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Simplified numerical method for nonlocal static and dynamic analysis of a graphene nanoplate

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Abstract. In this paper, the dynamic analysis of single-layer rectangular armchair graphene nanoplates has been considered. The theory of nonlocal elasticity for small scale effects and the Kirchhoff's theory for plates have been used to obtain the dynamic equation of graphene nanoplates. Discrete Least Squares Meshless (DLSM) method has been used to examine the response of single-layer graphene nanoplates with various boundary conditions. The validation of the results has also been carried out using dynamic analysis of single-layer rectangular armchair graphene nanoplates under stationary loads. Results revealed that DLSM method is an efficient mean to solve the problems of structural mechanics in nano-dimensions. In addition, in nanostructures, the small scale effects have considerable impacts that should be considered as well. An increase in nonlocal coefficient increases the deflection. Higher nonlocal coefficient leads to higher deflection intensity and vibration amplitude.

Keywords: dynamic analysis; graphene nanoplate; nonlocal elasticity theory; Discrete Least Squares Meshless (DLSM) method

1. Introduction

Nanotechnology is a modern and very functional science in all fields of engineering so that it has been significantly developed in recent decades. Nanostructured materials have interesting physical, electrical, and chemical properties.

Graphene plates are nano-layers which are created through putting carbon atoms together on a plate and in a crystal lattice with hexagonal structures. Due to the strong covalent force between carbon atoms, despite very low thickness, it has a tensile strength 200 times greater than steel.

Carbon nanoplates are divided into two categories of single-layer and multi-layer; in the multi-layer graphene nanoplates, there is a weak van der Waals bond force between each layer.

Single-layer plates are superior to graphene layers on many cases and applications, but in cases where the purpose is to increase the flexural strength, graphene multi-layers can be used. In terms of rolling, graphene nanoplates are divided into three categories of armchair, zigzag and chiral (asymmetrical) each of which gives unique structural properties to these graphene plates.

Graphene plates have many applications in different types of nano-actuators, nano-sensors,

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electrical batteries, nano-micro-electromechanical systems, atomic force microscopes, and high strength composites (Geim and Novoselov 2007, Pumera *et al*. 2010, Geim 2009, Katsnelson 2007, Rao *et al*. 2009, Heyrovska 2008, Choi *et al*. 2010, Geim and Kim 2008, Chen *et al*. 2008, Kuzmenko *et al*. 2008).

Interesting properties and characteristics of the graphene nanoplates have attracted the researchers. Researchers have tried to understand these properties with various theoretical and experimental methods. However, experimental investigations have many problems such as hard work, being time consuming and costly, but the theoretical and numerical methods have led to saving time and cost, and favorable results (Martel *et al*. 1998).

To study the mechanical properties of graphene plates, and in general, investigating materials on a small scale, the theory of classic continuous environments cannot take into consideration the empty spaces between atoms and atomic forces among particles at nanodimensions, and that is why this theory is not efficient to be used for studying the mechanical properties of the nanoplates.

In large structures, the strain creates stress in a point, but in nano-structures, the resultant stress depends on the strain field of the space around that point. Therefore, other methods must be used, of which, the experimental observations and molecular dynamics method can be mentioned (Eringen 1972, 1983, 2002, Eringen and Edelen 1972). But these methods are so costly, time consuming, and limited to a few number of atoms in the structure. Thus, for larger systems, different theories have been proposed, including the theories of strain gradient (Hutchinson and Fleck 1997), modified couple stress (Yang *et al*. 2002), micropolar (Wang *et al*. 2006) and nonlocal elasticity (Eringen 1983).

In the nonlocal theory of elasticity, the internal characteristic length (the way of bonding in the molecular network, particle size, etc.) and the external characteristic length (crack length, wavelength, etc.) of the nanostructure can be introduced in the equations. This method give rises to acceptable results which are close to atomic and molecular dynamics method (Wang *et al*. 2006, Wang and Varadan 2006).

Among the studies conducted on graphene plates, Ansari *et al*. (2010b), presented the vibration analysis of single-layer graphene plates using the equation of nonlocal elasticity, theory of classic plate, and considering the fourth order general differential equations. They studied the effects of surface stress on free vibration behavior of nanoplates, and concluded that for non-classic plate models, the effects of surface stress can be ignored for lower values of mode number and higher aspect ratios of dimensions.

Pradhan and Phadikar (2009) obtained equations of the theory of nonlocal elasticity using the theory of classical plates and the first order shear deformation theory, and obtained vibration of graphene plates. Bouadi *et al*. (2018) studied the new nonlocal higher order shear deformation for analysis of stability of single layer grapheme sheet by employing the nonlocal differential constitutive relations of Eringen.

Liu *et al*. (2016) presented the nonlocal static bending, buckling, free and forced vibrations of graphene nanosheets using the Kirchhoff plate theory and Taylor expansion approach. Ansari *et al*. (2010a) studied the vibration of multi-layer graphene plates with different boundary conditions in an elastic material, using the finite element method. Recently, several studies have been carried out on bending, buckling, and vibration of single-layer and multi-layer graphene plates by Sobhy (2014a, b), Alzahrani *et al*. (2013).

Fattahi *et al*. (2019) reviewed the application of nonlocal elasticity to determin vibration behavior of functionally graded nanoplate. Using the fourth order differential method, the elastic buckling behaviors of small-scale orthotropic plates were studied under biaxial stress by Murmu and Pradhan

(2009a ,b). Malekzadeh *et al*. (2011a, b), have studied the vibrational behavior and temperature buckling of arbitrary quadrilateral orthotropic nano-plates. Sakhaee-Pour *et al*. (2008) modeled vibrational behavior of zigzag and armchair single-layer graphene plates with various boundary conditions using molecular structural mechanics.

Duan and Wang (2009) reviewed the nonlinear bending and stretching of a circular graphene under a point load at the center, using simulation of molecular mechanics. Aghababaei and Reddy (2009) formulated the third order shear deformation of plate theory for the problem of bending and vibration using the theory of Eringen nonlocal elasticity.

Using nonlocal continuum mechanics, Babaei and Shahidi (2011) studied the elastic buckling behavior of quadrilateral single-layer graphene plates under biaxial pressure using Galerkin method. Wang *et al*. (2016) evaluated the nonlocal effect on the nonlinear dynamic characteristics of buckled parametric double-layered nanoplates.

Jomehzadeh and Saidi (2011) analyzed the three-dimensional vibration of graphene plates by decoupling the nonlocal elasticity equations. Wang *et al*. (2015b) investigated the homoclinic behaviors and chaotic motions of double layered viscoelastic nanoplates based on nonlocal theory and extended Melnikov method.

Murmu and Pradhan (2009a) employed nonlocal Timoshenko beam theory and the fourth-order differential method for stability analysis of monolayer carbon nanotubes embedded in the elastic medium.

Peddieson *et al*. (2003) studied the small-scale effects on nanoscale structures by using the nonlocal elasticity theory. They showed that the nonlocal continuum mechanics has a high potential to be used in nanotechnology applications. In another work, Behfar *et al*. (2006) examined and calculated the flexural modulus of two-layer graphene plates using analytical methods.

Sakhaee-Pour (2009) studied the elastic properties of single-layer graphene plates. Using analytical methods, he could obtain the Young's modulus, shear modulus, and Poisson’s ratio for various arrangements of carbon atoms in these plates. They showed that the effect of small scale is quite scientific and should be considered.

Reddy (2010) obtained nonlinear bending for nanotubes using beam relations; and also for orthotropic plates using the classic theory and the theory of first order shear and considering the nonlocal effects. Shen *et al*. (2010) investigated nonlinear vibration of single-layer graphene placed on the thermal environment for orthotropic rectangular plates with simple boundary conditions. In another work, Shen (2011) conducted a nonlinear analysis on thin films located on the elastic foundations and in thermal environment, with plates’ nonlocal model.

Pouresmaeeli *et al*. (2012) evaluated free vibrations of rectangular two-layer nano-plates placed in polymer environment. Dehshahri *et al*. (2020) analyzed the free vibrations of the nanoplates made of three-directional functionally graded material and reviewed small-scale effects on natural frequency.

Using semi-Galerkin technique, Ghannadpour and Moradi (2019) have conducted nonlocal nonlinear analysis of nano-graphene sheets under compression. Wang *et al*. (2019) studied the nonlocal nonlinear chaotic and homoclinic analysis of double layered forced viscoelastic nanoplates.

Farajpour *et al*. (2012) investigated the buckling of graphene single-layer plates under a variety of in-plane linear loads by nonlocal theory and squared differential method. To validate the results, they also solved the equations using power series and showed that the results of the two methods are in good agreement.

Zhang *et al*. (2015) investigated and analyzed the vibration of single-layer graphene plate based on continuous nonlocal model, using meshless method. Arash and Wang (2011) analyzed the vibration of single-layer and multi-layer graphene plates using analytical method. Using Galerkin meshless analysis, Naderi and Baradaran (2013) performed the static analysis of microscale nanoplates based on the theory of nonlocal plate with various boundary conditions. Alwar and Reddy (1979) carried out the static and dynamic analyses of plates with great displacements.

Rouhi *et al*. (2016) studied the effects of size on the free vibration analysis of nanoshells based on flat stress elasticity with various boundary conditions, and evaluated the effects of properties associated with surface. They concluded that the flat stress has a significant effect on frequency intensification of the nano-shells.

Using different time scale methods, Wu and Li (2017) conducted three-dimensional vibration analyses on single-layer graphene nanoplates and graphene plates surrounded by a material. Wang *et al*. (2015a) have conducted a nonlinear vibrational analysis on two-layer nanoplates with different boundary conditions.

Meshless discrete least squares method is also a new method based on moving least square (MLS) method; one of the advantages of which is elimination of integration. Point weighted discrete least squares meshless method was proposed by Firoozjaee and Afshar to solve the equations of elliptic type, and the impact of point weights was studied in improving the accuracy and convergence rate (2009). Using discrete squares meshless (DLSM) method, Ebrahimi and Firoozjaee (2020) reviewed numerical solution of bed load transport equations.

The main motivation of this study is to use numerical method DLSM for the first time to know Static and dynamic behavior of graphene nanoplates

The main motivation of this study is to use DLSM method for the first time to know the dynamic behavior of graphene nanoplates. For dynamic analysis of armchair graphene nanoplates, the theory of nonlocal elasticity was used in this study, assuming that the thickness of the nanoplates is very insignificant compared to other dimensions, and it is also assumed that displacements are small, and hence, the Kirchhoff's theory has been used to create dynamic equation of nanoplates; moreover, because of the difficulty of solving mechanical problems of nanostructures with complex boundary conditions, the numerical method of discrete meshless least squares has been applied using moving least square approximation to approximate shape functions; this method has not been used so far for mechanical analysis of nano-structures, but it has been used successfully here; the results of which may help well to understand the functional properties of this structure.

2. Theory of nonlocal elasticity

In the theory of nonlocal elasticity, stress at a point x of a continuous body is a function of strain in all x' points of the body (Eringen 1983), so nonlocal stress can be expressed as

$$\sigma_{ij} = \int_V \lambda(|x - x'|) C_{ijkl} \varepsilon_{kl} dv(x') \tag{1}$$

where σ_{ij} is the nonlocal stress, λ is the kernel function describing strain effects in x' for stress given in x. ε_{kl} and C_{ijkl} are the strain and elasticity tensor, respectively.

Using the Eq. (1) in the equation of motion, we have

$$\sigma_{ij,j} + f_i = \rho \frac{d^2 u_i}{dt^2} \tag{2}$$

f_i, u_i, t and ρ are body force, displacement component, time and mass density, respectively. Nonlocal constitutive relation is expressed as

$$\Gamma\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{3}$$

where Γ is the derivative operator. Therefore, the equation of motion can be written as follows.

$$(C_{ijkl}\varepsilon_{kl}) + \Gamma(f_i - \rho\frac{d^2u_i}{dt^2}) = 0 \tag{4}$$

Eq. (4) is a partial differential equation which is a simple form of partial integral equations, so, this form is used in nonlocal theory of elasticity.

In two-dimensional problems, we have

$$\Gamma = 1 - \mu\nabla^2 \tag{5}$$

Where

$$\lambda(|x|) = (2\pi L^2\tau^2)^{-1}k_0(|x|/L\tau) \qquad \tau = e_0a_0/L \tag{6}$$

where

$$\int_V \lambda(|x|)dv = 1 \tag{7}$$

∇^2 is the Laplacian operator, k_0 is Bessel function, a_0 and L are the internal and external characteristic lengths, respectively, and $\mu = (e_0a_0)^2$ is given where e_0 is estimated by the models of elasticity theory.

Using Eqs. (3) to (7), the differential form of the nonlocal elasticity takes the form

$$\left(1 - \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{8}$$

which finally leads to the following expression (Analooei *et al*. 2013, Bouadi *et al*. 2018)

$$\left(1 - \mu\nabla^2\right)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{9}$$

$$(1 - (e_0a_0)^2\nabla^2)\sigma_{ij} = \sigma^L \tag{10}$$

3. Equilibrium equation

Assuming a thin orthotropic rectangular plate with length of a, width of b, thickness of h, and vertical load of $p(x,y)$, according to Kirchhoff plate theory, the strain-displacement relationship of the plate in Cartesian coordinate system is written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -zw_{,xx} \\ -zw_{,yy} \\ -2zw_{,xy} \end{Bmatrix} \tag{11}$$

where $w$is the vertical displacement of the mid-plate, and z is the axis in the direction of the plate thickness. Equations pertaining to an orthotropic plate are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} - (e_0 a_0)^2 \nabla^2 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{12}$$

where $\bar{Q}_{\eta}(i, j = 1,2,6)$ (Naderi and Baradaran 2013) are

$$\begin{Bmatrix} \bar{Q}_{11} \\ \bar{Q}_{12} \\ \bar{Q}_{16} \\ \bar{Q}_{22} \\ \bar{Q}_{26} \\ \bar{Q}_{66} \end{Bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2 \\ c^2s^2 & c^4+s^4 & c^2s^2 & -4c^2s^2 \\ c^3s & cs^3-c^3s & -cs^3 & -2cs(c^2-s^2) \\ s^4 & c^2s^2 & c^4 & 4c^2s^2 \\ cs^3 & c^3s-cs^3 & -c^3s & 2cs(c^2-s^2) \\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2-s^2)^2 \end{bmatrix} \begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \end{Bmatrix} \tag{13}$$

and $s = sin\,\theta$, $c = cos\,\theta$ where θ is the off-axis angle which is 0° for armchair graphene nanoplates. Q_{ij} can be expressed in terms of engineering constants as

$$Q_{11} = \frac{E_1}{(1\text{-v}_{12}v_{21})}, \quad Q_{22} = \frac{E_2}{(1\text{-v}_{12}v_{21})}, \quad Q_{12} = \frac{v_{12}E_1}{(1\text{-v}_{12}v_{21})}, \quad Q_{66} = G_{12} \tag{14}$$

G_{12}, v_{12}, E_1, E_2 are respectively shear modulus, Poisson’s ratio, modulus of elasticity in the directions of x and y. Using the principle of virtual work, the plate equilibrium equation is obtained as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + p(x,y) = \rho h \ddot{w} - \frac{\rho h^3}{12}(\ddot{w}_{,xx} + \ddot{w}_{,yy}) \tag{15}$$

$\ddot{w}$ is the second derivative with respect to time.

Here, nonlocal moments M_x, M_y, and M_{xy} are satisfying

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} - (e_0 a_0)^2 \nabla^2 \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} M_x^L \\ M_y^L \\ M_{xy}^L \end{Bmatrix} \tag{16}$$

where M_x^L, M_y^L, and M_{xy}^L are the results of local moments and are defined as

$$\begin{Bmatrix} M_x^L \\ M_y^L \\ M_{xy}^L \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} W_{,xx} \\ W_{,yy} \\ W_{,xy} \end{Bmatrix} \tag{17}$$

where $D_{ij} = \int_{-h/2}^{h/2} z^2 \bar{Q}_{ij} dz (i, j = 1,2,6)$.

Using Eqs. (15)-(16), we have

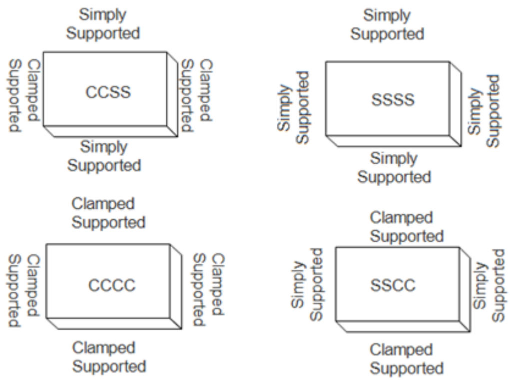


Fig. 1 Different boundary conditions of graphene nanoplates

$$\frac{\partial^2 M_x^L}{\partial x^2} + 2\frac{\partial^2 M_{xy}^L}{\partial x \partial y} + \frac{\partial^2 M_y^L}{\partial y^2} = (1-(e_0 a_0)^2 \nabla^2)\left[-p(x,y,t) + \rho h \ddot{w} - \frac{\rho h^3}{12}\left(\frac{\partial^2 \ddot{w}}{\partial x^2} + \frac{\partial^2 \ddot{w}}{\partial y^2}\right)\right] \tag{18}$$

Assuming small displacements, according to Kirchhoff's plate theory, $\frac{\partial^2 \ddot{w}}{\partial x^2}$, $\frac{\partial^2 \ddot{w}}{\partial y^2}$ and higher order derivatives can be ignored (Love 1888), and Eq. (18) is rewritten as

$$\frac{\partial^2 M_x^L}{\partial x^2} + 2\frac{\partial^2 M_{xy}^L}{\partial x \partial y} + \frac{\partial^2 M_y^L}{\partial y^2} = (1-(e_0 a_0)^2 \nabla^2)[-p(x,y,t)] + \rho h \ddot{w} \tag{19}$$

Boundary conditions on each side of the plate are expressed as

$$\begin{array}{lcl} Simply\ Supported & : & W = M_n = 0 \\ Clamped\ Supported & : & W = W_{,n} = 0 \end{array} \tag{20}$$

Here, n is the normal vector in boundary (Lu *et al*. 2007).

So, for dynamic analysis of plates with various boundary conditions, using DLSM (Discrete least squares meshless) method, Eq. (17) and Eq. (19) will be solved by combining the boundary conditions (20) according to Fig. 1.

4. Moving Least Square (MLS) shape functions

In discrete least squares meshless method, these functions are produced using MLS. Approximation using MLS has two prominent features causing its popularity: (1) the approximate field function is continuous and soft in the whole range of the problem; and (2) it has the ability to approximate with an arbitrary order of adaptation (GR 2003) where $\phi(X)$ is the approximated function at point $X = [x, y]$

$$\phi(X) = \sum_{i=1}^{mp} p_i(X) a_i(X) = P^T(X) a(X) \tag{21}$$

$p_i(X)$ is the vector of polynomial basic functions, and mp is the number of terms in the basic functions, and $a(X)$ is the vector of coefficients. First and second order polynomial basic functions in one-dimensional and two-dimensional modes are as follows: the first order basic function in one-dimensional mode is $P^T = [1 \quad x]$, the first order basic function in two-dimensional mode is $P^T = [1 \quad x \quad y]$, the second order basic function in one-dimensional mode is $P^T = [1 \quad x \quad x^2]$, and the second order basic function in two-dimensional mode is $P^T = [1 \quad x \quad y \quad x^2 \quad xy \quad y^2]$, in which x and y indicate the first and second components of the spatial coordinates, respectively. It should be noted that $a(X)$ in Eq. (21) is a function of X and is obtained using minimization of function $J(X)$, which has been shown as

$$J(X) = \sum_{j=1}^{n} w(X - X_j)(P^T(X_j) a(X) - \phi_j^h)^2 \tag{22}$$

$w(X - X_j)$ is the weight function, which is of the characteristics of the meshless method. Cubic spline weight function has been used here

$$w(X - X_j) = w(\bar{d}) = \begin{cases} 2/3 - 4\bar{d}^2 + 4\bar{d}^3 & \bar{d} \le 1/2 \\ 4/3 - 4\bar{d} + 4\bar{d}^2 - 4/3\bar{d}^3 & 1/2 \le \bar{d} \le 1 \\ 0 & \bar{d} > 1 \end{cases} \tag{23}$$

$\bar{d} = |X - X_j|/dw_j$ and dw_j are the radius of the impact of point X_j. Minimizing the J function, we have

$$\phi(X) = P^T(X) A^{-1}(X) B(X) \varphi^h \tag{24}$$

where φ^h is the nodal parameters vector, and $A(X)$,$B(X)$ is as follows

$$A(X) = \sum_{j=1}^{n} w(X - X_j)\, P(X_j) P^T(X_j) \tag{25}$$

$$\begin{aligned} B(X) = [&w(X - X_1)P(X_1), \\ &w(X - X_2)P(X_2), \dots, w(X - X_n)P(X_n)] \end{aligned} \tag{26}$$

According to Eq. (24), it can be written as follows

$$\phi(X) = N^T(X) \varphi^h \tag{27}$$

$$N^T(X) = P^T(X) A^{-1}(X) B(X) \tag{28}$$

Where, $N^T(X)$ represents the nodal shape function vector at point X, which is called the MLS shape function.

5. Discrete least squares meshless (DLSM) method

To provide a method of DLSM, the general form of the following differential equations is used to create the system of discrete equations. Partial differential equations and their boundary conditions are written as follows

$$\Im(\phi) + f = 0 \qquad in \quad \Omega \tag{29}$$

$$\phi - \bar{\phi} = 0 \qquad in \quad \Gamma_2 \tag{30}$$

$$\aleph(\phi) + \bar{t} = 0 \qquad in \quad \Gamma_1 \tag{31}$$

Ω is the problem domain, Γ_1 and Γ_2 are Dirichlet and Neumann boundary conditions, respectively, $\Im$ and $\aleph$are partial differential operators, $f$shows the external forces or the source term in the problem domain. The residue of the differential equation at a regular point of K is as follows

$$R_\Omega(X_k) = \Im(\phi(X_k)) + f(X_k) = \sum_{j=1}^{n} \Im(N_j(X_k))\phi_j + f\,(X_k) \tag{32}$$

$$K = 1, \ldots, m \tag{33}$$

The residue of Neumann boundary conditions at a regular point of K can be written as follows

$$R_1(X_k) = \aleph(\phi) - \bar{t}(X_k) = \sum_{j=1}^{n} \aleph(N_j(X_k))\phi_j - \bar{t}(X_k) \tag{34}$$

$$K = 1, \ldots, m_1 \tag{35}$$

and finally, the residue of Dirichlet boundary conditions at a regular point of K can be expressed as

$$R_2(X_k) = \phi - \bar{\phi}(X_k) = \sum_{j=1}^{n} N_j(X_k)\phi_j - \bar{\phi}(X_k) \tag{36}$$

$$K = 1, \ldots, m_2 \tag{37}$$

n is the number of nodal points, m is the number of sampling points in the domain, while m_1 and m_2 represent the number of sample points chosen on Dirichlet and Neumann boundaries. m_1 and m_2 are typically independent of the number of sample points of m. However, the higher number of these sample points, creates a better match to satisfy the boundary conditions. A penalty method is used in order to create the whole residue of the problem as follows (Firoozjaee and Afshar 2009, Ebrahimi and Firoozjaee 2020)

$$I = \sum_{k=1}^{m} R_{\Omega}^{2}\,(X_k) + \alpha_1 \sum_{k=1}^{m_1} R_1^2\,(X_k) + \alpha_2 \sum_{k=1}^{m_2} R_2^2\,(X_k) \tag{38}$$

Where α_1 and α_2 are the penalty coefficients for the Dirichlet and Neumann boundaries, respectively. Combining the above relations, the following relation is obtained

$$\begin{aligned} I = \sum_{k=1}^{m}\left(\sum_{j=1}^{n} \Im(N_j(X_k))\phi_j + f\,(X_k)\right)^2 + \alpha_1 \sum_{k=1}^{m_1}\left(\sum_{j=1}^{n} \aleph(N_j(X_k))\phi_j - \bar{t}(X_k)\right)^2 \\ +\alpha_2 \sum_{k=1}^{m_2}\left(\sum_{j=1}^{n} N_j(X_k)\phi_j - \bar{\phi}(X_k)\right)^2 \end{aligned} \tag{39}$$

Minimizing the function (39), and taking into account the nodal parameters of $(\phi_j,\ j = 1,2,3,\dots,n)$, and substituting in the Eq. (40), Eq. (41) and Eq. (42) are obtained.

$$k\phi = F \tag{40}$$

$$\begin{aligned} k_{ij} = \sum_{k=1}^{m} \Im(N_i(X_k))\,\Im(N_j(X_k)) + \alpha_1 \sum_{k=1}^{m_1} \aleph(N_i(X_k))\,\aleph(N_j(X_k)) \\ +\alpha_2 \sum_{k=1}^{m_2} N_i(X_k) N_j(X_k) \end{aligned} \tag{41}$$

$$F_i = -\sum_{k=1}^{m} \Im(N_i(X_k))\,f(X_k) + \alpha_1 \sum_{k=1}^{m_1} \aleph(N_i(X_k))\bar{t}(X_k) + \alpha_2 \sum_{k=1}^{m_2} N_i(X_k)\,\bar{\phi}(X_k) \tag{42}$$

The stiffness matrix of k is symmetric and can be easily used to solve the problem (Firoozjaee and Afshar 2009, Ebrahimi and Firoozjaee 2020).

Now, the mass matrix is obtained using the following equation (GR 2003)

$$MM_{ij} = \int_{Ar} (\rho N_i(X_k) N_j(X_k) h + N_{i,x}(X_k) N_{j,x}(X_k)\varpi + N_{i,y}(X_k) N_{j,y}(X_k)\varpi)\, dA_p \tag{43}$$

where A_p is the plate surface area, and ϖ is as

$$\varpi = \frac{\rho h^3}{12} \tag{44}$$

and using mass and stiffness matrices regardless of damping, and employing the Newmark method, we have addressed the dynamic analysis of graphene nanoplates using the DLSM method.

6. Dynamic analysis of graphene nanoplate under static loading

In this section, the meshless DLSM method is implemented for dynamic analysis of single-layer rectangular armchair graphene nanoplates with various boundary conditions under static loading. The exact analytical deflection of a thin graphene plate with dimensions of *a* and *b* and thickness of h under static loading is compared with the deflections from DLSM numerical method. Comparison revealed reliable results for DLSM method of the dynamic analysis.

Let

$$p(x, y, t) = p_{\lambda\mu}\, sin(\zeta_\lambda x)\, sin(\eta_\mu y) \tag{45}$$

where, $\zeta_\lambda = \pi/a$ and $\eta_\mu = \pi/b$.

As given in Eq. (45), the loading on the structure is of trigonometric type. This type of loading which is frequently used in dynamic analysis of structures, can be employed for defining any arbitrary loading pattern. Any arbitrary loading can be expressed in terms of Fourier expressions. Thus, the selected type of loading is of general type and application.

For a simply supported nanoplate, the accurate deflection equation is obtained using Eqs. (17)-(18) with respect to the static equations ($\ddot{w} = 0$) as follows (Naderi and Baradaran 2013)

$$w^{exact}(x, y) = \frac{(1 + (e_0 a_0)^2(\zeta_\lambda^2 + \eta_\mu^2))}{(D_{11}\zeta_\lambda^4 + 2(D_{12} + 2D_{66})\zeta_\lambda^2\eta_\mu^2 + D_{22}\eta_\mu^4} \times P_{\lambda\mu}\, sin(\zeta_\lambda x)\, sin(\eta_\mu y) \tag{46}$$

Specifications necessary to solve the problem of single-layer rectangular armchair graphene nanoplate dynamically and statically under static loading (45) with various boundary conditions, are: (Naderi and Baradaran 2013)

$$a = 9.519nm, \quad b = 4.844nm, \quad h = 0.129nm$$
$$e_0 a_0 = 0.67nm, \quad E_1 = 2434GPa, \quad E_2 = 2473GPa$$
$$\upsilon_{12} = 0.197, \quad G_{12} = 1039GPa, \quad \rho = 6316\frac{kg}{m^3}$$

The output results are provided in Table 1 using the Eq. (46) and DLSM method. Here S is the number of sampling points between each two main nodes and Nx and Ny are the number of main nodes in the direction of *x* and *y* coordinates

Table 1 Static analysis of the central node deflection in exact and approximate responses with different boundary conditions

Boundary conditions	$\boldsymbol{w^{exact}}$ $\boldsymbol{(a/2, b/2)}$, $\boldsymbol{nm}$ (Naderi and Baradaran 2013)	$(e_0 a_0 = 0.67nm,$ $Nx = Ny = 31, S = 3)$			EFG (Naderi and Baradaran 2013) $\boldsymbol{w(a/2, b/2), nm}$	Error analysis	
		α_1	α_2	DLSM $\boldsymbol{w(a/2, b/2), nm}$		$\left\lvert\frac{W_{DLSM} - W^{exact}}{W_{DLSM}}\right\rvert$	$\left\lvert\frac{W_{DLSM} - W_{EFG}}{W_{DLSM}}\right\rvert$
SSSS	-9.5914	10^7	10^0	-9.5338	-9.8817	0.006	0.036
CCCC	-	10^{14}	10^{14}	-2.7227	-2.9229	-	0.074
CCSS	-	10^{10}	10^{10}	-8.0697	-8.5005	-	0.053
SSCC	-	10^{11}	10^{11}	-2.5993	-2.9831	-	0.148

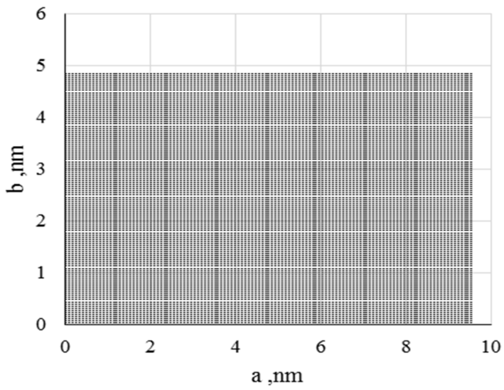


Fig. 2 Regular distribution of sampling points

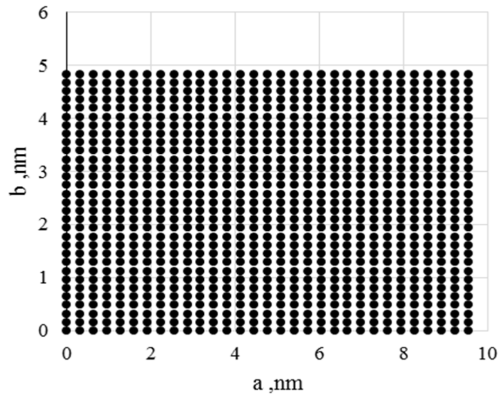


Fig. 3 Regular distribution of nodal points

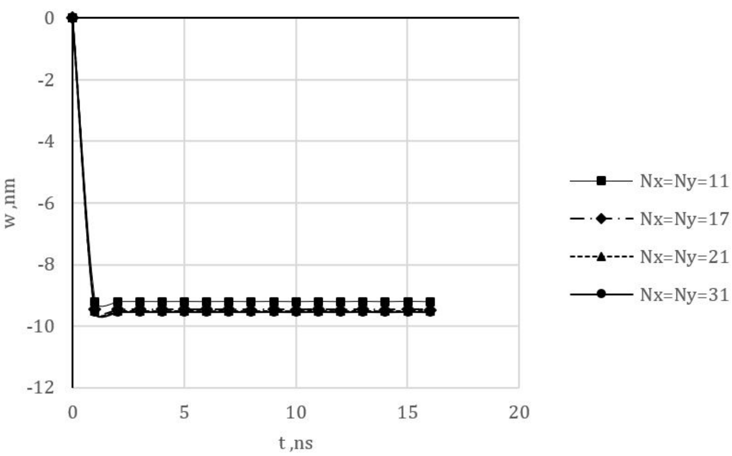


Fig. 4 Evaluation of the effect of increased number of nodal points on deflection of graphene nanoplates with SSSS boundary conditions $(e_0 a_0 = 0.67nm, S = 3, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 1ns)$

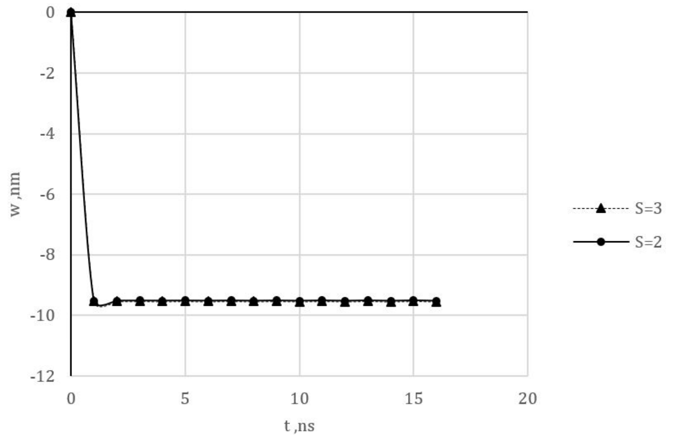


Fig. 5 Evaluation of the effect of increased number of sample points on deflection-time graphe of graphene nanoplates with SSSS boundary conditions $(e_0a_0 = 0.67nm, N_x = N_y = 31, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 1ns)$

Figs. 2 and 3 respectively indicate the sampling points and nodal points spread on a regular basis in the rectangular computational domain. A regular grid of sampling points is used to analyze the problem.

Also, the output results of dynamic analysis of nanoplates with the aforementioned characteristics are presented graphically. Fig. 4 shows the deflection-time graph, in which increased nodal points of the graph with fixed sampling points, deflection values approach the exact solution but with further increase in these points, the responses approach the more accurate response, and that is why the number of suitable nodal points for solution is $Nx = Ny = 31$, because with these nodal points, deflection value with SSSS boundary conditions converge to the deflection value exact solution. It should be noted that dt is indicative of the time step for dynamic solution of graphene nanoplates.

Fig. 5 shows the effect of increased sampling points with fixed node points on the deflection-time graph with SSSS boundary conditions, in which the rate of approaching more accurate results increases by increasing the sampling points.

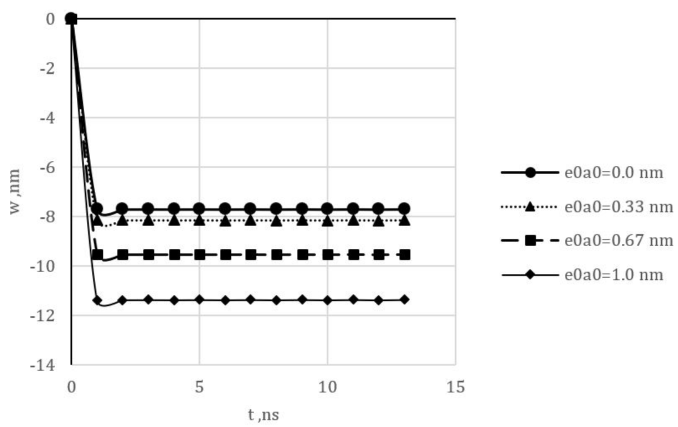


Fig. 6 Effect of increasing the nonlocal factor e_0a_0 on deflection of graphene nanoplates with SSSS boundary conditions $(N_x = N_y = 31, S = 3, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 1ns)$

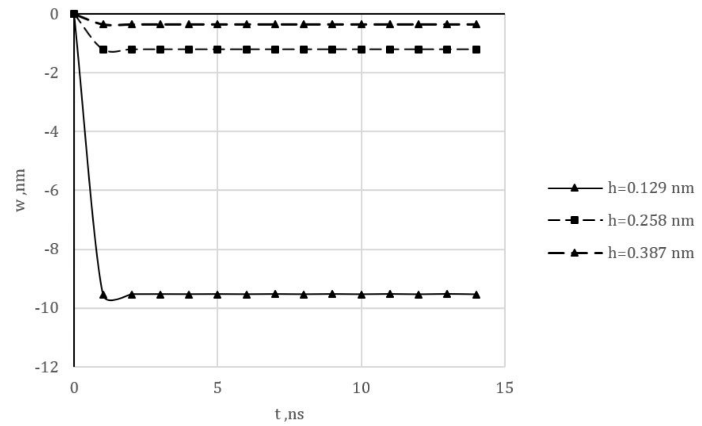


Fig. 7 Comparison of increased nanoplate thickness on deflection-time graph of the nanoplates with SSSS boundary conditions $(e_0a_0 = 0.67nm, N_x = N_y = 31, S = 3, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 1ns)$

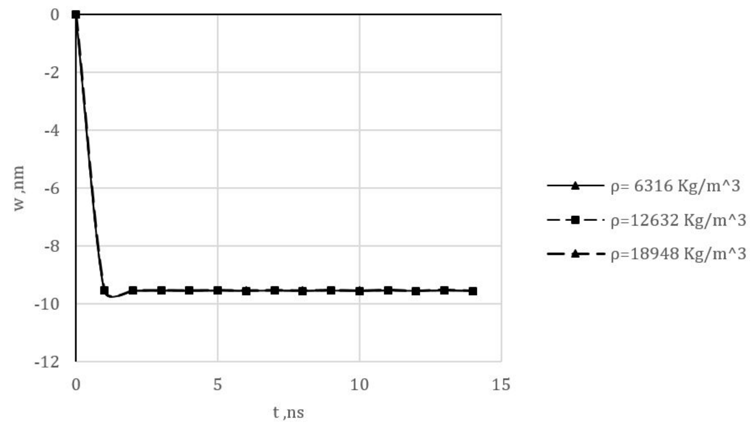


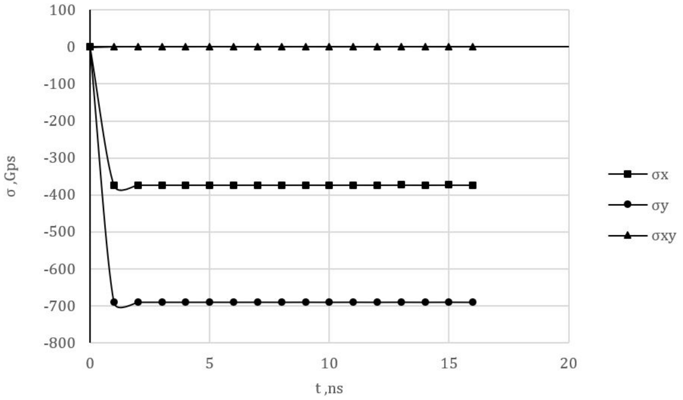
Fig. 8 Effect of increased nanoplate density on deflection-time graph for SSSS boundary conditions $(e_0a_0 = 0.67nm, N_x = N_y = 31, S = 3, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 1ns)$

Fig. 6 shows the effects of nonlocal factors on the deflection-time graph, in which the deflection increases by increasing the nonlocal coefficient. Increasing the effect of nonlocal coefficients in nano problems has had a significant impact on the deflection-time graph. While in the macro problems, due to insignificance of these nonlocal effects on the responses, the coefficients are ignored.

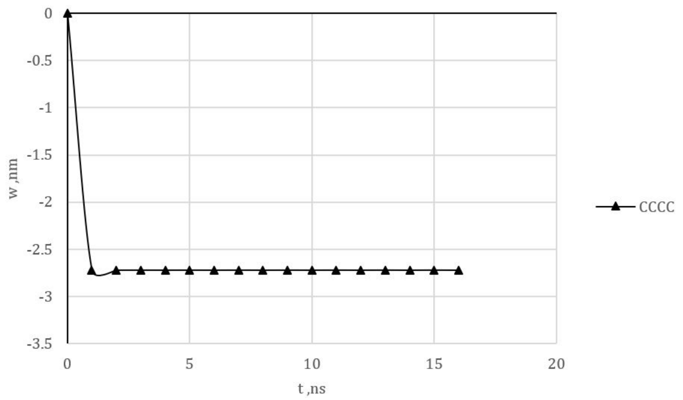
According to Fig. 7, assuming that all the specifications of graphene nanoplate are constant, only its thickness increase will cause the final deflection to be drastically reduced.

In Fig. 8, increasing the nanoplates density, the amount of deflection has not changed, but its oscillation has been increased.

Figs. 9 to 11 show the stress-time and deflection-time graphs for dynamic analysis of graphene nanoplates with various specifications and different boundary conditions with static loading, solved by DLSM method.

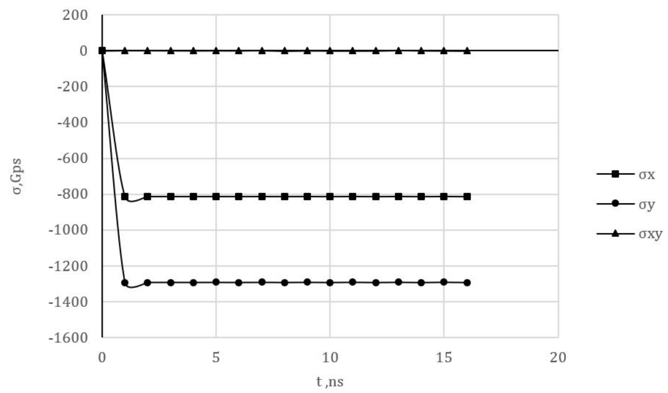


(I)



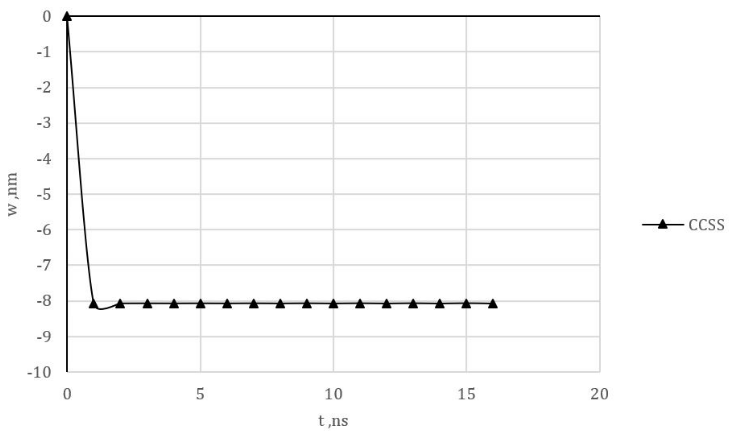
(II)

Fig. 9 (I) Stress-time graph and (II) deflection-time graph of the middle point of the graphene nanoplates with CCCC boundary conditions $(e_0a_0 = 0.67nm, N_x = N_y = 31, S = 3, \alpha_1 = 10^{14},\ \alpha_2 = 10^{14}, dt = 1ns)$



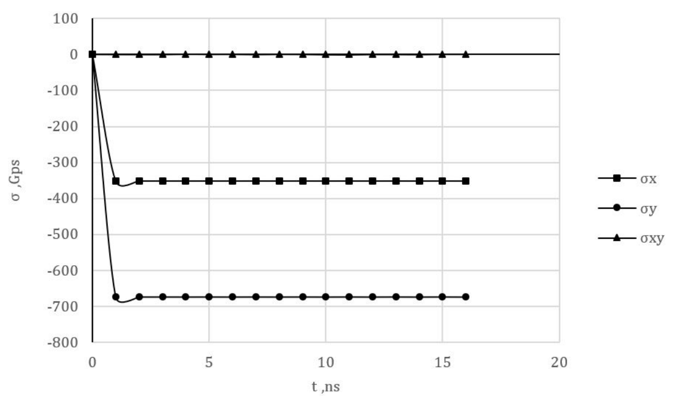
(I)

Fig. 10 (I) Stress-time graph and (II) deflection-time graph of the middle point of the graphene nanoplates with CCSS boundary conditions $(\boldsymbol{e_0a_0 = 0.67nm, N_x = N_y = 31, S = 3, \alpha_1 = 10^{10}, \alpha_2 = 10^{10},\ dt = 1ns})$

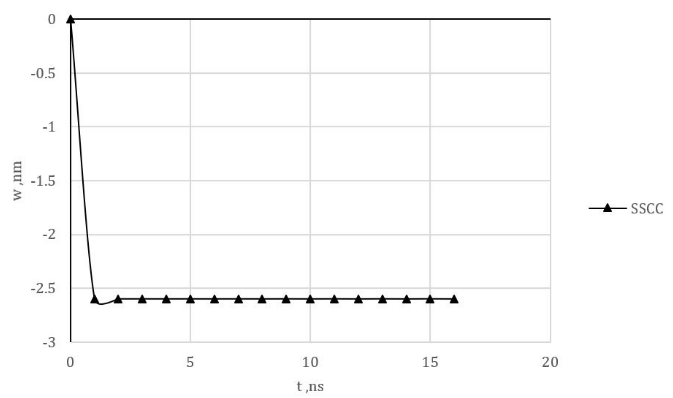


(II)

Fig. 10 Continued



(I)



(II)

Fig. 11 (I) Stress-time graph and (II) deflection-time graph of the middle point of the graphene nanoplates with SSCC boundary conditions $(e_0a_0 = 0.67nm, N_x = N_y = 31, S = 3, \alpha_1 = 10^{11}, \alpha_2 = 10^{11}, dt = 1ns)$

7. Dynamic analysis of graphene nanoplate under dynamic loading

In this section, the DLSM method is implemented for dynamic analysis of armchair single-layer rectangular graphene nanoplates with simply and clamped boundary conditions with dynamic loading

$$\begin{cases} p(x,y,t) = p_{\lambda\mu}\, sin(\zeta_\lambda x)\, sin(\eta_\mu y)\, sin(\frac{t\pi}{60}) & t \le 60 \\ 0 & t > 60 \end{cases} \quad (47)$$

where, t is time and its unit is picosecond (ps), $\zeta_\lambda = \pi/a$, $\eta_\mu = \pi/b$and $p_{\lambda\mu} = 1nN/nm^2$.

Specifications necessary to solve the problem of armchair single-layer rectangular graphene nanoplates dynamically under dynamic loading with SSSS and CCCC boundary conditions, are

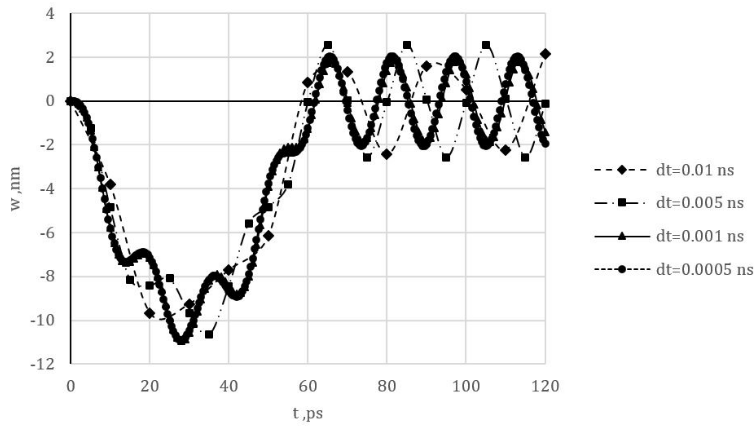


Fig. 12 Evaluation of the effect of time step(dt) on deflection-time graph of graphene nanoplates with SSSS boundary conditions $(e_0a_0 = 0.67nm, N_x = N_y = 31, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 0.001ns)$

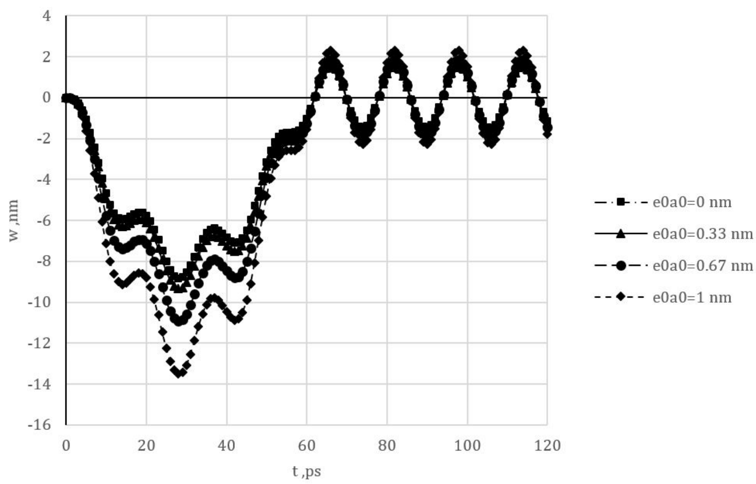


Fig. 13 Effect of increasing the nonlocal factor e_0a_0 on deflection of graphene nanoplates with SSSS boundary conditions $(N_x = N_y = 31, \alpha_1 = 10^7, \alpha_2 = 10^0, dt = 0.001ns)$

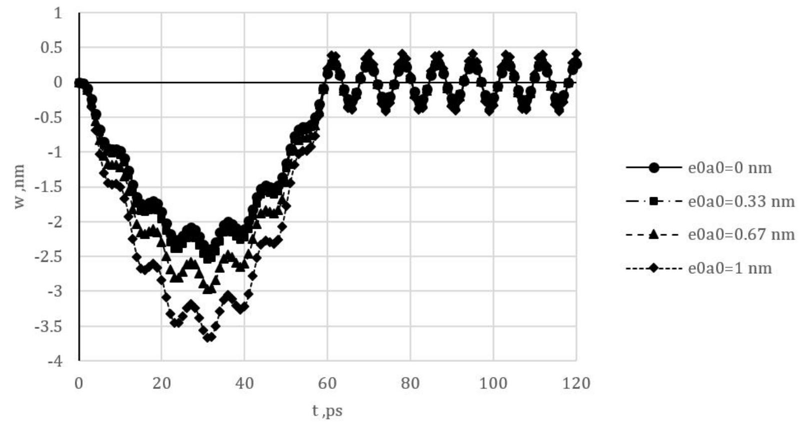


Fig. 14 Effect of increasing the nonlocal factor e_0a_0 on deflection of graphene nanoplates with CCCC boundary conditions $(N_x = N_y = 31, \alpha_1 = 10^{14}, \alpha_2 = 10^{14}, dt = 0.001ns)$

$$\begin{array}{lll} a = 9.519nm, & b = 4.844nm, & h = 0.129nm \\ e_0a_0 = 0.67nm, & E_1 = 2434GPa, & E_2 = 2473GPa \\ \upsilon_{12} = 0.197, & G_{12} = 1039GPa, & \rho = 6316\dfrac{kg}{m^3} \end{array}$$

The output results using the DLSM method are depicted in Figs. 12 to 14. According to Fig. 12, the best time step is dt = 0.001 ns; for smaller steps, results change negligibly. So according to this time step, and with SSSS and CCCC boundary conditions, the results were obtained.

Figs. 13 and 14 show the effects of nonlocal factors on the deflection-time graph. These effects include increased amplitude in the forced and free vibration.

8. Conclusions

According to the results extracted, the meshless DLSM method is an appropriate method to solve the dynamic problems at nano dimensions. Also, there is no need for integration to solve the problems, and it provides fairly accurate solution to all boundary conditions mentioned in the article. In addition, the increased nonlocal coefficient increases the deflection, and higher nonlocal coefficient leads to higher deflection intensity and amplitude vibration. Therefore, these factors are introduced in differential equations in nanoscale problems, while in the macro-scale problems, the impact of these factors are not significant, because they do not have considerable effect on the problem solutions. Increasing the thickness of nanoplates reduces deflection, and the rate of final deflection reduction is also increased, and an increase in the density of the nanoplates increases vibration, but does not make a difference in its deflection. Moreover, increased number of sampling points and nodes makes the nanoplates deflection closer to its real value, but if the number of points is more than a certain value, it approaches less to the exact response.

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References

Aghababaei, R. and Reddy, J.N. (2009), "Nonlocal third-order shear deformation plate theory with application to bending and vibration of plates", *J. Sound Vib.*, **326**(1-2), 277-289. https://doi.org/10.1016/j.jsv.2009.04.044

Alwar, R.S. and Reddy, B.S. (1979), "Large deflection static and dynamic analysis of isotropic and orthotropic annular plates", *Int. J. Non-Linear Mech.*, **14**(5-6), 347-359. https://doi.org/10.1016/0020-7462(79)90008-8

Alzahrani, E.O., Zenkour, A.M. and Sobhy, M. (2013), "Small scale effect on hygro-thermo-mechanical bending of nanoplates embedded in an elastic medium", *Compos. Struct.*, **105**, 163-172. https://doi.org/10.1016/j.compstruct.2013.04.045

Analooei, H.R., Azhari, M. and Heidarpour, A. (2013), "Elastic buckling and vibration analyses of orthotropic nanoplates using nonlocal continuum mechanics and spline finite strip method", *Appl. Mathe. Model.*, **37**(10-11), 6703-6717. https://doi.org/10.1016/j.apm.2013.01.051

Ansari, R., Rajabiehfard, R. and Arash, B. (2010a), "Nonlocal finite element model for vibrations of embedded multi-layered graphene sheets", *Computat. Mater. Sci.*, **49**(4), 831-838. https://doi.org/10.1016/j.commatsci.2010.06.032

Ansari, R., Sahmani, S. and Arash, B. (2010b), "Nonlocal plate model for free vibrations of single-layered graphene sheets", *Phys. Lett. A*, **375**(1), 53-62. https://doi.org/10.1016/j.physleta.2010.10.028

Arash, B. and Wang, Q. (2011), "Vibration of single-and double-layered graphene sheets", *J. Nanotech. Eng. Med.*, **2**(1). https://doi.org/10.1115/1.4003353

Babaei, H. and Shahidi, A.R. (2011), "Small-scale effects on the buckling of quadrilateral nanoplates based on nonlocal elasticity theory using the Galerkin method", *Arch. Appl. Mech.*, **81**(8), 1051-1062. https://doi.org/10.1007/s00419-010-0469-9

Behfar, K., Seifi, P., Naghdabadi, R. and Ghanbari, J. (2006), "An analytical approach to determination of bending modulus of a multi-layered graphene sheet", *Thin Solid Films*, **496**(2), 475-480. https://doi.org/10.1016/j.tsf.2005.08.317

Bouadi, A., Bousahla, A.A., Houari, M.S.A., Heireche, H. and Tounsi, A. (2018), "A new nonlocal HSDT for analysis of stability of single layer graphene sheet", *Adv. Nano Res.*, *Int. J.*, **6**(2), 147-162. https://doi.org/10.12989/anr.2018.6.2.147

Chen, J.H., Jang, C., Xiao, S., Ishigami, M. and Fuhrer, M.S. (2008), "Intrinsic and extrinsic performance limits of graphene devices on SiO_2", *Nature Nanotech.*, **3**(4), 206-209. https://doi.org/10.1038/nnano.2008.58

Choi, W., Lahiri, I., Seelaboyina, R. and Kang, Y.S. (2010), "Synthesis of graphene and its applications: a review", *Critical Rev. Solid State Mater. Sci.*, **35**(1), 52-71. https://doi.org/10.1080/10408430903505036

Dehshahri, K., Nejad, M.Z., Ziaee, S., Niknejad, A. and Hadi, A. (2020), "Free vibrations analysis of arbitrary three-dimensionally FGM nanoplates", *Adv. Nano Res.*, *Int. J.*, **8**(2), 115-134. https://doi.org/10.12989/anr.2020.8.2.115

Duan, W.H. and Wang, C.M. (2009), "Nonlinear bending and stretching of a circular graphene sheet under a central point load", *Nanotechnology*, **20**(7), 075702. https://doi.org/10.1088/0957-4484/20/7/075702

Erami, F.E. and Firoozjaee, A.R. (2020), "Numerical solution of bed load transport equations using discrete least squares meshless (DLSM) method", *Appl. Mathe. Model.*, **77**, 1095-1109. https://doi.org/10.1016/j.apm.2019.08.016

Eringen, A.C. (1972), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**(1), 1-16. https://doi.org/10.1016/0020-7225(72)90070-5

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710. https://doi.org/10.1063/1.332803

Eringen, A.C. (2002), *Nonlocal Continuum Field Theories*, Springer Science & Business Media.

Eringen, A.C. and Edelen, D.G.B. (1972), "On nonlocal elasticity", *Int. J. Eng. Sci.*, **10**(3), 233-248. https://doi.org/10.1016/0020-7225(72)90039-0
Farajpour, A., Shahidi, A.R., Mohammadi, M. and Mahzoon, M. (2012), "Buckling of orthotropic micro/nanoscale plates under linearly varying in-plane load via nonlocal continuum mechanics", *Compos. Struct.*, **94**(5), 1605-1615. https://doi.org/10.1016/j.compstruct.2011.12.032
Fattahi, A.M., Safaei, B. and Moaddab, E. (2019), "The application of nonlocal elasticity to determine vibrational behavior of FG nanoplates", *Steel Compos. Struct., Int. J.*, **32**(2), 281-292. https://doi.org/10.12989/scs.2019.32.2.281
Firoozjaee, A.R. and Afshar, M.H. (2009), "Discrete least squares meshless method with sampling points for the solution of elliptic partial differential equations", *Eng. Anal. Boundary Elem.*, **33**(1), 83-92. https://doi.org/10.1016/j.enganabound.2008.03.004
Geim, A.K. (2009), "Graphene: status and prospects", *Science*, **324**(5934), 1530-1534. https://doi.org/10.1126/science.1158877
Geim, A.K. and Kim, P. (2008), "Carbon wonderland", *Scientific American*, **298**(4), 90-97.
Geim, A.K. and Novoselov, K.S. (2007), "The rise of graphene", *Naturematerials*, **6**, 183-191. https://doi.org/10.1142/9789814287005_0002
Ghannadpour, S.A.M. and Moradi, F. (2019), "Nonlocal nonlinear analysis of nano-graphene sheets under compression using semi-Galerkin technique", *Adv. Nano Res., Int. J.*, **7**(5), 311-324. https://doi.org/10.12989/anr.2019.7.5.311
GR, L. (2003, November), "1013 Mesh Free Methods: Moving beyond the Finite Element Method", *Proceedings of The Computational Mechanics Conference*, The Japan Society of Mechanical Engineers, pp. 937-938. https://doi.org/10.1299/jsmecmd.2003.16.937
Heyrovska, R. (2008), "Atomic structures of graphene, benzene and methane with bond lengths as sums of the single, double and resonance bond radii of carbon", arXiv preprint arXiv:0804.4086.
Hutchinson, J.W. and Fleck, N. (1997), "Strain gradient plasticity", In: *Advances in Applied Meechanics*, **33**, 295-361.
Jomehzadeh, E. and Saidi, A.R. (2011), "Decoupling the nonlocal elasticity equations for three dimensional vibration analysis of nano-plates", *Compos. Struct.*, **93**(2), 1015-1020. https://doi.org/10.1016/j.compstruct.2010.06.017
Katsnelson, M.I. (2007), "Graphene: carbon in two dimensions", *Mater. Today*, **10**(1-2), 20-27. https://doi.org/10.1016/S1369-7021(06)71788-6
Kuzmenko, A.B., Van Heumen, E., Carbone, F. and Van Der Marel, D. (2008), "Universal optical conductance of graphite", *Phys. Rev. Lett.*, **100**(11), 117401. https://doi.org/10.1103/PhysRevLett.100.117401
Liu, J., Chen, L., Xie, F., Fan, X. and Li, C. (2016), "On bending, buckling and vibration of graphene nanosheets based on the nonlocal theory", *Smart Struct. Syst., Int. J.*, **17**(2), 257-274. https://doi.org/10.12989/sss.2016.17.2.257
Love, A.E.H. (1888), "XVI. The small free vibrations and deformation of a thin elastic shell", *Philosophical Transactions of the Royal Society of London.(A.)*, **179**, 491-546. https://doi.org/10.1098/rsta.1888.0016
Lu, P., Zhang, P.Q., Lee, H.P., Wang, C.M. and Reddy, J.N. (2007), "Non-local elastic plate theories", *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **463**(2088), 3225-3240. https://doi.org/10.1098/rspa.2007.1903
Malekzadeh, P., Setoodeh, A.R. and Beni, A.A. (2011a), "Small scale effect on the thermal buckling of orthotropic arbitrary straight-sided quadrilateral nanoplates embedded in an elastic medium", *Compos. Struct.*, **93**(8), 2083-2089. https://doi.org/10.1016/j.compstruct.2011.02.013
Malekzadeh, P., Setoodeh, A. and Beni, A.A. (2011b), "Small scale effect on the free vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates", *Compos. Struct.*, **93**(7), 1631-1639. https://doi.org/10.1016/j.compstruct.2011.01.008
Martel, R., Schmidt, T., Shea, H.R., Hertel, T. and Avouris, P. (1998), "Single-and multi-wall carbon nanotube field-effect transistors", *Appl. Phys. Lett.*, **73**(17), 2447-2449. https://doi.org/10.1063/1.122477
Murmu, T. and Pradhan, S.C. (2009a), "Buckling analysis of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity and Timoshenko beam theory and using DQM", *Physica E*:

Low-dimensional Systems and Nanostructures, **41**(7), 1232-1239. https://doi.org/10.1016/j.physe.2009.02.004

Murmu, T. and Pradhan, S.C. (2009b), "Buckling of biaxially compressed orthotropic plates at small scales", *Mech. Res. Commun.*, **36**(8), 933-938. https://doi.org/10.1016/j.mechrescom.2009.08.006

Naderi, A. and Baradaran, G. (2013), "Element free Galerkin method for static analysis of thin micro/nanoscale plates based on the nonlocal plate theory", *Int. J. Eng.*, **26**(7), 795-806.

Nowacki, W. (1974), "The Micropolar Thermoelasticity", In: *Micropolar Elasticity*, Springer, Vienna, pp. 105-168.

Peddieson, J., Buchanan, G.R. and McNitt, R.P. (2003), "Application of nonlocal continuum models to nanotechnology", *Int. J. Eng. Sci.*, **41**(3-5), 305-312. https://doi.org/10.1016/S0020-7225(02)00210-0

Pouresmaeeli, S., Fazelzadeh, S.A. and Ghavanloo, E. (2012), "Exact solution for nonlocal vibration of double-orthotropic nanoplates embedded in elastic medium", *Compos. Part B: Eng.*, **43**(8), 3384-3390. https://doi.org/10.1016/j.compositesb.2012.01.046

Pradhan, S.C. and Phadikar, J.K. (2009), "Nonlocal elasticity theory for vibration of nanoplates", *J. Sound Vib.*, **325**(1-2), 206-223. https://doi.org/10.1016/j.jsv.2009.03.007

Pumera, M., Ambrosi, A., Bonanni, A., Chng, E.L.K. and Poh, H.L. (2010), "Graphene for electrochemical sensing and biosensing", *TrAC Trends Anal. Chem.*, **29**(9), 954-965. https://doi.org/10.1016/j.trac.2010.05.011

Rao, C.N.R., Biswas, K., Subrahmanyam, K.S. and Govindaraj, A. (2009), "Graphene, the new nanocarbon", *J. Mater. Chem.*, **19**(17), 2457-2469. https://doi.org/10.1039/B815239J

Reddy, J.N. (2010), "Nonlocal nonlinear formulations for bending of classical and shear deformation theories of beams and plates", *Int. J. Eng. Sci.*, **48**(11), 1507-1518. https://doi.org/10.1016/j.ijengsci.2010.09.020

Rouhi, H., Ansari, R. and Darvizeh, M. (2016), "Size-dependent free vibration analysis of nanoshells based on the surface stress elasticity", *Appl. Mathe. Model.*, **40**(4), 3128-3140. https://doi.org/10.1016/j.apm.2015.09.094

Sakhaee-Pour, A. (2009), "Elastic properties of single-layered graphene sheet", *Solid State Commun.*, **149**(1-2), 91-95. https://doi.org/10.1016/j.ssc.2008.09.050

Sakhaee-Pour, A., Ahmadian, M.T. and Vafai, A. (2008), "Applications of single-layered graphene sheets as mass sensors and atomistic dust detectors", *Solid State Commun.*, **145**(4), 168-172. https://doi.org/10.1115/IMECE2007-43238

Shen, H.S. (2011), "Nonlocal plate model for nonlinear analysis of thin films on elastic foundations in thermal environments", *Compos. Struct.*, **93**(3), 1143-1152. https://doi.org/10.1016/j.compstruct.2010.10.009

Shen, L.E., Shen, H.S. and Zhang, C.L. (2010), "Nonlocal plate model for nonlinear vibration of single layer graphene sheets in thermal environments", *Computat. Mater. Sci.*, **48**(3), 680-685. https://doi.org/10.1016/j.commatsci.2010.03.006

Sobhy, M. (2014a), "Thermomechanical bending and free vibration of single-layered graphene sheets embedded in an elastic medium", *Physica E: Low-dimens. Syst. Nanostruct.*, **56**, 400-409. https://doi.org/10.1016/j.physe.2013.10.017

Sobhy, M. (2014b), "Generalized two-variable plate theory for multi-layered graphene sheets with arbitrary boundary conditions", *Acta Mechanica*. **225**(9), 2521. https://doi.org/10.1007/s00707-014-1093-5

Wang, Q. and Varadan, V.K. (2006), "Wave characteristics of carbon nanotubes", *Int. J. Solids Struct.*, **43**(2), 254-265. https://doi.org/10.1016/j.ijsolstr.2005.02.047

Wang, C.M., Tan, V.B.C. and Zhang, Y.Y. (2006), "Timoshenko beam model for vibration analysis of multi-walled carbon nanotubes", *J. Sound Vib.*, **294**(4-5), 1060-1072. https://doi.org/10.1016/j.jsv.2006.01.005

Wang, Y., Li, F., Jing, X. and Wang, Y. (2015a), "Nonlinear vibration analysis of double-layered nanoplates with different boundary conditions", *Phys. Lett. A*, **379**(24-25), 1532-1537. https://doi.org/10.1016/j.physleta.2015.04.002

Wang, Y., Li, F.M. and Wang, Y.Z. (2015b), "Homoclinic behaviors and chaotic motions of double layered viscoelastic nanoplates based on nonlocal theory and extended Melnikov method", *Chaos: An Interdiscipl. J. Nonlinear Sci.*, **25**(6), 063108. https://doi.org/10.1063/1.4922299

Wang, Y., Li, F.M. and Wang, Y.Z. (2016), “Nonlocal effect on the nonlinear dynamic characteristics of buckled parametric double-layered nanoplates”, *Nonlinear Dyn*., **85**(3), 1719-1733. https://doi.org/10.1007/s11071-016-2789-y

Wang, Y., Li, F. and Shu, H. (2019), “Nonlocal nonlinear chaotic and homoclinic analysis of double layered forced viscoelastic nanoplates”, *Mech. Syst. Signal Process*., **122**, 537-554. https://doi.org/10.1016/j.ymssp.2018.12.041

Wu, C.P. and Li, W.C. (2017), “Free vibration analysis of embedded single-layered nanoplates and graphene sheets by using the multiple time scale method”, *Comput. Mathe. Appl*., **73**(5), 838-854. https://doi.org/10.1016/j.camwa.2017.01.014

Yang, F.A.C.M., Chong, A.C.M., Lam, D.C.C. and Tong, P. (2002), “Couple stress based strain gradient theory for elasticity”, *Int. J. Solids Struct*., **39**(10), 2731-2743. https://doi.org/10.1016/S0020-7683(02)00152-X

Zenkour, A.M. and Sobhy, M. (2013), “Nonlocal elasticity theory for thermal buckling of nanoplates lying on Winkler–Pasternak elastic substrate medium”, *Physica E: Low-dimens. Syst. Nanostruct*., **53**, 251-259. https://doi.org/10.1016/j.physe.2013.04.022

Zhang, Y., Lei, Z.X., Zhang, L.W., Liew, K.M. and Yu, J.L. (2015), “Nonlocal continuum model for vibration of single-layered graphene sheets based on the element-free kp-Ritz method”, *Eng. Anal. Bound. Elem*., **56**, 90-97. https://doi.org/10.1016/j.enganabound.2015.01.020

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