

Seismic behavior of concrete gravity dams

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Abstract. Dams play a vital role in the development and sustainment in a country. Failure of dams leads to the catastrophic event with sudden release of water and is of great concern. Hence earthquake-resistant design of dams is of prime importance. The present study involves static, modal and transient analyses of dam-reservoir-foundation system using finite element software ANSYS 15. The dam and the foundation are modeled with 2D plane strain element “PLANE 42” and the reservoir by fluid acoustic element “FLUID 29” with proper consideration of fluid-structure interaction. An expression for the fundamental period of concrete dams is developed based on modal analysis. Seismic response of gravity dams subjected to earthquake acceleration is evaluated in terms of peak displacement and stress.

Keywords: gravity dam; earthquake; fundamental period; fluid structure interaction; reservoir

1. Introduction

A dam is the cornerstone in the development and management of water resources development of a river basin (International Commission on large Dams, www.icold-cigb.org). A gravity dam is a structure so proportioned that its own weight resists the external forces exerted upon it. This type of structure is most common and requires less maintenance. It usually consists of two sections; namely, the non-overflow section and the overflow section or spillway section. These are particularly suited across gorges with very steep side slopes.

Gravity dam can be constructed with ease on any dam site, where there exists a natural foundation strong enough to bear the enormous weight of the dam. Such a dam is generally straight in plan, although sometimes, it may be slightly curved. When suitable conditions are available, such dams can be constructed up to great heights. The highest gravity dam in the world is Grand Dixence Dam in Switzerland (285 m), followed by Bakra Dam in India (226 m); both are of concrete gravity type.

Many concrete gravity dams have been in service for over 50 years. The identified causes of failure of gravity dams are failure of foundation or abutments, inadequate spillway capacity, spillway design error, poor construction, poor maintenance, extreme rainfall, uneven settlement,

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acts of war, human or computer design errors, embankment slips, landslides into reservoir, defective materials, incorrect operation, earthquakes, etc.

India is a country with 5,100 large dams and 1,040 active faults covering 57% of land mass, thus making the built-up structures prone to earthquake. There is always a possibility that a severe earthquake in high seismic zones might affect the stability of dams (Patra 2014).

During past decades, several studies were conducted on the safety and stability of gravity dams under operating as well as seismic loadings. Linear analyses (Chopra and Gupta 1982) reveal that large tensile stresses in excess of the strength of the concrete would develop in the dam during strong earthquakes. Following this, many nonlinear analyses have been carried out to predict the occurrence and propagation of cracks. Skrikerud and Bachmann (1986) conducted studies related to the development of cracks in the body of gravity dams. Using the maximum tensile strength criteria, they simulated the crack propagation of the Koyna dam under strong earthquakes by incorporating the discrete crack approach in a finite element program.

2. Finite element model of dam-reservoir system

Dynamic analyses of buildings and dams are very complex phenomena. In order to solve these complex phenomena, mathematical models are generally adopted considering certain assumptions imposed on the physical problem. The discretized structural dynamic equation including dam and foundation subjected to ground motion can be formulated using finite element approach as

$$M_s \ddot{u}_e + C_s \dot{u}_e + k_s u_e = -M_s \ddot{u}_g + Q_{pe} \quad (1)$$

where M_s , C_s and K_s are respectively the mass, damping and stiffness matrices of the structure, u_e is the nodal displacement vector with respect to the ground and \dot{u}_e and \ddot{u}_e represent the velocity and acceleration vectors. \ddot{u}_g is the ground acceleration and the term Q_{pe} represents the nodal force vector associated with the hydrodynamic pressure produced by the reservoir (Khosravi and Heydari 2013).

The discretized wave equation is given by

$$M_f \ddot{p}_e + C_f \dot{p}_e + k_f p_e + p_w Q^T (\ddot{u}_g + \ddot{u}_e) = 0 \quad (2)$$

where M_f , C_f and K_f are the fluid mass, damping and stiffness matrices respectively and p_e is the nodal pressure. The term $p_w Q^T$ is referred to as the coupling matrix.

3. Fluid-structure interaction

Fluid structure Interaction (FSI) refers to the coupling of unsteady fluid flow and the structural deformation. It is a two-way coupling of pressure and deflection. There are various approaches for modelling FSI; the added mass approach, Eulerian approach and Lagrangean approach. Because of its simplicity, Westergaard's added mass approach has been frequently adopted for modelling Dam-liquid interaction. In Eulerian approach, the variables for measuring the response are pressure and velocity. The hydrodynamic pressure distribution is governed by the Wave equation

$$\nabla^2 \Phi(x, y, t) = \frac{1}{V_p^2} \ddot{\Phi}(x, y, t) \tag{3}$$

The relations between pressure P , velocity vector $\{v\}$ and velocity potential φ are $\{v\} = \nabla \Phi$ and $p = -\rho \dot{\Phi}$ where ρ is the density of fluid.

In Lagrangean approach, the behavior of the fluid and structure is represented by displacement. Hence, compatibility and equilibrium are automatically satisfied at the nodes along the interface between liquid and structure.

The coupled fluid structure foundation equation is

$$\begin{bmatrix} M_s & 0 \\ M_{sf} & M_f \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_e \\ \ddot{\mathbf{p}}_e \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_f \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_e \\ \dot{\mathbf{p}}_e \end{Bmatrix} + \begin{bmatrix} k_s & k_{fs} \\ 0 & k_f \end{bmatrix} \begin{Bmatrix} \mathbf{u}_e \\ \mathbf{p}_e \end{Bmatrix} = \begin{Bmatrix} -M_s \ddot{\mathbf{u}}_g \\ -M_{fs} \ddot{\mathbf{u}}_g \end{Bmatrix} \tag{4}$$

where, $k_{fs} = -Q$, $M_{fs} = p_{\omega} Q^T$. Eq. (4) expresses a second order linear differential equation having unsymmetrical matrices and may be solved by direct integration method. In general the dynamic equilibrium equations of the system modeled in finite element can be expressed as

$$M_c \ddot{\mathbf{u}}_c + C_c \dot{\mathbf{u}}_c + k_c \mathbf{u}_c = F(t) \tag{5}$$

where M_c , C_c and K_c are the coupled mass, damping and stiffness matrices respectively and $F(t)$ is the dynamic load vector.

4. Formulation of natural periods of concrete gravity dams

One of the objectives of the present work is to develop an expression for the natural period of dam-reservoir system. Concrete gravity dam-reservoir systems are idealized as two-dimensional sections in the plane normal to the dam axis. The dam is assumed to be homogeneous, isotropic, and linearly elastic having modulus of elasticity (E_d) equal to 2.74×10^{10} N/m², Poisson's ratio of 0.24 and density of 2446.5 Kg/m³. Reservoir water is assumed to have a density equal to 1000 Kg/m³, sonic velocity of 1440 m/s and wave reflection coefficient of 0.5. Dam is assumed to be fixed at the base. Reservoir length is taken as one and a half times the depth.

In order to find a simplified formula for natural period, a set of concrete gravity dams is selected. The dams that are taken into consideration are of varying height and base width. The heights, base width, top width, and height of reservoir of selected dams are listed in Table 1.

Table 1 Geometry of dams under study

Dam	Height (m)	Base width (m)	Top width (m)	Reservoir Height (m)
Grand Dixence	285	200	15	270.00
Bhakra	226	403	10	214.70
Three Gorges	181	115	40	171.95
Guangzho	200	410	20	190.00
Boyabat	195	135.8	10	185.25
Dworshak	219	193	13	208.05

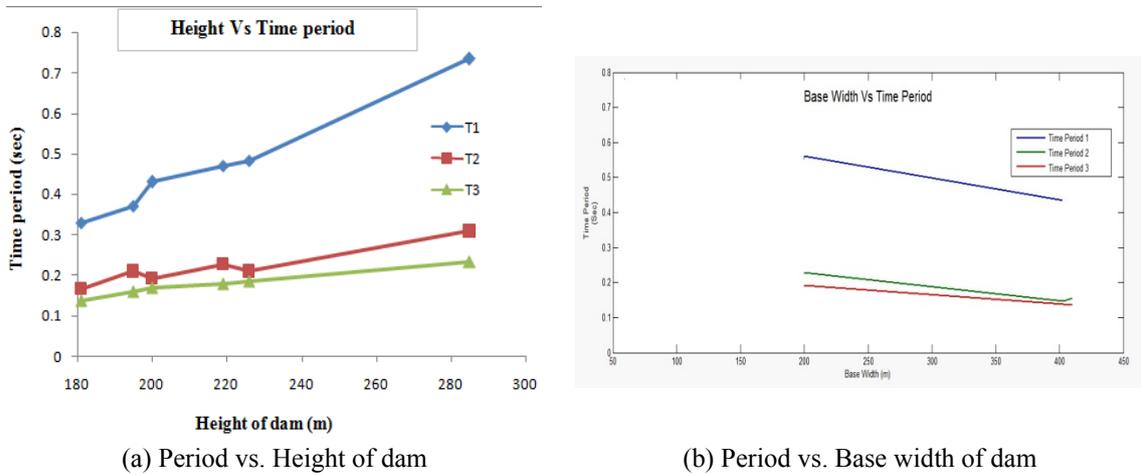


Fig. 1 Natural periods of Empty Dam

4.1 Natural periods of empty dam

Modal analysis is performed on the selected dams and the natural periods of vibration are extracted. The variation of period with respect to height of the dam and base width for first three modes is shown in Fig. 1. From the figure, it is clear that as the height increases the time period also increases for constant ratio of bottom width to height. As the base width of the dam increases the first fundamental mode period decreases. However, base width is found to have little influence for higher mode periods.

4.2 Natural periods of Dam-Reservoir System

Modal analysis on the dams with full reservoir is carried out. The results from modal analysis indicate that the water in the reservoir alters the dynamic characteristics of the structural system. It is found that there is an increase in periods of fundamental modes of vibration compared to that of empty dam and a corresponding reduction in the frequency of vibration.

Regression analysis is carried out on the results obtained from modal analysis. Modal analyses on dams with empty and full reservoir conditions show that the first fundamental natural time period is a function of height and base width of the dam. But the second fundamental natural time period depends only on the height of the dam. The following equations for time periods are obtained by regression analysis

$$\begin{aligned}
 T_{R1} &= 0.0035h^{1.3}b^{-0.30} - 0.41 \\
 T_{R2} &= 0.0041h^{0.86} - 0.48
 \end{aligned}
 \tag{6}$$

where T_{R1} and T_{R2} are respectively the first and second natural periods of dam with full reservoir condition, h is the height of dam and b is the base width of dam.

The graphical representation of periods as per the empirical formula (theoretical) and from modal analysis (actual value) is shown in Fig. 2. The values predicted by the proposed equations are in good agreement with modal analysis results.

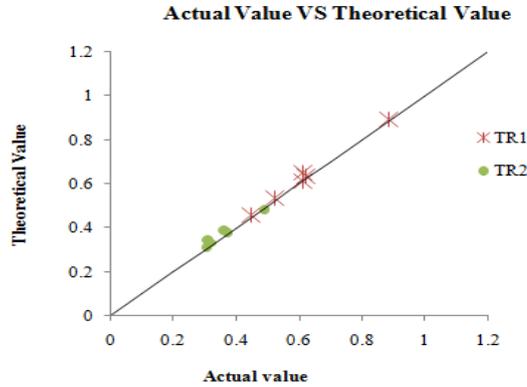


Fig. 2 Plot for first and second mode periods of dam (full reservoir)- comparison of empirical and modal analysis values

Table 2 Comparison of time periods obtained using empirical and modal analysis

Dam	Time Period (s)			
	Empirical (using Eq. (6))		From Modal Analysis	
	T_{R1} (s)	T_{R2} (s)	T_{R1} (s)	T_{R2} (s)
Pine Flat	0.406	0.207	0.396	0.205
Koyna	0.368	0.173	0.375	0.178

To check the accuracy of the equations predicting the time period of concrete gravity dams, another set of existing dams, viz., Pine Flat Dam and Koyna Dam are selected and the time periods are calculated using the proposed formula.

Time period obtained from modal analysis of the selected dams and that from the proposed equation is shown in Table 2. The periods obtained using Eq. (6) are matching well with modal analysis results. Hence the proposed equations can be used for calculating the natural periods of concrete gravity dams satisfactorily.

5. Seismic Analysis of concrete gravity dams

For performing transient analysis using earthquake time history records, Koyna dam (Maharashtra, India) is selected. This monolith is 103 m high and 70 m wide at its base. The depth of the reservoir at the time of Koynanagar earthquake (1967) was 91.75 m (Sarkar *et al.* 2007). The 6.5 magnitude shock hit near the site of Koyna dam; but, it didn't cause any major damage to the dam except some cracks which were quickly repaired. Some geologists believe that the earthquake was due to reservoir-triggered seismicity.

The non-overflow monolith of the dam is assumed to be in the plane strain condition. First order plane strain elements have been used to model the dam body. The dam is assumed to rest on a 350×140 m foundation. The bottom of the foundation is assumed to be fixed and the foundation is considered to be in the plane strain condition. First order plane strain elements have been used for modeling the foundation. Symmetric boundary conditions have been used at the ends of the

foundation block to simulate the unbounded nature of the foundation. The reservoir is assumed to be 140 m in length, and first-order acoustic elements have been used for modeling the reservoir. The acoustic element has four corner nodes with three degrees of freedom per node: translations in the nodal x and y directions and pressure. The translations, however, are applicable only at nodes that are on the interface.

The material properties adopted for the dam body are the modulus of Elasticity (E) equal to 32027 N/mm^2 , mass density (ρ) of 2643 kg/m^3 and Poisson's ratio (μ) of 0.2. Properties of rock foundation are modulus of elasticity (E_d) equal to 62054 N/mm^2 , mass density of 3300 Kg/m^3 and Poisson's ratio of 0.33. Properties of reservoir are bulk modulus (K_w) equal to 2250 N/mm^2 , mass density of 1000 Kg/m^3 , boundary admittance of 0.5 and sonic velocity of 1440 m/s .

5.1 Modal response of Dam-Reservoir-Foundation System

It is obvious that the foundation and water reservoir affect the modal frequencies and consequently the dynamic response of gravity dams during earthquakes. A parametric study is performed here to view the combined effect of foundation and reservoir on the dynamic response of the dam. To investigate the modal behavior of the dam, three different cases are taken as follows:

Model 1- Dam with fixed support and empty reservoir named "Fixed-Empty" (Fig. 3(a))

Model 2- Dam with foundation and empty reservoir, named "Mass-Empty" (Fig. 3(b))

Model 3- Dam with foundation and full reservoir, named "Mass-Fluid" (Fig. 3(c))

Eigen value analysis of the above three models are carried out and a comparative study on the modal frequencies is depicted in Fig. 4.

From the modal analysis results, it can be inferred that when the reservoir is empty and foundation is fixed (Fixed-Empty model), natural frequency of dam is maximum. Furthermore, a minimum value for the fundamental frequency is obtained when dam-reservoir-foundation interaction is considered (Mass-Fluid model), i.e., there is a 20% decrease of modal frequency for first mode and more than 50% decrease of modal frequencies of other higher modes if structure-foundation and structure-reservoir is considered. This is because when reservoir interaction is considered the water that is found near the structure causes increase in inertial force acting on the structure. The reservoir moves along with the displaced structure and thus the hydrodynamic force act on the structure. Hence the water in the reservoir leads to changes in the dynamic characteristics of the system by modifying the mode shapes and decreasing the frequency of vibration.

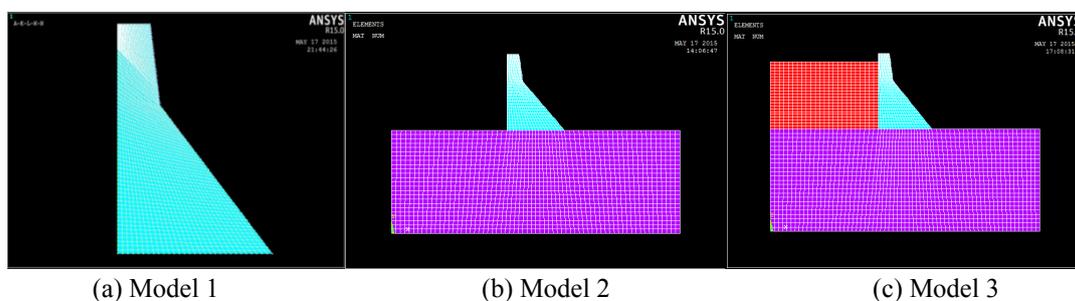


Fig. 3 Models for Seismic Analysis

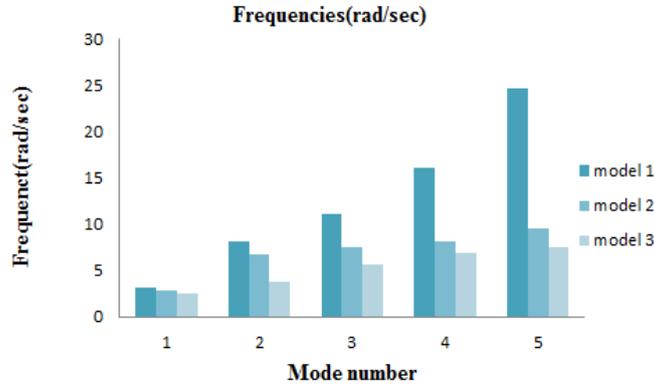


Fig. 4 Effect of reservoir and foundation on the modal frequencies of the dam

Table 3 Static Analysis Results (C-Compression and T –Tension)

Parameter	<i>Fixed-Empty</i>	<i>Mass-Empty</i>	<i>Mass-Fluid</i>
Stress at heel (N/mm ²)	3.05(C)	4.06 (C)	0.05 (T)
Stress at toe (N/mm ²)	0.11(C)	0.21(C)	1.02 (C)

When foundation interaction is considered it is found that there is a decrease in the natural frequency compared to that of dam with fixed support, which is because of a reduction in the stiffness and an increase in the mass of the vibrating system.

5.2 Static analysis of Dam-Reservoir-Foundation System

Static analysis is carried out for the three models considering the self weight of the dam and the hydrostatic pressure. The output in terms of the stresses at salient points is shown in Table 3.

From the static analysis results, it is found that in *Fixed-Empty* model, compressive stresses at the heel and toe are lesser than that of the *Mass-Empty* model, where foundation interaction is considered. In the *Mass-Fluid* model (Dam-Reservoir-Foundation System), due to hydrostatic forces acting on the structure, tensile stresses develop at the heel and the compressive stresses increase at the toe. However, the magnitudes are within permissible limits. Hence, the section is safe for static forces.

5.3 Linear time history analysis of dam-reservoir-foundation system

Linear time history analysis has been carried out on dam-reservoir-foundation system using four spectrum compatible time histories which are normalized to peak ground acceleration (PGA) of 0.6 g. Two modeling techniques are adopted for the fluid, viz., fluid finite element modeling technique and Westergaard’s added mass approach (Berrabah 2011). In fluid finite element modeling, the reservoir is modeled using fluid acoustic element FLUID 29. But in Westergaard’s added mass approach, fluid is modeled using SURF 153 element. Here, the added mass per unit area is taken as

$$M_a = \frac{7}{8} \rho_w \sqrt{h(h-y)} \tag{7}$$

ρ_w is the mass density of water, h is the height of reservoir and y is the depth of the node below the surface of water. The added mass is applied on the upstream face of the dam in addition to hydrostatic pressure.

Four natural ground acceleration time histories are selected from the strong motion database available in the website of Centre for Engineering Strong Motion Data, USA (www.strongmotioncenter.org). Their response spectra are generated using Seismospect and made compatible with IS 1893:2002 design spectrum for Type II soil using Seismomatch (www.seismosoft.com). The response spectra of ground motions along with the design spectrum are shown in Fig. 5. The earthquake records used for the study are shown in Fig. 6.

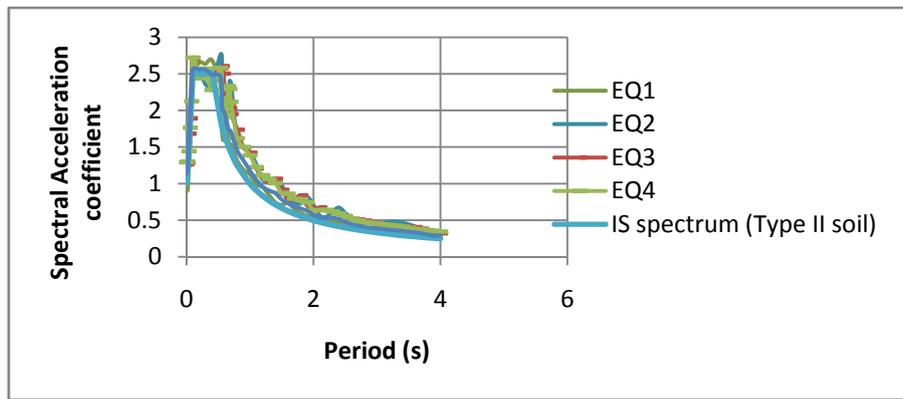


Fig. 5 Response spectra of accelerograms along with IS 1893 spectrum for Type II soil

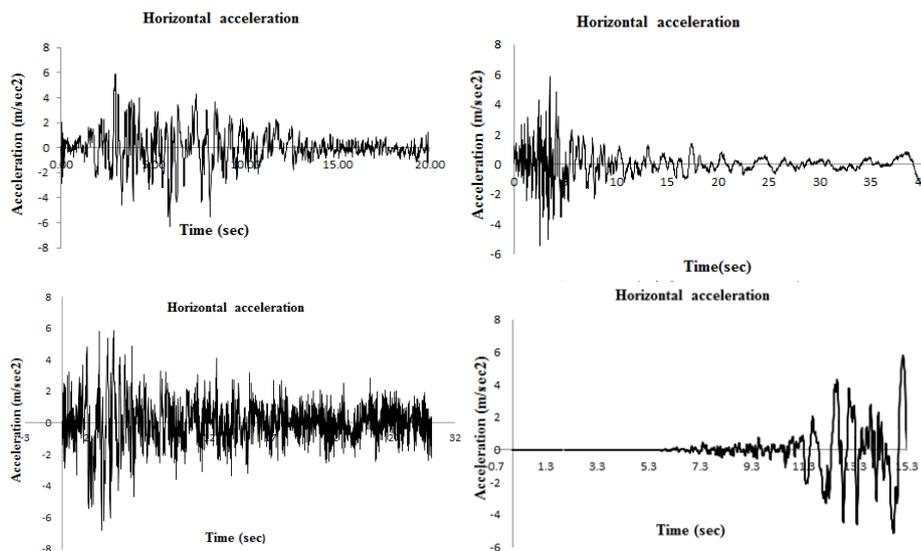


Fig. 6 Acceleration-time history of earthquake records made compatible with IS 1893 spectrum for Type II soil with a PGA of 0.6 g

Generally, two principal procedures are used for ground motion modification: *direct scaling* and *spectral matching*. The *direct scaling* procedure consists of determining a constant scale factor by which the amplitude of an accelerogram is increased or decreased. As the elastic response spectra correspond to linear response of single-degree-of freedom systems, the same scale factor applies to spectral accelerations at all periods. In contrast, *spectral matching* adjusts the frequency content of accelerograms until the response spectrum is within user-specified limits of a target response spectrum over a defined period band. The period range for spectral matching varies among code provisions.

As per ASCE 7-05, the ground motions shall be scaled such that the average value of the 5 percent damped response spectra for the suite of motions is not less than the design response spectrum for the site, for periods ranging from $0.2T$ to $1.5T$ where T is the natural period of the structure in the fundamental mode for the direction of response being analyzed. 5% damping is adopted for the concrete dam models under investigation.

The summary of peak values of responses such as crest displacement, stresses at heel etc. are shown in Fig. 7. It is found that reservoir modeling technique using Westergaards' added mass approach gives conservative values of response in terms of stresses and displacements. By adopting added mass approach, the displacement at crest increases by almost 1.85 times that obtained using FE fluid modeling technique, and the Von Mises stress at heel and neck is about 1.5 times that using FE fluid modeling technique of reservoir.

For the two fluid modeling techniques the same level of water is considered for the analysis, but the way of application is different. By fluid FE modeling technique of reservoir using FLUID29 element, the water effect is transmitted to the structural system as displacement and pressure. But, in reservoir modeling technique of added mass using SURF153 element, the water effect is applied as mass per unit area over the upstream face of dam structure, which increases the inertial force of the system, thereby attracting more seismic forces.

Maximum responses such as crest displacement (at 3.22 s), vertical stress and Von Mises stress at heel (9.1 s) due to Earthquake 1 are shown in Fig. 8.

Considering the responses due to time history analysis using four earthquakes, it is observed that maximum displacement occur at the crest of dam and higher value of stresses are found at heel of dam, neck of dam and the region opposite to neck in the upstream side.

It is also found that the maximum stress values found at these regions are above the material permissible tensile stress of 1.2 N/mm^2 and permissible compressive stress of 13 N/mm^2 . Thus

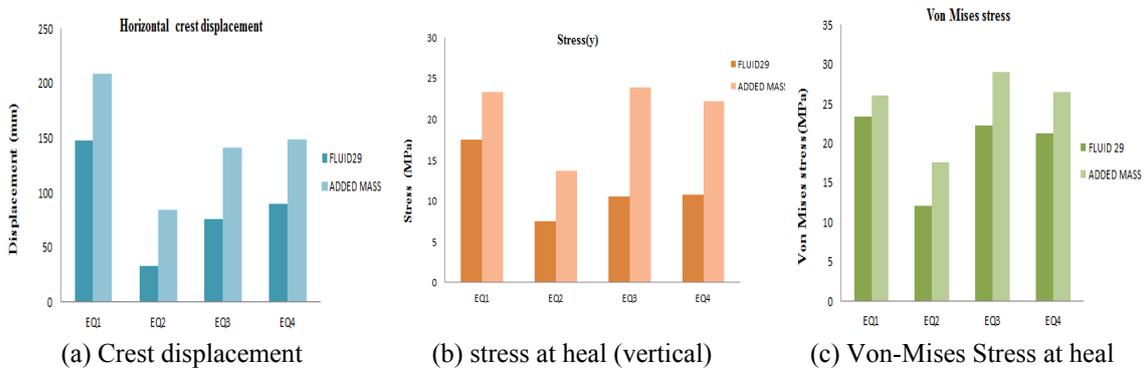
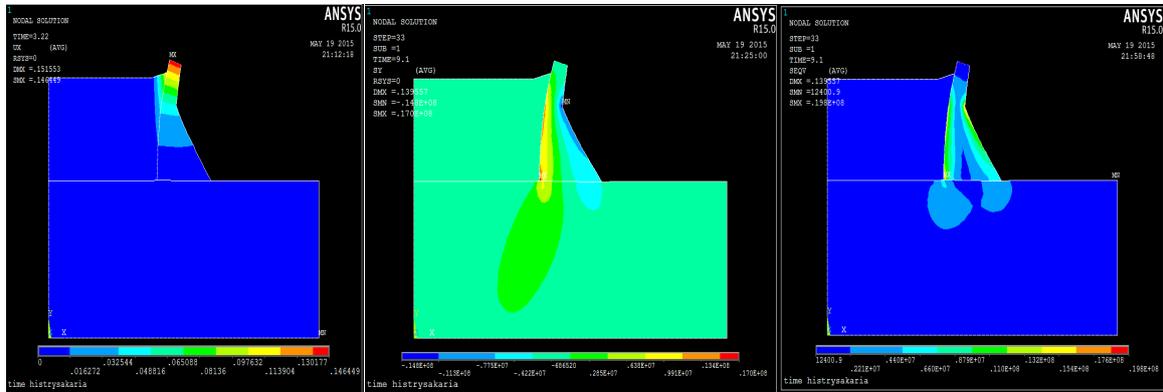


Fig. 7 Peak values of responses for different modeling techniques of reservoir



(a) Crest displacement at 3.2 s (b) stress at heal (vertical) at 9.1 s (c) Von-Mises Stress at heal at 9.1 s

Fig. 8 Peak values of responses due to earthquake 1 with fluid FE Modeling

there are chances of localized failure at these regions due to seismic action which may affect the overall stability of the dam in due course.

6. Conclusions

It is obvious that the presence of water in the reservoir, and the foundation affect the response of dams under seismic excitation. Two fluid modeling techniques are used in the present study; one is the standard added mass approach and the second one using fluid finite elements. Lagrangean approach is adopted for fluid modeling and hence, the equations of motion for the fluid and dam structures are similar. The nodes of the common boundary are constrained to be coupled in the normal direction, while movements are allowed in the tangential direction.

ANSYS uses an automated tetrahedral mechanical meshing method to produce an optimal mesh for the structural domain. Fixed support conditions as well as FE modeling of foundation are adopted as the boundary conditions. Fluid-structure interface boundaries are identified to allow for coupled mesh deformation resulting from fluid motion.

From the modal analysis results of different FE models, it can be inferred that when the reservoir is empty and foundation is fixed, the natural frequency of the dam is maximum. Furthermore, a minimum value for the fundamental frequency is obtained when dam-reservoir-foundation interaction is considered. There is about 20% decrease of modal frequency for first mode and more than 50% decrease of modal frequencies of higher modes if structure-foundation and structure-reservoir is considered.

Most dams are founded on rocks. Usually, analyses of dams are done assuming fixed support condition. However, from the present study, it is found that the dynamic response of dams is affected by the interaction of reservoir and foundation. Hence, it is necessary to consider both the dam-reservoir interaction and dam-foundation interaction for predicting the realistic behavior of dams under earthquake forces.

Reservoir modeling using Westergaards' added mass technique is easier and requires only less computing power. The analysis gives conservative values of responses compared to fluid FE modeling technique. FE modeling predicts the realistic effect of fluid-structure interaction, but it

takes more computational time.

From the time-history results, it is also observed that the response of dam may change with the type of earthquake depending on the band width and predominant frequency. Maximum displacement occurs at the crest of dam and the higher values of stresses are obtained at the heel, neck, and the region opposite to neck of the dam on the upstream side. The stress values due to seismic loading are found to be higher than the material permissible tensile and compressive stresses at these locations. Hence damages are likely to initiate and get distributed from these regions during earthquake.

References

- Akkose, M. and Simsek, E. (2010), "Non-linear seismic response of concrete gravity dams to near-fault ground Motions including Da-Water-Sediment-Foundation Interaction", *Appl. Math. Model.*, **34**(11), 3685-3700.
- Akpinar, U., Binici, B. and Arici, Y. (2014), "Earthquake stresses and effective damping in concrete gravity dams", *Earthq. Struct.*, **6**(3), 251-266.
- ASCE 7 (2005), *Minimum design loads for buildings and other structures*, American Society of Civil Engineers, USA.
- Altunisik, A.C. and Sesli, H. (2015), "Dynamic response of concrete gravity dams using different water modelling approaches: westergaard, lagrange and euler", *Comput. Concrete*, **16**(3), 429-448.
- Berrabah, T. (2011), "Modal behavior of Dam Reservoir Foundation System", *EJGE*, 1593-1601.
- Bhattacharjee, S.S. and Leger, P. (1996), "Seismic cacking and energy dissipation in concrete gravity dams", *J. Earthq. Eng. Struct. Dyn.*, **22**(11), 991-1007.
- Brijesh, S. and Agarwal, P. (2009), "Seismic response of high concrete gravity dam including dam-reservoir foundation effect", *J. South Asia Disaster Studies*, **2**, 41-57.
- BIS IS 1893 (Part 1)-2002: Indian Standard Criteria for Earthquake Resistant Design of structures, Part-1- General Provisions and Building (Fifth Revision), *Bureau of Indian Standards*, New Delhi.
- Chopra, A.K. and Gupta, S. (1982), "Hydrodynamic and foundation interaction effects in frequency response functions for concrete gravity dams", *Earthq. Eng. Struct. Dyn.*, **10**(1), 89-106.
- Hariri-Ardebilin, M.A., Seyed-Kolbadi, S.M. and Mirzabozorg, H. (2013), "A smeared crack model for seismic failure analysis of concrete gravity dams considering fracture energy effects", *Struct. Eng. Mech.*, **48**(1), 17-39.
- Hou, H. Wang, J. and Layton, A. (2012), "Numerical methods for fluid-structure interaction- A review", *Commun. Comput. Phys.*, **12**(2), 337-377.
- Khosravi, S.H. and Heydari, M.M. (2013), "Modeling of concrete dam including dam-wWater-rock interaction", *World Appl. Sci. J.*, **22**(4), 538-546.
- Khosravi, S.H. (2011), "Optimum of concrete gravity dams with dynamic constraints and considering Dam-reservoir-Foundation rock interaction", MS Thesis, Islamic Azad University of kerman.
- Lokke, A. (2013), "Earthquake analysis of concrete gravity dams", Master Thesis, Norwegian of Science and Technology.
- Lotfi, V. (2003), "Seismic analysis of concrete gravity dams by decoupled modal approach in time domain", *Electron. J. Struct. Eng.*, **3**, 102-116.
- Patra, P. (2014), "Development methodology for seismic design of concrete gravity dam", Thesis report, NIT Rourkela.
- Sances, D.J. and Gangadharan, S.N. (2010), "CFD fuel slosh modeling of Fluid-structure interaction in spacecraft propellant tanks with diaphragms", *51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Orlando, Florida.
- Sarkar, R., Paul, D.K. and Stempniewski, L. (2007), "Influence of reservoir and foundation on the nonlinear

- dynamic response of concrete gravity dams”, *J. Earthq. Technol.*, **44**(2), 377-389.
- Skrikerud, P.E. and Bachmann, H. (1986), “Discrete crack modeling for dynamically loaded unreinforced concrete structures”, *J. Earthq. Eng. Struct. Dyn.*, **14**(2), 297-315.
- Thanoon, H. (2008), “Nonlinear analysis of fracture mechanism of roller compacted concrete dam”, Ph.D. Thesis, University Putra, Malaysia.
- Valestos, A.S. and Yang, Y.J. (1976), “Dynamics of fixed base liquid storage tanks”, *Proceedings of US-Japan Seminar for Earthquake Engineering Research with Emphasis on Lifeline Systems*, Tokyo, Japan.
- Verma, G., Verma, M.K. and Tripathi, R.K. (2013), “Dynamic analysis of Hirakud Dam due to seismic forces”, *Global J. Res. Eng.*, **13**(1).