org/10.1007/s40430-019-2118-8

Belmahi, S., Zidour, M. and Meradjah, M. (2019), "Small-scale effect on the forced vibration of a nano beam embedded an elastic medium using nonlocal elasticity theory", *Adv. Aircr. Spacecr. Sci., Int. J.*, **6**(1), 1-18. https://doi.org/10.12989/aas.2019.6.1.001

Benferhat, R., Daouadji, T.H. and Rabahi, A. (2020), "Predictions of the maximum plate end stresses of imperfect FRP strengthened RC beams: study and analysis", *Adv. Mater. Res., Int. J.*, **9**(4), 265-287. https://doi.org/10.12989/amr.2020.9.4.265

Benferhat, R., Daouadji, T.H. and Rabahi, A. (2021), "Effect of air bubbles in concrete on the mechanical behavior of RC beams strengthened in flexion by externally bonded FRP plates under uniformly distributed loading", *Compos. Mater. Eng.*, **3**(1), 41-55. https://doi.org/10.12989/cme.2021.3.1.041

Bensattalah, T., Bouakkaz, K., Zidour, M. and Daouadji, T.H. (2018), "Critical buckling loads of carbon nanotube embedded in Kerr's medium", *Adv. Nano Res., Int. J.*, **6**(4), 339-356. https://doi.org/10.12989/anr.2018.6.4.339

Bensattalah, T., Zidour, M. and Daouadji, T.H. (2019a), "A new nonlocal beam model for free vibration analysis of chiral single-walled carbon nanotubes", *Compos. Mater. Eng., Int. J.*, **1**(1), 21-31. https://doi.org/10.12989/cme.2019.1.1.021

Bensattalah, T., Zidour, M., Daouadji, T.H. and Bouakaz, K. (2019b), "Theoretical analysis of chirality and scale effects on critical buckling load of zigzag triple walled carbon nanotubes under axial compression embedded in polymeric matrix", *Struct. Eng. Mech., Int. J.*, **70**(3), 269-277. https://doi.org/10.12989/sem.2019.70.3.269

Bensattalah, T., Hamidi, A., Bouakkaz, K., Zidour, M. and Daouadji, T.H. (2020), "Critical Buckling Load of Triple-Walled Carbon Nanotube Based on Nonlocal Elasticity Theory", *J. Nano Res.*, **62**, 108-119. https://doi.org/10.4028/www.scientific.net/jnanor.62.108

Civalek, Ö. and Avcar, M. (2020), "Free vibration and buckling analyses of CNT reinforced laminated non-rectangular plates by discrete singular convolution method", *Eng. Comput.* https://doi.org/10.1007/s00366-020-01168-8

Civalek, Ö., Dastjerdi, S., Akbaş, S.D. and Akgöz, B. (2021), "Vibration analysis of carbon nanotube-reinforced composite microbeams", *Mathe. Methods Appl. Sci.* https://doi.org/10.1002/mma.7069

Dai, H., Hafner, J.H., Rinzler, A.G., Colbert, D.T. and Smalley, R.E. (1996), "Nanotubes as nanoprobes in scanning probe microscopy", *Nature*, **384**(6605), 147-150. https://doi.org/10.1038/384147a0

Dehghan, M., Ebrahimi, F. and Vinyas, M. (2019), "Wave dispersion characteristics of fluid-conveying magneto-electro-elastic nanotubes", *Eng. Comput.*, **36**, 1687-1703. https://doi.org/10.1007/s00366-019-00790-5

Dehsaraji, M.L., Arefi, M. and Loghman, A. (2020), "Three dimensional free vibration analysis of functionally graded nano cylindrical shell considering thickness stretching effect", *Steel Compos. Struct., Int. J.*, **34**(5), 657-670. https://doi.org/10.12989/SCS.2020.34.5.657

Dharap, P., Li, Z., Nagarajaiah, S. and Barrera, E.V. (2004), "Nanotube film based on single-wall carbon nanotubes for strain sensing", *Nanotechnology*, **15**(3), 379-382. https://doi.org/10.1088/0957-4484/15/3/026

Dresselhaus, M.S., Dresselhaus, G. and Eklund, P.C. (1996), *Science of Fullerenes and Carbon Nanotubes*, Academic Press, New York, USA.

Dresselhaus, M.S., Lin, Y.M., Rabin, O., Jorio, A., Souza Filho, A.G., Pimenta, M.A., Saito, R., Samsonidze, G. and Dresselhaus, G. (2003), "Nanowires and nanotubes", *Mater. Sci. Eng.: C*, **23**(1-2), 129-140. https://doi.org/10.1016/s0928-4931(02)00240-0

Ebrahimi, F. and Barati, M.R. (2017), "Buckling analysis of nonlocal strain gradient axially functionally graded nanobeams resting on variable elastic medium", *Proceedings of the Institution of Mechanical Engineers, Part C: J. Mech. Eng. Sci.*, **232**(11), 2067-2078. https://doi.org/10.1177/0954406217713518

Ebrahimi, F. and Barati, M.R. (2019), "A nonlocal strain gradient mass sensor based on vibrating hygro-thermally affected graphene nanosheets", *Iran J. Sci. Technol. Trans. Mech. Eng.*, **43**, 205-220. https://doi.org/10.1007/s40997-017-0131-z

Eltaher, M.A., Agwa, M. and Kabeel, A. (2018), "Vibration analysis of material size-dependent CNTs using energy equivalent model", *J. Appl. Computat. Mech.*, **4**(2), 75-86. https://doi.org/10.22055/JACM.2017.22579.1136

Eltaher, M.A., Mohamed, N., Mohamed, S. and Seddek, L.F. (2019a), "Postbuckling of Curved Carbon Nanotubes Using Energy Equivalent Model", *J. Nano Res.*, **57**, 136-157. https://doi.org/10.4028/www.scientific.net/jnanor.57.136

Eltaher, M.A., Almalki, T.A., Almitani, K.H., Ahmed, K.I.E. and Abdraboh, A.M. (2019b), "Modal participation of fixed–fixed single-walled carbon nanotube with vacancies", *Int. J. Adv. Struct. Eng.*, **11**(2), 151-163. https://doi.org/10.1007/s40091-019-0222-8

Eltaher, M.A., Almalki, T.A., Almitani, K. and Ahmed, K.I. (2019c), "Participation Factor and Vibration of Carbon Nanotube with Vacancies", *J. Nano Res.*, **57**, 158-174. https://doi.org/10.4028/www.scientific.net/jnanor.57.158

Eltaher, M.A., Omar, F.-A., Abdalla, W.S., Kabeel, A.M. and Alshorbagy, A.E. (2020), "Mechanical analysis of cutout piezoelectric nonlocal nanobeam including surface energy effects", *Struct. Eng. Mech., Int. J.*, **76**(1), 141-151. https://doi.org/10.12989/sem.2020.76.1.141

Eringen, A.C. (1972), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**(1), 1-16. https://doi.org/10.1016/0020-7225(72)90070-5

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710. https://doi.org/10.1063/1.332803

Fenjan, R.M., Ahmed, R.A., Alasadi, A.A. and Faleh, N.M. (2019a), "Nonlocal strain gradient thermal vibration analysis of double-coupled metal foam plate system with uniform and non-uniform porosities", *Coupl. Syst. Mech., Int. J.*, **8**(3), 247-257. https://doi.org/10.12989/csm.2019.8.3.247

Fenjan, R.M., Ahmed, R.A. and Faleh, N.M. (2019b), "Investigating dynamic stability of metal foam nanoplates under periodic in-plane loads via a three-unknown plate theory", *Adv. Aircr. Spacecr. Sci., Int. J.*, **6**(4), 297-314. https://doi.org/10.12989/aas.2019.6.4.297

Fenjan, R.M., Moustafa, N.M. and Faleh, N.M. (2020), "Scale-dependent thermal vibration analysis of FG beams having porosities based on DQM", *Adv. Nano Res., Int. J.*, **8**(4), 283-292. https://doi.org/10.12989/anr.2020.8.4.283

Frankland, S. (2003), "The stress–strain behavior of polymer–nanotube composites from molecular dynamics simulation", *Compos. Sci. Technol.*, **63**(11), 1655-1661. https://doi.org/10.1016/s0266-3538(03)00059-9

Gafour, Y., Hamidi, A., Benahmed, A., Zidour, M. and Bensattalah, T. (2020), "Porosity-dependent free vibration analysis of FG nanobeam using non-local shear deformation and energy principle", *Adv. Nano Res., Int. J.*, **8**(1), 37-47. https://doi.org/10.12989/anr.2020.8.1.037

Ghandourh, E.E. and Abdraboh, A.M. (2020), "Dynamic analysis of functionally graded nonlocal nanobeam with different porosity models", *Steel Compos. Struct., Int. J.*, **36**(3), 293-305. https://doi.org/10.12989/scs.2020.36.3.293

Hadji, L. and Bernard, F. (2020), "Bending and free vibration analysis of functionally graded beams on elastic foundations with analytical validation", *Adv. Mater. Res., Int. J.*, **9**(1), 63-98. https://doi.org/10.12989/amr.2020.9.1.063

Hamad, L.B., Khalaf, B.S. and Faleh, N.M. (2019), "Analysis of static and dynamic characteristics of strain gradient shell structures made of porous nano-crystalline materials", *Adv. Mater. Res., Int. J.*, **8**(3), 179-196. https://doi.org/10.12989/amr.2019.8.3.179

Hamed, M.A., Mohamed, S.A. and Eltaher, M.A. (2020), "Buckling analysis of sandwich beam rested on elastic foundation and subjected to varying axial in-plane loads", *Steel Compos. Struct., Int. J.*, **34**(1), 75-89. https://doi.org/10.12989/scs.2020.34.1.075

Hamidi, A., Zidour, M., Bouakkaz, K. and Bensattalah, T. (2018), "Thermal and Small-Scale Effects on Vibration of Embedded Armchair Single-Walled Carbon Nanotubes", *J. Nano Res.*, **51**, 24-38. https://doi.org/10.4028/www.scientific.net/JNanoR.51.24

Hashim, H.A. and Sadiq, I.A. (2021), "A five-variable refined plate theory for thermal buckling analysis of composite plates", *Compos. Mater. Eng., Int. J.*, **3**(2), 135-155. https://doi.org/10.12989/cme.2021.3.2.135

Hernandez, E., Goze, C., Bernier, P. and Rubio, A. (1998), "Elastic Properties of C and $B_xC_yN_z$ Composite Nanotubes", *Phys. Rev. Lett.*, **80**, 4502. https://doi.org/10.1103/PhysRevLett.80.4502

Iijima, S. (1991), "Helical microtubules of graphitic carbon", *Nature*, **354**(6348), 56-58. https://doi.org/10.1038/354056a0

Jalaei, M.H. and Civalek, Ö. (2019), "On dynamic instability of magnetically embedded viscoelastic porous FG nanobeam", *Int. J. Eng. Sci.*, **143**, 14-32. https://doi.org/10.1016/j.ijengsci.2019.06.013

Jiang, H., Liu, B., Huang, Y. and Hwang, K.C. (2004), "Thermal expansion of single wall carbon nanotubes", *J. Eng. Mater. Technol.*, **126**(3), 265. https://doi.org/10.1115/1.1752925

Kachapi, S.H.H. (2020), "Nonlinear and nonclassical vibration analysis of double walled piezoelectric cylindrical nanoshell", *Adv. Nano Res., Int. J.*, **9**(4), 277-294. https://doi.org/10.12989/anr.2020.9.4.277

Kar, V.R., Panda, S.K. and Mahapatra, T.R. (2016), "Thermal buckling behaviour of shear deformable functionally graded single/doubly curved shell panel with TD and TID properties", *Adv. Mater. Res., Int. J.*, **5**(4), 205-221. https://doi.org/10.12989/amr.2016.5.4.205

Karami, B. and Janghorban, M. (2019b), "A new size-dependent shear deformation theory for free vibration analysis of functionally graded/anisotropic nanobeams", *Thin-Wall. Struct.*, **143**, 106227. https://doi.org/10.1016/j.tws.2019.106227

Karami, B. and Janghorban, M. (2019a), "On the dynamics of porous nanotubes with variable material properties and variable thickness", *Int. J. Eng. Sci.*, **136**, 53-66. https://doi.org/10.1016/j.ijengsci.2019.01.002

Karami, B., Shahsavari, D., Nazemosadat, S.M.R., Li, L. and Ebrahimi, A. (2018a), "Thermal buckling of smart porous functionally graded nanobeam rested on Kerr foundation", *Steel Compos. Struct., Int. J.*, **29**(3), 349-362. https://doi.org/10.12989/scs.2018.29.3.349

Karami, B., Shahsavari, D., Li, L., Karami, M. and Janghorban, M. (2018b), "Thermal buckling of embedded sandwich piezoelectric nanoplates with functionally graded core by a nonlocal second-order shear deformation theory", *Proceedings of the Institution of Mechanical Engineers, Part C: J. Mech. Eng. Sci.*, **233**(1), 287-301. https://doi.org/10.1177/0954406218756451

Karami, B., Shahsavari, D., Janghorban, M. and Li, L. (2021), "Free vibration analysis of FG nanoplate with poriferous imperfection in hygrothermal environment", *Struct. Eng. Mech., Int. J.*, **73**(2), 191-207. https://doi.org/10.12989/sem.2020.73.2.191

Katariya, P.V., Panda, S.K. and Mahapatra, T.R. (2017), "Nonlinear thermal buckling behaviour of laminated composite panel structure including the stretching effect and higher-order finite element", *Adv.*

Mater. *Res*., **6**(4), 349-361. https://doi.org/10.12989/amr.2017.6.4.349
Khazaei, P. and Mohammadimehr, M. (2020), "Vibration analysis of porous nanocomposite viscoelastic plate reinforced by FG-SWCNTs based on a nonlocal strain gradient theory", *Comput*. *Concrete*, *Int*. *J*., **26**(1), 31-52. https://doi.org/10.12989/cac.2020.26.1.031
Krishnan, A., Dujardin, E., Ebbesen, T.W., Yianilos, P.N. and Treacy, M.M.J. (1998), "Young's modulus of single-walled nanotubes", *Phys*. *Rev*. *B*, **58**(20), 14013-14019. https://doi.org/10.1103/physrevb.58.14013
Mahesh, V. and Harursampath, D. (2020), "Nonlinear vibration of functionally graded magneto-electro-elastic higher order plates reinforced by CNTs using FEM", *Eng*. *Comput*. https://doi.org/10.1007/s00366-020-01098-5
Mirjavadi, S.S., Forsat, M., Nia, A.F., Badnava, S. and Hamouda, A.M.S. (2020a), "Nonlocal strain gradient effects on forced vibrations of porous FG cylindrical nanoshells", *Adv*. *Nano Res*., *Int*. *J*., **8**(2), 149-156. https://doi.org/10.12989/anr.2020.8.2.149
Mirjavadi, S.S., Nikookar, M., Mollaee, S., Forsat, M., Barati, M.R. and Hamouda, A.M.S. (2020b), "Analyzing exact nonlinear forced vibrations of two-phase magneto-electro-elastic nanobeams under an elliptic-type force", *Adv*. *Nano Res*., *Int*. *J*., **9**(1), 47-58. https://doi.org/10.12989/anr.2020.9.1.047
Mohamed, N., Mohamed, S.A. and Eltaher, M.A. (2020), "Buckling and post-buckling behaviors of higher order carbon nanotubes using energy-equivalent model", *Eng*. *Comput*., **37**(4), 2823-2836. https://doi.org/10.1007/s00366-020-00976-2
Mohammadimehr, M., Firouzeh, S., Pahlavanzadeh, M., Heidari, Y. and Irani-Rahaghi, M. (2020), "Free vibration of sandwich micro-beam with porous foam core, GPL layers and piezo-magneto-electric facesheets via NSGT", *Comput*. *Concrete*, *Int*. *J*., **26**(1), 75-94. https://doi.org/10.12989/cac.2020.26.1.075
Pourmoayed, A., Fard, K.M. and Rousta, B. (2021), "Free vibration analysis of sandwich structures reinforced by functionally graded carbon nanotubes", *Compos*. *Mater*. *Eng*., *Int*. *J*., **3**(1), 1-23. https://doi.org/10.12989/cme.2021.3.1.001
Robertson, J. (2004), "Realistic applications of CNTs", *Materials Today*, **7**(10), 46-52. https://doi.org/10.1016/s1369-7021(04)00448-1
Rostami, R. and Mohammadimehr, M. (2020), "Vibration control of rotating sandwich cylindrical shell-reinforced nanocomposite face sheet and porous core integrated with functionally graded magneto-electro-elastic layers", *Eng*. *Comput*. https://doi.org/10.1007/s00366-020-01052-5
Safa, A., Hadji, L., Bourada, M. and Zouatnia, N. (2019), "Thermal vibration analysis of FGM beams using an efficient shear deformation beam theory", *Earthq*. *Struct*., *Int*. *J*., **17**(3), 329-336. https://doi.org/10.12989/eas.2019.17.3.329
Salvetat, J.-P., Bonard, J.-M., Thomson, N.H., Kulik, A.J., Forró, L., Benoit, W. and Zuppiroli, L. (1999), "Mechanical properties of carbon nanotubes", *Appl*. *Phys*. *A*: *Mater*. *Sci*. *Process*., **69**(3), 255-260. https://doi.org/10.1007/s003390050999
Sedighi, H.M. and Yaghootian, A. (2016), "Dynamic instability of vibrating carbon nanotubes near small layers of graphite sheets based on nonlocal continuum elasticity", *J*. *Appl*. *Mech*. *Tech*. *Phys*., **57**(1), 90-100. https://doi.org/10.1134/s0021894416010107
Shanab, R.A., Attia, M.A., Mohamed, S.A. and Mohamed, N.A. (2020), "Effect of microstructure and surface energy on the static and dynamic characteristics of FG Timoshenko nanobeam embedded in an elastic medium", *J*. *Nano Res*., **61**, 97-117. https://doi.org/10.4028/www.scientific.net/jnanor.61.97
Shariati, A., Barati, M. R., Ebrahimi, F., Singhal, A. and Toghroli, A. (2020), "Investigating vibrational behavior of graphene sheets under linearly varying in-plane bending load based on the nonlocal strain gradient theory", *Adv*. *Nano Res*., *Int*. *J*., **8**(4), 265-276. https://doi.org/10.12989/anr.2020.8.4.265
She, G.-L., Liu, H.-B. and Karami, B. (2020), "On resonance behavior of porous FG curved nanobeams", *Steel Compos*. *Struct*., *Int*. *J*., **36**(2), 179-186. https://doi.org/10.12989/scs.2020.36.2.179
Tadmor, E.B., Smith, G.S., Bernstein, N. and Kaxiras, E. (1999), "Mixed finite element and atomistic formulation for complex crystals", *Phys*. *Rev*. *B*, **59**(1), 235-245. https://doi.org/10.1103/physrevb.59.235
Tahouneh, V., Naei, M.H. and Mashhadi, M.M. (2020), "Influence of vacancy defects on vibration analysis of graphene sheets applying isogeometric method: Molecular and continuum approaches", *Steel Compos*.

Struct., *Int*. *J*., **34**(2), 261-277. https://doi.org/10.12989/scs.2020.34.2.261
Tayeb, B. and Daouadji, T.H. (2020), "Improved analytical solution for slip and interfacial stress in composite steel-concrete beam bonded with an adhesive", *Adv*. *Mater*. *Res*., *Int*. *J*., **9**(2), 133-153. https://doi.org/10.12989/amr.2020.9.2.133
Tayeb, T.S., Zidour, M., Bensattalah, T., Heireche, H., Benahmed, A. and Bedia, E.A. (2020), "Mechanical buckling of FG-CNTs reinforced composite plate with parabolic distribution using Hamilton's energy principle", *Adv*. *Nano Res*., *Int*. *J*., **8**(2), 135-148. https://doi.org/10.12989/anr.2020.8.2.135
Tersoff, J. and Ruoff, R.S. (1994), "Structural properties of a carbon-nanotube crystal", *Phys*. *Rev*. *Lett*., **73**(5), 676-679. https://doi.org/10.1103/physrevlett.73.676
Thanh, C.L., Nguyen, T.N., Vu, T.H., Khatir, S. and Abdel Wahab, M. (2020), "A geometrically nonlinear size-dependent hypothesis for porous functionally graded micro-plate", *Eng*. *Comput*. https://doi.org/10.1007/s00366-020-01154-0
Timesli, A. (2020), "Buckling analysis of double walled carbon nanotubes embedded in Kerr elastic medium under axial compression using the nonlocal Donnell shell theory", *Adv*. *Nano Res*., *Int*. *J*., **9**(2), 69-82. https://doi.org/10.12989/anr.2020.9.2.069
Timoshenko, S.P. (1921), "LXVI. On the correction for shear of the differential equation for transverse vibrations of prismatic bars", *The London*, *Edinburgh*, *and Dublin Philosophical Magazine and Journal of Science*, **41**(245), 744-746. https://doi.org/10.1080/14786442108636264
Tombler, T.W., Zhou, C., Alexseyev, L., Kong, J., Dai, H., Liu, L. and Wu, S.-Y. (2000), "Reversible electromechanical characteristics of carbon nanotubes underlocal-probe manipulation", *Nature*, **405**(6788), 769-772. https://doi.org/10.1038/35015519
Treacy, M.M.J., Ebbesen, T.W. and Gibson, J.M. (1996), "Exceptionally high Young's modulus observed for individual carbon nanotubes", *Nature*, **381**(6584), 678-680. https://doi.org/10.1038/381678a0
Wang, Q. (2005), "Wave propagation in carbon nanotubes via nonlocal continuum mechanics", *J*. *Appl*. *Phys*., **98**(12), 124301. https://doi.org/10.1063/1.2141648
Wang, Q., Varadan, V.K. and Quek, S.T. (2006), "Small scale effect on elastic buckling of carbon nanotubes with nonlocal continuum models", *Phys*. *Lett*. *A*, **357**(2), 130-135. https://doi.org/10.1016/j.physleta.2006.04.026.
Wang, C.M., Zhang, Y.Y. and He, X.Q. (2007), "Vibration of nonlocal Timoshenko beams", *Nanotechnology*, **18**(10), 105401. https://doi.org/10.1088/0957-4484/18/10/105401
Wilder, J.W.G., Venema, L.C., Rinzler, A.G., Smalley, R.E. and Dekker, C. (1998), "Electronic structure of atomically resolved carbon nanotubes", *Nature*, **391**(6662), 59-62. https://doi.org/10.1038/34139
Yaghoobi, H. and Taheri, F. (2020), "Analytical solution and statistical analysis of buckling capacity of sandwich plates with uniform and non-uniform porous-cellular core reinforced with graphene nanoplatelets", *Compos*. *Struct*., **252**, 112700. https://doi.org/10.1016/j.compstruct.2020.112700
Yao, X. and Han, Q. (2006), "Buckling analysis of multiwalled carbon nanotubes under torsional load coupling with temperature change", *J*. *Eng*. *Mater*. *Technol*., **128**(3), 419. https://doi.org/10.1115/1.2203102
Ye, L.H., Liu, B.G. and Wang, D.S. (2001), "Ab initio molecular dynamics study on small carbon nanotubes", *Chinese Phys*. *Lett*., **18**(11), 1496-1499. https://doi.org/10.1088/0256-307x/18/11/323
Yoon, J., Ru, C.Q. and Mioduchowski, A. (2004), "Timoshenko-beam effects on transverse wave propagation in carbon nanotubes", *Compos*. *Part B*: *Eng*., **35**(2), 87-93. https://doi.org/10.1016/j.compositesb.2003.09.002
Yuan, Y., Zhao, K., Zhao, Y. and Kiani, K. (2020), "Nonlocal-integro-vibro analysis of vertically aligned monolayered nonuniform FGM nanorods", *Steel Compos*. *Struct*., *Int*. *J*., **37**(5), 551-569. https://doi.org/10.12989/scs.2020.37.5.551
Zouatnia, N. and Hadji, L. (2019), "Effect of the micromechanical models on the bending of FGM beam using a new hyperbolic shear deformation theory", *Earthq*. *Struct*., *Int*. *J*., **16**(2), 177-183. https://doi.org/10.12989/eas.2019.16.2.177

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