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# Buckling assessment of dented truncated cones under external pressure

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**Abstract.** Notwithstanding a considerable body of references in the literature on the buckling response of conical shell structures, it seems imperative to provide further insight on the buckling response of locally imperfect steel cones. This paper contains different simulations including non-linear FE analysis and discusses the influence of dent imperfection on the buckling load of these structures subject to external pressure. Data of the present work are evaluated against available experimental results, codes and recommendations and the effect of the local damages is exhaustively set forth. It is also found that the employed FE program can reliably predict the structural response of locally damaged conical shells.

**Keywords:** conical shells; dent imperfection; external pressure; FE analysis

## 1. Introduction

Conical shells, as modern industrial structures, appear in a great many of different forms of engineering structures. As a few instances, in civil and structural engineering, these structures may appear as towers or silos' caps, as drilling machines seen in offshore structures and pressure hulls of submarines, or even in the form of spacecraft hulls and aircraft fuselages. Thus, investigation on the buckling behavior of a conical shell with different variables is required and of a great interest of researchers of the above-mentioned industries.

A significant threat to the structural integrity of thin-walled structures during the service life is mechanical damages caused by physical contacts. In recent years, Prabu and his co-authors have published a few papers on the effect of dent imperfections on different structures (Prabu *et al*. 2007, 2010, Raviprakash *et al*. 2012, Rathinam and Prabu 2013). The first author and his collaborators also published a significant body of experimental works in this regards which can be found in Refs. (Ghanbari Ghazijahani *et al*. 2014a, b, 2015a, b, c). In all these papers the effect of the local irregularities is expounded upon, which will be discussed in the coming sections in this paper.

Geometric imperfections take a great part in the buckling response of thin conical shell

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Structures under uniform peripheral pressure. Such structures normally buckle in a lobar manner or axisymmetrically under such loading (Ross *et al.* 1999). Holst and his co-workers studied the effect of imperfections caused by fabrication misfit both for perfect and geometrically imperfect shells (Holst *et al*. 1999). Shen and Chen investigated buckling behavior of both imperfect and perfect shells subjected to combined axial stresses and peripheral pressure (Shen and Chen 1991). They showed the effect of geometric parameters, loading and the imperfection parameters on the structural behavior of such structures. A "*volume-control*" testing method was studied on conical and cylindrical shells under peripheral pressure (MacKay and Van Keulen 2010). The implementation of the modern volume control method versus the conventional method of buckling evaluation was described in this work. Showkati and Golzan developed experiments on conical structures (truncated conical shells) considering normal fabrication-related geometrical imperfections (Golzan and Showkati 2008). They demonstrated that their gained critical loads were sometimes much less than the results obtained from FE and theoretical equations depending upon tapering ratio of the specimens and the geometry of the irregularities.

Experiments on conical reducers under uniform pressure was conducted on three steel specimens (Ghazijahani and Showkati 2011). The effect of length of frusta shells and the elastic boundary condition were examined in this paper. This study was followed by the authors as a few experimental and analytical works on the stability of the thin steel structures with normal fabrication-related imperfections under external pressure (Ghanbari *et al*. 2010, Ghanbari Ghazijahani and Showkati 2012, Ghanbari Ghazijahani and Showkati 2013, Ghanbari Ghazijahani and Showkati 2013, Imani *et al*. 2013, Ghanbari Ghazijahani and Zirakian 2014). In all these studies, the effect of small-amplitude imperfections was considered in determining the buckling load of such structures. Showkati *et al*. studied the effect of local weld-induced imperfection as the longitudinal depression along the weld line of shell structures (Maali *et al*. 2012, Fatemi *et al*. 2013, Ghazijahani and Showkati 2013, Niloufari *et al*. 2014). It was found that in some cases local imperfections introduced in such structures had strengthening effect on the buckling behavior. They strongly recommended that further studies are needed in order to generalize the effect of local imperfections to reach definitive conclusions.

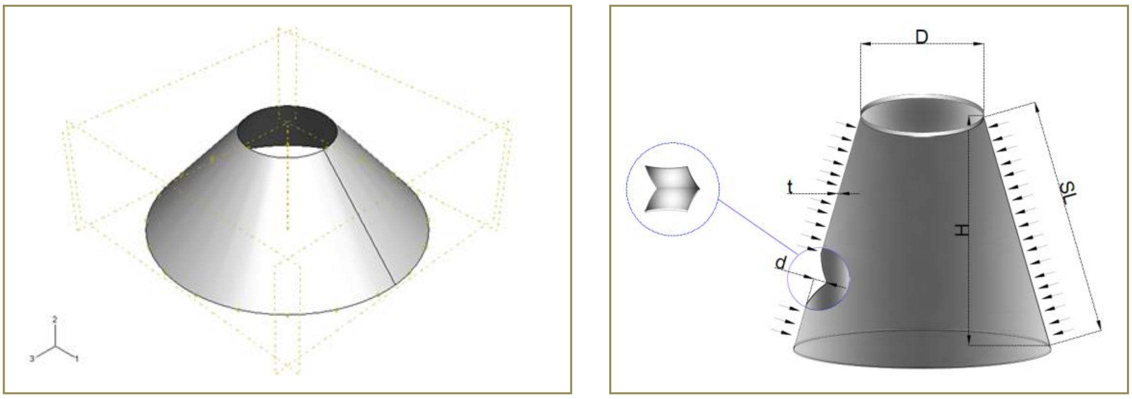
In the present research, perfect and dented shells were numerically simulated and the influence of dent imperfections on the critical buckling of conical shells is studied. It was found that the FE analysis is capable of being employed as a reliable tool in order to precisely predict the behavior of the damaged elements. Relevant comparisons were drawn to verify the results of the current numerical study and further discussions were made to elucidate the structural trend of such structures under different geometrical discontinuities.

## 2. Experimental research

Showkati and Golzan assessed the buckling of nominally perfect frusta shells with normal imperfections against external pressure (Golzan and Showkati 2008). It was found that the buckling of such shells was strongly dependent upon "*radius to thickness*" *(R/t)* and "*slant-length to radius*" *(L/R)* ratios. In this paper the aforementioned data were evaluated against the corresponding dented models which were simulated numerically as two types of frusta models (see Tables 1, 2 and Fig. 1). SC1 and SC3 in the mentioned reference are modeled in this study with the labels of SCS and SCL respectively.

Table 1 Geometry of the current models

| Model | Top radii (mm) | Bottom radii (mm) | Semi-vertex angle | *R/t* | *H* (mm) | *t* (mm) | *SL/R* | *R/r* |
|---|---|---|---|---|---|---|---|---|
| SCS | 100 | 300 | 41.81 | 500 | 223.6 | 0.6 | 1 | 3 |
| SCL | 100 | 300 | 19.47 | 500 | 565.7 | 0.6 | 2 | 3 |




(a) Conical shell model in FE simulation (b) geometry of conical shell under external pressure

Fig. 1 Schematic illustrations of models

## 3. Finite element analysis

<u>*General features and Type of analysis:*</u>

Structural analysis of this study were gained employing the well-known finite element program "*ABAQUS*". This program has revealed a good history of yielding accurate predictions compared with theoretical and experimental results, so that is extensively used in buckling and post-buckling investigations of the shell elements.

Bifurcation buckling analysis and nonlinear buckling were undertaken for each model. The influence of local imperfections on the geometry prior to the buckling is not normally considered in a bifurcation buckling analysis. In contrast, this effect is considered in a nonlinear analysis, which identifies the buckling load given an imperfect geometry.

Eigenvalue analysis is utilized to obtain proper estimations in such structures. As well, Eigenvalue studies may lead to a reasonable mesh pattern as convergence study is always definitely required to ensure the convergence of the eigenvalue estimates of the buckling. Note that mesh has to be adequate in any model so as to yield consistent buckling modes. "*Riks-Arc-Length*" is the other major phase of the study. This method is used to obtain a possible instability of the post-buckling behavior of such structures (Riks 1979). The numerical models of this study were based upon the nonlinear elasto-plastic response of such structures, i.e., material and geometric nonlinearity were both adopted. This method was employed which is commonly utilized in nonlinear analysis to reach the post-buckling deformations in simulations.

### *3.1 FE model characteristics*

*Used element:*

S4R element was used, which is commonly used for modeling such shells. S4R is an element with four nodes possessing five freedom degrees (Ozkan and Mohareb 2009). The three translations (orthogonally) and two components dimension change for a vector perpendicular to the shell surface is considered for this element, which in turn forms the rotations. Stress stiffening, deflections and nonlinear behavior are its main features (Ozkan and Mohareb 2009).

*Characteristics of models:*

Geometric non-linearity is considered, which yields quite promising results for the large displacements, and rotations. Uniform external pressure was modeled as perpendicular uniform forces to the body of the shells. Simply supported boundaries are adopted wherein only a hoop constraint is provided for the edges by limiting the displacement of a number of nodes on the top and bottom sections. The models were simulated to be free in the axial axis in a similar fashion to the experiments in Ref. (Golzan and Showkati 2008), although one end of the models was axially fixed to make the whole system structurally stable. The stress-strain data of the models and the geometric dimensions quite matched those of Ref. (Golzan and Showkati 2008). The yield stress of the material as well as the ultimate stresses and Young's modulus were 277 Mpa, 373 Mpa and 210 GPa respectively.

*Geometric Imperfections:*

Node perturbations as input data were provided in *ABAQUS* as the imperfections data in different coordinates. Dent imperfections in this research were modeled and studied in three different forms of: (i) horizontal, (ii) vertical, and (iii) inclined dent. The value of horizontal and vertical dents was 10 mm in all cases which was parallel to the one of the basic directions in the global coordinate system. The inclined dent was the same as the horizontal and vertical ones, but with the equivalent depth in two directions. The position of the dents in all cases was in the bottom half of the slant length centered at the 1/3 of the total height.

Table 2 Models specifications

| Model No. | Model | Imperfection |
|---|---|---|
| 1 | SCS | Undented (Eigenvalue) |
| 2 | SCL | Undented (Eigenvalue) |
| 3 | SCS | Undented |
| 4 | SCL | Undented |
| 5 | SCS | Vertical dent |
| 6 | SCL | Vertical dent |
| 7 | SCS | Horizontal dent |
| 8 | SCL | Horizontal dent |
| 9 | SCS | Inclined dent |
| 10 | SCL | Inclined dent |

## 4. Results and discussions

### 4.1 Buckling load and dent imperfections

Buckling loads obtained using FE analysis are tabulated in Table 3. The buckling loads of three different models, i.e., vertically and horizontally dented, as well as specimens with inclined dents are presented through this table. The critical loads obtained from the experimental and numerical study of Ref. (Golzan and Showkati 2008) are seen which is compared with the results of FE analysis. A very good agreement is found considering the results of different methods and the experimental data of Ref. (Golzan and Showkati 2008). The results of the experimental work are lower than the FE models, which is quite predictable as the geometrical imperfections of the models and other parameters - such as sliding of the supports, etc. - can definitely affect the experimental data. Given the reasonable difference obtained comparing the two studies, a validity of the present FE data can be readily inferred. It further appears that the effect of the dent was mild which could be owing to the fact that the localization of the imperfection may change the material properties of the damaged area so that the material enters the nonlinear stiffening zone, which may have a strengthening effect. The geometry of the dent itself can be of a considerable significant - when it comes to its effect on the capacity - as it is hypothesized that sharp-edge dents would act more like a stiffener as opposed to the gouge-shaped dents.

It is quite evident that, the effect of dent imperfection is higher for SCL in comparison with SCS indicating that longer specimens are highly affected by the local imperfections. Fig. 2 shows the decreasing effect of the dent imperfection for three different dents, wherein the effect of the slant length is also shown for the models. The greatest effect of the dent imperfection is seen for the specimen SCL with approximately 20% of the reduction in the capacity. It appears that, albeit only slightly different, the effect of different types of imperfections is more or less similar, while it is negligibly higher for the model including an inclined dent. In addition, the consistence of the results shows that FE analysis can reliably predict the buckling behavior of imperfect structures.

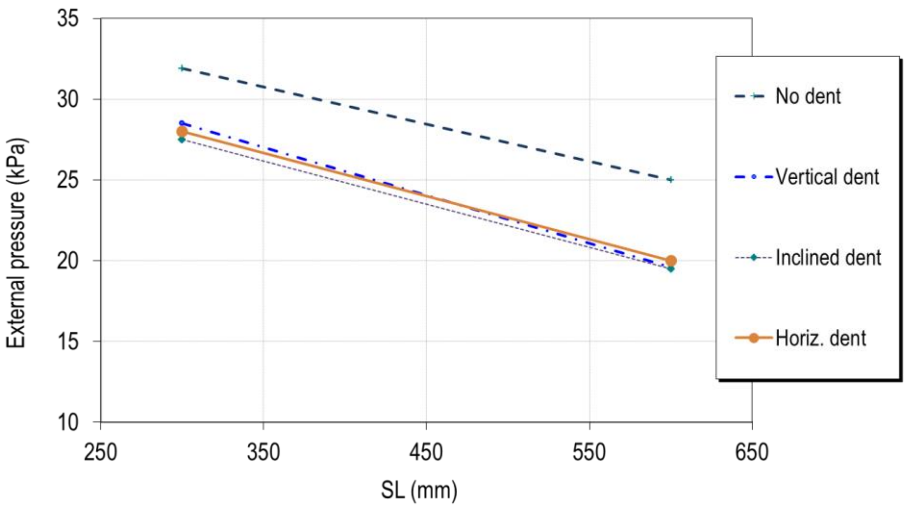



Fig. 2 Critical load of the truncated cones, intact and dented shells

Table 3 Comparison of obtained critical values under external pressure (kPa)

| Model | Eigenvalue | No dent | Vertical dent | Horizontal dent | Inclined dent | Ref. (Golzan and Showkati 2008), FE | Ref. (Golzan and Showkati 2008), Exp. |
|---|---|---|---|---|---|---|---|
| SCS | 28 | 31.9 | 28 | 28.1 | 27.5 | 28 | 25 |
| SCL | 20.5 | 25 | 19.6 | 20 | 19.5 | 22 | 14 |

Table 4 Descriptions of the parameters used in the theoretical predictions

| Parameter | Description |
|---|---|
| $E$ | Young's modulus of elasticity |
| $r$, $R$ | radii of the respective small and large ends of the cones |
| $r_m$ | $(r+R)/2$ |
| $t_e$ | $t\cos\alpha$ |
| $t$ | thickness of the shell |
| $\alpha$ | semi-vertex angle |
| $L_e$ | $(SL.\cos\alpha/2)\times(1+r/R)$ |
| $SL$ | slant length |
| $d$ | mean diameter of the cone |
| $\varepsilon$ | function of $2R_{mean}\cos\alpha/t$ and $L/(2R_{mean}\cos\alpha)$ |
| $R_{mean}$ | mean radius |
| $\beta_{\min}$ | function of $t/r_e$ and $SL/r_e$ |
| $r_e$ | $(r+R)/2\cos\alpha$ |

### 4.2 Comparing the results of this study with design recommendations and codes

The results of the present study were evaluated against different available codes and recommendations. Jawad, Venstel and Krauthammer, and Ross respectively proposed Eqs. (1), (2) and (3) to predict the buckling load of conical shell structures (Jawad 1994, Ventsel and Krauthammer 2001, Ross 2007)

$$P_{cr} = \frac{0.92E(t_e/R)^{2.5}}{L_e/R} \tag{1}$$

$$P_{cr} = 0.92\frac{Et}{SL.\cos\alpha}\left(\frac{t}{r_m}\right)^{1.5}(\cos\alpha)^{1.5} \tag{2}$$

$$P_{cr} = \frac{2.6E(t/d)^{2.5}}{H/d - 0.45(t/d)^{0.5}} \tag{3}$$

$$P_{mc} = \frac{Et\varepsilon \cos^3 \alpha}{R_{mean}} \quad (4)$$

$$P_{cr} = E \frac{t}{r_e} \beta_{\min} \quad (5)$$

*British Standards Institution* (*BSI, 2009*) and *European Recommendations ECCS-CECM-EKS* respectively proposed estimates as two formulas of Eq. (4) and Eq. (5), whereby good predictions are obtained for the buckling of a cone shell subject to peripheral pressure. In these equations, parameter $\varepsilon$ is a function of $2R_{mean}\cos\alpha/t$ and $L/(2R_{mean}\cos\alpha)$, and parameter $\beta_{\min}$ is a function of $t/r_e$ and $SL/r_e$ which are considered as the main geometric parameters of a cone (geometric aspect ratios). Table 4 provides explanations on the parameters used in the equations.

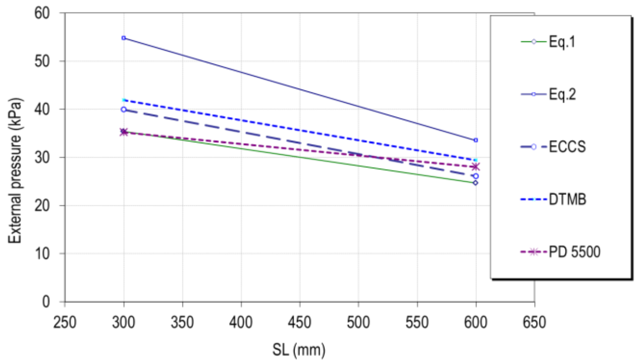



Fig. 3 Estimations of critical loads obtained from design codes and recommendations

Table 5 Comparison of the results with the models with and without a dent

| Model | Eq. (1)/perfect | Eq. (2)/perfect | ECCS/perfect | DTMB/perfect | PD5500/perfect |
|---|---|---|---|---|---|
| SCS | 1.110 | 1.718 | 1.251 | 1.313 | 1.103 |
| SCL | 0.987 | 1.340 | 1.044 | 1.176 | 1.120 |
| Model | Eq. (1)/Horiz. dent | Eq. (2)/Horiz. dent | ECCS/Horiz. dent | DTMB/Horiz. dent | PD5500/Horiz. dent |
| SCS | 1.264 | 1.957 | 1.425 | 1.496 | 1.257 |
| SCL | 1.234 | 1.675 | 1.305 | 1.470 | 1.400 |
| Model | Eq. (1)/vertical dent | Eq. (2)/vertical dent | ECCS/vertical dent | DTMB/vertical dent | PD5500/vertical dent |
| SCS | 1.242 | 1.923 | 1.400 | 1.470 | 1.235 |
| SCL | 1.266 | 1.718 | 1.338 | 1.508 | 1.436 |
| Model | Eq. (1)/inclined dent | Eq. (2)/inclined dent | ECCS/inclined dent | DTMB/inclined dent | PD5500/inclined dent |
| SCS | 1.287 | 1.993 | 1.451 | 1.524 | 1.280 |
| SCL | 1.266 | 1.718 | 1.338 | 1.508 | 1.436 |

Current results are compared to the results of the mentioned design equations and recommendations. Fig. 3 plots different theoretical predictions for the buckling load provided by the mentioned design codes and the other available references. Table 5 shows the comparison of the results of the intact models with the mentioned equations. Although all of the results are by-and-large in a quite acceptable range, PD 5500 and Eq. (1) provide best predictions for the *control model* (intact) so that the other models can be examined based upon this comparison.

It is noteworthy that the adverse effect of the local imperfection, i.e., dent, can be verified considering the results of these tables. It is shown through this Table that different shapes of the dents do not yield significant differences when it comes to the buckling load of such structures. It is quite evident that, the difference between different studies and methods - to reach the buckling load of shell structures - can be largely attributed to the effect of different imperfections, different boundary conditions and the other assumptions which are oftentimes considered differently in various theoretical and/or numerical simulations. As another point, one may note that the rate of discrepancies for the thin-walled structures may be higher in comparison with the other structures.

The reason is that thin-walled structures are quite vulnerable to the local buckling caused by the initial imperfections, which is thanks to the existence of low transverse strength. This weakness though, makes the surface less perfect whilst fabrication comparing with the other structures, which may lead to greater imperfections.

| | |
|---|---|
| ESSCRec, 2008 | $L=I_r$:$t_r=\overline{W}$<br>$t_r$<1% of $L_{mx}$ |
| ENV 1993-1-6, 2007 | $L_{mx}=L_g$:$t_r=\Delta w_0$<br>$t_r \le 0.006 L_{mx}$<br>OR (0.01 $L_{mx}$, 0.016 $L_{mx}$ for different classes) |
| DIN 18800-4 | $t_r$<1% of $L_{mx}$ |

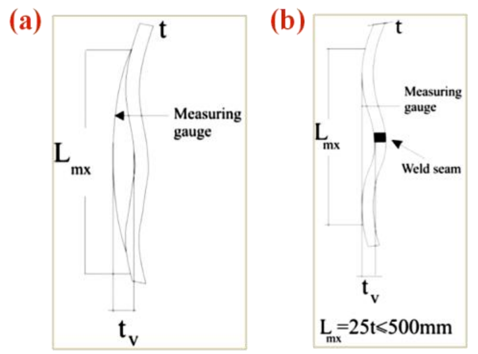



Fig. 4 Tolerated imperfections based on available design codes. (a) Dimple in vertical direction, (b) dimple of hoop direction (Maali *et al*. 2012).

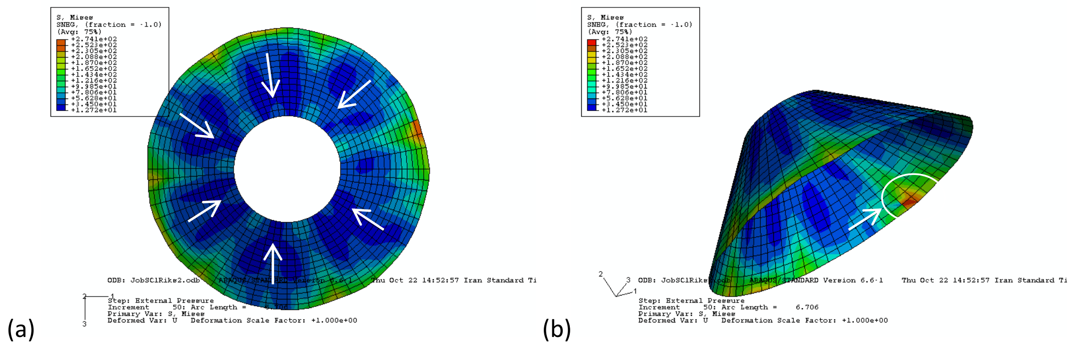



Fig. 5 Stress distribution and creation of the buckling lodes in the SCS model with no dent

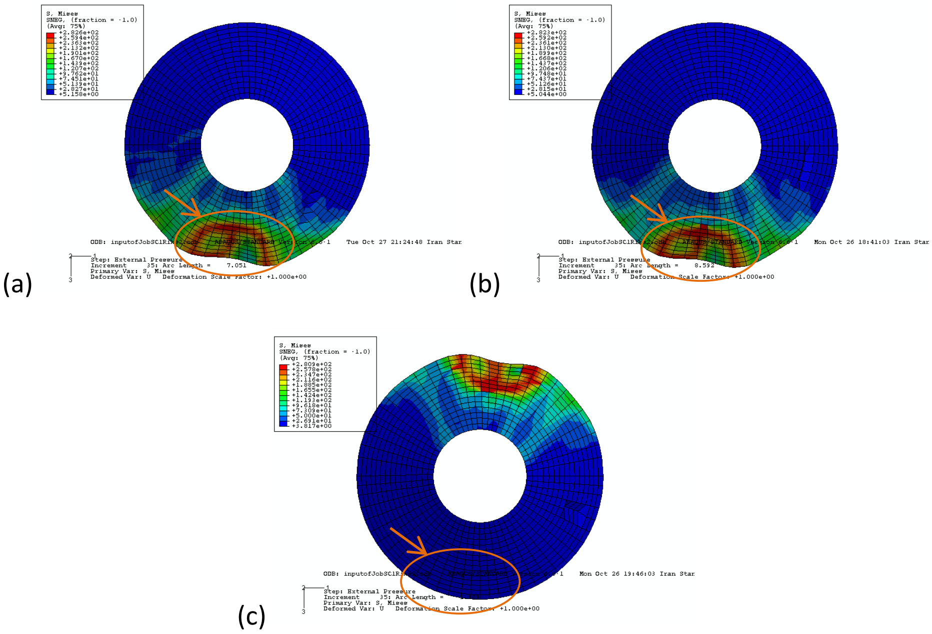



Fig. 6 Stress distribution in the zone of dent imperfection for the model SCS: Figures a to c represent inclined, horizontal and vertical dents respectively

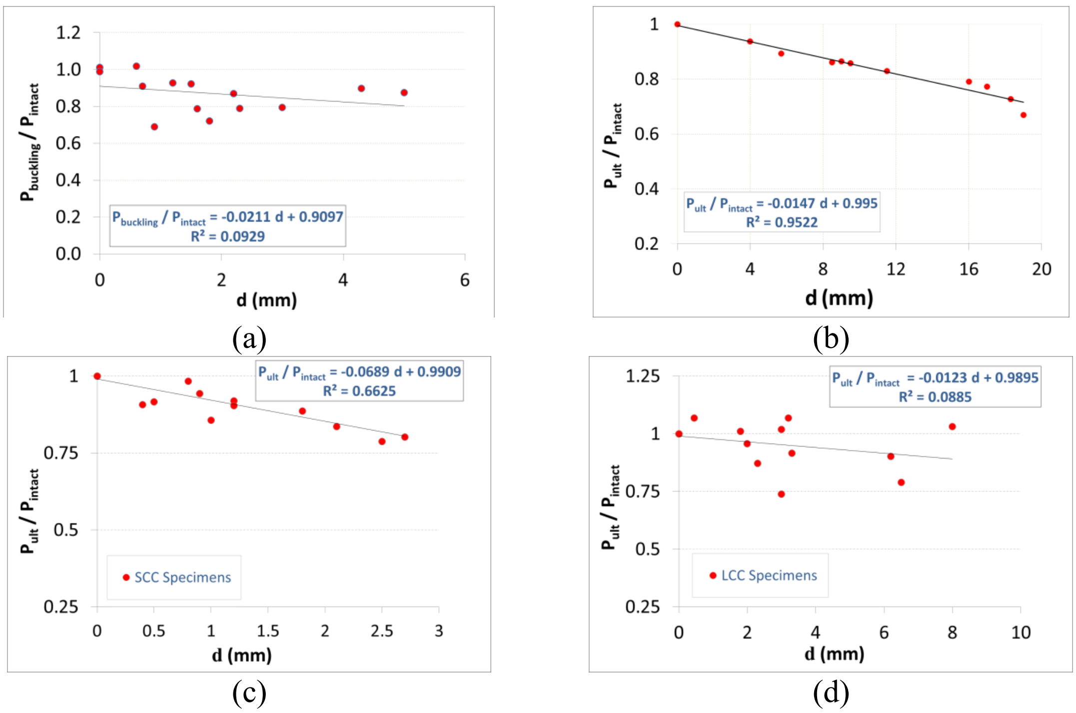



Fig. 7 Capacity ratio of the dented to the intact specimens (Ghanbari Ghazijahani *et al*. December 2015): (a) slender shells under external pressure (Ghanbari Ghazijahani *et al*. 2014), (b) tubes under compression (Ghanbari Ghazijahani *et al*. 2015), (c,d) slender shells under compression (Ghanbari Ghazijahani *et al*. 2014)

Fig. 4 shows the allowed magnitudes of the imperfections tolerated by different codes (Battles *et al*. 2000, Stevens and Criner 2000, Dagher *et al*. 2002), wherein different parameters of dent in different directions of longitudinal and circumferential were assessed. Figs. 5 and 6 show the buckled SCS model in the post-buckling stage. Although the imperfection values tolerated by the mentioned studies are much less than the values of local damages, no dramatic decrease was obtained for locally imperfect dented models if compared to the fabrication-related imperfections. It is fitting to mention that design codes includes a range of allowable geometrical imperfection in shells of revolutions, which are mostly based upon the small amplitude imperfections on the surface of the shell structures made during the fabrication process - and/or welding if at all. It should be noted that in view of the modest effect of the large imperfections obtained in this study and the experimental studies of the first author and his collaborators, it is worthwhile to reconsider the existing tolerances, although it may require more studies to further verify this point. In point of fact though, this remark was also reported recently in a few other studies as Refs. (Maali *et al*. 2012, Fatemi *et al*. 2013, Ghazijahani and Showkati 2013, Niloufari *et al*. 2014).

### *4.3 Decreasing trend of the dent in recent experimental works*

Fig. 7 compares the capacity of the dented specimens with the intact ones in three different studies (Ghanbari Ghazijahani *et al*. 2014a, b, 2015a). This figure indicates a mild decreasing effect of the dent imperfection especially for the elements against external pressure as opposed to the axial compression. The figure also demonstrates a good consistence of the results of this study that the dent had a reduction influence on the capacity, fairly to a same extent as the magnitudes seen in the experiments (Ghanbari Ghazijahani *et al*. 2014a, b, 2015a). The results of this section also support the above-mentioned reconsiderations of the codes. Further experimental studies would undoubtedly reveal a series of comprehensive advice on the idea generalized to different kinds of local imperfections with various depths and amplitudes.

## 5. Conclusions

Locally imperfect conical shells exposed to the peripheral pressure were studied in this research. The models were based on the previous experimental data in the literature, but the local damages were introduced to the present FE models so as to assess the effect of large damages on the capacity of such shell elements.

The detrimental impact of large local imperfection was obtained for the present models, which was in a modest reducing manner. The effects of imperfections with the same depth but different inclinations were almost similar. Dent imperfection had a higher effect in SCL - with approximately 20% of the reduction in the capacity - in comparison with SCS, which demonstrates that longer models were affected more by the local imperfections. Although the results of comparisons with all theoretical predictions were in a quite acceptable range, PD 5500 and Eq. (1) revealed the best agreement with the results of this study. Design codes mostly define tolerances for small amplitude imperfections created during the fabrication process, which may not be always fully applicable for all kinds of large (amplitude and depth) imperfections. Therefore, further experimental studies are undoubtedly needed to help generalize the idea to cover different types of local imperfections.

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