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A review of numerical approach for dynamic response of strain gradient metal foam shells under constant velocity moving loads

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Abstract. Dynamic characteristics of a scale-dependent porous metal foam cylindrical shell under a traveling load have been explored within this article based on a numerical approach. Within the material texture of the metal foams, uniform and non-uniform porosities may be dispersed. Based upon differential quadrature method (DQM) and Laplace transforms, the equations of motion for a shear deformable scale-dependent shell may be solved numerically. Scale-dependent shell modeling has been provided based upon strain gradient elasticity. Solving the equations will give the shell deflection as a function of load speed. Also, it is reported that shell deflection relies on the porosity dispersion and strain gradient influences.

Keywords: forced vibration; moving loads; metal foam; strain-gradient theory; DQM

1. Introduction

A new type of material in which porosities play a major role for defining the effective material properties is metal foam. The effective material properties for this material rely on the amount of porosity and also its type of dispersion within material texture (She *et al*. 2019). Such material provides reliable stiffness and low weight with broad application in many engineering structural components such as plates and shells. So, they can be used in civil, marine and mechanical engineering fields (Atmane *et al*. 2015, She *et al*. 2018a, 2018b). The variation of porosities in this material causes a notable difference between metal foams and other perfect metals. This Issue is proved by many researchers that mechanical properties of metal foam rely on porosity amount and dispersion (Mirjavadi *et al*. 2017, Chaabane *et al*. 2019). Also, when the porosity dispersion within the material is non-uniform, it can be stated that the material is functionally graded. A complete discussion about functionally graded material can be found in the recent works (Zarga *et al*. 2019, Zine *et al*. 2018, Medani *et al*. 2019, Mahmoudi *et al*. 2019, Tlidji *et al*. 2019, Zaoui *et al*. 2019). In this material, pores may be produced in a phase between ceramic and material (Addou *et al*. 2019). These researches reported that the gradation of material has great influence on mechanical properties and specially static/dynamic characteristics of structures made of graded materials (Aissani *et al*. 2015, Bouderba *et al*. 2016, Chikh *et al*. 2016, Yahiaoui *et al*. 2018, Achouri *et al*.

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2019, Rezaiee-Pajand *et al*. 2018).

Recent studies focus on engineering structures at nano-scales due to their involvement in nano-mechanical systems or devices. However, the main issue in these studies is to select an appropriate elasticity theory accounting for small scale impacts. The impact of size-dependency might be considered with the help of a scale parameter involved in non-local theory of elasticity (Berghouti *et al*. 2019, Boukhatem *et al*. 2019, Boutaleb *et al*. 2019). The word "non-local" means that the stresses are not local anymore. This is because we are talking about a stress field of nano-scale structure. Many authors are aware of these facts and they are using this theory to analysis mechanical characteristics of small size engineering structures. Strain gradients at nano-scale are observed by many researchers (Lam *et al*. 2003, Ebrahimi *et al*. 2016, Alimirzaei *et al*. 2019). Thus, nonlocal-strain gradient theory was introduced as a general theory which contains an additional strain gradient parameter together with nonlocal parameter (Zeighampour and Beni 2014, Akgöz and Civalek 2015, Nami and Janghorban 2014). The scale parameters used in nonlocal strain gradient theory can be obtained by fitting obtained theoretical results with available experimental data and even Molecular Dynamic (MD) simulations (Arefi and Zenkour 2016, Ansari *et al*. 2015, Li *et al*. 2015, Martínez-Criado 2016). Recently, it is shown that moving or travelling loads have great impacts on dynamic characteristics of small-scale structures. This is due to the reason that such loads lead to forced vibrations of the structure during the loading time (Simsek 2010, Abouelregal and Zenkour 2017, Khaniki and Hosseini-Hashemi 2017, Shahsavari e*t al*. 2017, Al-Maliki *et al*. 2019).

In this research, dynamic properties of a porous cylindrical shell exposed to travelling loads have been researched via first order shell theory based on a numerical trend. Considered theory confirms the shear deformations impacts and contains five field components in comparison to classic theory. Higher order theories are suitable for thicker structures needless of shear correction factor (Bedia *et al*. 2019, Addou *et al*. 2019, Boulefrakh *et al*. 2019, Boukhlif *et al*. 2019, Bourada *et al*. 2019, Draoui *et al*. 2019, Draiche *et al*. 2019, Khiloun *et al*. 2019, Mahmoudi *et al*. 2019). Some functions have been utilized in order to express the porosity-dependent material coefficients. The equations of shell have been represented with the form of ordinary equations via DQ method to derive time responses. Detailed impacts of dynamic loading parameters, strain gradient and porosities on time responses of graded porous shells are explored.

2. Formulation of porous shell

2.1 Effective properties of metal foam shells

A porous material, for instance a steel foam, might be placed in the category of lightweight materials and can be applied in several structures such as curved panels. Often, pore variation along the thickness of shells results in a notable alteration in every kind of material property. When the pore distribution inside the material is selected to be non-uniform, the metal foam might be defined as a functionally graded material since its properties obey some specified functions. Herein, the following types of the pore dispersion will be employed (Fenjan *et al*. 2019, Barati 2018):

• Uniform kind

$$E = E_2(1 - e_0\chi) \tag{1a}$$

$$G = G_2(1 - e_0\chi) \tag{1b}$$

$$\rho = \rho_2\sqrt{(1 - e_0\chi)} \tag{1c}$$

• Non-uniform kind

$$E(z) = E_2(1 - e_0\,cos\left(\frac{\pi z}{h}\right)) \tag{2a}$$

$$G(z) = G_2(1 - e_0\,cos\left(\frac{\pi z}{h}\right)) \tag{2b}$$

$$\rho(z) = \rho_2(1 - e_m\,cos\left(\frac{\pi z}{h}\right)) \tag{2c}$$

The most important factors in above relations are the greatest values of material properties E_2, G_2 and ρ_2. Also, there are two important factors related to pores and mass which are e_0 and e_m as

$$e_0 = 1 - \frac{E_2}{E_1} = 1 - \frac{G_2}{G_1}, \qquad e_m = 1 - \frac{\rho_2}{\rho_1} = 1 - \sqrt{1 - e_0} \tag{3}$$

Based on uniformly distributed pores, the following parameter is used in Eq. (1) as

$$\chi = \frac{1}{e_0} - \frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1 - e_0} - \frac{2}{\pi} + 1\right)^2 \tag{4}$$

2.2 First order shell theory

By employing first order shell model, the displacement components based on axial u, lateral v, bending w displacements may be introduced as (Barati 2018)

$$u_1(x,\theta,z,t) = u(x,\theta,t) + z\varphi_x(x,\theta,t) \tag{5}$$

$$u_2(x,\theta,z,t) = v(x,\theta,t) + z\varphi_\theta(x,\theta,t) \tag{6}$$

$$u_3(x,\theta,z,t) = w(x,\theta,t) \tag{7}$$

Finally, the strains based on the considered shell model have been obtained as

$$\begin{gathered}
\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \varphi_x}{\partial x}, \qquad \varepsilon_\theta = \frac{1}{R}(\frac{\partial v}{\partial \theta} + w + z\frac{\partial \varphi_\theta}{\partial \theta}) \\
\gamma_{x\theta} = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{z}{R}\frac{\partial \varphi_x}{\partial \theta} + z\frac{\partial \varphi_\theta}{\partial x} \\
\gamma_{zx} = \varphi_x + \frac{\partial w}{\partial x}, \qquad \gamma_{z\theta} = \varphi_\theta + \frac{1}{R}\frac{\partial w}{\partial \theta} - \frac{v}{R}
\end{gathered} \tag{8}$$

Five equations of motion are available for the cylindrical shell by considering that N_{ij} are membrane forces, M_{ij} are bending moments and Q_{ij} are shear forces (Barati 2018)

$$\frac{\partial N_{xx}}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} = I_0\frac{\partial^2 u}{\partial t^2} + I_1\frac{\partial^2 \varphi_x}{\partial t^2} \tag{9}$$

$$\frac{\partial N_{x\theta}}{\partial x}+\frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial\theta}+\frac{Q_{z\theta}}{R}=I_0\frac{\partial^2 v}{\partial t^2}+I_1\frac{\partial^2\varphi_\theta}{\partial t^2} \tag{10}$$

$$\frac{\partial Q_{xz}}{\partial x}+\frac{1}{R}\frac{\partial Q_{z\theta}}{\partial\theta}-\frac{N_{\theta\theta}}{R}=+I_0\frac{\partial^2 w}{\partial t^2}+q_{dynamic} \tag{11}$$

$$\frac{\partial M_{xx}}{\partial x}+\frac{1}{R}\frac{\partial M_{x\theta}}{\partial\theta}-Q_{xz}=I_1\frac{\partial^2 u}{\partial t^2}+I_2\frac{\partial^2\varphi_x}{\partial t^2} \tag{12}$$

$$\frac{\partial M_{x\theta}}{\partial x}+\frac{1}{R}\frac{\partial M_{\theta\theta}}{\partial\theta}-Q_{\theta z}=I_1\frac{\partial^2 v}{\partial t^2}+I_2\frac{\partial^2\varphi_\theta}{\partial t^2} \tag{13}$$

so that the travelling force has been selected as $q_{dynamic}=P_0\delta(x-v_P t)$ having the magnitude of P_0 and speed of v_P and

$$(I_0,I_1,I_2)=\int_{-h/2}^{h/2}(1,z,z^2)\rho\,(z)dz \tag{14}$$

Also, the below resultants should be defined

$$\{N_{xx},N_{\theta\theta},N_{x\theta}\}=\int_{-h/2}^{h/2}\{\sigma_{xx},\sigma_{\theta\theta},\sigma_{x\theta}\}\,dz \tag{15}$$

$$\{M_{xx},M_{\theta\theta},M_{x\theta}\}=\int_{-h/2}^{h/2}\{\sigma_{xx},\sigma_{\theta\theta},\sigma_{x\theta}\}\,zdz \tag{16}$$

$$\{Q_{xz},Q_{z\theta}\}=\kappa_s\int_{-h/2}^{h/2}\{\sigma_{xz},\sigma_{z\theta}\}\,dz \tag{17}$$

Based upon strain-gradient scale-dependent elasticity having size parameter l, the below stress-strain relation may be introduced

$$\begin{Bmatrix}\sigma_{xx}\\\sigma_{\theta\theta}\\\sigma_{x\theta}\\\sigma_{xz}\\\sigma_{z\theta}\end{Bmatrix}=[1-l^2\nabla^2]\frac{E(z)}{1-v^2}\begin{pmatrix}1 & v & 0 & 0 & 0\\ v & 1 & 0 & 0 & 0\\ 0 & 0 & (1-v)/2 & 0 & 0\\ 0 & 0 & 0 & (1-v)/2 & 0\\ 0 & 0 & 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}\varepsilon_{xx}\\\varepsilon_{\theta\theta}\\\gamma_{xy}\\\gamma_{xz}\\\gamma_{z\theta}\end{Bmatrix} \tag{18}$$

After integrating Eq. (18) in thickness direction, we get to the following relationships:

$$N_{xx}=[1-l^2\nabla^2][A_{11}\frac{\partial u}{\partial x}+B_{11}\frac{\partial\varphi_x}{\partial x}+\frac{A_{12}}{R}(\frac{\partial v}{\partial\theta}+w)+\frac{B_{12}}{R}\frac{\partial\varphi_\theta}{\partial\theta}] \tag{19}$$

$$M_{xx}=[1-l^2\nabla^2][B_{11}\frac{\partial u}{\partial x}+D_{11}\frac{\partial\varphi_x}{\partial x}+\frac{B_{12}}{R}(\frac{\partial v}{\partial\theta}+w)+\frac{D_{12}}{R}\frac{\partial\varphi_\theta}{\partial\theta}] \tag{20}$$

$$N_{\theta\theta}=[1-l^2\nabla^2][A_{12}\frac{\partial u}{\partial x}+B_{12}\frac{\partial\varphi_x}{\partial x}+\frac{A_{11}}{R}(\frac{\partial v}{\partial\theta}+w)+\frac{B_{11}}{R}\frac{\partial\varphi_\theta}{\partial\theta}] \tag{21}$$

$$M_{\theta\theta} = [1 - l^2\nabla^2][B_{12}\frac{\partial u}{\partial x} + D_{12}\frac{\partial \varphi_x}{\partial x} + \frac{B_{11}}{R}(\frac{\partial v}{\partial \theta} + w) + \frac{D_{11}}{R}\frac{\partial \varphi_\theta}{\partial \theta}] \quad (22)$$

$$N_{x\theta} = [1 - l^2\nabla^2][A_{66}(\frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}) + B_{66}(\frac{1}{R}\frac{\partial \varphi_x}{\partial \theta} + \frac{\partial \varphi_\theta}{\partial x})] \quad (23)$$

$$M_{x\theta} = [1 - l^2\nabla^2][B_{66}(\frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}) + D_{66}(\frac{1}{R}\frac{\partial \varphi_x}{\partial \theta} + \frac{\partial \varphi_\theta}{\partial x})] \quad (24)$$

$$Q_{xz} = [1 - l^2\nabla^2]\tilde{A}_{66}(\varphi_x + \frac{\partial w}{\partial x}) \quad (25)$$

$$Q_{\theta z} = \tilde{A}_{66}[1 - l^2\nabla^2](\varphi_\theta + \frac{1}{R}\frac{\partial w}{\partial \theta} - \frac{v}{R}) \quad (26)$$

where

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-v^2}dz, \quad B_{11} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-v^2}z\,dz, \quad D_{11} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)z^2}{1-v^2}, \quad \tilde{A}_{66} = k_s\int_{-h/2}^{h/2}\frac{E(z)}{2(1+v)}dz \quad (27)$$

Five equations of motion for metal foam shell based on strain gradient elasticity will be achieved by placing Eqs. (19)-(26) in Eqs. (9)-(13) by

$$[1 - l^2\nabla^2][A_{11}\frac{\partial^2 u}{\partial x^2} + B_{11}\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{A_{12}}{R}(\frac{\partial^2 v}{\partial x\partial \theta} + \frac{\partial w}{\partial x}) + \frac{B_{12}}{R}\frac{\partial^2 \varphi_\theta}{\partial x\partial \theta} + \frac{A_{66}}{R}(\frac{1}{R}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 v}{\partial x\partial \theta}) + \frac{B_{66}}{R}(\frac{1}{R}\frac{\partial^2 \varphi_x}{\partial \theta^2} + \frac{\partial^2 \varphi_\theta}{\partial x\partial \theta})] - I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^2 \varphi_x}{\partial t^2} = 0 \quad (28)$$

$$[1 - l^2\nabla^2][A_{66}(\frac{1}{R}\frac{\partial^2 u}{\partial x\partial \theta} + \frac{\partial^2 v}{\partial x^2}) + B_{66}(\frac{1}{R}\frac{\partial^2 \varphi_x}{\partial x\partial \theta} + \frac{\partial^2 \varphi_\theta}{\partial x^2}) + \frac{A_{12}}{R}\frac{\partial^2 u}{\partial x\partial \theta} + \frac{B_{12}}{R}\frac{\partial^2 \varphi_x}{\partial x\partial \theta} + \frac{A_{11}}{R^2}(\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial w}{\partial \theta}) + \frac{B_{11}}{R^2}\frac{\partial^2 \varphi_\theta}{\partial \theta^2} + \frac{\tilde{A}_{66}}{R}(\varphi_\theta + \frac{1}{R}\frac{\partial w}{\partial \theta} - \frac{v}{R})] - I_0\frac{\partial^2 v}{\partial t^2} - I_1\frac{\partial^2 \varphi_\theta}{\partial t^2} = 0 \quad (29)$$

$$[1 - l^2\nabla^2][\tilde{A}_{66}(\frac{\partial \varphi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}) + \frac{\tilde{A}_{66}}{R}(\frac{\partial \varphi_\theta}{\partial \theta} + \frac{1}{R}\frac{\partial^2 w}{\partial \theta^2} - \frac{1}{R}\frac{\partial v}{\partial \theta}) - \frac{A_{12}}{R}\frac{\partial u}{\partial x} - \frac{B_{12}}{R}\frac{\partial \varphi_x}{\partial x} - \frac{A_{11}}{R^2}(\frac{\partial v}{\partial \theta} + w) - \frac{B_{11}}{R^2}\frac{\partial \varphi_\theta}{\partial \theta}] - I_0\frac{\partial^2 w}{\partial t^2} = q_{dynamic} \quad (30)$$

$$[1 - l^2\nabla^2][B_{11}\frac{\partial^2 u}{\partial x^2} + D_{11}\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{B_{12}}{R}(\frac{\partial^2 v}{\partial x\partial \theta} + \frac{\partial w}{\partial x}) + \frac{D_{12}}{R}\frac{\partial^2 \varphi_\theta}{\partial x\partial \theta} + \frac{B_{66}}{R}(\frac{1}{R}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 v}{\partial x\partial \theta}) + \frac{D_{66}}{R}(\frac{1}{R}\frac{\partial^2 \varphi_x}{\partial \theta^2} + \frac{\partial^2 \varphi_\theta}{\partial x\partial \theta}) - \tilde{A}_{66}(\varphi_x + \frac{\partial w}{\partial x})] - I_1\frac{\partial^2 u}{\partial t^2} - I_2\frac{\partial^2 \varphi_x}{\partial t^2} = 0 \quad (31)$$

$$[1 - l^2\nabla^2]B_{66}\left(\frac{1}{R}\frac{\partial^2 u}{\partial x\partial \theta} + \frac{\partial^2 v}{\partial x^2}\right) + D_{66}\left(\frac{1}{R}\frac{\partial^2 \varphi_x}{\partial x\partial \theta} + \frac{\partial^2 \varphi_\theta}{\partial x^2}\right) + \frac{B_{12}}{R}\frac{\partial^2 u}{\partial x\partial \theta} + \frac{D_{12}}{R}\frac{\partial^2 \varphi_x}{\partial x\partial \theta} + \frac{B_{11}}{R^2}(\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial w}{\partial \theta}) + \frac{D_{11}}{R^2}\frac{\partial^2 \varphi_\theta}{\partial \theta^2} - \tilde{A}_{66}(\varphi_\theta + \frac{1}{R}\frac{\partial w}{\partial \theta} - \frac{v}{R})] - I_1\frac{\partial^2 v}{\partial t^2} - I_2\frac{\partial^2 \varphi_\theta}{\partial t^2} = 0 \quad (32)$$

3. Solution by differential quadrature method

In the present chapter, Differential Quadrature Method (DQM) has been utilized for solving the governing equations for porous FG shells. According to DQM, at an assumed grid point (x_i, y_j) the derivatives for function F are supposed as weighted linear summation of all functional values within the computation domains as

$$\frac{d^n F}{dx^n}\Big|_{x=x_i} = \sum_{j=1}^{N} c_{ij}^{(n)} F(x_j) \tag{33}$$

where

$$C_{ij}^{(1)} = \frac{\pi(x_i)}{(x_i - x_j)\ \pi(x_j)} \qquad i,j = 1,2,\dots,N, \qquad i \neq j \tag{34}$$

in which $\pi(x_i)$ is defined by

$$\pi(x_i) = \prod_{j=1}^{N} (x_i - x_j), \qquad i \neq j \tag{35}$$

and when $i = j$

$$C_{ij}^{(1)} = c_{ii}^{(1)} = -\sum_{k=1}^{N} C_{ik}^{(1)} \qquad i = 1,2,\dots,N, \qquad i \neq k, \qquad i = j \tag{36}$$

Then, weighting coefficients for high orders derivatives may be expressed by

$$C_{ij}^{(2)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(1)}, \qquad C_{ij}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(2)} = \sum_{k=1}^{N} C_{ik}^{(2)} C_{kj}^{(1)}$$

$$C_{ij}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(3)} C_{kj}^{(1)} \qquad i,j = 1,2,\dots,N \tag{37}$$

$$C_{ij}^{(5)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(4)} C_{kj}^{(1)}, \qquad C_{ij}^{(6)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(5)} = \sum_{k=1}^{N} C_{ik}^{(5)} C_{kj}^{(1)}$$

According to presented approach, the dispersions of grid points based upon Gauss-Chebyshev-Lobatto assumption are expressed as

$$x_i = \frac{a}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right] \qquad i = 1,2,\dots,N, \tag{38}$$

Next, the displacement components may be determined by

$$u = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}(t)\, cos(n\theta) \tag{39}$$

$$v = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}(t)\, sin(n\theta) \tag{40}$$

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}(t)\, cos(n\theta) \tag{41}$$

$$\varphi_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Phi_{mn}(t)\, cos(n\theta) \tag{42}$$

$$\varphi_\theta = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Theta_{mn}(t)\, sin(n\theta) \tag{43}$$

where U_{mn}, V_{mn}, W_{mn}, Φ_{mn} and Θ_{mn} are oscillation amplitudes. The boundary conditions are

$$w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^4 w}{\partial x^4} = 0 \qquad \text{for S} - \text{S} \tag{44}$$

Now, one can express the modified weighting coefficients for all edges simply-supported as

$$\begin{aligned} &\bar{C}_{1,j}^{(2)} = \bar{C}_{N,j}^{(2)} = 0 && i = 1, 2, \ldots, M \\ &\bar{C}_{i,1}^{(2)} = \bar{C}_{1,M}^{(2)} = 0 && i = 1, 2, \ldots, N \end{aligned} \tag{45}$$

and

$$\bar{C}_{ij}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(1)} \bar{C}_{kj}^{(2)} \qquad \bar{C}_{ij}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(1)} \bar{C}_{kj}^{(3)} \tag{46}$$

Placing Eqs. (39)-(43) into Eqs. (28)-(32) and performing several simplifications results in the below relation as

$$[K]\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Phi_{mn} \\ \Theta_{mn} \end{Bmatrix} + [M]\begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \\ \ddot{\Phi}_{mn} \\ \ddot{\Theta}_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F(t) \\ 0 \\ 0 \end{Bmatrix} \tag{47}$$

As an efficient tool, the well-known Laplace transform technique can be used to derive the shell deflection as a function of load speed. Further studies on forced vibration of a porous strain gradient shell will be performed based on below normalized factors

$$\begin{aligned} &V* = \frac{V_p}{V_{cr}}, \quad V_{cr} = \frac{\omega_n L}{\pi}, \\ &t^* = \frac{V_p t}{L}, \quad \bar{W} = W\frac{100 E_c h^3}{P_0 L^3}, \quad \lambda = \frac{l}{L} \end{aligned} \tag{48}$$

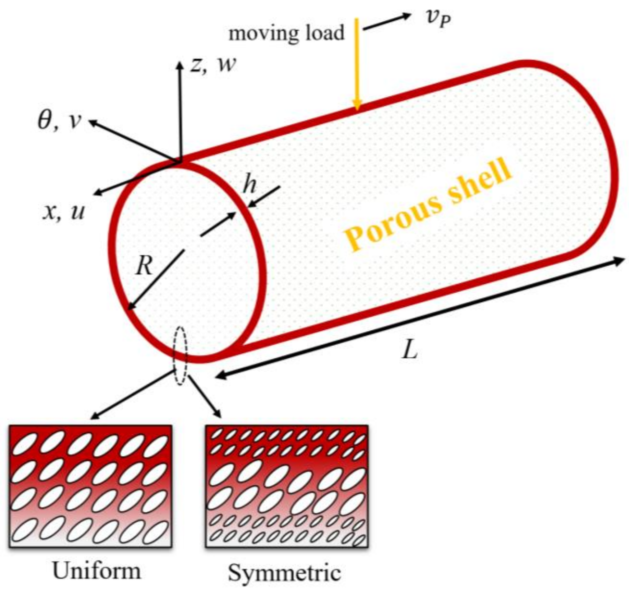


Fig. 1 Geometry of a cylindrical shell made of metal foams

Table 1 Frequency verification for a strain-gradient shell ($l = h$)

	ω_{11}		ω_{22}	
h/R	SGT (Zeighampour and Beni 2014)	present	SGT (Zeighampour and Beni 2014)	present
0.02	0.1980	0.1982	0.2795	0.2797
0.05	0.2110	0.2112	0.3953	0.3958

4. Discussions on results

Thorough the present section, results are provided for forced vibration investigation of scale-dependent metal foam shells formulated by a first order theory and strain gradient elasticity. The small-size foam shell under a travelling load has been depicted in Fig. 1. Table 1 provides validation study for vibrational frequency of a small-scale shell with the results obtained by Zeighampour and Beni (2014). Accordingly, the present formulation and DQ solution is capable of giving accurate results of micro size shells. In this research, obtained results based on metal foam material are presented using the below properties: $E_2 = 200\ GPa$, $\rho_2 = 7850\ kg/m^3$, $v = 0.33$.

In Fig. 2, the variation of normalized deflections of a metal foam small-dimension shell versus loading time of travelling load is represented for several stain gradients (λ) coefficients. By selecting λ = 0, the deflections and vibrational properties based upon classic shell assumption will be derived. Actually, selecting $\lambda = 0$ gives the deflections in the context of classic elasticity theory and discarding strain gradients impacts. It can be understood from Fig. 2 that normalized deflection of system will reduce with strain gradient coefficient. This observation is valid for all ranges of dimensionless time. So, forced vibration behavior of the shell system is dependent on the

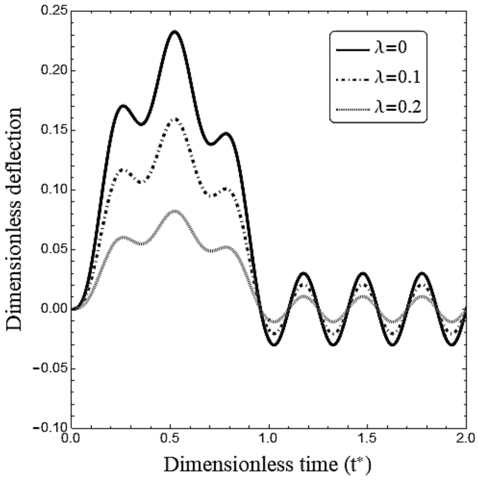
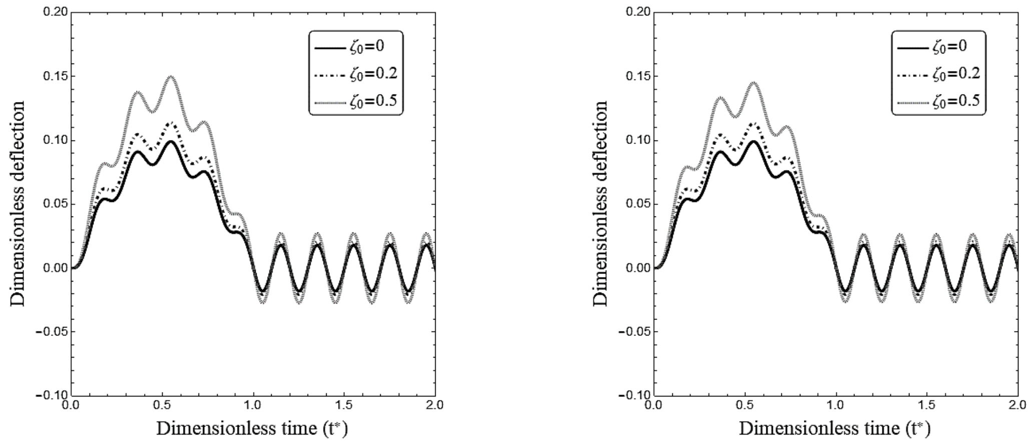


Fig. 2 Strain gradient influence on dynamical response of the metal foam shell (L/h = 30, R/h = 5, $\zeta_0 = 0.5$, $V^* = 0.15$)



(a) Uniform porosities (b) Symmetric porosities

Fig. 3 Porosity influence on dynamical response of the metal foam shell ($V^* = 0.1$, R/h = 5, L/h = 30, $\lambda = 0.1$)

scale effects. Note that up to $t^* = 1$, the shell has forced vibrations due to travelling load, however, after this value the shell has free vibrations.

In Fig. 3 one can see the response curves of metal foam shell system with different porosity coefficients and dispersions. Speed of traveling load is selected to be constant for this figure. It can be understood from Fig. 3 that shell deflection of system will increase with pore coefficient. But, this variation relies on the type of pore dispersion in thickness of cylindrical shell. Uniform pore type gives higher shell deflections than other pore types. This is due to the reason that the cylindrical shell is with uniform porosities is more flexible than a cylindrical shell with symmetric porosities. This issue is also shown in Fig. 4 when the porosity coefficient is selected as $\zeta_0 = 0.7$. This figure shows a good comparison between obtained results based on two porosity distributions.

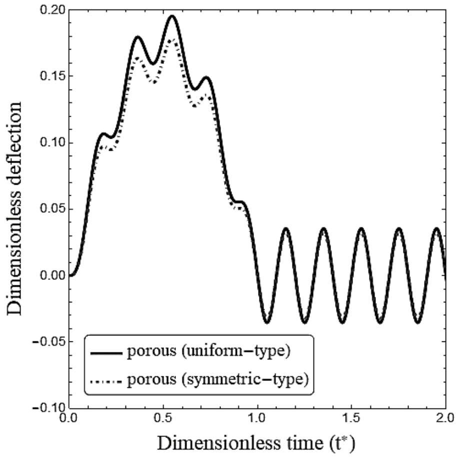


Fig. 4 Porosity type influence on dynamical response of the metal foam shell (ζ_0 = 0.7, V^* = 0.1, R/h = 5, L/h = 30, λ = 0.1)

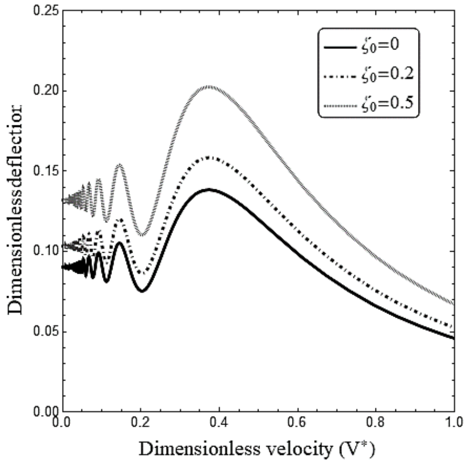


Fig. 5 Load speed influence on dynamical response of the metal foam shell with symmetric porosities (t^* = 0.5, R/h = 5, L/h = 30, λ = 0.1)

Fig. 5 illustrates the dynamical deflections change of a porous metal foam shell according to speed factor (V^*) at a fixed non-dimensional time t^* = 0.5 and different porosity coefficients (ζ_0). Based on above discussion, when the magnitudes of porosity coefficient are greater, the non-dimensional deflections are larger. An important finding is that the dynamical deflections have decreasing or increasing tends in low speeds of traveling load. Actually, the porous shell has considerable number of oscillations at low speeds of travelling load. But, the dynamical deflections are reducing at greater values of load speed.

Fig. 6 demonstrates the dynamical deflections changes of a porous metal foam shell according to normalized load speed (V^*) when the non-dimensional time is t^* = 0.5 and various values for radius-to-thickness ratio (R/h) have been considered. For this figure, uniform porosities dispersion having coefficient value ζ_0 = 0.5 has been assumed. It is evident that via enlargement of radius-to-thickness ratio, the dynamical deflections of the microshell in contact with travelling load become

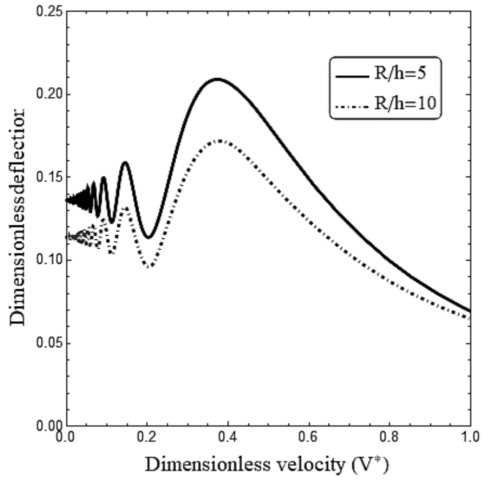


Fig. 6 Shell radius influence on dynamical response of the metal foam shell (t^* = 0.5, L/h = 30, λ = 0.1)

lower for every value of load speed. This finding is owning to the reason that the geometries of microshell can prominently change the stiffness as well as dynamical behaviors.

5. Conclusions

This article focused on forced vibration characteristic of metal foam shells modeled by strain gradient and first order shell theories. Scale-dependent shells were considered to be porosity-dependent accounting for different pore types. It was understood that dynamic deflection of system reduced with strain gradient coefficient. It was also found that dynamic deflection of system might increase with pore coefficient. Also, uniform pore type gave greatest shell deflections among considered pore types. An important finding was that the dynamical deflections have decreasing or increasing tends in low speeds of traveling load. Actually, the porous shell has considerable number of oscillations at low speeds of travelling load. But, the dynamical deflections were reducing at greater values of load speed.

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