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Thermo-mechanically induced finite element based nonlinear static response of elastically supported functionally graded plate with random system properties

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Abstract. The present work proposes the thermo mechanically induced statistics of nonlinear transverse central deflection of elastically supported functionally graded (FG) plate subjected to static loadings with random system properties. The FG plate is supported on two parameters Pasternak foundation with Winkler cubic nonlinearity. The random system properties such as material properties of FG material, external loading and foundation parameters are assumed as uncorrelated random variables. The material properties are assumed as non-uniform temperature distribution with temperature dependent (TD) material properties. The basic formulation for static is based on higher order shear deformation theory (HSDT) with von-Karman nonlinear strain kinematics through Newton-Raphson method. A second order perturbation technique (SOPT) and direct Monte Carlo simulation (MCS) are used to compute the nonlinear governing equation. The effects of load parameters, plate thickness ratios, aspect ratios, volume fraction, exponent, foundation parameters, and boundary conditions with random system properties are examined through parametric studies. The results of present approaches are compared with those results available in the literature and by employing direct Monte Carlo simulation (MCS).

Keywords: integrated design; evaluation; current practice; integrated platform; online survey; designers

1. Introduction

The functionally graded materials have attracted much attention in the many engineering applications from last decade due to high temperature resistance and maintain structural integrity by gradation of composition along the thickness direction through the appropriate volume fraction change (Birman and Byrd 2007, Koizumi 1997, Suresh and Mortensen 1998).

The FG materials are being increasingly used in many engineering sectors such as in aerospace for spacecraft antennas and thermal barrier coating, in nuclear for making a wall of fission

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reactors, in biomedical for making artificial bones and teeth, in automobiles for engine inner parts of piston and cylinder and cutting tools, and other allied fields such as solar receiver systems, heat exchanger tubes, turbine blades etc. (Aboudi *et al.* 1994, Matsuzaki 1993).

The FG structural components supported on elastic foundation are also being increasingly used in large transportation aircraft runways, launching pads of missiles and tops, suspension systems in automobiles, foundation of deep wells, ship and bridge structures parts etc. The exact modeling of elastic foundation is one of the areas of the interesting area of research. For design prospective, two parameters Pasternak elastic foundations with Winkler cubic nonlinearity is one of the most appropriate foundation models (Ying *et al.* 2008, Fallah *et al.* 2008).

When a structure is subjected to temperature change, thermally induced compressive stresses are developed in the constraint edges due to mismatch thermal expansion coefficients. Such thermal stresses oppose the mechanical stresses and finally lower the stiffness. Ultimately it reduces the strength and stability of the plate. Hence, the effect of thermal loading on the structural performance is one of the essential parameters and plays a significant role in the structural integrity and stability of the structures (Sankar and Tzeng 2002, Shi-Rong *et al.* 2006).

The study of transverse central deflection of FG materials plate resting on elastic foundation subjected to static and dynamic loadings is highly important for optimal and reliable performance of overall structures.

The large number of random system parameters in terms of exact design specification and materials gradation is involved during manufacturing and fabrication of FG materials as compared to conventional isotropic and homogeneous materials. These random system parameters are inherent in nature and yield the material and geometrical uncertainties. The present of these types of system randomness may have effect on the structural performance and finally affects the reliability of final design.

Aside from this, during application, the structures are constantly subjected to different types of mechanical loadings and hence it may be assumed as an independent random variable. Similarly, the modeling of foundation parameters by assuming as separate random system parameters to obtain the accurate response of supported elastic foundation is also a matter of concern in practice. Hence, the quantification of system randomness at various levels using stochastic approach is extremely important for reliable and safe design of the overall structure.

Several literatures are available on the large deflection response of FG materials panels subjected to static thermo-mechanical loadings acting simultaneously or separately using various deterministic approaches. In this direction, several micromechanics models have been developed to calculate effective properties of macroscopically homogeneous composite materials. The FGMs plates are usually used in high temperature in which significant variations in physical properties of the constituent materials occur as temperature changes (see Reddy and Chin 1998), and are expected to failure from large amplitude deflection and/or excessive stresses induced by thermal or mechanical loads. It is therefore important to account for the nonlinearity in deformation subjected to temperature dependent material properties. A considerable amount of literatures are available in this direction, namely, Shen and Wang (2010) evaluated the nonlinear transverse central deflection response of FGM plate supported by an elastic foundation in thermal environment using HSDT through semi-analytical method. They presented the finite element model based on third-order shear deformation theory for static and dynamic analysis of the FGM plates. Ma and Wang (2003) investigated the axisymmetric large deflection, bending and thermal post-buckling of a FGMs circular plate under mechanical, thermal and combined thermal-mechanical loadings, based on the classical nonlinear of von Karman plate theory. Yang and Shen (2003) presented nonlinear bending response of shear deformable functionally graded plate subjected to thermo-mechanical loads, based on Reddy's higher order shear deformation plate theory using the semi analytical method. Ferreira *et al.* (2005) presented a static analysis of functionally graded material plate using third-order shear deformation theory based on mesh less methods. Ghannadpour and Alinia (2006) presented a large deflection analysis of rectangular functionally graded plates under pressure loads using the Von-Karman theory with potential energy. Na and Kim (2006) presented nonlinear bending of clamped rectangular FGM plate subjected to a transverse uniform pressure and thermal loads using a 3-D finite element method, in this study the thermal loads were assumed as uniform, linear and sinusoidal temperature rises across the thickness direction. Yang *et al.* (2005) presented the bending response of shear deformable FG materials plate with system randomness using first-order shear deformation plate theory (FSDT) combined with FOPT. Khabbaz *et al.* (2009) presented the energy concept with the first and third-order shear deformation theories (FSDT and TSDT) for nonlinear analysis of FGM plates under pressure loads. Alinia and Ghannadpour (2009) studied the nonlinear analysis of pressure loaded FGM plates based on classical plate theory. Shen and Wang (2010) presented the nonlinear bending of simply supported FGM plates subjected to combined thermo-mechanical loadings resting on elastic foundations using temperature dependent material properties. Singha *et al.* (2011) presented the finite element analysis of functionally graded plates to evaluate the transverse central deflection under transverse distributed load using FSDT considering the exact neutral surface position through Newton-Raphson iteration method. Praveen and Reddy (1998) evaluated the transverse central of functionally graded ceramic, metal plates in thermal environment using finite element method combined with first order shear deformation theory using von-Karman nonlinearity. Huang and Shen (2004) evaluated the nonlinear transverse central deflection and free vibration response of functionally graded plate subjected to thermo mechanical loadings using HSDT with von-Karman nonlinearity through semi analytical approach. Wang and Shen (2013) examined the nonlinear dynamic response of sandwich FGM plate resting elastic foundation in thermal environment using HSDT through semi-analytical method. Shen (2007) presented the nonlinear thermal bending response of simply supported, shear deformable FGM plates subjected to combined action of thermal and electrical loading due to heat conduction based on higher order shear deformation theory. Zhang (2014) evaluated the transverse central deflection response of FG materials plate resting on two-parameter elastic foundations using on physical neutral surfaces and high-order shear deformation theory.

All the above mentioned literatures are based on deterministic study which gives only mean structural response and unaccounted the effect of random system properties on the structural performance for higher reliability and safety.

The studies related to the stochastic response of FG and other material structures subjected static loading are very limited due to complexities involved in quantification of the random system properties. In this direction, Yang *et al.* (2005) presented the elastic buckling response of shear deformable FG materials plate with uncertain system randomness using FSDT combined with FOPT. Onkar *et al.* (200, 2006) presented the nonlinear bending and post buckling of composite laminates with random material properties under random loading based on FOPT. Singh *et al.* (2001, 2003, 2008) evaluated the vibration and bending of composite laminate plates supported with or without elastic foundation using C^0 finite element method based on HSDT in conjunction with FOPT. Lal *et al.* (2007, 2009) presented the effect of random material properties on nonlinear free vibration and buckling response of laminated composite plates supported with and without elastic foundation in the thermal environment using HSDT based C^0 nonlinear FEM based on the direct iterative method in conjunction with FOPT. Shaker *et al.* (2008) evaluated the free vibration of functionally graded plates using stochastic finite element method (SFEM) combined with HSDT through first and second order reliability method. Jagtap *et al.* (2012) evaluated the stochastic nonlinear bending response of elastically supported FGM plate with system randomness in thermal environment using direct iterative based nonlinear FEM in conjunction with first order perturbation theory (FOPT). Pandit *et al.* (2010) presented the stochastic finite element method for free vibration of core sandwich plate using with mean centered FOPT through higher-order zigzag theory with random material properties. A C^0 finite element method combined MCS with hypercube sampling technique using third order shear deformation theory are used to handle the material and structural uncertainties by Chandrashekhar and Ganguli (2009, 2010); Murugan *et al.* (2008). Liu *et al.* (1986) evaluated the statistics of deflection response of spring mass system using a stochastic finite element method based on perturbation and MCS techniques. Chang and Chang (1994) investigated the statistical dynamic responses of a nonuniform beam by using the finite element method in conjunction with perturbation technique and Monte Carlo simulation considering uncertain Young's modulus of elasticity. Kitipornchai *et al.* (2006) presented a stochastic model to obtain second order statistics of fundamental frequency of FG materials laminates in thermal environments using first-order shear deformation plate theory (FSDT) in conjunction with FOPT. Ibrahim *et al.* (2007) evaluated random transverse deflection and thermal bucking response in terms of the mean and variance of a FG plate subjected to combined thermal and acoustic loads with random acoustic pressure using a finite element method based on the thin theory through Newton-Raphson method via Newmark direct time integration. Onkar and Yadav (2005) proposed stochastic FOPT to examine the mean and variance of nonlinear transverse central deflection response of laminated composite plate with random material properties under random external loading using Kirchoff-Love plate theory with von-Karman nonlinearity. Singh and Lal (2010) proposed similar FOPT based stochastic model combined with a conventional finite element method (FEM) to obtain the mean and coefficient of variance of post buckling and nonlinear free vibration analysis of laminated composite plate resting on two parameters elastic foundation with Winkler cubic nonlinearity through HSDT. Lal *et al.* (2012a, 2012b, 2013) evaluated the mean and coefficient of variance (COV) of initial and post buckling analysis of laminated composite and functionally graded plates subjected to thermo-mechanical loadings using C0 nonlinear FEM based on HSDT combined with FOPT. Jagtap *et al.* (2011, 2013) evaluated the second order statistics of nonlinear free vibration and post buckling analysis of FGM shell panels using HSDT combined with direct iterative based nonlinear FEM combined with FOPT. Shegokar and Lal (2013a, 2014) proposed a stochastic FEM model based on FOPT and MCS to evaluate mean and COV of thermo-electro-mechanically induced buckling and vibration response of the FGM beam with random system properties using HSDT. Talha and Singh (2014) proposed stochastic FEM based on FOPT to obtain mean and COV of post buckling response of FGM plate with random material properties in thermal environment using HSDT. Kumar *et al.* (2014) proposed the stochastic FEM based on FOPT to obtain the hygro-thermo-mechanically induced nonlinear transverse central deflection response of laminated composite plate supported by an elastic foundation with random system properties using HSDT with von-Karman nonlinearity through micromechanics approach. Lal *et al.* (2015) evaluated the thermally induced post buckling response of piezoelectric laminated composite plate resting on elastic foundation with random system properties using micromechanical approach using C^0 finite element methods combined with second order perturbation method.

It is accomplished from the above mentioned literatures that the studies of thermo mechanically

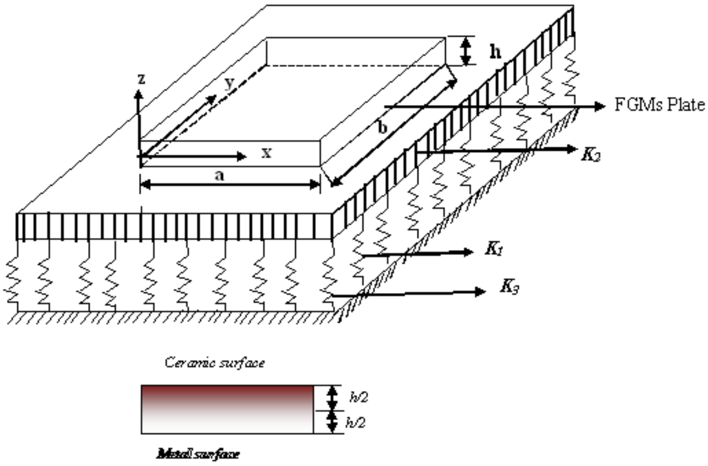


Fig. 1 Geometry of FGM plate resting on elastic foundation

induced nonlinear bending analysis of elastically supported FGM plates subjected to static transverse uniformly distributed mechanical loadings in thermal environment involving randomness in random system properties are rarely available. For the optimum performance and accurate prediction of response it is obligatory to understand the thermo mechanically induced transverse central deflection response through stochastically. The main objective of this paper is to evaluate statistics in terms of mean and coefficient of variance (COV) of thermo mechanically induced nonlinear transverse central deflection response of FG materials plate supported by an elastic foundation with random system parameters.

2. Mathematical formulations

Consider a rectangular FGM plate consists of metal and ceramic at the top and bottom layer having length *a*, width *b*, and total thickness *h*, defined in (*x*, *y*, *z*) system with *x*- and -*y* axes located the middle plane and its origin placed at the corner of the plate. The plate is assumed to be attached to the elastic foundation excluding any separation takes place in the process of deformation as shown in Fig. 1. The interaction between the plate and the supporting foundation follows the two parameter model (Pasternak-type) with Winkler cubic nonlinearity as (Shen *et al.* 2010)

$$p = K_1 w + K_3 w^3 - K_2 \nabla^2 w \tag{1}$$

Where p is the foundation reaction per unit area, and $\nabla^2 = \partial,^2_{x2} + \partial,^2_{y2}$ is second order Laplace differential operator. The parameters K_1, K_2 and K_3 are linear normal, shear and nonlinear normal spring stiffness foundations, respectively. This model is simply known as Winkler type when K_2=0 (Lal *et al.* 2007). The symbol comma ($,_x$) denotes as partial differential with respect to x.

The properties of the FGM plate are assumed to vary through the thickness of the plate only, such that the top surface z=h/2 is ceramic-rich and the bottom surface z=-h/2 is metal reach as shown in Figs. 2(a)-(d). The effective mechanical and thermal properties of the FGMs plate of an arbitrary point within the plate domain are expressed as (Jagtap *et al.* 2012).

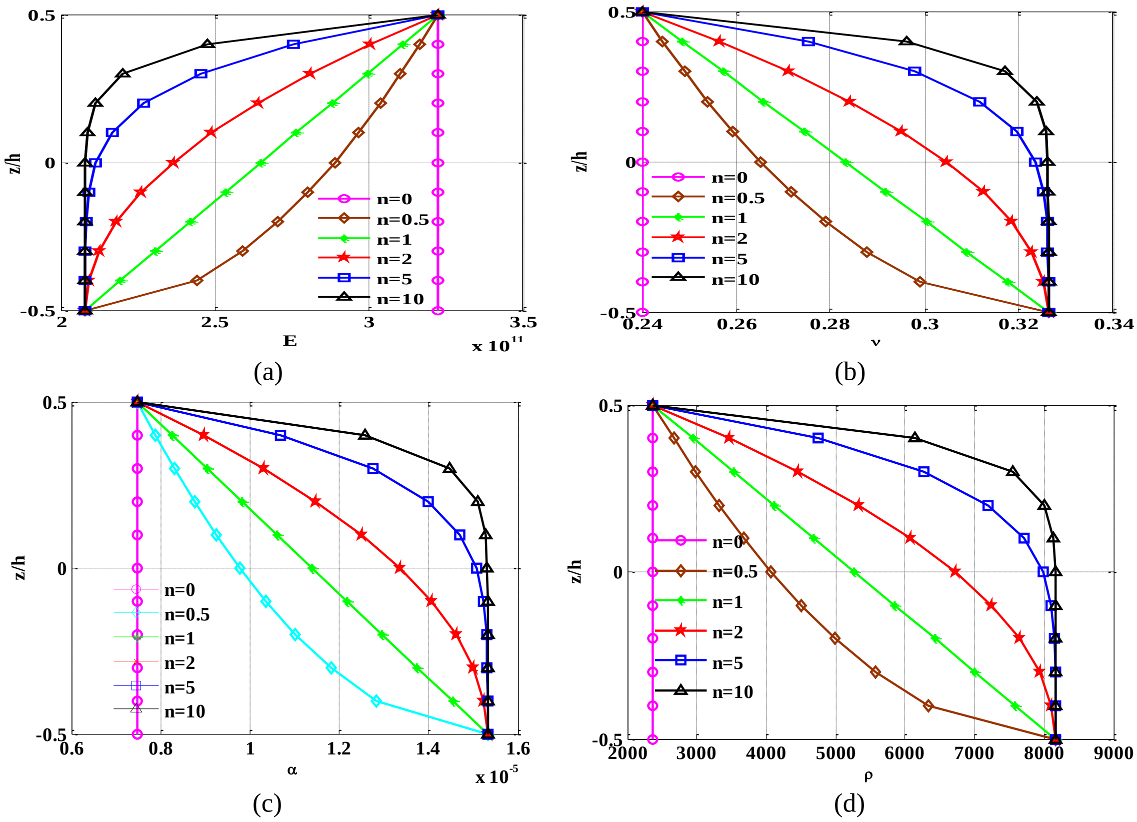


Fig. 2 Variation of (a) Young's modulus model, (b) Poisson's ratio, (c) Density and (d) Thermal expansion coefficient of SUS304-Si3N4 FGM beam along the thickness for various volume fraction index

$$
\begin{aligned}
E(z,T) &= E_b(T) + \left[E_t(T) - E_b(T)\right]V_c(z) \\
\alpha(z,T) &= \alpha_b(T) + \left[\alpha_t(T) - \alpha_b(T)\right]V_c(z) \\
\rho(z,T) &= \rho_b(T) + \left[\rho_t(T) - \rho_b(T)\right]V_c(z) \\
k(z,T) &= k_b(T) + \left[k_t(T) - k_b(T)\right]V_c(z)
\end{aligned}
\tag{2}
$$

Where, t and b represent to the ceramic and metal constituents, respectively. With E, α, ρ and k are the effective young modulus, thermal expansion coefficient, density and thermal conductivity respectively. The elastic material properties vary through the plate thickness according to the volume fractions of the constituents. Power-law distribution is commonly used to describe the variation of material properties, which is expressed as (Jagtap *et al.* 2012, Shen 2009),

$$P(z) = (P_c - P_m)\left(\frac{2z+h}{2h}\right)^n + P_m, \tag{3a}$$

$$V_C(z) = \left(0.5 + \frac{z}{h}\right)^n, \quad -h/2 \le z \le h/2,\ 0 \le n \prec \infty \tag{3b}$$

Where P denotes the effective material property, P_m and P_c represents the properties of the metal and ceramic, respectively, V_c is the volume fraction of the ceramic and n is the volume fraction exponent and is always positive. The effective material properties of the plate, including Young's modulus E, density ρ vary according to Eq. (2) and v is assumed to be constant due to weakly dependent on temperature change.

2.1 Displacement field model

In the present analysis, the assumed displacement field based on Reddy's HSDT having C^1 continuity by the satisfaction of conditions that the transverse shear stresses vanish at the top and bottom of the plate and nonzero elsewhere is modified by C^0 continuity by considering derivatives of the out-of-plane displacements as separate degree of freedom (Shaker *et al.* 2008, Jagtap *et al.* 2012, William *et al.* 1992, Singh and Lal 2010, Lal *et al.* 2013, Lal *et al.* 2012a, Shankara and Iyengar 1996). The modified displacement based on C^0 continuity field, along the X, Y, and Z directions for an arbitrary plate is now written as (Singh *et al.* 2001).

$$\begin{aligned}\bar{u} &= u + f_1(z)\psi_x + f_2(z)\phi_x;\\ \bar{v} &= v + f_1(z)\psi_y + f_2(z)\phi_y;\\ \bar{w} &= w;\end{aligned} \tag{4}$$

Where $\bar{u}$, $\bar{v}$, and $\bar{w}$ denote the displacements of a point along the (x, y, z) coordinates, u, v, and w are corresponding displacements of a point on the mid plane. ψ_x and ψ_y are the rotations of normal to the mid plane about the y-axis and x-axis respectively, with $\phi_x = \partial w/\partial x$ and $\phi_y = \partial w/\partial y$. The parameter $f_1(z)$ and $f_2(z)$ are defined as $f_1(z) = C_1 z - C_2 z^3;\ f_2(z) = -C_4 z^3$ with $C_1 = 1; C_2 = C_2 = \dfrac{4}{3h^2}$

The displacement vector for the modified C^0 continuous model is denoted as

$$\{\Lambda\} = \begin{bmatrix} u & v & w & \phi_y & \phi_x & \psi_y & \psi_x \end{bmatrix}^T \tag{5}$$

2.2. Strain displacement relations

For the structures considered here, the relevant total strain vector consisting of strains in terms of mid-plane deformation, rotation of normal and higher order terms associated with the displacement for FGM including thermal strain is expressed as (Shegokar and Lal 2013a, 2013b)

$$\{\varepsilon\} = \{\varepsilon_l\} + \{\varepsilon_{nl}\} - \{\bar{\varepsilon}_t\} \tag{6}$$

Where $\{\varepsilon_l\}$, $\{\varepsilon_{nl}\}$ and $\{\bar{\varepsilon}_t\}$ are the linear and nonlinear strain vectors, thermal strain vector, respectively.

Using Eq. (6) the linear strain vector can be obtained using linear strain displacement relations (Singh *et al.* 2008). Assuming that the strains are much smaller than the rotations (in the von-Karman sense), one can obtain nonlinear strain vector $\{\varepsilon_{nl}\}$ as (Lal *et al.* 2007).

$$\{\varepsilon_{nl}\} = \frac{1}{2}\left[A_{nl}\right]\{\phi\} \tag{7}$$

$$\text{Where}\ \{A_{nl}\} = \frac{1}{2}\begin{bmatrix} w_{,x} & 0 \\ 0 & w_{,y} \\ w_{,x} & w_{,y} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \ \text{and} \quad \{\phi\} = \begin{Bmatrix} w_{,x} \\ w_{,y} \end{Bmatrix}, \tag{7a}$$

The thermal strain vector $\{\bar{\varepsilon}_t\}$ as given in Eq. (6) is represented as

$$\{\bar{\varepsilon}_t\} = \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\varepsilon}_{xy} \\ \bar{\varepsilon}_{yz} \\ \bar{\varepsilon}_{zx} \end{Bmatrix} = \Delta T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

Where α_1, α_2 and α_{12} are coefficients of thermal expansion along the *x*, *y*, and *x*-*y* directions, respectively. These parameters can be obtained from the thermal coefficients in the longitudinal (α_l) and transverse (α_t) directions of the ceramic and metal using transformation matrix and ΔT is the uniform and non-uniform temperature change. For the transversely non-uniform temperature rise, the temperature field, along the thickness should be determined by solving the boundary value problem of thermal conduction, given by (Shegokar and Lal 2013).

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}z}\left(k(z,T)\frac{\mathrm{d}T}{\mathrm{d}z}\right) = 0\ ,(-h/2 < z < h/2) \\ &T(h/2) = T_t\ ,T(-h/2) = T_b \end{aligned} \tag{9}$$

The temperature field for non uniform temperature change is expressed as

$$\Delta T = T(z) - T_0 \tag{9a}$$

Where, T_0 is initial temperature and T(z) is expressed as (Shen 2009, Jagtap *et al.* 2011, Niranjan and Lal 2013)

$$T(z) = T_b + (T_t - T_b)\eta(z) \tag{9b}$$

Where, T(z) is the temperature distribution along z direction, *t* and *b* are referred as top and bottom surface η(z) is the parameters defined as (Jagtap *et al.* 2011)

$$\eta(Z) = \frac{1}{c}\left[\begin{aligned} &\left(0.5+\frac{z}{h}\right) - \frac{k_{tb}}{(n+1)k_b}\left(0.5+\frac{z}{h}\right)^{n+1} + \frac{k_{tb}^{\ 2}}{(2n+1)k_b^{\ 2}}\left(0.5+\frac{z}{h}\right)^{2n+1} - \frac{k_{tb}^{\ 3}}{(3n+1)k_b^{\ 3}}\left(0.5+\frac{z}{h}\right)^{3n+1} \\ &+ \frac{k_{tb}^{\ 4}}{(4n+1)k_b^{\ 4}}\left(0.5+\frac{z}{h}\right)^{4n+1} - \frac{k_{tb}^{\ 5}}{(5n+1)k_b^{\ 5}}\left(0.5+\frac{z}{h}\right)^{5n+1} \end{aligned}\right] \tag{10}$$

Where

$$C = 1 - \frac{k_{tb}}{(n+1)k_b} + \frac{k^2_{\ tb}}{(2n+1)k^2_{\ b}} - \frac{k^3_{\ tb}}{(3n+1)k^3_{\ b}} + \frac{k^4_{\ tb}}{(4n+1)k^4_{\ b}} - \frac{k^5_{\ tb}}{(5n+1)k^5_{\ b}}$$

Here *k*, *z*, *n* indicate the thermal conductivity, distance from central axis and volume fraction, respectively with $k_{tb}=k_t-k_b$.

2.3 Constitutive relations

The constitutive relationship between stress and strain vectors in the plane stress state for an isotropic layer accounting thermal effect can be written as (Lal *et al.* 2009)

$$\{\sigma\} = \left[Q_{ij}\right]\{\varepsilon\} \tag{11}$$

Where $\{Q_{ij}\}$, $\{\sigma\}$ and $\{\varepsilon\}$ are transformed stiffness matrix, stress and strain vectors of the isotropic lamina, respectively. For FGM material the elastic constant (Q_{ij}) are defined as

$$Q_{11} = Q_{22} = \frac{E(z,T)}{1-\nu^2},\ \ Q_{12} = \frac{\nu E(z,T)}{1-\nu^2},\ \ Q_{44} = Q_{55} = Q_{66} = \frac{E(z,T)}{2(1+\nu)} \tag{12}$$

2.4 Strain energy of the plate

The strain energy (Π_1) of the FG material plates considering linear and nonlinear strain can be expressed as

$$\Pi_1 = \frac{1}{2}\int_{\text{A}} \left\{\varepsilon_{\text{l}} + \varepsilon_{\text{nl}}\right\}^{\text{T}} \left[\sigma\right] \text{dA} \tag{13}$$

Substituting Eq. (6)-(7a) in Eq. (13) can be written as

$$\begin{aligned}\Pi_1 = &\frac{1}{2}\int_A \{\bar{\varepsilon}_l\}^T [D]\{\bar{\varepsilon}_l\} dA + \frac{1}{2}\int_A [\bar{\varepsilon}_l]^T [D_3]\{A\}\{\theta\} dA \\ &+ \frac{1}{2}\int_A \{A\}^T \{\theta\}^T [D_4]\{\bar{\varepsilon}_l\} dA + \frac{1}{2}\int_A \{A\}^T \{\theta\}^T [D_5]\{A\}\{\theta\} dA\end{aligned} \tag{14}$$

Where (D), (D_3), (D_4) and (D_5) are the FG material stiffness matrices as given in Appendix (A.1) and

$$\{\bar{\varepsilon}\} = \left(\varepsilon_1^0 \quad \varepsilon_2^0 \quad \varepsilon_6^0 \quad k_1^0 \quad k_2^0 \quad k_6^0 \quad k_1^2 \quad k_2^2 \quad k_6^2 \quad \varepsilon_4^0 \quad \varepsilon_5^0 \quad k_4^2 \quad k_5^2\right) \tag{14a}$$

2.5 Strain energy due to foundation

Strain energy due to elastic foundation having a shear deformable layer with Winkler cubic nonlinearity is expressed as (Lal *et al.* 2012b)

$$\Pi_2 = \frac{1}{2}\int_A pw dA \tag{15}$$

Substituting value of p from Eq. (1) into Eq. (15), and applying variation principle the strain energy due to foundation can be written as (Shen

$$\Pi_2 = \frac{1}{2}\int_A \left\{K_1 w^2 + \frac{1}{2}K_3 w^4 + K_2\left[\left(w,_x\right)^2 + \left(w,_x\right)^2\right]\right\} dA \tag{16}$$

Eq. (16) can be written in matrix form as

$$\Pi_2 = \frac{1}{2}\int_A \begin{Bmatrix} w \\ w,_x \\ w,_y \end{Bmatrix}^T \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_2 \end{bmatrix} \begin{Bmatrix} w \\ w,_x \\ w,_y \end{Bmatrix} dA + \int_A \begin{Bmatrix} w \\ w,_x \\ w,_y \end{Bmatrix}^T \begin{bmatrix} \frac{1}{2}K_3 w^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} w \\ w,_x \\ w,_y \end{Bmatrix} dA \tag{16a}$$

2.6 Potential energy due to thermal stresses

The potential energy (Π_2) storage by thermal load (uniform and non-uniform change in temperature) across the thickness is written as

$$\begin{aligned} \Pi_3 &= \frac{1}{2}\int_A \left[N_x (w,_x)^2 + N_y (w,_y)^2 + 2N_{xy}(w,_x)(w,_y) \right] dA \\ &= \frac{1}{2}\int_A \begin{Bmatrix} w,_x \\ w,_y \end{Bmatrix}^T \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} \begin{Bmatrix} w,_x \\ w,_y \end{Bmatrix} dA \end{aligned} \tag{17}$$

Where, N_x, N_y and N_{xy} are pre-buckling thermal stresses acting along x, y and shear directions, respectively.

2.7 External work done due to transverse mechanical load

The potential due to applied external mechanical loading $q(x,y)$ is given by (Huang and Shen 2004, Wanga and Shen 2013)

$$\Pi_4 = -W_q = \int_A q(x,y)\; w\, dA \tag{18}$$

Where, $q(x, y)$ is the intensity of distributed transverse static load corresponding to each degree of freedom (DOF) which is defined as

$$q(x,y) = \frac{QE_m h^4}{b^4} \tag{19}$$

Where Q and E_m are represented as load parameter and Young's modulus in transverse direction, respectively.

3. Finite element model

3.1 Strain energy of the plate

In the present study, a C^0 nine-noded isoparametric finite element with 7 DOFs per node is employed. In this type of element, the displacement vector and the element geometry are expressed as

$$\{q\} = \sum_{i=1}^{NN} \varphi_i \{q\}_i \,; \tag{20}$$

And

$$x = \sum_{i=1}^{NN} \varphi_i x_i; \quad y = \sum_{i=1}^{NN} \varphi_i y_i \tag{21}$$

Where φ_i the interpolation function for the i^{th} node, $\{q\}$ is the vector of unknown displacements for the i^{th} node, NN is the number of nodes per element and x_i and y_i are Cartesian Coordinate of the i^{th} node.

The linear mid plane strain vector as given in Eq. (7) can be expressed in terms of mid plane displacement field and then the energy is computed for each element and then summed over all the elements to get the total strain energy. Following this, and using Eq. (21), Eq. (14) can be written as

$$\Pi_1 = \sum_{e=1}^{NE} \Pi_1^{(e)} \tag{22}$$

Where, NE is the number of elements and $\Pi^{(e)}$ is the elemental total potential energy. Following the assembly procedure, Eq. (22) can be further written as

$$\Pi_1 = \frac{1}{2}\{q\}^T \left[K_l + K_{nl}(q)\right]\{q\} - \{q\}^T \{F^T\} \tag{23}$$

Where

$$\left[K_{nl}(q)\right] = \frac{1}{2}\left[K_{nl1}(q)\right] + \left[K_{nl2}(q)\right] + \frac{1}{2}\left[K_{nl3}(q)\right]$$

Where $[K_l]$, and $[K_{nl}(q)]$ are the global linear and nonlinear stiffness matrices defined in Appendix (A.2). The parameters $\{q\}$ and $\{F^T\}$ are the global displacement and thermal load vectors and defined in the Appendix (A.3).

3.2 Foundation analysis

Similarly, using finite element model Eqs. (20)-(21), Eq. (16) after the assembly procedure can be written as

$$\Pi_2 = \sum_{e=1}^{NE}\left(\Pi_3^{(e)}\right) = \left\{q^{(e)}\right\}\left[K_{fl} + K_{fnl}(q)\right]^{(e)}\left\{q^{(e)}\right\} = \{q\}\left[K_{fl} + K_{fnl}(q)\right]\{q\} \tag{24}$$

Where (K_{fl}) and $(K_{fnl}(q))$ is global linear and nonlinear foundation stiffness matrices, respectively and defined in Appendix (A.4).

3.3 Thermal buckling analysis

Using finite element model Eq. (20), Eq. (17) after the assembly procedure can be written as

$$\Pi_3 = \sum_{e=1}^{NE} \Pi_2^{(e)} = \frac{1}{2}\lambda\{q\}^T \left[K_g\right]\{q\} \tag{25}$$

Where λ and (K_g) are defined as the critical thermal buckling load parameter and the global geometric stiffness matrix defined in Appendix (A.5), respectively.

3.4 Work done due to external transverse load

Using finite element model Eq. (21), Eq. (18) may be written as

$$\Pi_4 = \sum_{e=1}^{NE} \Pi_4^{(e)}$$

Where

$$\Pi_4^{(e)} = -\int_{A^{(e)}} \{q\}^{(e)T} \{P_M\}^{(e)}\, dA\,; \tag{26}$$

With

$$\{P_M\}^{\{e\}} = \begin{pmatrix} 0 & 0 & q & 0 & 0 & 0 & 0 \end{pmatrix}^{T\{e\}} \tag{27}$$

Adopting Gauss quadrature integration numerical rule, the element stiffness and geometric stiffness matrices, load vectors, respectively, can be obtained by transforming expression in x, y coordinate system to natural coordinate system (ξ, η).

4. Governing equation

The governing equation can be derived using Variational principle, which is a generalization of the principle of virtual displacement (Zhang *et al.* 1996). For the bending analysis, the minimization of the first variation of total potential energy Π ($\Pi_1+\Pi_2+\Pi_{3+}$ Π_4) with respect to displacement vector is given by (Reddy and Chin 1998).

$$\delta\ (\Pi_1+\Pi_2+\Pi_3+\Pi_4) = 0 \tag{28}$$

By substituting Eqs. (23), (26) in Eq. (28) and after simplification once obtains as (Shen and Wang 2010, Yang and Shen 2003, Singha *et al.* 2011)

$$\left[K(q)\right]\{q\} = \{F\}, \tag{29}$$

With $[K] = \left[K_l + K_{nl}(q) + K_{fl} + K_{fnl}(q)\right]$ and $\{F\} = \{P_M\} + \{P_T\}$

In the given Eq. (29), the stiffness matrix [K], displacement vector {q} and force vector {F} are random in nature, being dependent on the system properties. In deterministic environment, the solution of Eq. (29) can be obtained using conventional procedure such as iterative, incremental, and Newton Rapson methods, etc. However, in random environment, it is not possible to obtain the solution using above mentioned numerical methods. Further analysis is required to obtain the complete solution of Eq. (29).

For this purpose novel probabilistic procedure based on C^0 nonlinear finite element method using HSDT with von Karman nonlinearity combined with SOPT and MCS through Newton-Raphson method are proposed to obtain the mean and (COV) of transverse central deflection of elastically supported FGM plate in the thermal environment.

5. Solution approach

In this paper, the deterministic solution of Eq. (29) is solved using C^0 nonlinear FEM combined with Newton-Raphson approach using SOPT and MCS methods to evaluate the nonlinear transverse central deflection of FGM elastically supported plate which is described below.

5.1 A newton-raphson method for the solution of nonlinear governing equation

After assembling the element stiffness matrices and force vectors, a new system of nonlinear algebraic equations from Eq. (29) can be written as (William *et al.* 1992, Reddy and Chin 1998)

$$\left[K(q)\right]\{q\}=\{F\} \tag{30}$$

This nonlinear system should be linearized to be solved and to get the nodal displacements $\{q\}$. The Newton-Raphson iterative linearization method is used in this study for evaluation of nonlinear analysis.

In the Newton Raphson procedure, the linearized element Eq. (30) is written in the following form as

$$\{q\}^{i}=\{q\}^{(i-1)}-\left[T\{q\}^{(i-1)}\right]^{-1}\left\{R\left(\{q\}^{(i-1)}\right)\right\} \tag{31}$$

Where the residual

$$\left\{R\left(\{q\}^{(i-1)}\right)\right\}=\left(K\{q\}^{(i-1)}\right)-\{F\} \tag{32}$$

The tangent stiffness matrix $\left[T\{q\}^{(i-1)}\right]$ element is calculated using the definition given as

$$\left[T\{q\}^{(i-1)}\right]=\left(\frac{\partial\{R(\{q\})\}}{\partial\{q\}}\right)^{(i-1)} \tag{33}$$

The next step is to divide the load into small increments as discussed below.

The force vector in Eq. (32) can be written as

$$\{F\}=\sum_{i=1}^{N}\{\Delta F_i\} \tag{34}$$

Where $\{\Delta F_{\mathrm{i}}\}$are the incremental forces applied for i^{th} iteration.

The displacement vector $\{q\}$ for the first step and second step can be written as

$$\left[K\left(\{q_0\}\right)\right]\{q_1\}=\{\Delta F_1\} \tag{35}$$

$$\left[K\left(\{q_1\}\right)\right]\{q_2\}=\{\Delta F_1\}+\{\Delta F_2\} \tag{36}$$

This process is continuing until $\{F\}$ is converged.

In both methods, direct and Newton-Raphson, the first iteration can be calculated using linear stiffness matrix, i.e., assume $\{q\}^{(i-1)}$ =0, and calculate $\{q\}^{i}$ using Eq. (31) or Eq. (32).Then calculate the residual and repeat iteration process till reach a sufficient residual. At the exact solution, the residual equals zero.

5.2 Solution approach of stochastic finite element method

In the present paper, two methodologies such as SOPT and direct MCS are adopted to quantify the statistics of structural response. The SOPT is based on a Taylor series expansion to formulate the linear relationship between some characteristics of the random response and random structural parameters on the basis of SOPT. The applicability of this technique is limited. It is because of it depends on low order polynomial i.e., where the coefficients of variations (COV) of input random variables are small. As the number of input variables becomes large, this method becomes inaccurate and inefficient (Zhang *el al.* 1996, Halder and Mahadevan 2000, Kitipornchai *et al.* 2006, Lal *et al.* 2015). The detail explanation of this method is given by second order perturbation method (SOPT) as discussed in next Section 5.3.

The MCS is adopted to quantify the structural response randomness on the basis of direct use of computer and simulate the experiments by generating of random numbers of the random system properties. In such simulated experiments, a set of random numbers of random system parameters is generated first to present the statistical uncertainties in the random system parameters. These random numbers are substituted into the response equation to obtain again a set of random number which reflects the uncertainties in structural response. A sufficient set of random number is generated for the mean and standard deviation and coefficient of variance of response. However, MCS is computationally expensive and sometimes suffers from prohibitive computational inefficiency. Therefore, MCS is used in limited cases. For the evaluation of MCS results, 5,000 random numbers sample based on convergence study is used to simulate the results.

5.3 Second order perturbation technique for stochastic response

For the present stochastic static analysis problem, it is assumed that the randomness in the input system parameters is small. The governing Eq. (29) can be written in the most general form as

$$[K^*]\{q^*\} = \left\{F^*\right\} \tag{37}$$

Wher $[K^*]$, $\{q^*\}$ and $\{F^*\}$are represented as the random stiffness matrix, random displacement vector and random force vector, respectively.

The operating random system variables in Eq. (37) can be expanded using Taylor series about the mean values of random variables as up to second order without loss of generality (Halder and Mahadevan 2000)

$$\begin{aligned} \left[K^*\right] &= [K_0] + \sum_{i=1}^{N}\left[K_i^{*I}\right]\alpha_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[K_{ij}^{*II}\right]\alpha_i\alpha_j \\ \left\{F^*\right\} &= \{F_0\} + \sum_{i=1}^{N}\left\{F_i^{*I}\right\}\alpha_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[F_{ij}^{*II}\right]\alpha_i\alpha_j \\ \left\{q^*\right\} &= \{q_0\} + \sum_{i=1}^{N}\left\{q_i^{*I}\right\}\alpha_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[q_{ij}^{*II}\right]\alpha_i\alpha_j \end{aligned} \tag{38}$$

Where $[K_0]$, $\{F_0\}$and$\{q_0\}$ are the mean values of respective tensors.·

The symbol $(\ )_i^{*I}$ and $(\ )_i^{*II}$ represent the first and second order derivatives evaluated at α=0, e.g.,

$$K_{ij}^{*II} = \left.\frac{\partial^2 K}{\partial\alpha_i\partial\alpha_j}\right|_{\alpha=0} \quad K_i^{*I} = \left.\frac{\partial K}{\partial\alpha_i}\right|_{\alpha=0} \tag{39}$$

Where α_i, and α_j are the random system parameters.

Substituting Eq. (38) in Eq. (37) and collecting the similar order of terms, following equations are obtained

$$\{q_0\} = \left[K_0^{-1}\right]\{F_0\} \tag{40}$$

$$\{q_i^I\} = \left[K_0^{-1}\right]\left\{F_i^{*I} - \left[K_i^{*I}\right]\{q_0\}\right\} \tag{41}$$

$$\{q_{ij}^{II}\} = \left[K_0^{-1}\right]\left\{F_{ij}^{*II} - \left[K_i^{*I}\right]\{q_j^{*I}\} - \left[K_j^{*I}\right]\{q_i^{*I}\} - \left[K_{ij}^{*II}\right]\{q_0\}\right\} \tag{42}$$

Obviously, Zeroth order Eq. (40) is the deterministic and gives the mean response. The first order Eq. (41) and second order Eq. (42) on the other hand represents its random counterpart and solution of this equation provides the statistics of the nonlinear bending response, which can be solved using the probabilistic methods like perturbation technique, Monte Carlo simulation, Newman's expansion technique etc.

From these mean and covariance matrix of deflection $\{q\}$ can be obtained as as (Halder and Mahadevan 2000, Jagtap *et al.* 2012, Kumar *et al.* 2014).

$$\langle q\rangle \approx \{q_0\} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\{q_{ij}^{II}\}Cov\left[\alpha_i,\alpha_j\right] \tag{43}$$

$$Cov[q,q] \approx \sum_{i=1}^{N}\sum_{j=1}^{N}\{q_i^I\}\left(\{q_j^I\}\right)^T Cov\left[\alpha_i,\alpha_j\right] \tag{44}$$

After $Cov[\alpha_i,\alpha_j]$ is substituted in terms of correlation coefficients ρ_{ij} in Eq. (45), the final expression for $Cov[q,q]$ is obtained as (Jagtap *et al.* 2012, Kumar *et al.* 2014, Singh and Lal 2010).

$$Cov\left[\{q\},\{q\}\right] = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\left.\frac{\partial\{q\}}{\partial\alpha_j}\right|_{\alpha=0}\left(\rho_{ij}\sigma_{\alpha_i}\sigma_{\alpha_j}\right)\left.\frac{\partial\{q^T\}}{\partial\alpha_j}\right|_{\alpha=0}\right] \tag{45}$$

Where $[\sigma_\alpha] = \begin{bmatrix} \sigma_{b1} & ... & ... & 0 \\ 0 & \sigma_{b2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & ... & ... & \sigma_{bm} \end{bmatrix}$ and $\left[\rho_{ij}\right] = \begin{bmatrix} 1 & \rho_{12} & ... & \rho_{1m} \\ \rho_{21} & 1 & ... & \rho_{2m} \\ ... & ... & .. & ... \\ \rho_{m1} & \rho_{m2} & ... & 1 \end{bmatrix}$

Where $[\sigma_\alpha]$, $[\rho_{ij}]$ and m are the standard deviation (SD) of input random variables, the correlation coefficient matrix and number of random variables, respectively. In the present analysis, the uncorrelad Gaussian random variables are taken into consideration. Therefore, covariance is equal to the variance.

The variance of the deflection of random variables b_i (i=1, 2…,R) and correlation coefficients can beexpressed as (Halder and Mahadevan 2000, Shegokar and Lal 2014).

$$\text{var}\{q\} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\left(\frac{\partial\{q\}}{\partial b_i^R}\right)[\sigma_\alpha]\left[\rho_{ij}\right][\sigma_\alpha]\left(\frac{\partial\{q\}}{\partial b_i^R}\right)^T\right] \tag{46}$$

It is to be noted that to estimate the second order variance, the information on the third and fourth order moments of the input random variables must be available. However, in most cases, this information is not available. Therefore, the use of second order mean and the first order variance is considered adequate for most practice engineering applications (Halder and Mahadevan 2000). The square root of variance is known as standard deviation (SD). The coefficient of variation (COV) of deflection is evaluated by the ratio of SD to expected mean of the deflection.In the present study, the expected mean of deflection and corresponding variance can be evaluated by Eq. (43).

6. Results and discussion

A Stochastic based SOPT and MCS are used to evaluate the statistics of nonlinear transverse central deflection of elastically supported FG material plate through Newton-Raphson method. A nine nod-ed Lagrangian isoparametric element with 63 degrees of freedom per element in the present HSDT model has been used throughout the study. In the present study, Coefficient of variance (COV) of random system parameters are taken as 0.1 i.e., 10% from their mean values. However, higher COV would be valid for higher dispersion in random system parameters, keeping in mind the limitation of the perturbation technique (Halder and Mahadevan 2000, Zhang *et al.* 1996).

The basic random system input variables (b_i) such as E_c, E_m, υ_c, υ_m, n, Q, α_c, α_m, k_c, k_m, k_1, k_2 and k_3 are sequenced and defined as

$$b_1 = E_c,\ b_2 = \upsilon_c,\ b_3 = E_m,\ b_4 = \upsilon_m,\ b_5 = n,\ b_6 = Q,\ b_7 = \alpha_c,\ b_8 = \alpha_m,\ b_9 = k_c,\ b_{10} = k_m,\ b_{11} = k_1,\ b_{12} = k_2 \text{ and } b_{13} = k_3$$

Where E_c, E_m, υ_c, υ_m, n, Q, α_c, α_m, k_c and are k_m Young's modulus, Poisson's ratios, volume fraction, exponent, thermal expansion coefficient, thermal conductivity of ceramic and metal, respectively and applied uniformly distributed transverse load. The terms k_1, k_2 and k_3 are known as dimensionless, linear spring, shear and nonlinear spring foundation parameters, respectively.

In the present study, three combinations of support boundary conditions, namely, simply supported (SSSS), clamped (CCCC) and two opposite edges are clamped and simply supported (CSCS) are taken into account. The constraints of these boundary conditions are written as:

All edges simply supported (SSSS)

$$v = w = \phi_y = \psi_y = 0,\ at\ x = 0, a; \quad u = w = \phi_x = \psi_x = 0\ at\ y = 0, b\ ;$$

All edges clamped (CCCC)

$$u = v = w = \psi_x = \psi_y = \phi_x = \phi_y = 0,\ at\ x = 0, a \quad and\ y = 0, b;$$

Two opposite edges clamped and other two simply supported (CSCS)

$$u = v = w = \psi_x = \psi_y = \phi_x = \phi_y = 0,\ at\ x = 0 \quad and\ y = 0;$$

$$v = w = \phi_y = \psi_y = 0,\ at\ x = a \quad u = w = \phi_x = \psi_x = 0,\ at\ y = b\ ;$$

In this study, the following dimensionless mean non lineartransverse centraldeflection (W_0) and foundation parameters (k_1, k_2 and k_3) are defined as (unless otherwise stated)

$$W_O = \frac{\bar{W}_0}{h}\ ,\ k_1 = K_1 a^4 / E_m h^3; \quad k_2 = K_2 a^2 / E_m h^3; \quad k_3 = K_3 a^4 / E_m h$$

Table 1 The material properties of ZrO_2/Ti-6Al-4V FGMs with TD material properties Reddy and Chin CD (1998)

Types of material	Properties	P_0	P_{-1}	P_1	P_2	P_3
ZrO_2	E(Pa)	244.27e+9	0	-1.371e-3	1.214e-6	-3.681e-6
	α (1/K)	12.766e-6	0	-1.491e-3	1.006e-5	-6.778e-11
Ti-6Al-4V	E(Pa)	122.56e+9	0	-4.586e-4	0	0
	α (1/K)	7.5788e-6	0	6.638e-4	3.147e-6	0

Table 2 Comparison and convergences study of the transverse central deflection of simply supported FGMs (Al/ZrO_2) square plate for various volume fraction index with different mesh size having b/h=5

Mesh size	Volume fraction index (n)				
	Ceramic	0.5	1	2	Metal
Present (2×2)	0.0039	0.0047	0.0053	0.0061	0.0087
Present (3×3)	0.0238	0.0293	0.0329	0.0371	0.0512
Present (4×4)	0.0212	0.0262	0.0294	0.0331	0.0458
Present (5×5)	0.0218	0.0252	0.0278	0.0326	0.0433
Present (6×6)	0.0224	0.0261	0.0306	0.0346	0.0448
Ferreira *et al.* (2005)	0.0205	0.0262	0.0294	0.0323	0.0443
Percentage Difference†	3.3018	0.0	0.0	2.4767	3.2751

†Percentage Difference is evaluated in between Present (4×4) and Ferreira *et al.* (2005)

Where $\bar{W}_0$ is the dimensional nonlinear transverse central deflection of FGM plate.
The material properties are position dependent and can be expressed as (Shen and Wang 2010)

$$P = P_t V_t(z) + P_b V_b(z)$$

Where P_t and P_b represent the temperature dependent properties (TD) of the top and bottom faces of the plate, respectively and can be expressed as

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3)$$

The material properties such as P_0, P_{-1}, P_1, P_2, P_3, and T considered for present analysis is shown in Table 1. The value of temperature T is taken as 300K for the whole of the analysis unless otherwise stated.

For the temperature independent material properties (TID) the value of P_{-1}, P_1, P_2,and P_3, are equal to zero. It is noted that, the results computed in this paper is for ZrO2/Ti-6Al-4V material plate unless otherwise stated. The temperature dependent (TD) material property of functionally graded materials is given in Table 1.

6.1 Comparative study for statistics of nonlinear transverse central deflection

The accuracy and convergence of present deterministic approach for transverse central deflection of simply supported FGMs (Al/ZrO_2) square plate subjected to a uniform transverse load is shown in Table 2 with numerical results of (Ferreira *et al.* 2005). Based on established approach and analyses of the foregoing sections, it is acknowledged that (4×4) mesh is founded

Table 3 Effect of individual random system properties and elastic foundations on the expected mean and COV, $\{b_i(i=1 \text{ to } 13)=0.1\}$ of transverse central deflection of FGM square simply supported plates resting on elastic foundation in thermal environment, for b/h=55, Q=50, ΔT=100K, and n=1

	FOPT			SOPT			MCS		
FP b_i	k_1=100, k_2=0, k_3=0	k_1=100, k_2=10, k_3=0	k_1=100, k_2=10, k_3=100	k_1=100, k_2=0, k_3=0	k_1=100, k_2=10, k_3=0	k_1=100, k_2=10, k_3=100	k_1=100, k_2=0, k_3=0	k_1=100, k_2=10, k_3=0	k_1=100, k_2=10, k_3=100
$b_1=E_c$	0.1243 (1.864)	0.0501 (1.289)	0.0468 (1.249)	0.1225 (1.864)	0.0500 (1.289)	0.0467 (1.249)	0.1276 (1.865)	0.06070 (1.2906)	0.05751 (1.25025)
$b_2=\upsilon_c$	0.0148	0.0062	0.0059	0.0147	0.0062	0.0059	0.00253	0.0066	0.0063
$b_3=E_m$	0.1280	0.0706	0.0747	0.1261	0.0704	0.0744	0.1289	0.0718	0.0788
$b_4=\upsilon_m$	0.0078	0.0032	0.0030	0.0078	0.0032	0.0030	0.0082	0.0043	0.0039
$b_5=n$	0.0128	0.0067	0.0064	0.0128	0.0067	0.0064	0.0149	0.0074	0.0069
$b_6=Q$	0.0618	0.0622	0.0622	0.0616	0.0621	0.0621	0.0638	0.0640	0.0641
$b_7=\alpha_c$	0.0165	0.0114	0.0110	0.0165	0.0114	0.0110	0.0172	0.0121	0.0120
$b_8=\alpha_m$	0.0036	0.0025	0.0024	0.0036	0.0025	0.0024	0.0038	0.0029	0.0027
$b_9=k_c$	0.0019	0.0013	0.0013	0.0019	0.0013	0.0013	0.0022	0.0015	0.0015
$b_{10}=k_m$	0.0078	0.0052	0.0050	0.0078	0.0052	0.0050	0.0088	0.0062	0.0060
$b_{11}=k_1$	0.0271	0.0187	0.0181	0.0271	0.0187	0.0181	0.0289	0.0198	0.0189
$b_{12}=k_2$	0.0	0.0425	0.0411	0.0	0.0424	0.0411	0.0	0.0431	0.0421
$b_{13}=k_3$	0.0	0.0	0.0073	0.0	0.0	0.0073	0.0	0.0	0.0082

*Values shown in bracket is the dimensionless mean of transverse central deflection

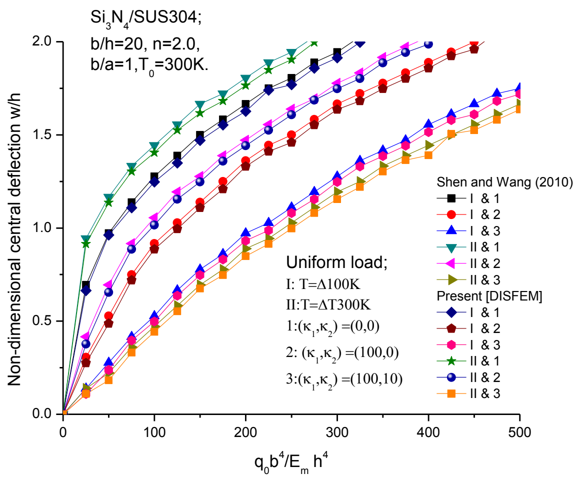


Fig. 3 Validation study of nonlinear transverse central deflection of FGM square plates resting on elastic foundation subjected to uniform pressure and temperature rise

good agreement with percentage difference is less than 3% as compared to the published literature and results are convergences for higher number of elements. Therefore, (4×4) mesh is

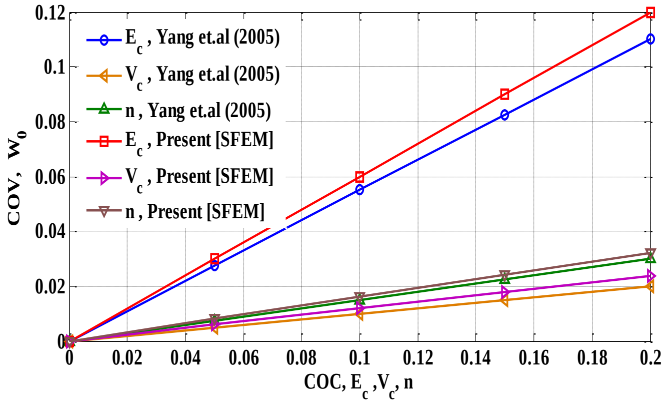


Fig. 4 Validation study of COV on nonlinear transverse central deflection of square FGM simply supported plate for random change in $b_1=E_c$, $b_3=E_m$, and $b_5=n$

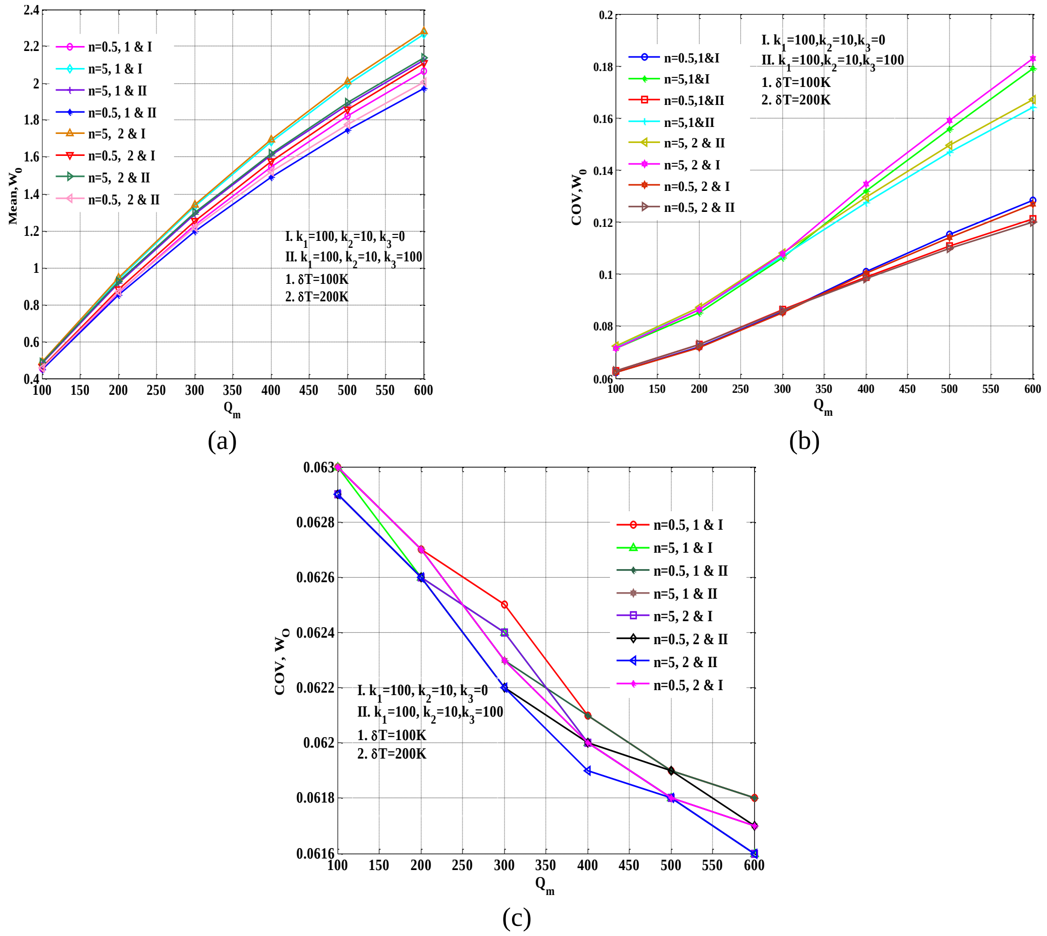


Fig. 5 The effect of temperature change, foundation parameters, volume fraction index and load parameters with random system properties on the (a) expected mean (b) COV,{b_i (i=1,..,5)=0.1} and (c) COV{b_i(i=6)=0.1} of transverse central deflection of square FGM simply supported plate resting on elastic foundation in thermal environments

taken into consideration in the present static study.

Table 3 presents the effect of individual uncorrelated random system properties $\{b_i(i=1$ to $13)=0.1\}$ with foundation parameters (k_1=100, k_2=0, k_3=0, k_1=100, k_2=10, k_3=0, *and* k_1=100, k_2=10, k_3=100) on the mean and COV of dimensionless mean and corresponding COV of transverse central deflection of FGM square simply supported elastically supported plates subjected to static loading in thermal environments using SOPT and MCS. The dimensionless mean and corresponding COV of transverse central deflection is highly affected by the random change in the COV of E_c, E_m, Q and k_2. The strict control of these parameters is therefore required, for high reliability of FGM plate. The mean and corresponding COV evaluated by FOPT, SOPT and MCS are in good agreement among one another which shows the efficacy of present stochastic approaches. The foundation parameters decrease the mean and increase the COV of transverse central deflection due to increase of overall stiffness.

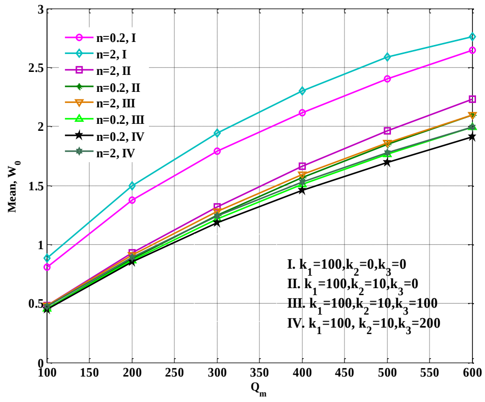
Fig. 3 shows the effect of the mechanical load parameters on thermo mechanically induced dimensionless mean nonlinear transverse central deflection of elastically supported FG Material (Si_3N_4/SUS304) square plate subjected to uniform static lateral pressure having a uniform temperature rise, b/h=20, n=2 with published results. The nonlinear results using present C^0 FEM are compared with published results of (Shen and Wang 2010) using the semi analytical approach. The results using both of the approaches are in good agreements. As the foundation parameters increases, the mean transverse central deflection decreases. It is because of foundation parameters increases the stiffness of the plate. Similarly, with the increase of temperate increment the mean transverse central deflection increases. It is because of thermal stresses oppose the mechanical stresses and ultimately stiffness of the plate decreases.

Fig. 4 shows the comparison study of individual effect of random material properties on the variance (COV) of the transverse nonlinear central deflection of simply supported square Al_2O_3-Ni FG materials plate with published results of (Yang *et al.* 2005), for n=2, a/h=10. The present results for various random variables using SOPT using HSDT are in good agreement with the published results using the semi analytical method through the first order perturbation technique (FOPT).

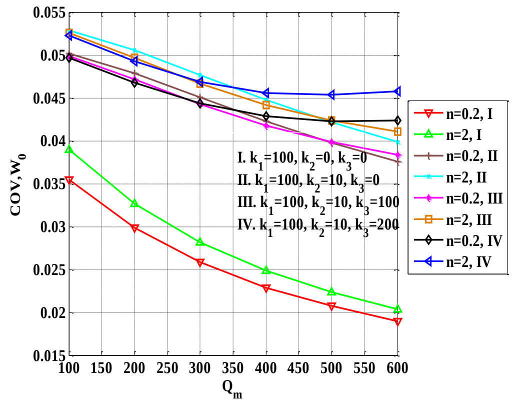
6.2 Parametric study for statistics of nonlinear bending response

The effect of temperature change, foundation parameters, volume fraction exponents and load parameters with random system properties on the (a) expected mean (b) COV, $\{b_i\ (i=1,..,5)=0.1\}$ and (c) COV $\{b_i\ (i=6)=0.1\}$ of transverse central deflection of square elastically supported FG materials simply supported plate resting on elastic foundation in thermal environments is shown in Fig. 5, for b/h=65. For the same foundation parameters and temperature increments, with the increase of volume fraction exponent, the mean transverse deflection increases and corresponding COV with random change in all material and load parameters increases. It is because of with the increase of volume fraction exponent the volume of the metal part of the FG materials increases. For the same foundation parameters and volume fraction index, with the increase of temperature, the mean and corresponding COV increases due to increment of deflection. It is also expected that for temperature change and volume fraction index, as the foundation parameters increases, the mean decreases and corresponding COV increases with random change in material and load parameters. As the load parameter increases, the mean and corresponding COV with random change in material properties increases while a random change in foundation parameter decreases.

Fig. 6 shows the effect of foundation parameters, volume fraction index and load parameters

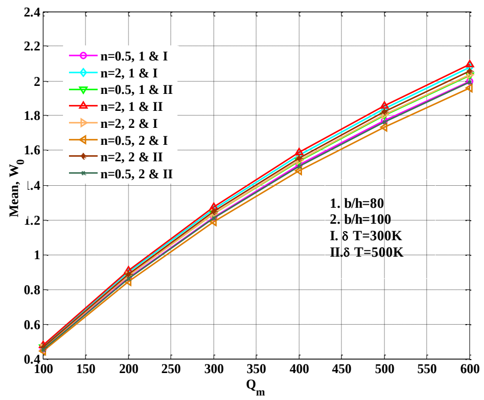


(a)

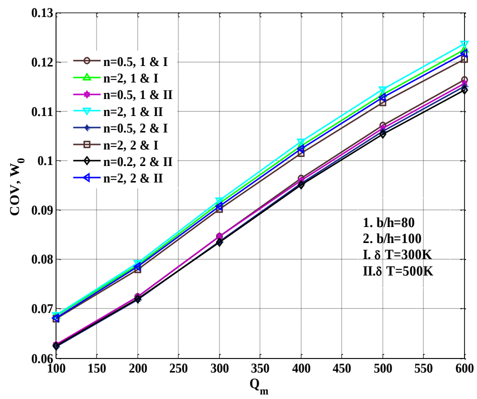


(b)

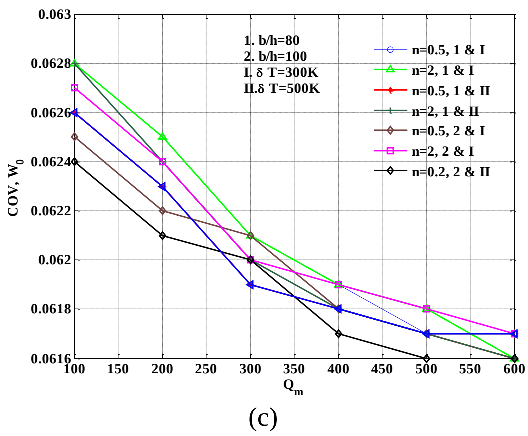
Fig. 6 The effect of foundation parameters, volume fraction index and load parameters with random system properties on the (a)expected mean, and (b) COV{bi (i=11,12,13)=0.1 of transverse central deflection of FGM square simply supported plate resting on elastic foundations in thermal environments



(a)

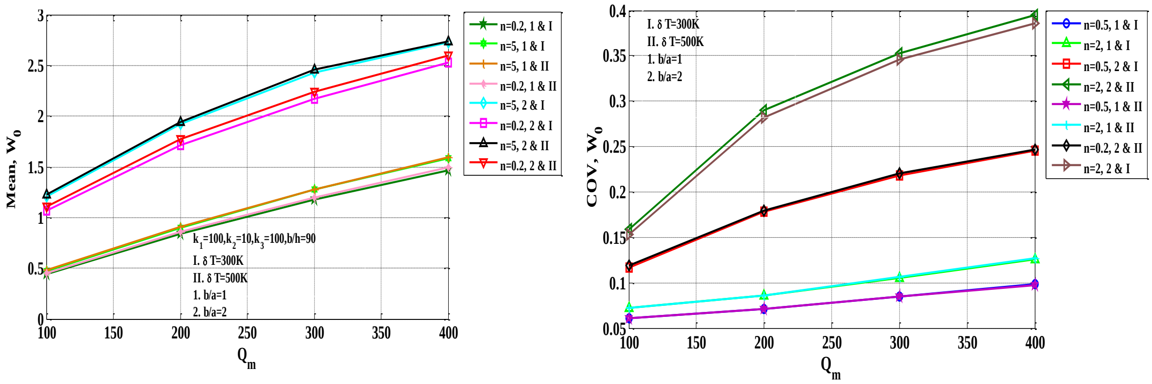


(b)



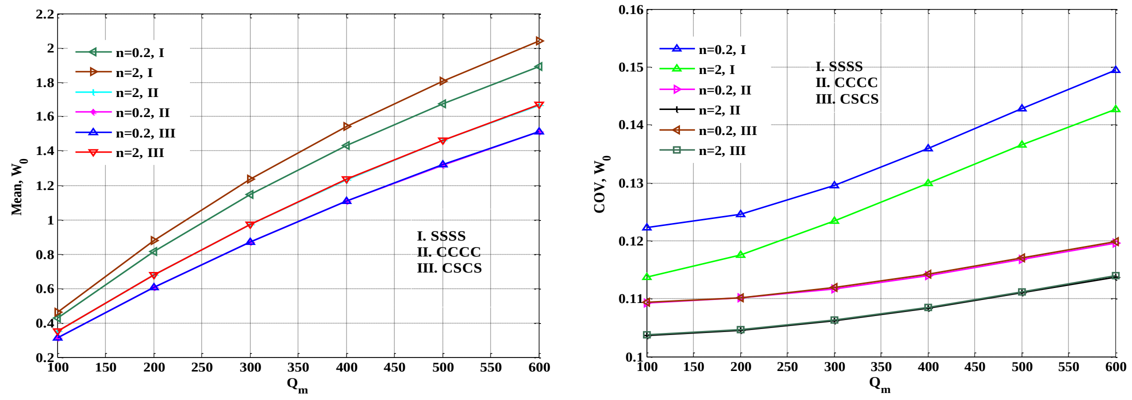
(c)

Fig. 7 The effect of plate thickness ratio, temperature change, volume fraction index, load parameters with random system properties on the (a) expected mean (b) COV, $\{b_i(i=1...5)=0.1\}$, and (c) COV, $\{b_i(i=6)=0.1\}$ of transverse central deflection of FGM square simply supported plate resting on a nonlinear elastic foundation (k_1=100, k_2=10, k_3=100) in thermal environments



(a) (b)

Fig. 8 The effect of plate aspect ratios, temperature change, volume fraction index, and load parameters with random system properties on the (a) expected mean (b) COV, $\{b_i(i=1\ldots5)=0.1\}$ of transversecentral deflection of FGM simply supported plate resting on elastic foundation in thermal environments

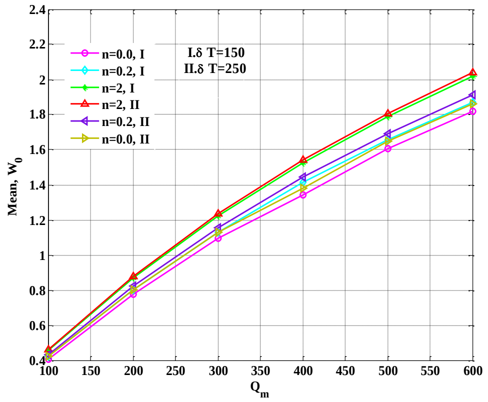


(a) (b)

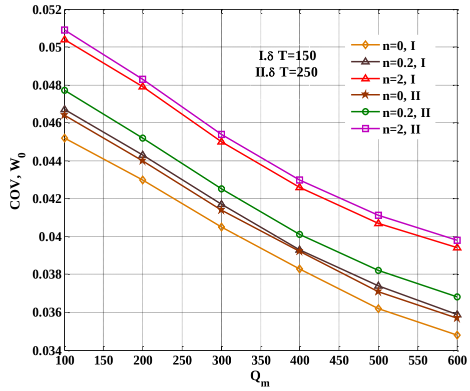
Fig. 9 The effect of support conditions, volume fraction index and load parameters with random system properties having (a) expected mean (b) COV, $\{b_i(i=1\ldots13)=0.1\}$of transverse central deflection of FGM square simply supported plate resting on elastic foundation in the thermal environment

with random system properties on the (a) expected mean (b) COV$\{b_i$ (i=11,12 and 13)=0.1$\}$ of transverse central deflection of FGM square simply supported plate resting on elastic foundations in thermal environments, b/h=75, ΔT=400K. For the same volume fraction, exponent and load parameters, with the increment of foundation parameter, the expected mean decreases and corresponding COV with random change in foundation parameters increases. For the same foundation parameter and load parameter, with the increase of volume fraction, exponent, the expected means and corresponding COV with random change in foundation parameters increases. Similarly, for the same foundation parameter and volume fraction, exponent, with the increase of foundation parameter, the expected mean increases and corresponding COV with random change in foundation parameters decreases.

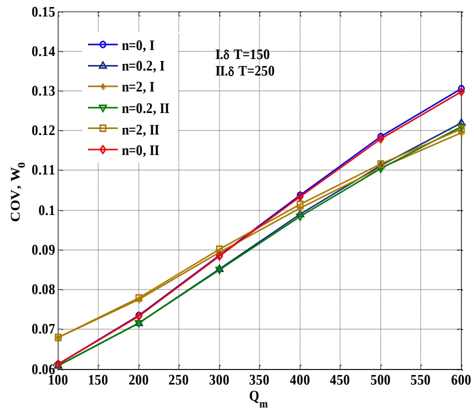
The effect of the plate thickness ratio, temperature change, volume fraction indices, and load parameters with random system properties on the (a) expected mean (b) COV, $\{b_i(i$=1...5)=0.1$\}$ (c)



(a)



(b)



(c)

Fig. 10 The effect of volume fraction index, temperature change, and load parameters with random system properties on the (a) expected mean (b) COV, $\{b_i(i=1\ldots5)=0.1\}$, and (c) COV, $\{b_i(i=11,12,13)=0.1$ of transverse central deflection of FGM square simply supported plates resting on elastic foundation in thermal environments

COV, $\{b_i(i=6)=0.1\}$ of transverse central deflection of square simply supported FGM plate resting on elastic foundation (k_1=100, k_2=10, k_3=100) in thermal environments is shown in Fig. 7. For the same temperature change, volume fraction, exponent and load parameters, with the increase of plate thickness ratio, the mean transverse central deflection decreases and corresponding COV increases. All other effects are already explained in the previous figure discussion.

Fig. 8 shows the effect of plate aspect ratios (b/a), temperature change, volume fraction index, and load parameters with random system properties on the (*a*) expected mean (*b*) COV, $\{b_i(i=1\ldots5)=0.1\}$ of transverse central deflection of simply supported FGM plate resting on elastic foundation (k_1=100, k_2=10, k_3=100) in thermal environments for b/h=90. For the same temperature change, volume fraction index and load parameters, with the increase of the plate aspect ratio, mean and corresponding COV with random change in material properties increases. The effect of volume fraction on the mean and corresponding COV is more sensitive for rectangular plate as compared to square plate.

The effect of support conditions (namely SSSS, CCCC, and CSCS), volume fraction index and

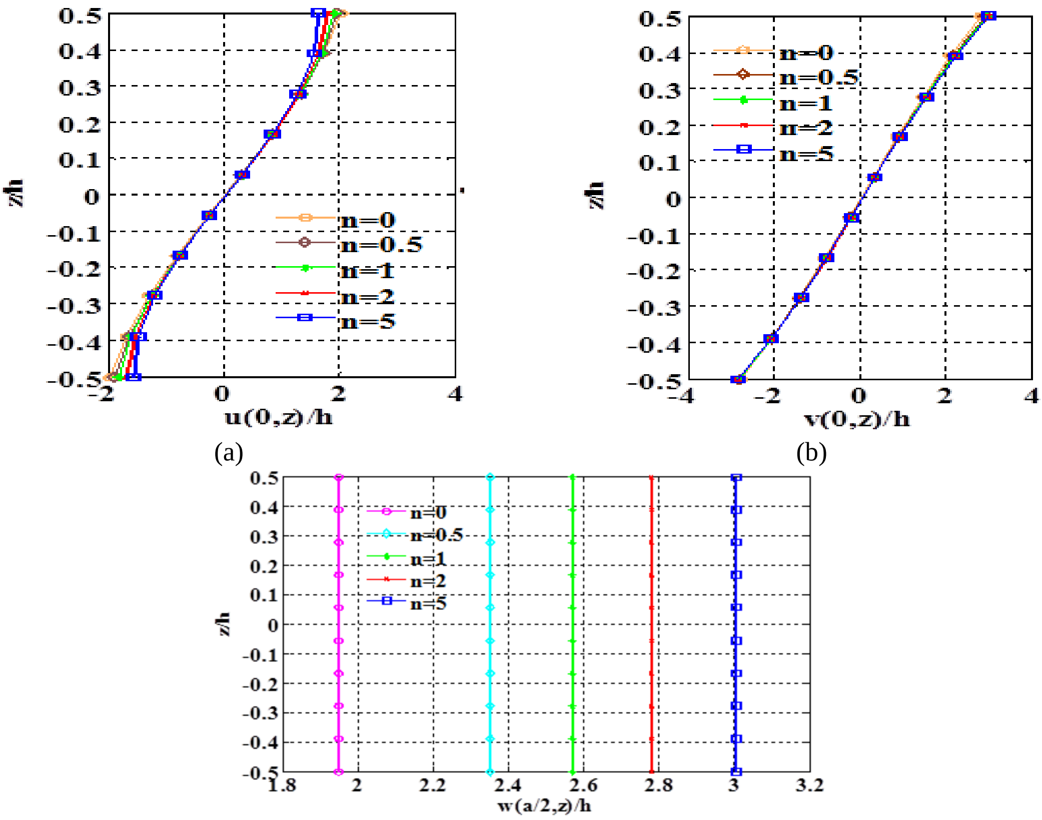


Fig. 11 Through the thickness variation of in-plane displacements in (a) x, (b), y, and (c) z directions of simply supported FGM (ZrO_2/TI-6Al-4V) plate with various volume fraction index

load parameters with random system properties having on the (a) expected mean (b) COV, $\{b_i(i=1\ldots13)=0.1\}$ of transverse central deflection of square simply supported FG material plate resting on elastic foundation (k_1=100, k_2=10, k_3=100), for b/h=85 and ΔT=150K in thermal environment is shown in Fig. 9. For the same volume fraction exponent and load parameter, among the given different boundary conditions simply supported boundary is most sensitive to the mean and COV with random change in all system properties. This is due to decreased effect of boundary constraints which significantly decreases the stiffness of the plate.

The effect of volume fraction index, temperature change, and load parameters with random system properties on the (a) expected mean (b) COV, $\{b_i\ (i=1\ldots5)=0.1\}$, and (c) COV, $\{b_i(i=11,12,13)=0.1\}$ of transverse central deflection of FGM square simply supported plates resting on elastic foundation(k_1=100, k_2=10, k_3=100) for b/h=95 in thermal environments as shown in Fig. 10. For the same volume fraction index and load parameters, as the temperature increases, the mean and corresponding COV with random change all FGM material properties increase. For the same temperature change and load parameters, as the volume fraction increases the mean and corresponding COV with random change all FGM material properties increases while random changes in foundation parameters decreases. It is because of the plate stiffness is higher for metallic plate and lower for ceramic plate.

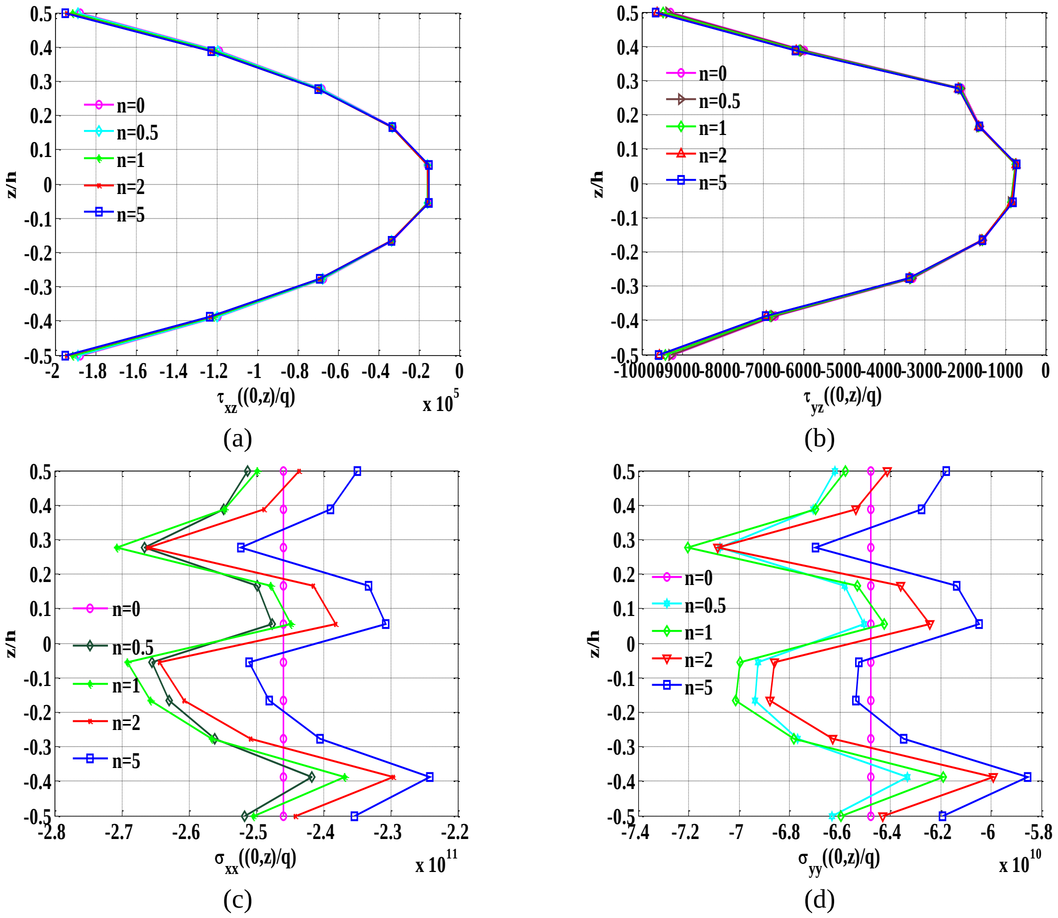


Fig. 12 Through the thickness variation of shear stresses (a) τ_{xy}, (b) τ_{yz}, and normal stresses (c) σ_{xx} and (d) σ_{yy} of simply supported FGM (ZrO_2/ TI-6Al-4V) plate with various volume fraction index

Fig. 11(a)-(c) show variation of displacements in x, y, and z axis across the thickness of simply supported FGM (ZrO_2/TI-6Al-4V) plate with various volume fraction index and having TD material properties, UT, a/h=20, ΔT=100 K and Q=100. It is clear that very little deformation has been occurred in the u-and v-directions, respectively. No deformation is observed in the w direction due to independent of thickness.

Variation of transverse shear stress and in plane stress across the thickness of simply supported FGM (ZrO_2/TI-6Al-4V) plate with various volume fraction index with TD material properties, uniform temperature distribution, a/h=20, ΔT=100 K and Q=100 is shown in Fig. 12(a)-(d). With the increase of volume fraction index, the direct stresses (σ_{xx} and σ_{yy}) are highly sensitive while, shear stresses (τ_{xy}, and τ_{yz}) are least sensitive.

7. Conclusions

The stochastic finite element method using SOPT and independent MCS methods combined with Newton-Raphson approach via HSDT is adopted to evaluate the second order statistics in terms of the mean and COV of nonlinear transverse central deflection of the elastically supported FGM plate subjected uniformly distributed time dependent in the thermal environment. The following conclusions are drawn based on observation of the present study:

• The FG materials plate resting on elastic foundation is more sensitive to mean and COV with random change in Young modulus of respective ceramic and metal, external mechanical loading and shear foundation parameter. The proper and strict control of above random system parameters is required for high reliability and safety of the design which is extremely important for aerospace and other high sensitive applications.

• The increment in temperature, volume fraction index and load parameters make the plate more sensitive to mean and COV of transverse central deflection by random change in all system properties. The proper controls of these parameters are required for high reliability point of view.

• The effect of shear foundation plays a significant role in supporting the elastic foundation. For reliable and safe design, thick, clamped supported square plate is most desirable.

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TK

Appendix

$$[D]=\begin{bmatrix} A1_{ij} & B_{ij} & E_{ij} & 0 & 0 \\ B_{ij} & D1_{ij} & F1_{ij} & 0 & 0 \\ E_{ij} & F1_{ij} & H_{ij} & 0 & 0 \\ 0 & 0 & 0 & A2_{ij} & D2_{ij} \\ 0 & 0 & 0 & D2_{ij} & F2_{ij} \end{bmatrix}, [D_3]=\begin{bmatrix} A1_{ij} & 0 \\ B_{ij} & 0 \\ E_{ij} & 0 \\ 0 & A2_{ij} \\ 0 & D2_{ij} \end{bmatrix}, [D_4]=[D_3]^T \text{ and } [D_5]=\begin{bmatrix} A1_{ij} & 0 \\ 0 & A2_{ij} \end{bmatrix} \quad \text{(A.1)}$$

$$\text{With } \left(A1_{ij}, B_{ij}, D1_{ij}, E_{ij}, F1_{ij}, H_{ij}\right)=\int_{-h/2}^{h/2} Q_{ij}\left(1, z, z^2, z^3, z^4, z^6\right) dz\,; \quad (i,j=1,2,6)$$

$$\left(A2_{ij}, D2_{ij}, F2_{ij}\right)=\int_{-h/2}^{h/2} Q_{ij}\left(1, z^2, z^4\right) dz\,; \quad (i,j=4,5) \quad \text{(A/1a)}$$

$$[K_l]=\int_A [B_l]^T [D][B_l]\, dA,$$

$$\left[K_{nl}\{q\}\right]=\int_A [B_{nl}]^T [D_1][B_l]\, dA+\frac{1}{2}\int_A [B_l]^T [D_2][B_{nl}]\, dA+\frac{1}{2}\int_A [B_{nl}]^T [D_3][B_{nl}]\, dA \quad \text{(A.2b)}$$

$$\{q\}=\sum_{e=1}^{NE}\{q\}^{(e)}, \ \left[F^T\right]=\sum_{i=1}^{n}\int_{A^{(e)}}\left[\left[B_{1i}{}^{(e)}\right]^T\left[N^T\right]+\left[B_{b1i}{}^{(e)}\right]^T\left[M^T\right]+\left[B_{b2i}{}^{(e)}\right]^T\left[P^T\right]\right] dA \quad \text{(A.3)}$$

$$\left[K_{fl}\right]=\frac{1}{2}\int_A \left[B_f\right]^T\left[D_f\right]\left[B_f\right] dA, \ \left[K_{fnl}\right]=\frac{1}{2}\int_A \left[B_f\right]^T\left[D_{fnl}\right]\left[B_f\right] dA \quad \text{(A.4)}$$

$$[K_G]=\int_A [B_{nl}]^T \{N\}\, dA=\int_A [G]^T\left[\overline{N}\right][G]\, dA \quad \text{(A.5)}$$