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Topology, shape, and size optimization of truss structures using modified teaching-learning based optimization

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Abstract. In this study, teaching-learning based optimization (TLBO) is improved by incorporating model of multiple teachers, adaptive teaching factor, self-motivated learning, and learning through tutorial. Modified TLBO (MTLBO) is applied for simultaneous topology, shape, and size optimization of space and planar trusses to study its effectiveness. All the benchmark problems are subjected to stress, displacement, and kinematic stability constraints while design variables are discrete and continuous. Analyses of unacceptable and singular topologies are prohibited by seeing element connectivity through Grubler's criterion and the positive definiteness. Performance of MTLBO is compared to TLBO and state-of-the-art algorithms available in literature, such as a genetic algorithm (GA), improved GA, force method and GA, ant colony optimization, adaptive multi-population differential evolution, a firefly algorithm, group search optimization (GSO), improved GSO, and intelligent garbage can decision-making model evolution algorithm. It is observed that MTLBO has performed better or found nearly the same optimum solutions.

Keywords: meta-heuristic algorithms; truss design; topology; shape, and size optimization; structural optimization

1. Introduction

A truss is a two or three-dimensional structure composed of linear members connected at nodes to sustain load and subjected to tension or compression. Truss optimization has become a fast emerging research field of structural optimization (SO) since the last two decades. SO can be separated in to three categories: topology, shape, and size (TSS). Size optimization deals with the optimum cross-sectional areas of the elements (link, plate, etc.) while shape optimization works by varying the positions of nodes or boundary contours. Topology optimization, on the other hand, works on addition and removal of elements, nodes, and material (Christensen and Klarbring 2009). Due to the presence of stress, displacement, and kinematic stability constraints, SO problems become more challenging for optimization methods. Hence, an efficient optimization method is required to solve such problems and researchers are continuously investigating in this area.

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Literature stated two different approaches to response TSS optimization. Hajela *et al.* (1993) used a two-stage approach, while Deb and Gulati (2001) used a single-stage approach to answer such problems. Moreover, the two-stage approach reduces search space but may not offer a global optimum solution, if the optimum topology does not recognize during the first stage (Wu and Tseng 2010). On the other hand, the single stage approach requires more computational efforts because it deals with simultaneous TSS, but is capable to achieve a global optimum solution (Ahrari *et al.* 2014). This work also focused on improving the efficiency of TSS by avoiding large number of unwanted analysis of kinematic instable and invalid structures. A ground structure method is adopted for this study, which is a set of all possible connection between nodes (Dorn 1964). In this method, the difficulties arise due to matrix singularity and unnecessary analysis, FEA model is reformed to resolve these difficulties. In this article, we used the single-stage approach, the ground structure method and the restructuring of FEA model for TSS optimization.

Genetic Algorithm (GA) has been applied to solve SO problems by many researchers such as; Grierson and Pak (1993), Rajan (1995), Deb and Gulati (2001), Kawamura *et al.* (2002), Tang *et al.* (2005), Balling *et al.* (2006), Richardson *et al.* (2012) with kinematic stability repair (KSR) and Rahami *et al.* (2008). On the other hand, contemporary and more auspicious meta-heuristic algorithms other than GA have been used in considerable ranges of SO. Simulated Annealing (SA) (Hasançebi and Erbatur 2002), Ant Colony Optimization (ACO) (Luh and Lin 2008), Group Search Optimizer (GSO) and Improved GSO (Li and Liu 2011), Adaptive multi-population differential evolution (AMPDE) (Wu and Tseng 2010), Harmony Search (HS) (Martini 2011), An Intelligent Garbage Can Decision-Making Model Evolution Algorithm (IGCMEA) (Kuo *et al.* 2012), Firefly Algorithm (FA) (Miguel *et al*. 2013), and Tejani *et al.* (2016a, 2016b, 2016c) have been employed in SO problems.

Teaching-learning based optimization (TLBO) is a meta-heuristic algorithm developed by Rao *et al.* (2011, 2012a) which is based on the influence of a teacher on the outcomes of learners. Rao and Patel (2012), Patel and Savsani (2014), Degertekin and Hayalioglu (2013), Camp and Farshchin (2014), Tejani *et al.* (2016b), and Savsani *et al*. (2017) introduced some progressive improvements in TLBO to enhance its exploration and exploitation capacities. Different modifications of TLBO has proved its capabilities for single and multiple objective optimization problems. The ability to improve TLBO has encouraged to formulate modified TLBO (MTLBO) for TSS problems, so in this article, MTLBO is investigated for the optimization of benchmark SO problems from literature.

The rest of article is articulated as follows: Section 2 presents the brief introduction of TLBO. Section 3 contributes improvements proposed in TLBO. Section 4 shows problem formulation. Section 5 contains testing of proposed method by series of benchmark problems and Section 6 concludes this article.

2. Teaching-learning based optimization (TLBO)

TLBO was proposed by Rao *et al.* (2011, 2012a), which is based on the natural wonder of teaching and learning philosophy. TLBO algorithm prerequisites only common governing parameters like population size and number of generations for its operation unlike GA which mutation, crossover, selection rate, etc., Particle swarm optimization (PSO) requiring inertia weight, social and cognitive parameters, Artificial bee colony (ABC) depending on number of employed and onlooker bees, HS requiring harmony memory rate, pitch adjusting rate and

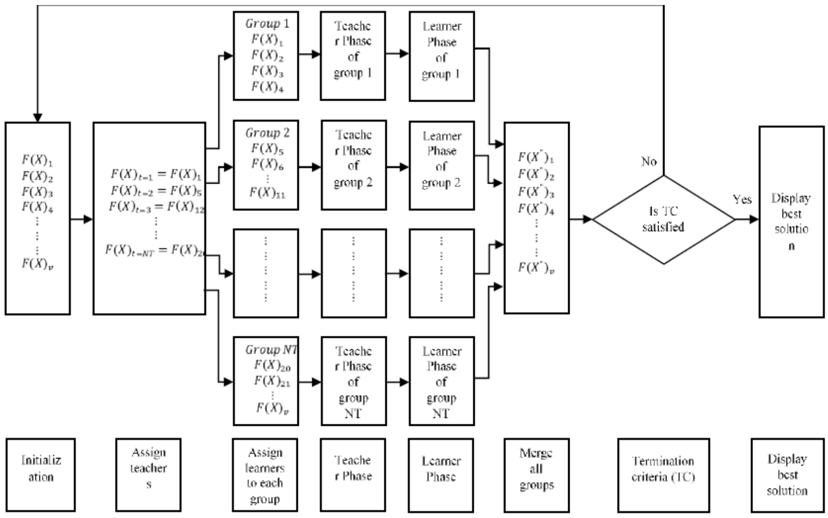


Fig. 1 Schematic diagram of the MTLBO algorithm

improvisation rate (Rao *et al.* 2012b). In this way, TLBO is a parameter-free, population-based meta-heuristic algorithm.

The quality of a teacher has a strong impact on the learners in a class. A good teacher inspires the learners and assists them to refine their knowledge. Hence, each learner follows the teacher and improves his or her knowledge. Similarly, each learner also interacts with other learners of the class to improve knowledge. The functioning of TLBO is alienated into two phases, 'teacher phase' and 'learner phase'. TLBO can be summarized as follows:

Step I: Define the problem and initialize the optimization parameters.

Step II: Initialize the population and define termination criterion.

Step III: Teacher phase: Select the best solution as a teacher, who assists the learners in the class to improve their performance.

Step IV: Learner phase: Learners improves their grades by mutual interaction among themselves.

3. Modified-teaching learning based optimization (MTLBO)

In the TLBO algorithm, a teacher and interactive learners improve knowledge of the learners by traditional classroom teaching. Improvements like a number of teachers (NT), an adaptive teaching factor, a self-motivated learning, and a learning through tutorial are described as follows:

3.1 Number of teachers (NT)

The original TLBO algorithm is based on a single teacher, who teaches the learners and tries to improve their knowledge. It may be possible that the efforts of the teacher scattered or at the other end, students might be less responsive, which will decrease the possibility of learning. To response this problem, TLBO is improved by introducing multiple teachers for the group of learners. In this modification, the entire class is divided into diverse groups of learners based on their grades and a discrete teacher is assigned to each group of learners. Hence, each teacher works to improve the grades of assigned learners and if the performance of the group reaches up to the assigned teacher, this group is allocated to a superior teacher. Mathematical form of this modification is given in Eqs. (1)-(2).

3.2 Adaptive teaching factor

In the basic TLBO, the teaching factor is decided through heuristic step and it can be either one or two. This means learners learn nothing or all the things taught by the teacher. However, in actual practice learners may learn in any proportion from the teacher. Hence, teaching factor (TF) is improved to an adaptive teaching factor, is given by Eq. (4).

3.3 Learning through tutorial

In modern teaching-learning activity, learners assign interactive projects, problems and tutorials during the tutorial hours. Accordingly, learners learn by discourse over with their fellow learners and even with the teacher while solving it. Since the learners can improve their knowledge by discussion with other learners or the teacher, this search mechanism is merged into the teacher phase of TLBO. This modification is given in Eq. (5).

3.4 Self-motivated learning

In TLBO, the grades of the learners are improved either by learning from the teacher or by interacting with the other learners. However, self-motivated learners may improve their knowledge by self-learning. Thus, the self-motivated learning to advance the knowledge of learners is adopted in the improved TLBO algorithm, which again upsurges the exploration and exploitation capacity of TLBO. This modification is shown by Eq. (6).

Graphical representation of the MTLBO algorithm is shown in Fig. 1.

MTLBO can be summarized as follows.

Step I: Define the problem and initialize the optimization parameters.

Step II: Initialize the population and define termination criterion.

Step III: Select the best solution as a chief teacher.

$$F(X)_{best} = F(X)_1,\ Hence\ F(X)_{chief\ teacher} = F(X)_{best} \tag{1}$$

Step IV: Select the other teachers based on the chief teacher and rank them.

$$F(X)_t = F(X)_1 - rand * F(X)_1 \tag{2}$$

where, other teachers (t) = 2,3,…,NT

Step V: Assign the group of learners to the teachers according to their fitness value.

Step VI: Keep the elite solutions of each group.

Step VII: Determine the mean result of each group of learners in each subject (i.e., $(M_u)_t$), where u is subjects offered to each learner.

Step VIII: For each group, calculate the difference between the current mean (DM) and the corresponding result of the teacher of that group for each subject with the use of the adaptive teaching factor.

$$DM_u = r*(X_u - T_F*M_u) \tag{3}$$

$$(T_F)_g = \begin{cases} \left(\dfrac{X_{total-v}}{X_{total-vbest}}\right)_g ; if \rightarrow X_{total-vbest,g} \neq 0 \\ 1, if \rightarrow X_{total-vbest,g} = 0 \end{cases} \tag{4}$$

Where, generation number = g; learners = v; subjects = u

Step IX: Tutorial hours, each group update the learners' knowledge with the help of the teacher's knowledge.

$$\left(X'_{u,v}\right)_t = \begin{cases} \left(X_{u,v} + DM_u\right)_t + rand(X_q - X_v)_t, if F(X)_q < F(X)_v \\ \left(X_{u,v} + DM_u\right)_t + rand(X_v - X_q)_t, if F(X)_q > F(X)_v \end{cases} \tag{5}$$

Where, q is randomly selected learner; $q \neq v$

Step X: For each group, update the learners' knowledge by utilizing the knowledge of some other learners and by self-learning.

$$\left(X''_{u,v}\right)_t = \begin{cases} X'_{u,v} + rand(X'_{u,v} - X'_{u,p})_t + rand(X_{teacher} - E_F X'_{u,v})_t, if F(X)_q < F(X)_v \\ X'_{u,v} + rand(X'_{u,p} - X'_{u,v})_t + rand(X_{teacher} - E_F X'_{u,v})_t, if F(X)_q > F(X)_v \end{cases} \tag{6}$$

Where, E_F =exploration factor = round (1+rand); p is randomly selected learner; $p \neq v$

Step XI: Replace the worst solution of each group with an elite solution.

Step XII: Remove the identical solutions randomly.

Step XIII: Merge all the groups.

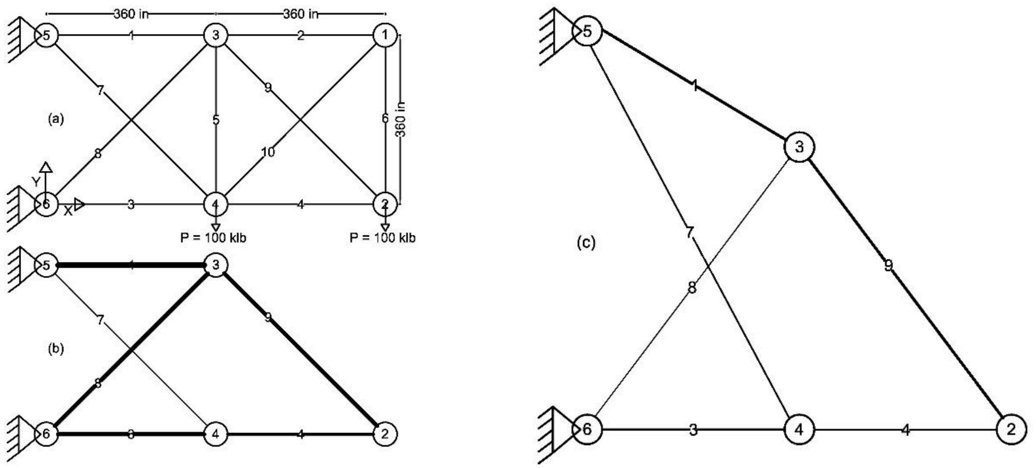
Step XIV: Termination criterion: Repeat the procedure from Step III until the termination criterion is satisfied.

4. Problem formulation

The objective of this study is to minimize the weight of a truss, $f(X)$ by satisfying all stated constraints. In addition, a constraint removal method is applied to handle impact of the removed elements on constraints. Formulation of an optimization problem employed by Deb and Gulati (2001) is adopted in this study for rational comparison. SO problem is formulated as follows

$$\begin{gathered} \text{Find, } X = \{A_1, A_2, \ldots, A_m, \zeta_1, \zeta_2, \ldots, \zeta_n\} \\ \text{to minimize }, f(X) = \sum_{i=1}^{m} B_i A_i \rho_i L_i \\ Where, B_i = \begin{cases} 0; if A_i < \text{Critical area} \\ 1; if A_i \geq \text{Critical area} \end{cases} \\ \text{Subjected to} \\ g_1 : \text{Truss is acceptable to the user} \\ g_2 : \text{Truss is kinematic stable} \\ g_3 : \text{Stress constraints}, \left|B_i \sigma_i\right| - \left|\sigma_i^{\max}\right| \leq 0 \\ g_4 : \text{Displacement constraints}, \left|\delta_j\right| - \left|\delta_j^{\max}\right| \leq 0 \\ g_5 : \text{Size constraints}, A_i^{critical} \leq A_i \leq A_i^{upper} \\ g_6 : \text{Shape constraints}, \zeta_j^{lower} \leq \zeta_j \leq \zeta_j^{upper} \\ \text{where, } 1,2, \ldots, \text{m and j} = 1,2, \ldots, \text{n} \end{gathered} \tag{7}$$

Where, A_i, ρ_i, L_i, E_i, *and* σ_i denote cross-sectional area, density, length, modules of elasticity



(a) (b)

Fig. 2 10-bar truss: (a) Ground structure, (b) Size and topology optimized truss, and (c) TSS optimized truss

and stress of element ‘i’ respectively. δ_j *and* ξ_j are real value of nodal displacements and coordinates of node ‘*j*’ respectively. B_i is a topological bit, which is 0 for absence and 1 for presence of an element ‘*i*’. The superscript, ‘max’ and ‘min’ signifies maximum and minimum allowable values respectively. The upper and lower bond of the design variables are selected in such a way that, $A^{lower} = -A^{max}$ and $A^{upper} = A^{max}$ to offer equal probability for the element existence and the critical area is a user-defined term, which is used to eliminate the element if the area is less than critical area. While the discrete design variables may take any integer values within [$-D$, D], where, D = total available discrete areas, the positive integer of design variable denotes element cross-sectional number and zero or negative integer of design variable signifying removal of the element.

A truss structure is called invalid (g_1) if truss is having the absence of loaded nodes, support nodes, and undeleted nodes (Li and Liu 2011). In this article, kinematic stability (g_2) is reviewed in two steps as per Deb and Gulati (2001) and the steps of this method are listed as follows:

Step (I). Grubler's criterion (Ghosh and Mallik 1986) to examine Degree of Freedom (DOF) of the truss and

Step (II). Positive definiteness of global stiffness matrix to examine singularity of the truss.

Penalty function approach is applied to handle all stated constraints. For no violation of the constraints, the penalty becomes zero; otherwise, penalty is intended by following criteria

$$f_{Penalized}(X) = \begin{cases} 10^9 \text{ if } g_1 \text{ is violated} \\ 10^8 \text{ if } g_2 \text{ is violated with DOF} \\ 10^7 \text{ if } g_2 \text{ is violated with positive definiteness} \\ f(X) + 10^5 * \sum\left(\left|\langle g_3(X)\rangle\right| + \sum\left|\langle g_4(X)\rangle\right|\right) \text{ otherwise} \end{cases} \quad (8)$$

5. Benchmark problems

This section introduces eight widely studied benchmark problems to evaluate the effectiveness

Table 1 Design parameters of the 10-bar truss

Design variables: A_i, y_1, y_3, y_5; where i=1,2,…,10
Loading condition: P_{y2} = –100 Klb, P_{y4} = –100 Klb
Stress and displacement constraints: σ_i^{max} = *25 Ksi*, δ_j^{max}= 2 in Where j =1, 2, 3, and 4
Shape constrains: 180 in ≤ y_1, y_3, y_5 ≤ *1000 in*
Size variables: Continuous area: (0.09, 30) in^2; Discrete area: (1, 30) in^2 in increments of 1 in^2

Table 2 Sensitivity analysis of the population size and maximum number of generations

Method	Population size	Maximum number of generations	Min	Max	Mean	SD
MTLBO	20	100	4898.7570	7998.3790	5248.5116	759.9478
MTLBO	25	80	4899.1370	7654.6020	5318.3561	766.3081
MTLBO	50	40	4898.4330	6259.4960	4960.1750	267.8459
MTLBO	100	20	4906.2140	7024.8300	5006.3941	241.7650

of MTLBO algorithm. Material properties such as modulus of elasticity (E) and weight density (ρ) of the elements are assumed as 10^4 Ksi and 0.1 lb in^{-3} respectively for all problems. MTLBO and TLBO algorithms are functioned for 100 independent runs to consider stochastic nature of a meta-heuristic and the best results are stated. All problems are optimized by doing comparison among the combinations of the population size, number of generations and NT to find the best combination in the proposed algorithms. All computer-generated results are compared with state-of-the-art algorithms reported in the literature. The programs are coded in MATLAB (R2013a) software and the FEM. A quad core 2 GHz CPU is used to quantify the computational effort in terms of CPU time in seconds (s) for this work.

5.1 10-bar truss

The ground structure of the first benchmark truss with load and boundary conditions is presented in Fig. 2(a). The design parameters such as design variables, loading condition, constraints and design variable bond are tabulated in Table 1. This is a well-known benchmark problem in this field and has been studied by many researchers by considering continuous and discrete areas. Moreover, this work considered three independent conditions for this problem described as follows.

5.1.1 Size and topology optimization of 10-bar truss using continuous design variables

In this problem, the aim is to do size and topology optimization of the 10-bar truss using ten contentious size variables. The best suited population size and maximum number of generations are investigated based on sensitivity analysis. Thus, the 10-bar truss problem is analysed for population size as 20, 25, 50, 100 and maximum number of generations as 100, 80, 40, 20 by assuming NT as 5. The results shown in Table 2 revels the proposed algorithm performs the best with the population size and the maximum number of generations being 50 and 40 respectively. Therefore, the population size and the maximum number of generations are considered as 50 and

Table 3 Comparison of optimized designs found for size and topology of the 10-bar truss with continuous size variables

	Proposed work		Deb and Gulati (2001)	Luh and Lin (2008)	Kuo *et al.* (2012)
Design variable	MTLBO	TLBO	GA	ACO	IGCMEA
A_1	29.9996	29.9675	29.68	29.81	27.98
A_3	22.3087	22.1303	22.07	22.24	22.21
A_4	15.1592	15.0880	15.3	15.15	15.58
A_7	6.0709	6.0817	6.09	6.08	5.93
A_8	21.1883	21.3281	21.44	21.329	22.65
A_9	21.2484	21.2972	21.29	21.24	21.14
Weight (lb)	4898.4330	4898.4412	4899.15	4899.11	4899.12
Max stress (ksi)	23.295	23.2535	23.222	23.26	24.37
Max disp. (in)	2	2	1.9999	1.9999	1.99
FE	4000	4000	49500	41000	270000

Table 4 Statistical results of the 10-bar truss with continuous size variables

Method	NT	Min	Max	Mean	SD	Mean Time (s)
TLBO	-	4898.4410	7997.3330	5166.0800	681.0892	10.6085
MTLBO	1	4898.4830	6260.5640	4930.7950	173.0177	13.9259
MTLBO	2	4898.5200	6272.8820	4947.6170	227.8065	13.5532
MTLBO	3	4898.5970	6741.4430	4948.9930	263.4199	13.6562
MTLBO	4	4898.6680	7490.7220	5011.0960	446.3459	13.4585
MTLBO	5	4898.4330	6259.4960	4960.1750	267.8459	13.1733

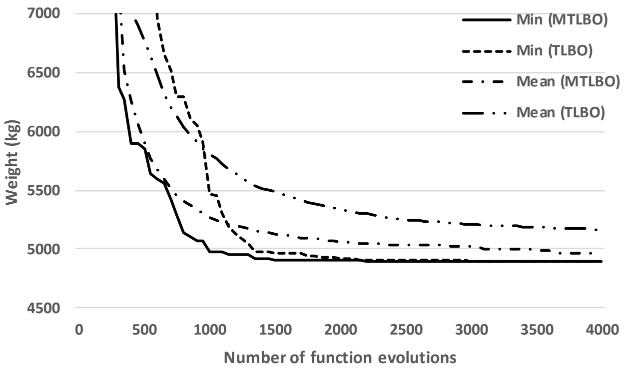


Fig. 3 Schematic diagram of MTLBO algorithm

40 respectively. In this problem, the effect of NT is investigated by considering the values from 1 to 5. The results are obtained for 100 independent runs and the best result obtained in these runs are displayed in Table 3. The result table summarizes that MTLBO and TLBO designed the optimum weight of 4898.433 lb and 4898.4412 lb respectively. The results show that MTLBO and

Table 5 Comparison of optimized designs found for size and topology of the 10-bar truss with continuous size variables

	Proposed work		Hajela and Lee (1995)	Deb and Gulati (2001)	Richardson *et al.* (2012)	Miguel *et al.* (2013)
Design variable	MTLBO	TLBO	GA	GA	KSR & GA	FA
A_1	30	30	28	30	30	30
A_3	24	24	24	24	24	24
A_4	16	16	16	16	16	16
A_7	6	6	6	6	6	6
A_8	20	20	21	20	20	20
A_9	21	21	22	21	21	21
Weight (lb)	4912.849	4912.849	4942.7	4912.849	4912.849	4912.85
FE	1500	3100	-	49500	15400	30000

Table 6 Comparison of optimized designs found for TSS of the 10-bar truss

	Proposed work		Tang *et al.* (2005)	Rahami *et al.* (2008)	Miguel *et al.* (2013)
Design variable	MTLBO	TLBO	Improved GA	Force method & GA	FA
A_1	11.1443	11.9136	13.5	11.5	11.5
A_3	9.9176	9.2231	7.97	11.5	11.5
A_4	7.1067	6.1069	7.22	5.74	7.22
A_5	-	-	1.62	-	-
A_7	5.8943	5.9454	4.49	5.74	5.74
A_8	3.4374	3.6882	3.13	3.84	2.88
A_9	12.6447	12.8588	13.5	13.5	13.5
y_3	520.1796	551.0741	527.9	506.42.3	
y_5	828.7986	826.7102	888.8	789.7306	
Weight (lb)	2691.283	2727.159	2813.8	2723.05	2705.16
Max stress (Ksi)	18.497	18.3453	18.5	19.1463	19.1
Max disp. (in)	2	2	1.9998	2	2
FE	5000	5000	4400	4000	50000

TLBO reported better results than former results reported in the literature with no violation of constraints. In addition, MTLBO and TLBO consumed only 4000 FE, which is much lesser compared with references shown in Table 3. Moreover, MTLBO shown better results than TLBO. It should be noted that all approaches have identified the same topology as depicted in Fig. 2(b).

Fig. 3 equates the convergence graph of best and mean weight obtained using MTLBO and TLBO. It is observed from the convergence graph of MTLBO that the min and mean results of the objective function converge nearly within 2000 FE and 3000 FE respectively. It also indicates that MTLBO converges much faster than TLBO. Table 4 presents the relative statistical results of

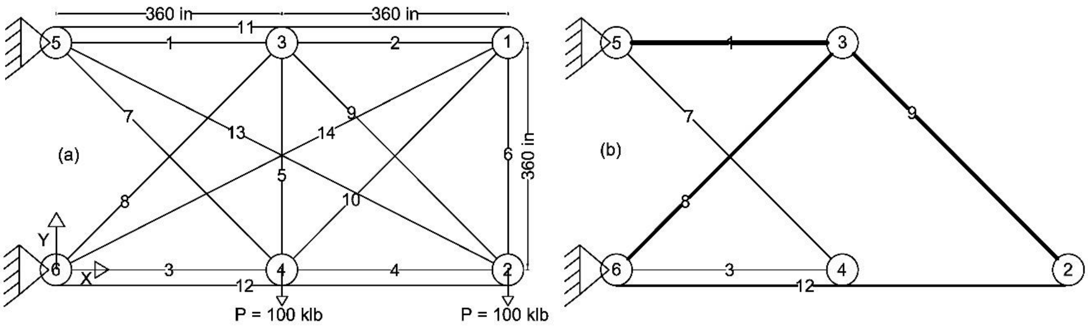


Fig. 4 14-bar planar truss: (a) Ground structure and (b) Size and topology optimized truss

Table 7 Comparison of optimized designs found for size and topology of the 14-bar truss

	Proposed work		Deb and Gulati (2001)		Luh and Lin (2008)	Wu and Tseng (2010)
Design variable	MTLBO	TLBO	GA	GA	ACO	AMPDE
A_1	28.8651	28.8651	28.286	28.189	28.876	29.046
A_3	5.4346	5.4346	5.172	5.219	5.428	5.533
A_7	7.6126	7.6126	7.821	7.772	7.617	7.545
A_8	20.3519	20.3519	20.054	20.31	20.549	20.549
A_9	20.3241	20.3241	20.446	20.65	20.265	20.65
A_{12}	14.4055	14.4055	14.845	14.593	14.308	14.342
Weight (lb)	4730.438	4730.5099	4733.443	4731.65	4730.824	4730.68
Max stress (Ksi)	18.5773	18.7117	-	19.161	18.423	18.701
Max disp. (in)	2	2	-	2	2	2
FE	10000	10000	-	85050	41000	40000

MTLBO and TLBO obtained. It is identified from the assessment that all statistical results of MTLBO are superior to TLBO. However, TLBO required less computational time than MTLBO. In this investigation, best solutions (min), worst solutions (max), average solutions (mean) and standard deviation (SD) are nearly similar, however optimal value of NT is empirical in nature.

5.1.2 Size and topology optimization of 10-bar truss using discrete design variables

In this problem, the aim is to do size and topology optimization of the 10-bar truss using ten discrete size variables as shown in Table 1. The results are obtained for 100 independent runs and the best results obtained in these runs are shown in Table 5 and compared with those found in the previous studies. The results illustrate that MTLBO and TLBO required 1500 FE (only 5.86% FE used by Deb and Gulati 2001) and 3100 FE respectively with similar optimum weight of 4912.849 lb. Topology obtained using the proposed method is depicted in Fig. 2(b), which is similar to the previous study.

5.1.3 TSS optimization of 10-bar truss

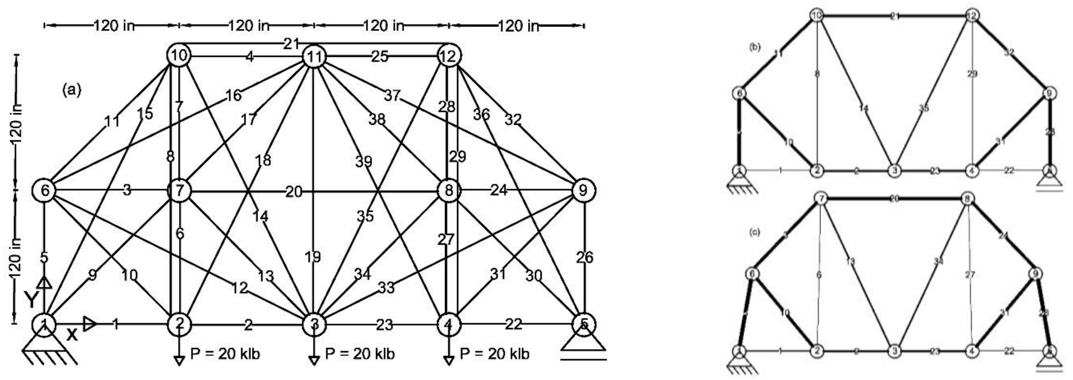


Fig. 5 39-bar truss: (a) Ground structure, (b) Size and topology optimized truss and (c) TSS optimized truss

Table 8 Design parameters of the 39-bar two-tier truss

Design variables: G_i, $x_6 = -x_9$, $y_6 = -y_9$, $x_7 = -x_8$, $y_7 = -y_8$, $x_{10} = -x_{12}$, $y_{10} = -y_{12}$, y_{11}; where i=1,2,…,21
Loading condition: $P_{y2} = -20\ Klb$, $P_{y3} = -20\ Klb$, $P_{y4} = -20\ Klb$
Stress and displacement constraints: $\sigma_i^{max} = 20\ Ksi$, $\delta_j^{max} = 2\ in$
Shape constrains: -120 in $\leq \xi(x, y) \leq 120$ in for nodes 6 to 12 with respect to the origin.
Size variables: Continuous area: (0.05, 2.25) in^2

In this problem, the 10-bar truss is considered for TSS using ten continuous size and three continuous shape variables as depicted in Table 1. In this problem, proposed algorithms are measured for population size and maximum generations as 50. Table 6 compares the final structure found by different methods together with the corresponding weight for 100 independent runs. Table 6 summarizes that MTLBO and TLBO give trusses with optimum weight of 2691.283 lb and 2727.159 lb respectively. The results show that MTLBO and TLBO reported better results than the former results reported in literature with no violation of constraints. However, the results of TLBO is slightly heavier than results of Force method & GA and FA. In addition, MTLBO and TLBO used 5000 FE. MTLBO and TLBO have identified the same topology as depicted in Fig. 2(c).

5.2 14-bar truss

The ground structure of the second benchmark truss is presented in Fig. 4(a). This problem is considered for size and topology optimization using fourteen continuous size variables. In this problem, population size and maximum generations are considered as 50 and 100 respectively, this results in 10000 FE. Table 7 compares the best designs found for 100 independent runs. Table 7 shows that MTLBO and TLBO give the best weight of 4730.438 lb and 4730.5099 lb respectively. The results show that MTLBO and TLBO reported better results than the former results reported in literature without violation of the constraints. It should be noted that all approaches reported identical topology as depicted in Fig. 4(b).

5.3 39-bar two-tier truss

The ground structure of the third benchmark truss with loads and boundary conditions is shown

Table 9 Comparison of optimized designs found for size and topology of the 39-bar truss

	Proposed work		Deb and Gulati (2001)		Luh and Lin (2008)		Wu and Tseng (2010)	Miguel *et al.* (2013)
Design variable	MTLBO	TLBO	GA	GA	ACO	ACO	AMPDE	FA
A_1, A_{22}	0.0502	0.1652	0.050	-	0.051	0.050	0.05	0.0500
A_2, A_{23}	0.7501	0.7720	1.001	0.751	0.751	1.001	0.750	0.7524
A_3, A_{24}	-	-	0.050	0.051	-	-	-	-
A_5, A_{26}	1.5000	1.4424	1.501	1.502	1.502	1.500	1.500	1.5001
A_7, A_{28}	-	-	-	0.052	-	-	-	-
A_8, A_{29}	0.2501	0.3079	-	0.251	0.250	-	0.250	0.2504
A_9, A_{30}	-	-	0.052	0.051	-	-	-	-
A_{10}, A_{31}	1.0607	1.0062	1.416	1.061	1.062	1.415	1.060	1.0647
A_{11}, A_{32}	1.0607	1.0180	-	1.063	1.063	-	1.060	1.0612
A_{12}, A_{33}	-	-	-	-	-	0.053	-	-
A_{13}, A_{34}	-	-	0.050	-	-	-	-	-
A_{14}, A_{35}	0.5591	0.5594	-	0.559	0.560	-	0.559	0.5604
A_{15}, A_{36}	-	0.1136	-	-	-	-	-	-
A_{16}, A_{37}	-	-	1.118	-	-	1.119	-	-
A_{19}	-	-	1.002	-	-	1.002	-	-
A_{21}	1.0000	1.0071	-	1.005	1.000	-	1.000	1.0016
Weight (lb)	193.22	200.883	198	196.546	193.474	196.195	193.199	193.547
Max stress (Ksi)	19.9997	19.9869	20	20	20	20	20	19.999
Max disp. (in)	1.4400	1.4427	-	-	-	-	-	-
FE	20000	20000	-	-	303600	303600	32300	50000

in Fig. 5(a). The design parameters are tabulated in Table 8. The elements are grouped into 21 groups (G) by considering symmetry about the middle vertical plane. In this problem, the proposed algorithms consider controlling parameters are as population size of 100 and the maximum number of generations is 100. To check effectiveness of MTLBO on this large-scale problem, two independent conditions considered as described below:

5.3.1 Size and topology optimization of 39-bar two-tier truss

In this problem, the 39-bar truss considered for size and topology optimization by considering twenty-one continuous size variables. Table 9 shows the design variables and corresponding truss weights for the best designs over 100 runs obtained by this study. Table 9 illustrates that that MTLBO and TLBO ending the best weight of 193.22 lb and 200.883 lb respectively. The results show that MTLBO reported better results than the former results reported in literature without violation of the constraints; however, the MTLBO result is a bit heavier than the results of AMPDE. Moreover, TLBO generated heavies truss among the tabulated results. MTLBO and TLBO consumed only 20000 FE, while AMPDE used 32300 FE. Final topology generated using

Table 10 Comparison of optimized designs found for TSS for the 39-bar truss with continuous size variables

	Proposed work		Deb and Gulati (2001)	Luh and Lin (2008)	Wu and Tseng (2010)	Miguel *et al.* (2013)
Design variable	MTLBO	TLBO	GA	ACO	AMPDE	FA
A_1, A_{22}	0.2670	0.3803	0.595	0.327	0.163	0.2947
A_2, A_{23}	0.9485	1.1110	1.166	1.095	1.509	1.0648
A_3, A_{24}	1.1413	1.1220	-	-	-	-
A_4, A_{25}	-	-	-	-	-	1.1914
A_5, A_{26}	1.5330	1.6524	1.615	1.538	0.895	-
A_6, A_{27}	0.1116	-	-	-	-	-
A_7, A_{28}	-	1.9231	-	-	-	-
A_8, A_{29}	-	-	0.051	0.081	0.170	1.2555
A_{10}, A_{31}	1.1383	1.2475	1.155	1.221	1.123	-
A_{11}, A_{32}	-	0.0661	0.504	1.259	1.135	-
A_{12}, A_{33}	-	0.0523	-	-	-	-
A_{13}, A_{34}	0.5376	0.5405	-	-	-	-
A_{14}, A_{35}	-	-	1.293	0.525	0.543	0.0505
A_{15}, A_{36}	-	-	-	-	-	1.5324
A_{19}	-	-	-	-	-	0.9609
A_{20}	1.1478	1.8097	-	-	-	-
A_{21}	-	0.9586	1.358	1.256	1.106	-
A_6	21.7303	26.8357	49.05	24.02	12.804	
y_6	124.5157	127.1930	124.11	111.53	118.052	
x_7	157.2741	238.3424	-	-	-	
y_7	209.9612	238.0225	-	-	-	
x_{10}	-	238.3456	183.54	179.38	155.519	
y_{10}	-	231.6299	177.43	191.77	216.909	
Weight (lb)	188.6255	206.494	192.19	188.732	188.426	191.304
Max stress (Ksi)	19.9968	19.9072	19.989	19.992	20	19.983
Max disp. (in)	1.3777	1.5141	-	-	-	-
FE	20000	20000	504000	453600	137200	50000

MTLBO is illustrated in Fig. 5(b).

5.3.2 TSS optimization of 39-bar two-tier truss

In this problem, the aim is to perform TSS optimization of the two-tire truss by means of twenty-one continuous size and seven shape variables. Table 10 illustrates the results obtained by the proposed algorithms and those of the previously reported results. The results signify that MTLBO and TLBO designed the optimum weight of 188.6255 lb and 206.494 lb respectively. The results show that MTLBO reported better results than the former results reported in literature with

Table 11 Design parameters of the 25-bar truss

Design variables: G_i, x_4, y_4, z_4, x_8, y_8 where i=1,2,…,8
Loading condition: P_{x1} = 1 *Klb*, $P_{y1} = P_{z1} = P_{y2} = P_{z2}$ –10 *Klb*, P_{x3} = 0.5 *Klb*, P_{x6} = 0.6 *Klb*
Stress and displacement constraints: $\sigma_i^{max} = 40\ Ksi$, δ_j^{max} = 0.35 *in* of nodes 1,2,…,6 in all directions
Shape constrains:
20 $in \le x_4 = x_5 = -x_3 = -x_6 \le 60$ *in*
40 $in \le y_3 = y_4 = -y_5 = -y_6 \le 80$ *in*
90 $in \le z_3 = z_4 = z_5 = z_6 \le 130$ *in*
40 $in \le x_8 = x_9 = -x_7 = -x_{10} \le 80$ *in*
100 $in \le y_7 = y_8 = -y_9 = -y_{10} \le 140$ *in*
Size variables: Discrete area: [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4] in^2

Table 12 Element grouping and nodal coordinates of the 25-bar and 39-bar trusses

Node numbers (coordinates)	
1(-37.5,0,200), 2(37.5,0,200), 3(-37.5,37.5,100), 4(37.5,37.5,100), 5(37.5,-37.5,100), 6(-37.5,-37.5,100), 7(-100,100,0), 8(100,100,0), 9(100,-100,0), 10(-100,-100,0) in	
Member (end nodes)	
Groups	Member (end nodes)
G_1	1(1-2)
G_2	2(1-4), 3(2-3), 4(1-5), 5(2-6)
G_3	6(2-5), 7(2-4), 8(1-3), 9(1-6)
G_4	10(3-6), 11(4-5), 12(3-4), 13(5-6)
G_5	14(3-10), 15(6-7), 16(4-9), 17(5-8)
G_6	18(3-8), 19(4-7), 20(6-9), 21(5-10)
G_7	22(3-7), 23(4-8), 24(5-9), 25(6-10)
G_8	26(5-7), 27(6-8), 28(3-9), 29(4-10)
G_9	30(3-5), 31(4-6)
G_{10}	32(1-7), 33(1-10), 34(2-9), 35(2-8)
G_{11}	36(2-7), 37(2-10), 38(1-8), 39(1-9)

no violation of the constraints; however, truss generated using MTLBO is a bit heavier than AMPDE. It should be noted that MTLBO and TLBO consumed only 20000 FE, while AMPDE used 137200 FE. The optimized truss by using MTLBO is depicted in Fig. 5(c).

5.4 25-bar 3-D truss

A 25-bar space truss is considered as the fourth benchmark truss. The ground structure of this truss is displayed in Fig. 6(a). This truss is considered for TSS optimization using eight discrete size and five continuous shape variables. The design parameters are presented in Table 11. The element are clustered into eight groups (G_i,i=1,2,…,8) by considering symmetry in both the *x-z* and y-z planes as specified in Table 12. MTLBO and TLBO are functioned for population size and number of generations as 50 and 100 respectively. Therefore, the maximum number of FE is of 10000. The results are obtained for 100 independent runs and best results obtained in this runs are

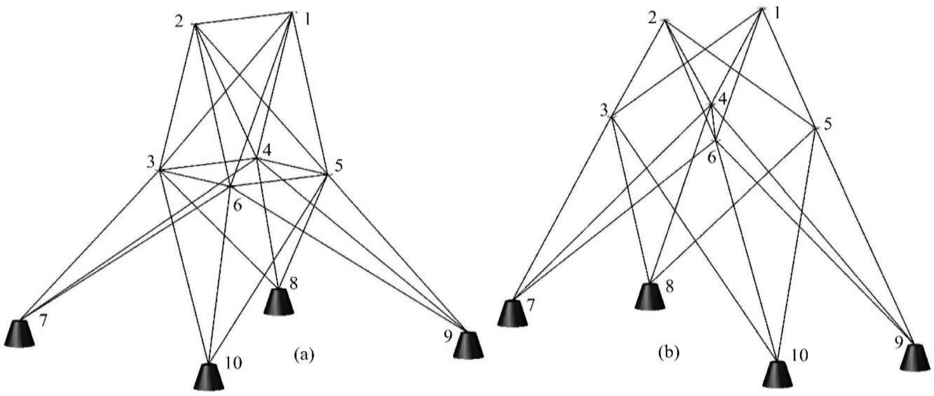


Fig. 6 25-bar space truss: (a) Ground structure and (b) TSS optimized truss

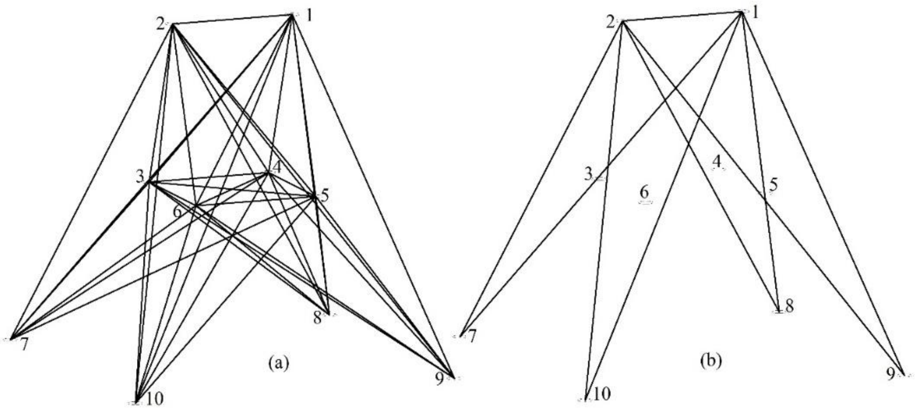


Fig. 7 39-bar space truss: (a) Ground structure and (b) Size and topology optimized truss

tabulated in Table 13. The results state that MTLBO and TLBO designed the optimum weight of 114.3473 lb and 117.5496 lb respectively. It shows that MTLBO reported better results than former results reported in literature without violation of the constraints. The final geometry from using MTLBO is illustrated in Fig. 6(b).

5.5 39-bar 3-D truss

For the fifth benchmark truss, the 39-bar 3-D truss is considered for size and topology optimization. The ground structure of this truss is shown in Fig. 7(a). The design parameters are presented in Table 14. The elements of the space truss are clustered into 11 groups (G_i,i=1,2,…,11) by considering symmetry in both the x-z and y-z planes as specified in Table 12. MTLBO and TLBO are functioned for population size and number of generations of 50, thus FE is 5000. Table 15 compares the final trusses found by the different methods. As per the result table, it is observed that MTLBO and TLBO designed the optimum weight of 2.3759 lb and 2.4534 lb respectively. The results show that MTLBO reported better results than the former results reported in literature without constraints violation. The final geometry from using MTLBO is illustrated in Fig. 7(b).

Table 13 Comparison of optimized designs found for TSS of the 25-bar truss

	Proposed work		Tang *et al.* (2005)	Li and Liu (2011)		Rahami *et al.* (2008)	Miguel *et al.* (2013)
Design variable	MTLBO	TLBO	Improved GA	DT and GSO	DT and IGSO	Force method and GA	FA
G_1	-	0.1	-	0.1	-	-	-
G_2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
G_3	0.9	1.0	0.9	1.1	1	0.9	1.1
G_6	0.1	0.1	0.1	0.1	0.1	0.1	0.1
G_7	0.1	0.1	0.1	0.2	0.2	0.1	0.1
G_8	1.0	1.0	1	0.9	0.9	1	0.9
x_4	38.7715	37.4817	39.91	33.743	36.026	38.7913	38.5
y_4	48.4319	47.0827	61.99	50.597	59.044	66.111	64.35
z_4	112.3517	109.9540	118.23	128.847	20.085	112.9787	112.87
x_8	66.5760	65.1895	53.13	42.5	46.717	48.7924	49.13
y_8	139.0357	132.6323	138.49	128.956	134.817	138.891	134.94
Weight (lb)	114.3473	117.5496	114.74	120.455	118.2341	114.3701	116.58
Max stress (Ksi)	17.52	15.5879	17.353	-	15.7831	17.7531	19.7911
Max disp. (in)	0.35	0.35	0.35	-	0.35	0.35	0.35
FE	10000	10000	6000	-	-	10000	60000

Table 14 Design parameters of the 39-bar 3-D truss

Design variables: G_i, where i=1,2,…,11
Loading condition: $P_{z1} = P_{z2} = -0.5$ *Klb*
Stress and displacement constraints: $\sigma_i^{max} = 40$ *Ksi*, $\delta_j^{max} = 0.35$ *in* of nodes 1,2,…,6 in all directions
Size variables: Continuous area: (0.005, 3) in^2

Table 15 Comparison of optimized designs found for size and topology of the 39-bar 3-D truss

	Proposed work		Deb and Gulati (2001)	Luh and Lin (2008)
Design variable	MTLBO	TLBO	GA	ACO
G_1	0.0065	-	0.166	0.005
G_{10}	0.0191	0.0166	0.409	0.015
G_{11}	0.0053	0.0087	0.071	0.015
Weight (lb)	2.3759	2.4534	47.93	2.82
Max stress (Ksi)	12.5215	12.0139	0.625	13.05
Max disp. (in)	0.3499	0.3499	0.0175	0.3397
FE	5000	5000	-	303600

Table 16 Statistical results of MTLBO and TLBO for 100 independent runs

Truss	Optimization	Size variables	Method	NT	Min	Max	Mean	SD	Mean Time (s)
10-bar 2D	Topology and size	Discrete	TLBO	-	4912.849	286880.1	8128.279	28162.66	4.7643
			MTLBO	2	4912.849	8000.553	5026.289	396.7332	6.4794
	TSS	Continuous	TLBO	-	2727.159	4235.037	3183.984	351.4563	5.7146
			MTLBO	5	2691.283	3842.735	2909.567	219.5284	7.8409
14-bar 2D	Topology and size	Continuous	TLBO	-	4730.51	7402.702	5409.697	773.9349	12.7786
			MTLBO	4	4730.438	7257.155	5071.669	628.528	17.5099
39-bar 2D	Topology and size	Continuous	TLBO	-	200.8831	324.5473	244.9857	28.1407	32.2087
			MTLBO	5	193.2200	274.3575	218.3570	17.2825	41.9073
	TSS	Continuous	TLBO	-	206.494	365.0063	247.9759	28.2394	34.5491
			MTLBO	4	188.6255	274.2484	216.6767	18.1232	43.3672
25-bar 3D	TSS	Discrete	TLBO	-	117.5496	170.4828	136.9960	10.9735	15.7981
			MTLBO	2	114.3473	150.3912	121.7046	6.6813	18.8152
39-bar 3D	Topology and size	Continuous	TLBO	-	2.4534	26.8657	7.6158	4.9813	8.2861
			MTLBO	3	2.3759	7.5490	3.4994	1.1824	9.1693

Table 16 shows comparative statistical results of weight for MTLBO and TLBO. It is observed from the results that all statistical results of MTLBO in terms of best solution (min), worst solution (max), average solution (mean) and standard deviation (SD) are better than TLBO. However, TLBO required less computational time than MTLBO. This study indicates that results of MTLBO are reliable and overall better than the results of TLBO.

6. Conclusions

This article presented four effective advances in TLBO to speed up the search procedure and to improve its convergence rate. Multiple teachers, adaptive teaching factor, self-motivated learning and learning through tutorial are incorporated to improve the effectiveness of TLBO in the search procedure. It is observed that selection of NT, population size and the maximum number of generations play a significant role for the best performance of the algorithm. The effectiveness of MTLBO is demonstrated by considering the benchmark structural optimization problems to design planner and space trusses subjected to stresses, displacement and kinematic stability constraints. The results illustrate that min, max, mean and SD of weight achieved by using MTLBO are superior to TLBO. Moreover, extra computational time needed due to this improvement is negligible. Results by using MTLBO are observed better or nearly equal as compared to results of literature, with fewer FE and without violation of constraints. The fast convergence rate of MTLBO concludes the high proficiency and stability for simultaneous TSS optimization of trusses.

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