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An approach to a novel modelling of structural reinforced glass beams in modern material components

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Abstract. In modern buildings, glass is considered a structurally unsafe material due to its brittleness and unpredictable failure behavior. The possible use of structural glass elements (i.e., floors, beams and columns) is generally prevented by its poor tensile strength and a frequent occurrence of brittle failures. In this study an innovative modelling based on an equivalent thickness concept of laminated glass beam reinforced with FRP (Fiber Reinforced Polymer) composite material and of glass plates punched is presented. In particular, the novel numerical modelling applied to an embedding Carbon FRP-rod in the interlayer of a laminated structural glass beam is considered in order to increase both its failure strength, together with its post-failure strength and ductility. The proposed equivalent modelling of different specimens enables us to carefully evaluate the effects of this reinforcement. Both the responses of the reinforced beam and un-reinforced one are evaluated, and the corresponding results are compared and discussed. A novel equivalent modelling for reinforced glass beams using FRP composites is presented for FEM analyses in modern material components and proved estimations of the expected performance are provided. Moreover, the new suggested numerical analysis is also applied to laminated glass plates with wide holes at both ends for the technological reasons necessary to connect a glass beam to a structure. Obtained results are compared with an integer specimen. Experimental considerations are reported.

Keywords: carbon FRP-rod; laminated glass plate; reinforced glass beam; structural glass modelling

1. Introduction

In the last decade the need of realizing buildings utilizing transparent load-bearing elements is considerably increased, with an excellent aesthetic value of transparent and translucent structural components (Richards 2006). These aspects are all present in the so-called structural glass, which is characterized by a brittle behavior and a time-decreasing strength due to surface damages. The break of a glass specimen usually occurs due to the growth of a superficial defect rather than inside. Furthermore, depending on environmental conditions, glass can present failures after the

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initial application of load. The unpredictability of failures related to the fragility of this material and the lack of an accurate numerical method for determining the strength of glass leads to adopt large safety factors and expensive tests on several glass prototypes. In (Badalassi *et al*. 2014) the calibration of partial safety factors of annealed glass elements under several loads (i.e., wind, snow and anthropic loads) has been conducted in agreement with the probabilistic analyses in paradigmatic case studies. In cited work, the conducted analysis showed that, with a probability of collapse compatible with second-class elements according to the classification of the Italian CNR code, the partial safety coefficient can be between 2.3 and 2.55. Moreover, partial material factors of float glass structures are calibrated in (Ballarini *et al*. 2016) according to the semi-probabilistic methods obtaining partial safety coefficients between 1,59 and 3,36. A comparison with results obtained with the full probabilistic approach on the same paradigmatic case studies is also presented. Numerous studies have been conducted on the development of new methods and approaches for the study of glass breakage stress considering different parameters (Beason *et al*. 1984, Fischer-Cripps *et al*. 1995, Overend *et al*. 2007, Veer *et al*. 2011, Bedon *et al*. 2018, Foti *et al*. 2020), going through glass failure prediction models, which relates the probability of glass failure to surface flaw characteristics including all known factors which significantly affect the strength of glass, such as load duration, surface area, etc. (Beason *et al*. 1984). The predicting failure probability for both short- and long-term stresses has been dealt with modified crack growth analytical models based on the statistical failure theory and linear elastic fracture; the differences between available failure prediction models were analyzed to obtain a wide range of strength and thickness values in the glass design, developing a growth model (Fischer-Cripps *et al*. 1995, Overend *et al*. 2007, Veer *et al*. 2011). A hidden damage approach was also used to investigate the heterogeneous distribution of microcracks on the edge glass surface caused by a hidden damage of glass, using an analytical approach based on a statistical analysis via a multilinear Weibull pattern. A careful attention has also been reserved to extreme loading configurations for glass facades and the analysis methods for the design of windows/facades, being the most fragile and vulnerable components of buildings, due to their relatively low value of tensile strength and mostly brittle behavior of glass, if compared to an amount of interacting structural and non-structural components (Bedon *et al*. 2018).

In the last years further risk models have been developed which focus on the damage state evaluation of different structure typologies (i.e., slender buildings, historical buildings) via innovative methods of structural monitoring (Carnimeo *et al*. 2014, 2015a, b, c, Foti *et al*. 2018).

It must be underlined that, because the brittleness of glass material makes quite difficult to use glass monolithic panels, more glass layers have been consequently bound together through interlayers of a polymeric material to leave connected all fragments after any glass failure (Aşık *et al*. 2003, Aşık *et al*. 2006, Norville *et al*. 1998, Dural *et al*. 2016, Ivanov 2007). Therefore, sudden collapses can be avoided using a laminated safety glass usually made with a plastic interlayer to replace the monolithic glass and prevent from effects of possible glass brittle failures (fail-safe response). The experimental laminated glass data have been complemented by theoretical models that take into account all parameters affecting laminated glass behavior including temperature, thickness and arrangement of the interlayer. Analytical models have been used with the Finite Element Method (FEM) for glass beams and plates under specific boundary conditions (Aşık *et al*. 2003, Aşık *et al*. 2006, Norville *et al*. 1998, Dural *et al*. 2016, Ivanov 2007). The elastic theory is shown to be applicable for the single glass layer with an additional differential equation describing interaction with the PolyVinyl Butyral (PVB)-layer, whereas the time-dependent behavior of a laminated-glass simply supported beam was also analyzed in Galuppi (2012).

It has to be pointed out that the use of a PVB interlayer presents very interesting aspects (e.g. transparency, good adhesion grade with glass etc.), but its model results highly complex due to the strongly nonlinear behavior of this material. Stiffness and strength of laminated glass depend upon a shear coupling among glass plies through the polymer, as shown in Galuppi *et al*. (2014, 2020), Timmel *et al*. (2007), Chen *et al*. (2016) and Martín *et al*. (2020). This aspect was considered by defining the so-called effective thickness of laminated glass, i.e., the thickness of a monolith with equivalent bending properties in Galuppi *et al.* (2014) and Timmel *et al*. (2007). Aiming at increasing the strength avoiding unexpected failures of laminated glass, a number of relevant studies on mechanical performance of structural reinforced laminated glass elements was experimentally and numerically developed by Wiechert (1893), Martens *et al*. (2015), Weller *et al.* (2010), Overend *et al*. (2014), Premrov *et al*. (2014), Belis *et al*. (2009), Speranzini *et al*. (2015), Cagnacci *et al*. (2009), Ølgaard *et al*. (2009), Louter *et al*. (2012), Slivanský (2012), Bedon *et al.* (2014), Bedon *et al*. (2018, 2020), Speranzini *et al*. (2014), obtaining maximum benefits with steel rods, whereas positive effects are also achieved with Glass FRP and Carbon FRP tendon rods.

Moreover, when designing high load bearing elements made of glass, connections are needed in the presence of large spans. In order to ensure connections between the structural glass beams and the corresponding support structure, point-fixings are needed for all bolted connections (i.e., holes). Unfortunately, limited results have been until now obtained in research investigations concerning residual stresses in connections (Bernard *et al*. (2009), Watson *et al*. (2013), Santarsiero *et al*. (2017), Bedon *et al*. (2018), Pourmoghaddam *et al*. (2018), Zienkiewicz *et al*. 1977).

In the present work, the concept of equivalent thickness is introduced, aiming at developing a novel numerical modeling of a reinforced glass beam with carbon fibre rod together with a pierced structural glass plate. Their structural behaviors are subsequently determined using FEM-analyses.

For this purpose, the numerical responses of stress and strain distributions of both glass beams with embedded carbon fiber rod referred to the non-reinforced element and of the structural glass plate in the presence/absence of perimetral holes are accurately investigated. Then, the highly performing numerical behaviors are developed to be inserted in a safety glass design.

2. Laminated glass beam modelling

The model proposed in this work for analyzing a composite beam formed by three thin glass layers and two PVB-layers is shown in Fig. 1.

By considering a composite beam composed by two thin glass layers and one PVB-layer, the linear analysis valid for determining the behavior of this composite beam follows the differential procedure reported in Aşık *et al*. (2005), where the analytic solution is given for a simply supported laminated glass beam subjected to a uniformly distributed load q. Considering the presented mathematical model, it can be noted that the non-linear term shown in the following differential equation has been proved to be null:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right)+h_t\frac{d}{dx}(Gb\gamma_{xz})=q$$

$$\frac{dN_1}{dx}=Gb\gamma_{xz}$$

$$\frac{dN_2}{dx}=-Gb\gamma_{xz}$$

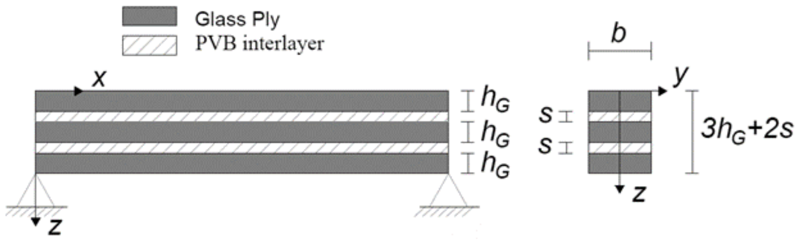


Fig. 1 Laminated glass beam with double PVB-interlayer

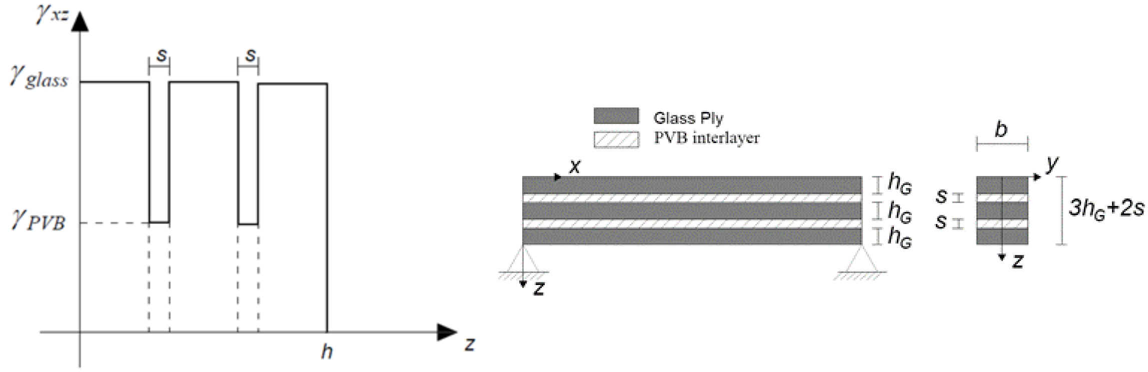


Fig. 2 Shear strain function for laminated glass beam with double PVB-interlayers

where:

- E= modulus of elasticity of glass;
- G= shear modulus of interlayer;
- C top and bottom glass ply;
- $h_t = \frac{h_1}{2} + \frac{h_2}{2}$ with h_1 and h_2 thickness of glass layer 1 and 2, respectively.

This implies that a simply supported beam will always behave in a linear mode independently from any lateral deformation.

The shear strain function $\gamma_{xz} = \frac{u_1 + u_2 - \frac{dw}{dx}\left(\frac{h_1}{2} + \frac{h_2}{2} + s\right)}{s}$ takes into account the difference displacements in x-z plane between glass and PVB faces in contact. Thus, this function considers the transverse shear that is generated from the lower stiffness of the visco elastic-interlayer of laminated glass (stiff glass and a soft interlayer). In this paper the main hypotesis is to consider the multi-layer structure given by three glass layers and two PVB-components as a unique connected material and a novel analytical model of a monolithic PVB-interlayer beam is herein adequately developed and adopted.

Based on all remarks made regarding the single PVB-interlayer beam, the shear strain qualitative trend for a double PVB-interlayer case is herein proposed (Fig. 2).

Furthermore, the PVB-interlayers play a crucial role in defining shear-coupling of glass plies and increasing the bending capacity of the laminated glass beam. Such aspect, that is the most studied in the structural glass field, reveals to be a drawback that can be afforded via an elastic modelling of the PVB material, as suggested in Galuppi *et al.* (2020). Thus, in this work a novel

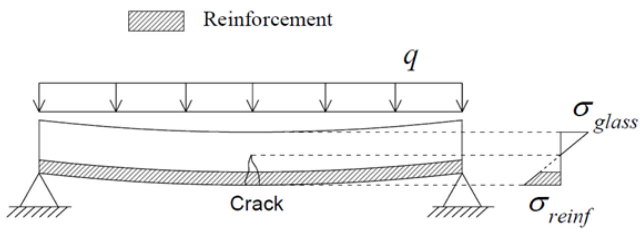


Fig. 3 Schematic overview of distribution of stress after glass failure

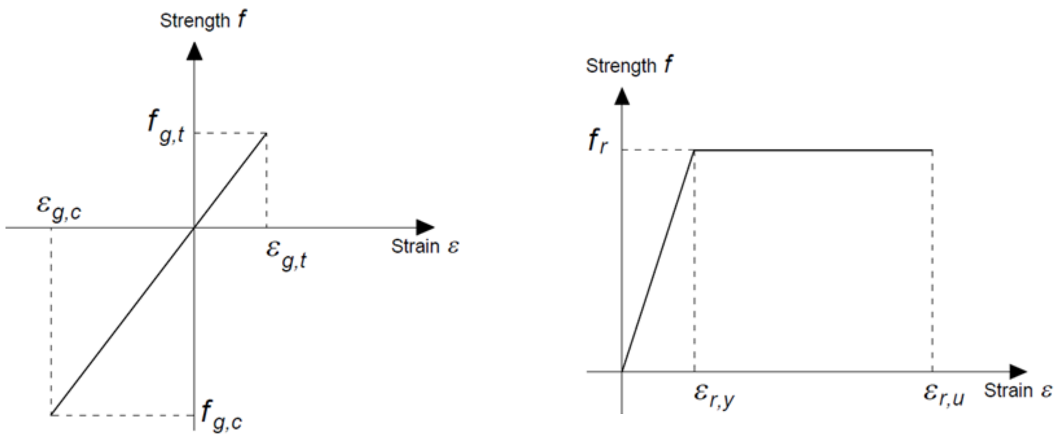


Fig. 4 Schematic representation of the stress-strain diagrams adopted for the analytical prediction method; (a) Glass; (b) Reinforcement.

modelling approach based on the response of laminate considered as a monolithic glass element with a proper equivalent thickness h_E is presented, where this equivalent thickness h_E of laminated glass can be computed by accurately processing the values of three glass plies of the same thickness h_G together with the values of two PVB layers of thickness s, properly combined via Young's moduli E_{PVB} and E_G as expressed in Eq. (1):

$$h_E = \sqrt[3]{h_G^3 + 3h_G(h_G + s)^2 + \frac{E_{PVB}}{3E_G}s^3} \tag{1}$$

In this relation, it has been assumed that a full bonding occurs between the glass and the PVB interlayer due to loading.

The innovative numerical modelling conducted in this paper focuses on the prediction of the structural response of both reinforced glass beams and punched ones. Since the structural concept of the FRP-rod reinforced glass beam can be considered comparable to the reinforced concrete beam, in this paper the reinforced concrete theory is consequently supposed to be valid also for reinforced glass models as shown in Fig. 3.

For the sake of a better comprehension, fundamental concepts of the reinforced concrete theory are briefly reported. On the basis of the reinforced concrete theory, the relations used to describe the F-δ (force-displacement) are implemented to reinforced glass beams. It is assumed that the glass responses are completely linear and elastic and that the reinforcement response is elastic perfectly-plastic, as shown in Fig. 4.

Table 1 Material properties of glass and CFRP used in the FE Numerical Model

		Glass	CFRP
Density	[Kg/m^3]	2500	1619
Elastic modulus	[N/mm^2]	70 x 10^3	218 x 10^3
Poisson's ratio	[-]	0.22	0.3
Tensile strength	[N/mm^2]	45	2780
Compression strength	[N/mm^2]	221	-
Coefficient of thermal expansion	[K^{-1}]	9 x 10^{-6}	-

3. Numerical study of laminated glass beams

In the field of fail-safe design, float glass beams can be reinforced with materials characterized by high tensile strength to limit or completely cancel out the negative brittle effects of glass. In particular, by including steel or composite material (i.e., FRP), elements with the same bending strength can be realized. The behavior of a glass beam reinforced with FRP rod is similar to the one of reinforced concrete beams. In fact, when a load produces the cracking of glass, all tensile stresses can be absorbed by the FRP component, whereas the glass continues to support the compression stresses. In this way, the whole advantage of the high tensile strength of the reinforcement can be taken and the transparency of the beam can be preserved.

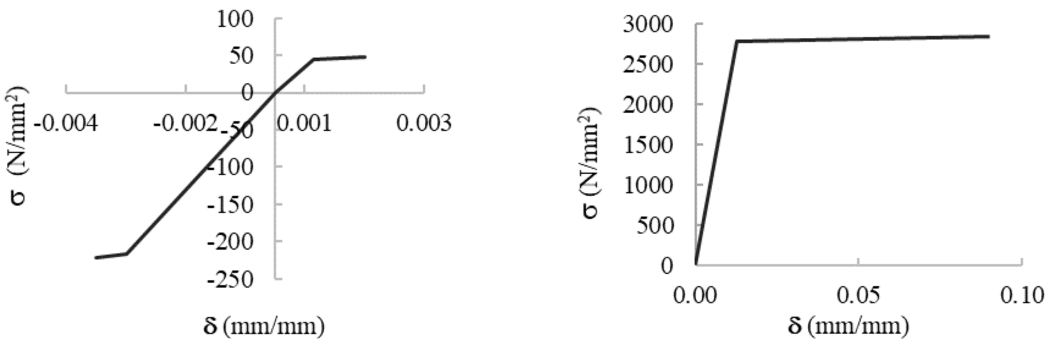
The present research is intended to investigate the behavior of the structural glass beams by embedding a Carbon FRP-rod as a reinforcement. In this regard, a numerical modelling is carried out on reinforced and un-reinforced glass beams. The analyses were conducted on laminated beams under a load acting parallel and orthogonal to the lamination plane. Moreover, in the application of laminate glass beams it is frequently necessary to realize the holes for the connection housing with other elements. Different ways of fixing glass panels (i.e., metallic frames, structural silicone adhesives) are available, but the point-fixing of the panels through metallic supports results an efficient available technology. Therefore, the numerical study also concerns the analysis of stress concentration in the glass beams with holes when the load is applied orthogonally to the lamination plane (out-of-plane loading).

3.1 Materials

In the following sections the materials and geometry used in the beam model are discussed. The specimens are made of three glass sheets with a 10 mm standard thicknesses. They are hold in place by an interlayer of PVB-material between the three layers of glass. The interlayer keeps the layers of the glass bonded even when broken. Ordinary float glass has been applied to all specimens. The outside glass sheet has a length greater than the internal glass sheet, so as to allow lodging of the CFRP-bar for the reinforced specimen.

The laminated glass stiffness is the result of the combination of the two materials (glass and PVB) with different modulus of elasticity. It is possible to model a laminated glass element determining an equivalent thickness of the beam that considers the mechanical characteristics of each component.

The properties of the adopted materials (glass and carbon rods) are listed in Table 1. The equivalent thickness h_E is equal to 16.6 mm according to Eq. (1).



(a) Glass (b) CFRP

Fig. 5 Stress-strain relationship diagram

Fig. 5(a) shows the stress-strain diagram of a glass type utilized for the non-linear behavior of the beams. Similarly, in Fig. 5(b) the stress-strain diagram of the CFRP-rod used as reinforcement of beam is reported.

It must be specified that the material utilized for the specimens is tempered glass. The ultimate stress for the tempered glass was determined in accordance with prEN 13474-3 norme. In particular, the allowable stress of prestressed glass material, whichever composition is:

$$f_{g;d} = \frac{k_{mod} k_{sp} f_{g;k}}{\gamma_{M;A}} + \frac{k_v (f_{b;k} - f_{g;k})}{\gamma_{M;V}} \quad (2)$$

where:

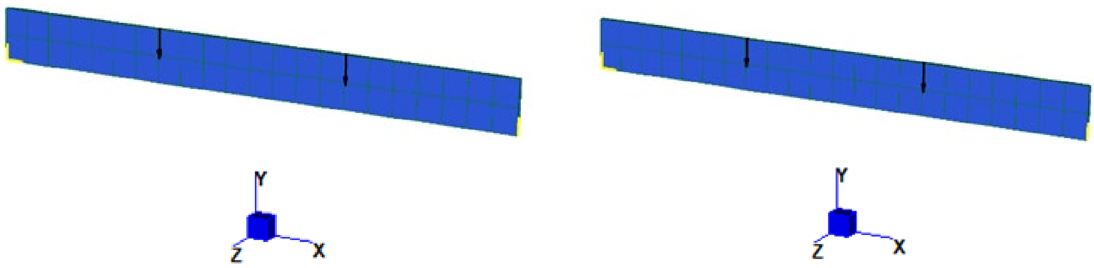
- $f_{g;k}$ is the characteristic value of the bending strength ($f_{g;k}$ =45 N/mm^2);
- $\gamma_{M;A}$ is the material partial factor for tempered glass ($\gamma_{M;A}$ =1.8) ;
- k_{sp} is the factor for the glass surface profile (k_{sp}=1 for float glass);
- k_{mod} is the factor for the load duration (k_{mod} =0.57 for the action of daily temperature variation with 11 hours extreme peak duration);
- $\gamma_{M;V}$ is the material partial factor for the surface prestress ($\gamma_{M;V}$ =1.2);
- $f_{b;k}$ is the characteristic value of the bending strength of prestressed glass ($f_{b;k}$ =120 N/mm^2 for thermally toughened safety glass);
- kv is the factor for strengthening of prestressed glass (k_v=1).

The tensile strength of the prestressed glass is $f_{g;d}$ = 72 N/mm^2.

3.2 FEM analysis

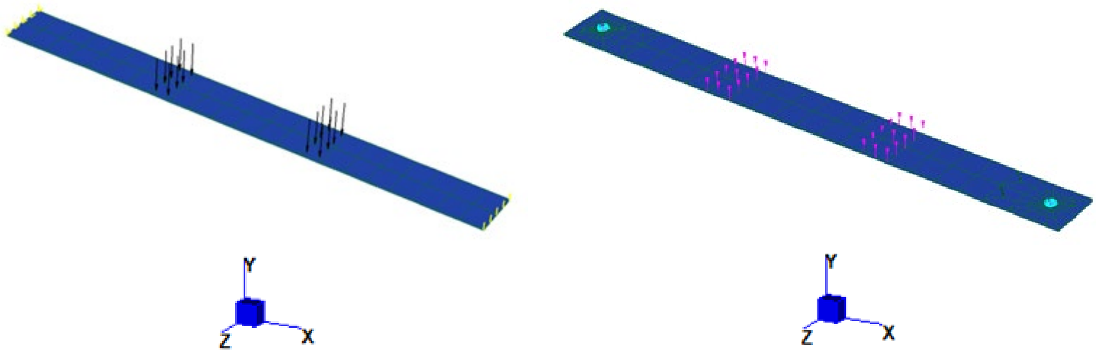
The analysis of the glass beams behavior has required a detailed study of several load-patterns. For this reason, the glass beam has been modelled by considering the load applied in the lamination plane (sheet-configuration) and orthogonal to the lamination plane (plate-configuration). The typologies of the specimens investigated in this research are the following:

- Unreinforced laminated glass sheet;
- CFRP laminated glass sheet;
- Laminated glass plate;
- Laminated glass plate with holes.



(a) Un-reinforced (b) Reinforced

Fig. 6 Numerical model of a laminated glass sheet



(a) Without holes (b) With holes

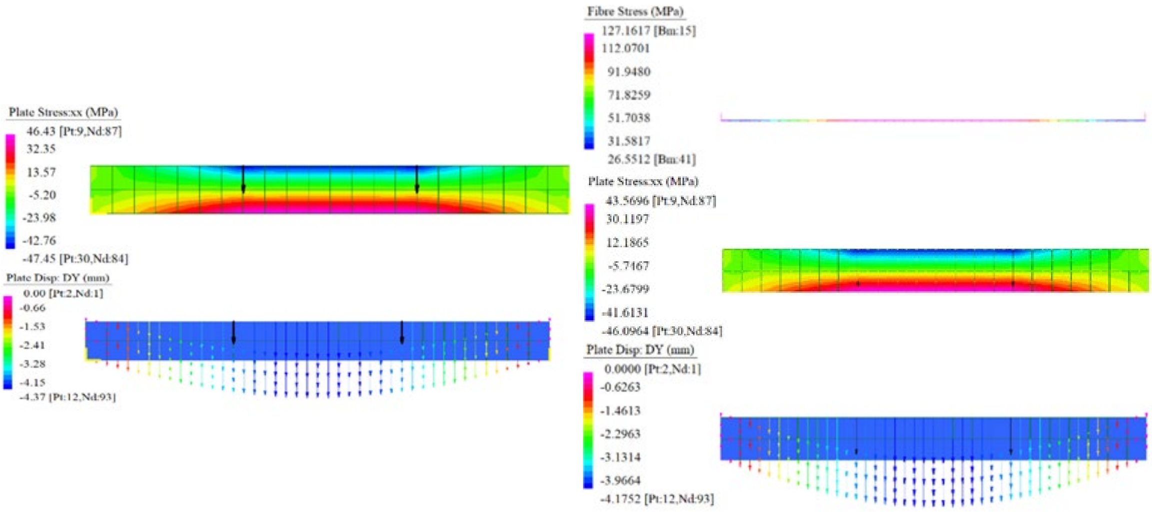
Fig. 7 Numerical model of a laminated glass plate

In order to analyse the behavior in the different cases, four beams with equivalent thickness and identical dimensions (0.3 x 3.0 m) have been considered. Two concentred loads (22 kN) have been applied at the third and at the two-third points of the span as shows in Fig. 6. Under these load conditions, the ultimate stress value, as well as its ultimate displacement have been evaluated in the beam configuration for un-reinforced (Fig. 6(a)) and reinforced (Fig. 6(b)) cases.

All behaviors of unreinforced beams or beams reinforced with CFRP-rod and the effect on the structural response of a plate in the presence of holes that cross the laminated glass section are evaluated.

The stress distribution and the ultimate displacement are evaluated in the case of plate with and without holes (holes diameter d=55 mm). In the case of plate without holes, a distributed load equal to 24 daN/cm^2 has been applied at the third and at the two-third points of the plate as shown in Fig. 7(a). A vertical displacement equal to 103 mm, instead, has been applied at the third and at the two-third points (Fig. 7(b)) of the plate with holes.

Both linear and non-linear analyses have been carried out to provide the overall response of the loaded element and verify flexural behaviors of considered beams. The numerical simulations of the 4-point static bending test of the composite sandwich beams have been carried out using a Finite Element Method (FEM) by mean of the commercial simulator STRAUS 7. The skin and the core material are modelled as 8-node layered solid plate elements with the previously described mechanical properties. The FEM-analysis has been carried out by modelling the specimen and the loading set-up.



(a) Un-reinforced (b) CFRP-reinforced

Fig. 8 Stress and displacement distribution in the glass beam

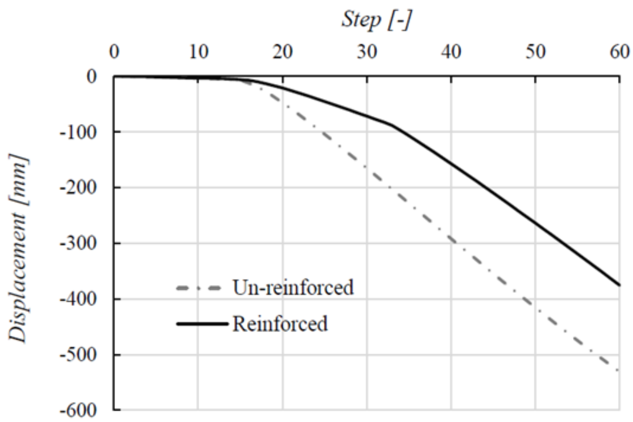


Fig. 9 Displacement curves for the un-reinforced and reinforced glass beams

4. Results and discussions

Both linear and non-linear analyses have been conducted for the sheet configuration (un-reinforced and reinforced) and for the plate configuration (without and with holes).

Fig. 8 shows the distribution of the maximum stresses in the different materials both without reinforcing (Fig. 8(a)) and with CFRP-reinforced (Fig. 8(b)) glass beam. It can be observed how the effect of the reinforcement redistributes the levels of stresses with a decrease of the maximum displacement in the CFRP- beam.

A different stress distribution results from the non-linear analysis carried out for several specimens. The non-linear analysis has been performed with an iterative increase procedure (Mondkar and Powell 1977) to identify the displacement of the control point.

In particular, the curves in Fig. 9 show the values of the displacement referred to load

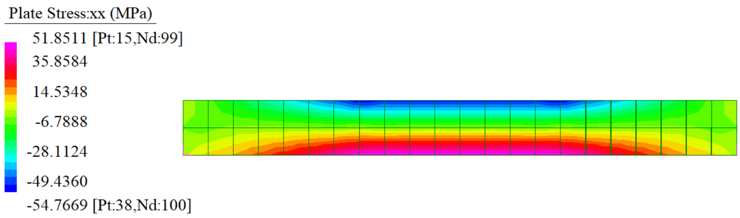


Fig. 10 Non-linear analysis: xx-stress distribution

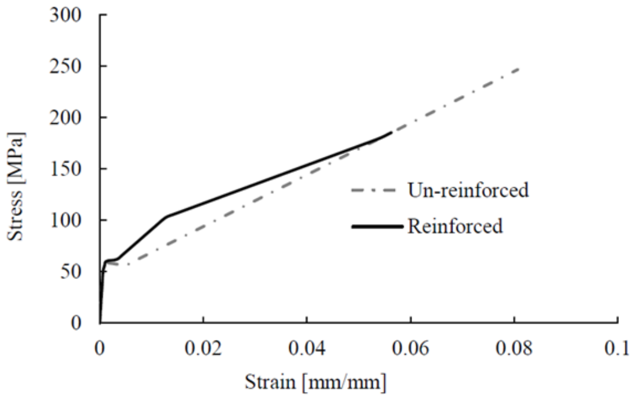


Fig. 11 Stress-strain curves for the un-reinforced and reinforced glass sheets

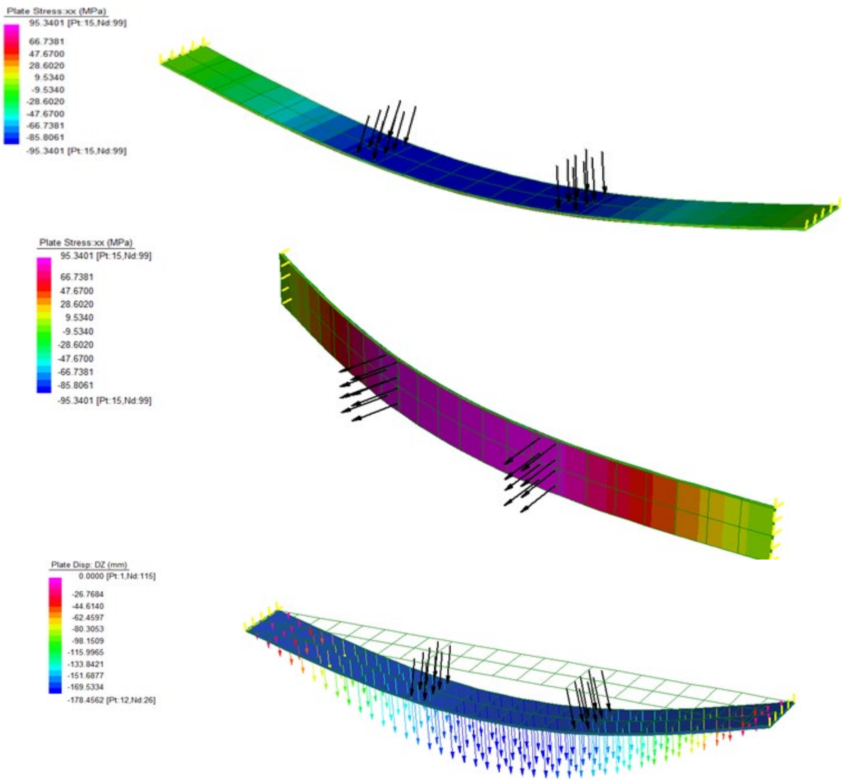


Fig. 12 Results of the FEM analysis on the glass plate without holes

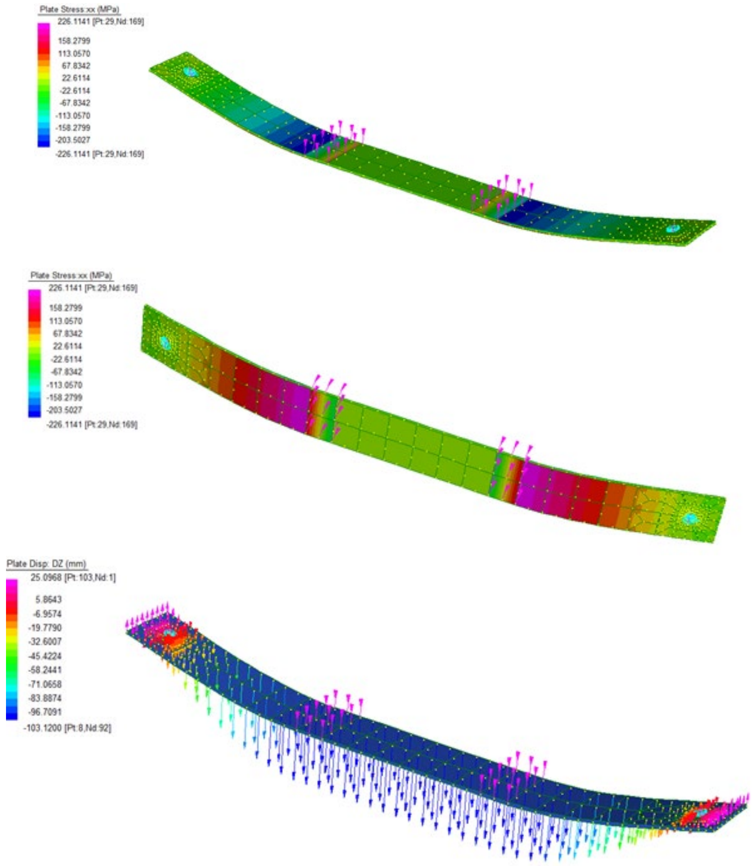


Fig. 13 Results of the FEM analysis on the glass plate with holes

increment for each step during the analysis. It can be noted that the slope curve of the un-reinforced beam increases significantly for the vertical displacement. The variation of the curve slope in the FRP-specimen occurs after the 13th load step (stress level equal to the glass tensile strength as shown in Fig. 10) (see also Table 1).

In Fig. 11 the stress-strain curves for the specimens (i.e., un-reinforced and reinforced glass sheet) are represented. It is possible to notice that after 50 MPa (corresponding to step 13), the stress-strain curve shows a jump and the behavior of the two sheets is not the same anymore. In the case of reinforced sheet, a different behavior in terms of glass stresses was observed with respect to the un-reinforced sheet. Furthermore, a significant difference has been detected in terms of ultimate strain if compared to the un-reinforced sheet.

Similarly, to the laminated glass beam reinforced with CFRP-rod, a numerical analysis of a glass plate has been carried out. The load-pattern has been applied orthogonally to the plane, as shown in Fig. 7(a). In particular, the distributed load was applied at the third and at the two-third points of the plate. The FEM-analysis results show that the stress values of the plate were lower than the ultimate compressive and tensile strength (Fig. 12).

The glass plate has been then modelled with two circular holes. In this case, a vertical displacement has been applied at the third and at the two-third points of the span. Unlike in

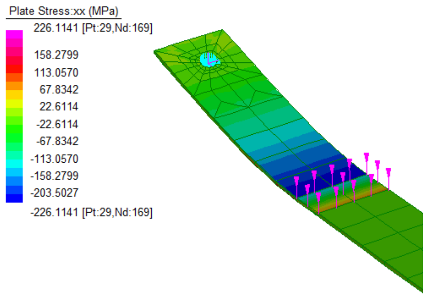


Fig. 14 Results of the FEM analysis on the glass plate with holes

previous case, in the glass plate with the holes the stresses are greater than the ultimate tensile strength of glass (Fig. 13). Therefore, the presence of two holes significantly varies the structural response of the glass plate. The area around the hole can be affected by failure mechanism (Fig. 14).

The herein reported numerical results demonstrate significant behaviors of reinforced beams used in analytical modelling and of beams with holes. In particular, the structural behavior of the analyzed specimens result affected by the embedded reinforcement and by holes. The reported numerical analysis enables to evaluate the expected stress-strain variation range at the elastic limit of the considered specimens.

5. Conclusions

Glass components can be applied as architectural and structural elements in buildings. In this paper, an innovative modelling based on an equivalent thickness concept of laminated glass beam formed by two PVB layers and three glass plies has been presented and detailed. The novel equivalent modelling carried out in this research wants to highlight that the brittleness of glass is a fundamental characteristic for the design of structural glass elements. In modern design, this gap can be overcome by considering alongside techniques such as a stratification of glass and design criteria, such as fail-safe design, have been also introduced, allowing the use of glass in big and relevant architectural structures. In this scenario, the novel solution proposed in this paper consists in the use of CFRP-glass composite structures by combining a brittle material (i.e., glass) with another material that offers ductility to the system (i.e., CFRP-rod). Embedded carbon fiber reinforced rod at the bottom face of structural glass beams has been proposed to be used. Such technique reveals very promising, because it can provide a transparent and yet ductile structural element with a safe failure mode. The behavior of an ordinary glass beam has been compared to a beam reinforced with CFRP-bar. It has been noticed a decrease of the stress level and the displacement of the glass sheet for the sample with embedded reinforcement (CFRP-rod) by the FEM-analyses conducted for the two types of investigated beams.

Furthermore, the effect of holes on the glass plate has been also investigated by comparing the behaviors of the glass plates without and with holes, since in structural applications it is generally necessary to perforate glass plates, since holes are the elements that allow the connection of any plate with other structural components. The conducted study shows that the presence of holes makes particularly brittle every glass area around them.

The completeness of the innovative modelling proposed in this preliminary work has to be subsequently validated by experimental data, beside the reported numerical analysis. However, experimental considerations can be made by comparing the behaviors of the beams considered in this work with those interesting hybrid beams made of glass and several fibers experimentally studied in Speranzini *et al*. (2015), Bedon *et al*. (2017), Corradi *et al*. (2019), where the analysed beams are composed by laminated glass sheets and glass FRP, CFRP and steel rods. In particular, the attention has been focused on the sample tested in Bedon *et al*. (2017), which consists of a two-ply glass assembly with three round CFRP rods differently than specimens analysed in the herein presented study. The structural behavior of the CFRP beam specimens reveal some potentials for the post-fracture performance enhancement of the un-reinforced beams after the first cracking peaks. In detail in Bedon *et al*. (2017) analogous collapse configurations have been experimentally observed for GFRP and CFRP beam specimens, similarly to what happens when applying steel rods. Moreover, in Corradi *et al*. (2019) all experimental results demonstrate that steel reinforcements are not able to produce any increase of the load-capacity at the elastic limit. However, the steel fiber reinforcement can guarantee the development of a post-elastic phase, and this has positive effects on the overall structural safety.

Taking into account all these preliminary experimental considerations, experimentations considering CFRP-glass beams with different patterns of reinforcement and glass plates with circular holes are planned, in order to validate the proposed innovative modelling and numeric analysis and for obtaining laboratory evidence on the post-cracking capacity of beams.

Thus, four-bending tests are planned in the next future on specimens made of three-ply glass assembled with two PVB-layer to evaluate the post-fracture assessment and the ductility of beam specimens to compare experimental data with the reported analytic/numerical investigations which reveal the importance of the proposed modelling.

Satisfying results have been herein determined with the proposed novel equivalent model, despite being the present investigation quite complex. In order to obtain an extensive study, many aspects will need to be explored in the future, such as more accurate evaluations of the PVB physical properties, the parameters of glass and the geometric parameters (i.e., beam size and hole size), that, of course, significantly affect each structural response. In the propose innovative modelling this preliminary results will be furtherly validated by planned experimental tests.

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