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# An investigation of non-linear optimization methods on composite structures under vibration and buckling loads

Mustafa Akbulut[1a], Abdulhamit Sarac[2b] and Ahmet H. Ertas[*3]

*[1]TUBITAK Marmara Research Center, Kocaeli 41400, Turkey*
*[2]TUBITAK National Metrology Institute, Kocaeli 41400, Turkey*
*[3]Department of Mechanical Engineering, Faculty of Engineering & Natural Sciences, Bursa Technical University, Bursa 16330, Turkey*

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**Abstract.** In order to evaluate the performance of three heuristic optimization algorithms, namely, simulated annealing (SA), genetic algorithm (GA) and particle swarm optimization (PSO) for optimal stacking sequence of laminated composite plates with respect to critical buckling load and non-dimensional natural frequencies, a multi-objective optimization procedure is developed using the weighted summation method. Classical lamination theory and first order shear deformation theory are employed for critical buckling load and natural frequency computations respectively. The analytical critical buckling load and finite element calculation schemes for natural frequencies are validated through the results obtained from literature. The comparative study takes into consideration solution and computational time parameters of the three algorithms in the statistical evaluation scheme. The results indicate that particle swarm optimization (PSO) considerably outperforms the remaining two methods for the special problem considered in the study.

**Keywords:** Benchmarking; Heuristic optimization algorithms; structural optimization; laminated composites; buckling load; fundamental frequencies

## 1. Introduction

In many engineering fields, fiber reinforced plastic composite plates are employed, and the demand for their use especially in aerospace industry is continuously rising. In aircraft industry where high specific strength and low cost are desired, the structural parts such as wings, ailerons, and tails are made of high-tech fiber-reinforced plastic composites and all these structures are subject to heavy operational conditions including buckling and vibration. In special cases both design parameters should be considered in the same problem, which may be formulated as a multi-objective optimization procedure in order to find out the fittest design configurations.

For the solution of multi-objective optimization problems dealing with high number of

*Corresponding author, Associate Professor, E-mail: ahmeth.ertas@btu.edu.tr
[a] Ph.D., E-mail: mustafaakbulut2002@hotmail.com
[b] Ph.D., E-mail: ahamitsarac@gmail.com


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optimization variables, heuristic optimization algorithms shine as the most suitable options. At this stage, the decision is to be made which method is the appropriate for the problem considered. In this study, the three most widely utilized heuristic optimization algorithms namely, simulated annealing (SA), genetic algorithm (GA) and particle swarm optimization (PSO) are put under the scope to determine the suitable optimization method especially for multi-objective structural composite design involved with buckling load and natural frequency. Some of these optimization procedures have also been applied to the design of laminated composite plates subject to buckling and vibration loading conditions. The objective function of all these problems deal with intricate expressions of non-linear nature making the use of linear programming and other gradient based analytic methods very limited, therefore in many of these optimization studies, heuristic algorithms have been used such as Genetic Algorithms (Riche and Haftka 1993, Soremekun *et al.* 2001, Apalak *et al.* 2008, Kang *et al.* 2008, Wu *et al.* 2012, Fabro *et al.* 2020), Simulated Annealing (Erdal and Sonmez 2005, Kayikci and Sonmez 2012), Particle Swarm Optimization (PSO) (Ghashochi and Sadr 2102a, b) or artificial bee colony algorithm (Topal and Ozturk 2014). In a limited number of research studies, analytical optimization methods such as Powell's optimization method (Sun 1989), Kuhn-Tucker Optimality conditions for maximum fundamental frequency (Narita and Zhao 1998), Golden section optimization method for buckling optimization (Walker and Reiss 1998), application of steepest ascent method in response surface of maximum buckling load and minimum weight search (Goldfeld *et al.* 2005), Newton's optimization (Chronopoulos 2015), The modified feasible direction method (MFD) for buckling load maximization (Topal and Uzman 2007) and a simplified formulation of buckling load maximization through integer programming (Haftka and Walsh 1992) have also been employed.

In an optimization problem, any objective function has its own special characteristics in terms of easiness to be minimized or maximized, local extremum properties, and the extent of solution domain etc. In structural engineering problems, in many cases, ideal configuration of parameters dealing with stress, displacement, buckling or natural frequencies are sought. In this respect, two cases having qualitatively and quantitatively dissimilar were chosen, i.e. buckling load and natural frequency, thus forming a generic objective function that can be applied to many challenging problems. Although there are a large number of studies attempting to optimize composite plates subject to buckling and vibration loading conditions, as given above, there exist very limited number of researches (Badallo *et al.* 2013, Razvan 2016, Bloomfield *et al.* 2010, Karakaya and Soykasap 2009) trying to compare the performance of heuristic algorithms for multi-objective structural optimization of composite plates, especially for the ones involving specifically buckling and vibration loading conditions. Badallo *et al.* (2013) conducted a comparative study of three common Genetic Algorithms on a composite stiffened panel considering three different strategies for the initial population and concluded that NSGA-II and AMGA seem the most suitable algorithms in terms of solution, computational time and number of generations to minimize the mass and to maximize the critical buckling load. Razvan (2016) compared between the performance of GA and PSO in a typical truss bar structural optimization problem and found out that Genetic Algorithms are superior to the particle swarm optimization for structural optimization problems, at least in what concerns truss structures. That shows the clear performance difference between two heuristic optimization methods. Bloomfield et al. (2010) applied Genetic Algorithm, Ant Colony and Particle Swarm Optimization for the optimization of a simply supported composite laminate subject to strength and buckling constraints and demonstrated that ant colony outperforms the other algorithms for inherently discrete sets of ply orientations and particle swarm optimization algorithm is the most convenient for continuous problems. As given in these references, for continues and discrete type of problems, different algorithms exhibit divers behaviors. Karakaya and Soykasap (2009) evaluated the performance of genetic algorithm (GA) and generalized pattern search algorithm (GPSA) for buckling load maximization of a composite structure and concluded that GA is more efficient for that type of specific problem. As to a benchmark study concerning the performance of GA, SA and PSO algorithms on the multi-objective optimization problem of a composite plate subject to buckling and vibration, as much as the authors know, there is not any specific research study. That fact constitutes the basic motivation for this research study. Since each optimization problem has its own objective functions having a peculiar form of intricacy depending on the problem definition, it is worthy of consideration to measure the performance of various heuristic algorithms based on that specific problem i.e. multi-objective optimization of composite plate subject to buckling and vibration in our case.

Fiber-reinforced composite structures have been widely investigated for the buckling and vibration analyses. In some of these studies only buckling load maximization (Riche and Haftka 1993, Soremekun *et al.* 2001, Erdal and Sonmez 2005, Topal and Ozturk 2014) are carried out, while others deal with just free vibration frequency maximization (Apalak *et al.* 2008, Ghashochi and Sadr 2102b, Zhao 1998) or both of them (Kang *et al.* 2008, Kam and Chang 1993, Sahoo and Singh 2014, Fazzolari and Carrera 2011, Oveys and Fazilati 2012, Dawe and Wang 1995, Ferreira *et al.* 2011, Shojaee *et al.* 2012). Buckling analysis of certain studies are based on classical lamination theory (Sun 1989, Walker and Reiss 1998, Goldfeld *et al.* 2005, Smerdov 2000, Walker and Hamilton 2005a, Adali and Duffy 1990) or numerical studies such as finite element (Walker and Hamilton 2005b, Lindgaard and Lund 2011, Lund 2009), semi-analytical finite difference approach (Khani *et al.* 2012), non-lineer finite element method (Lindgaard and Lund 2011) or other analytical solutions such as Rayleigh–Ritz (Wu *et al.* 2012) or Galerkin (Fazzolari and Carrera 2011). In all these studies various kinds of geometries are considered such as plane composite plates (Walker and Hamilton 2005a, Adali and Duffy 1990, Walker and Hamilton 2005b), composite plates consisting of hybrid laminates (Adali and Duffy 1990), cylindrical shells (Sun 1989, Smerdov 2000), buckling of variable angle tow (VAT) placed composite laminates (Wu *et al.* 2012), cylindrical laminated composite structures using steered fiber tows (Khani *et al.* 2012) or buckling of laminated conical Shells (Goldfeld *et al.* 2005). Mainly in-plane compressive loads (Erdal and Sonmez 2005, Topal and Ozturk 2014, Dawe and Wang 1995, Walker and Hamilton 2005a, Adali and Duffy 1990, Lund 2009), and rarely axial-torsional (Walker and Reiss 1998, Goldfeld *et al.* 2005, Diaconu *et al.* 2002) and hydro static pressure (Goldfeld *et al.* 2005) type loadings are employed for problem definitions. In order to handle the buckling and natural frequency calculations, numerous alternative solution approaches are taken into account by many researchers (Sahoo and Singh 2014, Fazzolari and Carrera 2011, Oveys and Fazilati 2012, Dawe and Wang 1995, Ferreira *et al.* 2011, Shojaee *et al.* 2012). Sahoo and Singh (2014) developed a new novel zigzag theory in combination with an efficient finite element model for the free vibration response and stability analysis of the laminated composite and sandwich plates. Fazzolari and Carrera (2011) addressed an accurate free-vibrations and linearized buckling analysis of anisotropic laminated plates with different lamination schemes employing methods such as Rayleigh-Ritz, Galerkin and Generalized Galerkin. Oveys and Fazilati (2012) applied the third order shear deformation theory of plates for the development of two versions of finite strip method (FSM) in order to investigate the buckling strength and also free vibration behavior of isotropic and layered composite plates containing cutouts. Dawe and Wang (1995) has developed the spline finite strip method for the prediction of buckling stresses and natural frequencies of vibration of rectangular composite laminated plates of arbitrary lamination and with general boundary conditions. Ferreira *et al.* (2011) used the radial basis function collocation method to analyze buckling loads and free vibrations of isotropic and laminated plates. Shojaee *et al.* (2012) benefitted from the NURBS-based isogeometric finite element method for analysis of natural frequencies and buckling phenomena for the thin laminated composite plates and examined several laminated composite plates with different geometrical configurations and boundary conditions considering various aspect ratios.

In the last several decades, optimal design of fiber reinforced composite plates has become a subject of many research studies. At this point, the researchers need to know about the performance of the search algorithms from which they benefit, in terms of solution accuracy, reliability, time consumption etc. for the specific problems they deal with. For that purpose, in this work, a comparative study is carried out to be able to comment on the performance of three most widely utilized heuristic search algorithms i.e. simulated annealing (SA), genetic algorithm (GA) and particle swarm optimization (PSO). The composite plate considered in this study is modeled as a symmetric composite structure. The governing equation for buckling is derived based on the classical lamination theory and vibration analysis is found with the help of first-order shear deformation plate theory. An analytical solution of buckling analysis is used, while finite element method is selected for numerical results of fundamental frequencies. Although there are various studies dealing with both buckling load and natural frequency maximization, the present study applies it with the help of Multi –Objective Design Index approach to conduct a comparative study. In addition, the effect of various in-plane loading ratios, a relatively large number of distinct fiber angles and different intervals between available orientation angles are investigated on the basis of the most appropriate heuristic algorithm determined.

## 2. Formulation

### 2.1 Buckling analysis

The composite structure considered is a panel simply supported on four sides with a length of $a$ and width of $b$ as illustrated in Fig. 1. The panel is subject to in-plane compressive loads $N_x$ and $N_y$ in the x and y directions respectively. The laminate is symmetric, balanced about the mid-plane and made of 64 layers with the thickness $t$.

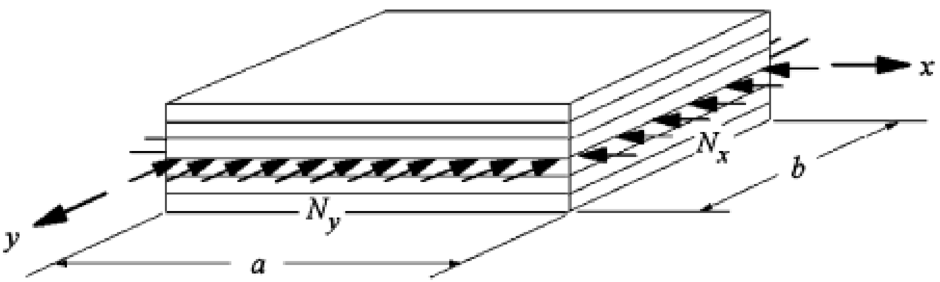



Fig. 1 A symmetric laminate under compressive biaxial loads

The laminate buckles into $m$ and $n$ half-waves in the $x$ and $y$ directions, respectively, when the loads reach the critical values of $N_x$ and $N_y$. $N_{cr}$ is the critical load before buckling. In order to make MODI non dimensional and unit-less, $N_0$ is the maximum critical load for the given fiber direction interval, plate aspect ratio and load conditions for $\lambda = 0$, that means for given fiber direction interval, For instance if it excepted an interval as 0°-90°, and if it is tried to find the maximum critical load that can be reached, without taking into consideration fundamental frequency that's why $\lambda = 0$, we do the same for fundamental frequency, thus when we reach the highest critical load $N^*$ will be 1. The ratio will make it dimensionless. The governing equation regarding the transverse deflection of the plate (Gibson 1994) under classical lamination theory is given in Eq. 1.

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=-N\frac{\partial^2 w}{\partial x^2} \tag{1}$$

where $D_{ij}$ are bending stiffness, $w$ is transverse deflection in terms of $x$ and $y$. $N$ is the in-plane compressive loads. The following form is assumed to be a solution of the Eq. 1;

$$w(x,y)=w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}1 \tag{2}$$

Substituting Eq. 2 into Eq. 1, after some mathematical operations, the solution for the critical buckling load is obtained as follows; (which each $m$ and $n$ integer combinations correspond to a new solution);

$$\lambda_b(m,n)=\pi^2\left[\frac{m^4 D_{11}+2(D_{12}+D_{66})(rmn)^2+(rn)^4 D_{22}}{(am)^2 N_x+(ran)^2 N_y}\right] \tag{3}$$

where $\lambda_b$ is the maximum critical load before buckling, $r$ is the plate aspect ratio and defined as $a/b$. The critical buckling load is the smallest value of $\lambda_b$ under any combination of pair ($m$, $n$), which should be greater than one to avoid immediate failure.

$$\lambda_b=\min\lambda_b(m,n) \tag{4}$$

Taking $\{m, n\}=2$ was shown to result in a good estimate of buckling load capacity. Accordingly, the smallest of $\lambda_b(1,1)$, $\lambda_b(1,2)$, $\lambda_b(2,1)$ and $\lambda_b(2,2)$ was taken as the critical buckling load (Gibson 1994).

### *2.2 Free vibration analysis*

The composite structure is modeled as a symmetric balanced plate which is described by the first-order shear deformation theory for free vibration analysis.

Constitutive equations for the plate are given as follows;

$$\begin{aligned} u(x,y,z)&=u_0(x,y)+z\phi_x(x,y)\\ v(x,y,z)&=v_0(x,y)+z\phi_y(x,y)\\ w(x,y,z)&=w_0(x,y) \end{aligned} \tag{5}$$

where, $u_0$, $v_0$ and $w_0$ are the mid-plane displacements in $x$, $y$ and $z$ directions, respectively. $\phi_x$ and $\phi_y$ represent the rotations of transverse normal about $y$ and $x$ axes, respectively. The constitutive equations for the composite structure are expressed as;

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} \tag{6}$$

$$\{Q\} = \begin{Bmatrix} Q_{yz} \\ Q_{xz} \end{Bmatrix} \begin{bmatrix} S_{44} & S_{45} \\ S_{45} & S_{55} \end{bmatrix} \{\gamma\} \tag{7}$$

where the laminate stiffness is defined as

$\left(A_{ij}, B_{ij}, D_{ij}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\overline{Q}_{ij}\right)_k \left(1, z, z^2\right) dz$ with $\left(i, j = 1, 2, 6\right)$, and

$S_{ij} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\overline{Q}_{ij}\right)_k dz$ with $\left(i, j = 4, 5\right)$,

$\varepsilon$ , $\gamma$ are strains and $\kappa$ bending curvatures.

### *2.2.1 Derivation of the governing equation for the free vibration of the plate*

The principle of virtual work is employed to derive the governing equations, i.e.

$$\delta W = \delta W_{\text{int}} - \delta W_{ext} = 0 \tag{8}$$

where $\delta W_{\text{int}}$ represents the internal virtual work and $\delta W_{\text{ext}}$ is the external virtual work.

$$\delta W_{\text{int}} = \int_A \left( \{\delta \varepsilon_m\}^T \{N\} + \{\delta \kappa\}^T \{M\} + \{\delta \gamma\}^T \{Q\} \right) dA \tag{9}$$

$$\delta W_{\text{int}} = \{\delta d\}^T [K] \{d\} \tag{10}$$

where $\{d\} = [uv\phi_x\phi_y w]^T$ is the displacement vector and $[K]$ is the linear stiffness matrix,
On the other hand, the external virtual work is given by

$$\delta W_{ext} = \int_A \begin{bmatrix} -I_0\left(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right) \\ -I_1\left(\ddot{\phi}_x \delta u + \ddot{u}\delta\phi_x + \ddot{\phi}_y \delta v + \ddot{v}\delta\phi_y\right) - I_2\left(\ddot{\phi}_x \delta\phi_x + \ddot{\phi}_y \delta\phi_y\right) \end{bmatrix} dA \tag{11}$$

$$\delta W_{ext} = -\{\delta d\}^T [M] \{\ddot{d}\} \tag{12}$$

where $I_0$, $I_1$, $I_2$ are the moment of inertias, and $[M]$ is the mass matrix. By substituting Eq. 10 and Eq. 12 into Eq. 8, we obtain the equation of motion as;

$$[M]\{\ddot{d}\} + [K]\{d\} = \{0\} \tag{13}$$

Eigenvalue problem for the vibration analysis is given as;

$$\left[[K] - \omega^2 [M]\right]\{\Theta\} = \{0\} \tag{14}$$

where $w$ and $\{\Theta\}$ are the natural frequency and the vibration mode shape, respectively.

## 3. Optimization procedure and algorithms considered

### *3.1 Optimization scheme*

The multi-objective optimization problem is formulated by means of a weighted summation method. The objective function is constructed as a summation of weighted ratios of critical buckling load and non-dimensional fundamental frequencies, which is called as Multi-objective design index MODI. The optimization parameters are the orientation angles and the thicknesses of each lamina.

$$MODI(\theta_1, \theta_2, ..., \theta_n, t_1, t_2, ..., t_n) = \xi N^* + \lambda W^* \tag{15}$$

where $N^* = N_{cr}/N_0$ and $W^* = \omega/\omega_0$, $N_0$ is the maximum critical load for the given fiber direction interval, plate aspect ratio and load conditions for $\lambda$=0, and $\omega_0$ is the maximum non-dimensional fundamental frequencies under the same conditions for $\xi = 0$, therefore $\lambda$, $\xi \geq 0$ and, $\xi + \lambda = 1$.

### *3.2 Genetic Algorithm (GA)*

The GA is based on the principles of natural selection. It mimics the process of survival of the fittest principle in nature by trying to maximize the fitness function. The population, which represents the optimization variable sets, is updated after each learning cycle through three evolutionary processes, i.e. selection, crossover and mutation. These create the new generation of solution variables. Because of its capability to handle high number of variables and complicated objective functions, GA has been used in the structural design of fiber reinforced composite plates very frequently. The fundamental theorem of the GA was introduced by Holland (1992). Callahan and Weeks (1992) was the pioneer to show that GA can be a viable alternative to traditional search algorithms in the design of composite laminates. Kogiso *et al.* (1994), used GA with local improvement to optimized laminated composite plate for buckling load maximization. As given in earlier sections, many researchers (Riche and Haftka 1993, Soremekun *et al.* 2001, Apalak *et al.* 2008, Kang *et al.* 2008, Wu *et al.* 2012) benefitted from GA in buckling and vibration optimization of composite panels.

Since each heuristic algorithm has its own properties peculiar to its own structure, which differs from those of others, it is extremely hard to use standard values to keep coherence among all. It should be pointed out that even in GA itself there are various selection, crossover and mutation strategies along with different probability coefficients which slightly affect the performance of the algorithm. Therefore, in this study an attempt is made to utilize certain standard values regarding the algorithm parameters given as follows; population size: 50; selection rate: 50%; crossover probability: 80%; Gaussian Mutation with probability of 50%.

### *3.3 Particle Swarm Optimization (PSO)*

The particle swarm optimization (PSO) algorithm is a kind of swarm intelligence techniques, which are inspired by the social behavior of flocking animals such as swarms of birds or fish school. This population based stochastic optimization algorithm was first developed by Eberhart and Kennedy (1995). The approach can be considered as a distributed behavioral algorithm that operates as a multi-dimensional search. Due to its natural ability to converge faster, PSO algorithm is very suitable for solving multi-objective optimization problems (Parsopoulos and Vrahatis 2002). PSO is a population based algorithm having many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The Algorithm starts with a population of random solutions and searches for optimal configuration by updating generations. However, unlike Genetic Algorithms, PSO has no evolution tools such as crossover or mutation. PSO trades on a population of individuals to explore promising regions of the search space. The individual behavior is affected either by the best-local or best-global individual. The population is referred as a swarm and individuals are called particles. The particles move in a multi-dimensional search space with adaptable velocity. In PSO, the particles have a memory of the best position in the past and the best position ever attained by the particles. This property makes it possible to search the multi-dimensional space faster.

Let us consider an optimization problem with nn dimensional design space. Assume that there are $MM$ particles in a swarm and $i^{th}$ particle in a swarm is represented as a vector $X_i$, which $X_i \in R_n$.

$$X_i = (x_{i1}, x_{i2}, \ldots, x_{inn})^T, \qquad i = 1, 2, \ldots, MM \tag{16}$$

The velocity of the particle moving in the nn-dimensional search space is

$$V_i = (v_{i1}, v_{i2}, \ldots, v_{inn})^T, \qquad i = 1, 2, \ldots, MM \tag{17}$$

and the best position encountered by the particle is

$$B_i = (b_{i1}, b_{i2}, \ldots, b_{inn})^T, \qquad i = 1, 2, \ldots, MM \tag{18}$$

Let us assume that the particle j attains the best position in the current iteration (l) then the position and the velocity of the particles are adapted using the following equations.

$$V_i(l+1) = wV_i(l) + c_1 r_1 (B_i(l) - X_i(l)) + c_2 r_2 (B_i(l) - X_i(l)) \tag{19}$$

$$X_i(l+1) = X_i(l) + V_i(l+1) \tag{20}$$

where $w$ is the inertia weight, $c_1$, $c_2$ represent positive acceleration constants and $r_1$, $r_2$ are uniformly distributed random numbers $r_1$, $r_2 \in [0,1]$. The first term in the above equation, relates to the current velocity of the swarm, the second term represents the local search while the third term represents the global search pointing towards the optimal solution.

The inertia weight ($w$) is employed to control the impact of the previous history of velocities on the current velocity of each particle. Thus, the parameter w regulates the tradeoff between global and local exploration ability of the swarm. It is an acceptable approach to initially set the inertia to a large value, in order to make better global exploration of the search space and gradually decrease the weight to get more refined solutions.

### *3.4 Simulated Annealing (SA)*

Simulated Annealing is a heuristic search algorithm to locate global extremums to large optimization problems. It was first proposed as an optimization technique by Kirkpatrick *et al.* (1983). SA is based on an analogy of thermal annealing of critically heated solids and is an iterative search method inspired by the annealing of metals. According to the principles of this search N number of initial configurations is randomly created within the design domain by randomly selecting values for the design variables. The objective function of the problem is calculated for each randomly created configuration. The probability of accepting a newly created configuration depends on a probability function whose value is a function of temperature *(T)*.

$$A_t = \left\{ \begin{matrix} 1 & if \ f_t \le f_h \\ \exp\left( \left( f_h - f_t \right) / T_j \right) & if \ f_t \rangle f_h \end{matrix} \right\} \tag{21}$$

Here $f_h$ is the highest cost in the current set. This means every new design having a cost lower than the cost of the current design is accepted. But, if the cost is higher, the trial configuration may be accepted depending on the value of $A_t$. If it is greater than a randomly generated number, *Pr*, the trial configuration is accepted, otherwise it is rejected. Uphill moves are occasionally accepted with above mentioned probability, which enables the algorithm to escape local minima. Iterations during which the value of the temperature (or control) parameter, $T_j$, is kept constant are called $j^{th}$ Markov chain (or inner loop). After a certain number of iterations, the temperature parameter, *T*, is reduced, a new inner loop begins. As Eq. (21) implies, when $T_{jS}$, decreased, the probability that a worse configuration is accepted becomes lower. At low values of temperature parameter, acceptability becomes low; thus, acceptance of worse configurations is unlikely, just as the atoms become stable, and do not tend to change their arrangements at low temperatures in an annealing process.

SA was utilized in numerous structural optimization problems (Erdal and Sonmez 2005, Kayikci and Sonmez 2012, Hasancebi *et al.* 2010, Akbulut and Sonmez 2008, Ertas and Sonmez 2010, Ertas 2013a, Ertas 2013b). In structural design, Hasancebi *et al.* (2010) used SA to find the optimum design of fiber composite structure problems with multiple global optima. Erdal and Sonmez (2005) maximized buckling load capacity using simulated annealing method. Akbulut and Sonmez (2008) benefitted from direct simulated annealing (DSA) to minimize thickness of laminated composite plates, subject to in-plane loading. Ertas and Sonmez (2010, 2014) and Ertas and Sonmez (2011) used the SA to design fiber composite structure for maximum fatigue life and strength, respectively. Ertas (2013a) and Ertas (2013b) also used the PSO to design fiber composite structure for maximum fatigue life.

## 4. Model validation

### *4.1 Fundamental frequency*

A simply supported cross-ply square laminated composite plate is studied with different modulus ratios ($E_{11}/E_{22}$). The four-layer plate symmetrically laminated with [0°/90°/90°/0°] is used and each layer has the same thickness. The following parameters are used in the computation: $L$=10 $m$, $t$= 1–2 $m$, $E_{22}$= 1.0 $GPa$, $G_{12}$=$G_{13}$=0.6 $E_{22}$, $G_{23}$= 0.5, $E_{22}$, $\nu_{12}$=$\nu_{23}$=$\nu_{13}$=0.25 and $\rho = 1 kg/m^3$. Table 1 shows the non-dimensionalized fundamental frequencies of laminates $\bar{\omega} = \omega_{11} b^2 \sqrt{\rho/(E_{22} t^2)}$ as a function of modulus ratios ($E_{11}/E_{22}$). Two different kinds of length/thickness ratios are considered. It can be seen that the primary frequency increases with the ratios $E_{11}/E_{22}$ and $L/t$. Since it is square plate, $L$ corresponds to the dimension of one side in this study, $t$ was used to denote the thickness. The present results are quite close to those obtained by other researchers. Slight differences result from the numerical settings of finite element computations.

Table 1 The non-dimensional fundamental frequencies $\varpi = \omega_{11} b^2 \sqrt{\rho/(E_{22}t^2)}$ of a simply supported cross-ply square laminated composite plate with [0°/90°/90°/0°] and different modulus ratios ($E_{11}/E_{22}$)

| ($E_{11}/E_{22}$) | $L/t$ | Present | Lei *et al.* | Cui *et al.* | Bletzinger *et al.* | Reddy | Dai *et al.* |
|---|---|---|---|---|---|---|---|
| 3 | 5 | 6.362 | 6.578 | 6.526 | 6.574 | 6.570 | 6.360 |
| | 10 | 8.079 | 7.254 | 7.192 | 7.249 | 7.242 | 7.157 |
| 10 | 5 | 7.354 | 8.349 | 8.283 | 8.338 | 8.271 | 8.080 |
| | 10 | 10.803 | 9.893 | 9.810 | 9.882 | 9.842 | 9.670 |
| 20 | 5 | 7.775 | 9.645 | 9.573 | 9.633 | 9.526 | 9.440 |
| | 10 | 12.495 | 12.309 | 12.211 | 12.297 | 12.218 | 12.115 |
| 30 | 5 | 7.955 | 10.416 | 10.341 | 10.404 | 10.326 | 10.238 |
| | 10 | 13.393 | 13.988 | 13.879 | 13.974 | 13.864 | 13.799 |
| 40 | 5 | 8.055 | 10.951 | 10.873 | 10.937 | 10.854 | 10.789 |
| | 10 | 13.965 | 15.259 | 15.142 | 15.244 | 15.107 | 15.068 |

Table 2 The optimal designs for $N_x/N_y$=1, $a/b$=2, 64 plies with possible angles of $0_2$, ±45, $90_2$

| Design | Buckling Load Factor |
|---|---|
| $[90_{10}/\pm45_4/90_6/\pm45_4]_s$ | 3973.01 |
| $[\pm45/90_8/\pm45/90_{20}]_s$ | 3973.01 |
| $[90_8/\pm45_2/90_6/\pm45_4/90_2/\pm45_2]_s$ | 3973.01 |
| $[\pm45/90_{16}/\pm45_5/90_4]_s$ | 3973.01 |

### *4.2 Buckling analysis*

In order to check the validity of the optimization procedure and buckling load factor formulation, the problem studied by Riche and Haftka (1993) and Soremekun *et al.* (2001) were considered. A graphite epoxy plate was chosen with the elastic properties of $E_{11}$=127.59 *GPa*, $E_{22}$=13.03 *GPa*, $G_{12}$=6.41 *GPa* and $\nu_{12}$= 0.28. The dimensions of the composite panel were $a$=50.8 *cm*, $b$= 25.4*cm*, and $t$= 0.127*cm*. Table 2 presents various design configurations with possible angles of $0_2$, ±45, $90_2$. The optimization procedure using PSO in this case is shown to be able find the same global designs as given in the literature.

Table 3 lists the optimum fiber orientations and corresponding buckling load factors found for various loading ratios and possible angle configurations. The present study offers almost the same results as determined by Erdal and Sonmez (2005), who had to use SA to locate the optimum solutions. It is clearly demonstrated that the PSO method can precisely locate the outputs of Simulated Annealing Optimization Method.

## 5. Results and discussions

Having verified the results of both fundamental frequency and the buckling load calculations along with the PSO optimization scheme separately, the comparative study among SA, GA and PSO in a general case example is carried out. Since there are various objective function scenarios with different Multi Objective Design Index, a MODI with $\lambda$ =0.5 and $\xi$ =0.5 was chosen for a simply supported panel which is symmetric, balanced about the mid-plane and made of 64 layers with a thickness $t$. The optimum results obtained for the above stated configuration are presented for each Algorithm in Tables 4, 5, 6.

Table 3 The Comparison of PSO and SA Outputs for various loading ratios and angle configurations

| Optimum Possible Fiber Orientations | Buckling Load Factor | |
|---|---|---|
| | Present Study | Erdal and Sonmez (2005) |
| | $N_x/N_y$=1, $a/b$=2, 64 plies with possible angles of 0, 30, 60, 90 | |
| $[90_3/60_5/90_9/60_7/90_5/60_3]_s$ | 4079.89 | 4080.08 |
| | $N_x/N_y$=2, $a/b$=2, 64 plies with possible angles of 0, 30, 60, 90 | |
| $[60_{12}/90_3/60_5/90_9/60_3]_s$ | 6379.31 | 6379.35 |
| | $N_x/N_y$=4, $a/b$=2, 64 plies with possible angles of 0, 30, 60, 90 | |
| $[60_6/30_3/60_9/30_4/60_3/30_7]_s$ | 8026.91 | 8026.83 |
| | $N_x/N_y$=4, $a/b$=2, 64 plies with possible angles of 0, 15, 30, 45, 60, 75, 90 | |
| $[60_4/45_4/60_4/45_{15}/60_5]_s$ | 8439.86 | 8440.27 |

Table 4 Simulated Algorithm (SA) results for 15 different iteration runs

| Iteration | MODI | Time (min) | Maximum Buckling Load | Non-dimensional fundamental frequency |
|---|---|---|---|---|
| 1 | 0.980039 | 607 | 4100.84 | 5.37 |
| 2 | 0.978853 | 823 | 4056.42 | 5.42 |
| 3 | 0.978439 | 574 | 4056.82 | 5.41 |
| 4 | 0.973734 | 805 | 4004.71 | 5.43 |
| 5 | 0.975098 | 625 | 4050.48 | 5.39 |
| 6 | 0.971126 | 558 | 3955.01 | 5.47 |
| 7 | 0.974436 | 709 | 4025.20 | 5.41 |
| 8 | 0.978659 | 430 | 4068.61 | 5.40 |
| 9 | 0.97498 | 977 | 4016.41 | 5.43 |
| 10 | 0.975432 | 406 | 4075.15 | 5.36 |
| 11 | 0.975976 | 671 | 4040.10 | 5.41 |
| 12 | 0.975133 | 752 | 4018.72 | 5.43 |
| 13 | 0.975677 | 444 | 4001.92 | 5.46 |
| 14 | 0.979249 | 351 | 4035.78 | 5.45 |
| 15 | 0.97583 | 441 | 4010.40 | 5.45 |

Table 5 Genetic Algorithm (GA) results for 15 different iteration runs

| Iteration | MODI | Time (min) | Maximum Buckling Load | Non-dimensional fundamental frequency |
|---|---|---|---|---|
| 1 | 0.984087 | 364 | 4080.18 | 5.45 |
| 2 | 0.983856 | 414 | 4077.75 | 5.45 |
| 3 | 0.982229 | 417 | 4071.73 | 5.44 |
| 4 | 0.982116 | 421 | 4089.70 | 5.41 |
| 5 | 0.981587 | 434 | 4095.07 | 5.40 |
| 6 | 0.984107 | 436 | 4079.56 | 5.45 |
| 7 | 0.983347 | 392 | 4084.87 | 5.43 |
| 8 | 0.983527 | 414 | 4085.69 | 5.43 |
| 9 | 0.980705 | 406 | 4083.21 | 5.40 |
| 10 | 0.983206 | 437 | 4079.37 | 5.44 |
| 11 | 0.975745 | 401 | 4052.65 | 5.39 |
| 12 | 0.982371 | 411 | 4070.53 | 5.44 |
| 13 | 0.984099 | 474 | 4080.81 | 5.45 |
| 14 | 0.983238 | 446 | 4079.10 | 5.44 |
| 15 | 0.983655 | 485 | 4078.20 | 5.44 |

Table 6 Particle Swarm Optimization (PSO) Algorithm results for 15 different iteration runs

| Iteration | MODI | Time (min) | Maximum Buckling Load | Non-dimensional fundamental frequency |
|---|---|---|---|---|
| 1 | 0.983455 | 222 | 4079.45 | 5.44 |
| 2 | 0.984031 | 319 | 4080.24 | 5.45 |
| 3 | 0.983707 | 199 | 4080.47 | 5.44 |
| 4 | 0.983583 | 246 | 4080.63 | 5.44 |
| 5 | 0.983486 | 175 | 4081.15 | 5.44 |
| 6 | 0.983963 | 189 | 4082.18 | 5.44 |
| 7 | 0.983737 | 164 | 4077.30 | 5.45 |
| 8 | 0.983513 | 211 | 4084.26 | 5.43 |
| 9 | 0.98351 | 207 | 4085.42 | 5.43 |
| 10 | 0.984015 | 252 | 4079.46 | 5.45 |
| 11 | 0.984152 | 200 | 4079.93 | 5.45 |
| 12 | 0.983478 | 157 | 4080.29 | 5.44 |
| 13 | 0.983422 | 231 | 4087.06 | 5.43 |
| 14 | 0.98408 | 229 | 4079.60 | 5.45 |
| 15 | 0.984048 | 188 | 4078.42 | 5.45 |

Table 7 Statistical results of Multi-Objective Problem

| Parameter | Optimization Method | Mean | Median | Standard Deviation |
|---|---|---|---|---|
| Objective Function | GA | 0.982525 | 0.983238 | 0.0021 |
| | PSO | 0.983745 | 0.983707 | 0,0003 |
| | SA | 0.976177 | 0.975677 | 0.0024 |
| Maximum Buckling Load | GA | 4079.23 | 4079.56 | 9.29 |
| | PSO | 4081.06 | 4080.29 | 2.56 |
| | SA | 4034.44 | 4035.78 | 34.64 |
| Non-dimensional Fundamental frequency | GA | 5.43 | 5.44 | 0.0185 |
| | PSO | 5.44 | 5.44 | 0.006 |
| | SA | 5.42 | 5.42 | 0.03085 |
| Time (min.) | GA | 423 | 417 | 30.48 |
| | PSO | 213 | 207 | 40.62 |
| | SA | 612 | 607 | 173.90 |

Table 7 presents the statistical outputs of the above defined optimization problem. The comparison among the three algorithms is based on the objective functions and computer run-times. In order to get rid of numerical fluctuations, each algorithm was run fifteen times for each optimization method and mean, median and standard deviation values were calculated for each series of samples. It is clearly observed that Particle Swarm Optimization Algorithm (PSO) provided the highest objective function value which is ~0.984 among all the three, along with average run-time of 213 minute. Standard deviation values indicate that there is a considerable margin between PSO and the remaining two algorithms. With an average value of 0.982, GA gave the second acceptable objective value along with relatively higher duration 423 min., even though it resulted in a reduced standard deviation of 30.48, which can be considered to be more reliable compared to the remaining two. All the data obtained suggests that PSO yielded the highest objective value and the shortest run-time.

### *5.1 Statistical analysis*

To explore the difference between the groups of data pertaining to MODI and time, the statistical analysis was performed and the statistical metrics were calculated accordingly. The direct method to determine whether the data are normally distributed is the normality analysis that is composed of several tests such as Shapiro-Wilk test (Shapiro and Wilk 1965, Massey 1951), Kolmogorov-Smirnov test (Massey 1951, Razali *et al.* 2011), skewness and kurtosis analysis (Doane and Seward 2011), and descriptive graphics.

#### *5.1.1 Normality analysis*

Normal distribution of the data gives direct information about the type of statistical analysis that needs to be carried out. Provided that the data are normally distributed, the parametric analysis is utilized. Non-parametric analysis (e.g. Mann-Whitney U Test) is employed if the data are not approximately normally distributed. We performed normality analysis for MODI and time data associated with each optimization algorithm (i.e. SA, GA, and PSO) by employing software packages, Microsoft Excel and IBM SPSS Statistics. Means and standard deviations of each data set were calculated, and the histograms were obtained in order to determine graphically whether the data are normally distributed. As a descriptive statistical analysis, skewness and kurtosis of the data sets were also calculated to exhibit the deviation from the normal distribution. The skewness is an indicator of symmetry or asymmetry in the histograms, and the kurtosis is essentially the sharpness of the peak of a frequency distribution that is in principle normal distribution herein. The MODI data of SA were approximately normally distributed with a skewness of -0.122 (SE=0.580), and a kurtosis of -0.153 (SE=1.121) as shown in Fig. 2(a). The histogram associated with the MODI of GA exhibits that the data were not normally distributed as illustrated in Fig. 2(b). The skewness and kurtosis of the data were calculated as -2.564 (SE=0.580) and 7.693 (SE= 1.121), respectively. However, the skewness and kurtosis pertaining to the MODI of PSO were calculated as 0.250 (SE=0.580) and -1.824 (SE= 1.121), which are suitable for normal distribution, the histogram exhibits non-normal distributed behavior as shown in Fig. 2(c).

Statistics such as Q-Q (quantile-quantile) plots and box plots clearly show that the MODI data of GA are not normally distributed. Whereas the box plot associated with MODI of SA is not symmetric, the box plot for MODI of PSO is nearly symmetric. The box plots need to be as symmetric as possible for normally distributed data. A Shapiro-Wilk's test ($p>0.05$) shows that while MODI data of SA are normally distributed, MODI of GA and PSO are not normally distributed. Finally, in order to determine the normality of the data fully, Kolmogorov-Smirnov test was carried out. Whereas MODI of SA and PSO are normally distributed, MODI data of GA are not normally distributed according to Kolmogorov-Smirnov test.

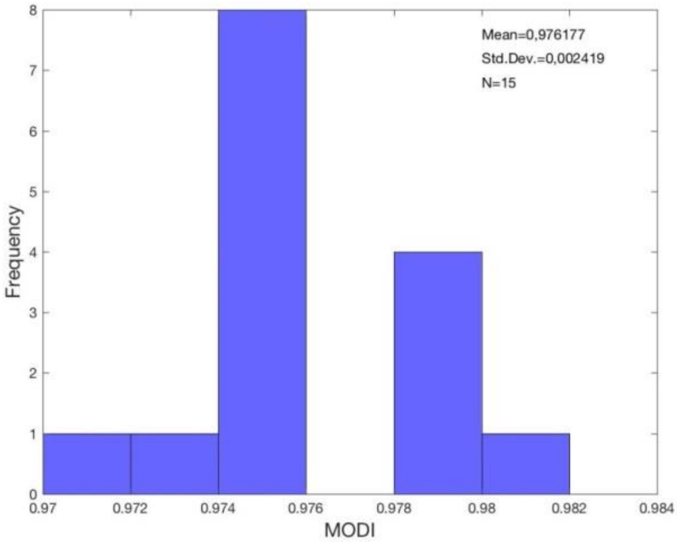



Fig. 2(a) MODI Histogram of SA Optimization Algorithm

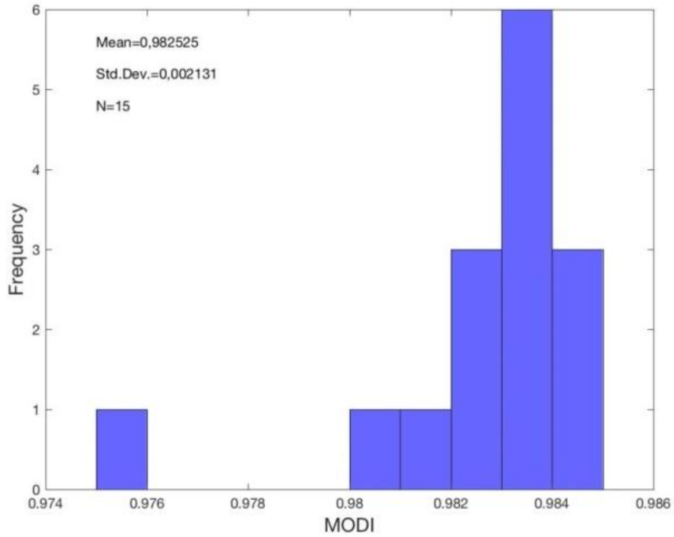



Fig. 2(b) MODI Histogram of GA Optimization Algorithm

Similar analysis was carried out for the time data of SA, GA, and PSO, respectively. The time data were approximately normally distributed for both SA and GA. The skewness and kurtosis for the time data of SA were calculated as 0.385 (SE=0.580) and -0.533 (SE=1.121). Similarly, the skewness and kurtosis for the time data of GA were calculated as 0.340 (SE=0.580), and 0.687 (SE=1.121), respectively. The skewness and kurtosis values are well aligned with the symmetry features in the histograms as illustrated in Fig. 3(a) and Fig. 3(b). The histogram for the time data of PSO is not symmetric with a skewness of 1.159 (SE=0.580), and a kurtosis of 2.285 (SE=1.121) as shown in Fig. 3(c). Both Shapiro-Wilk and Kolmogorov-Smirnov tests show that all of the time data sets (i.e. SA, GA, and PSO) are approximately normally distributed. Similarly, the other descriptive statistical graphs such as Q-Q plots and box plots show that the time data sets for three distinct algorithms are almost normally distributed.

The normality analysis shows that MODI data of SA, time data of SA, GA, and PSO can be accepted as approximately normally distributed data, however, the MODI data of GA and PSO are not normally distributed. We can conclude that since the time data for the three algorithms are normally distributed, a parametric analysis should be applied to these data sets. A non-parametric analysis should be carried out to the MODI data of the three algorithms that can be classified as non-normal distributions. We can now conduct parametric or non-parametric analysis to detect whether there is a significant difference between the data sets.

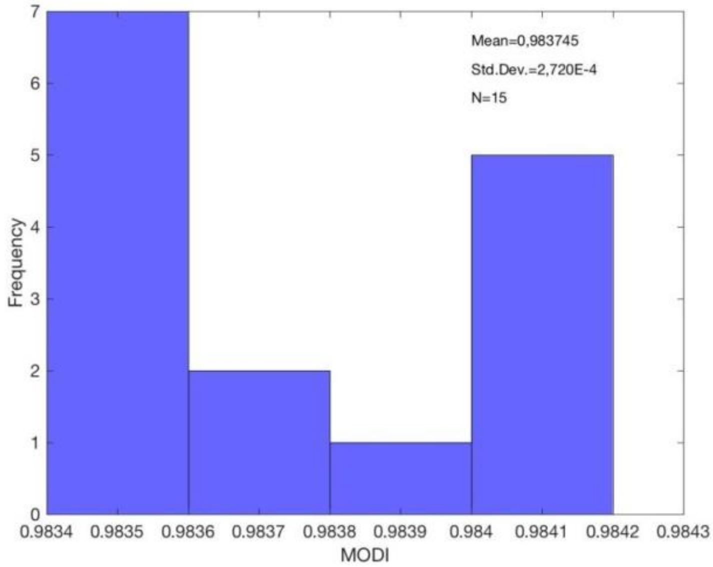



Fig. 2(c) MODI Histogram of PSO Algorithm
Fig. 2 MODI Histograms of SA, GA and PSO

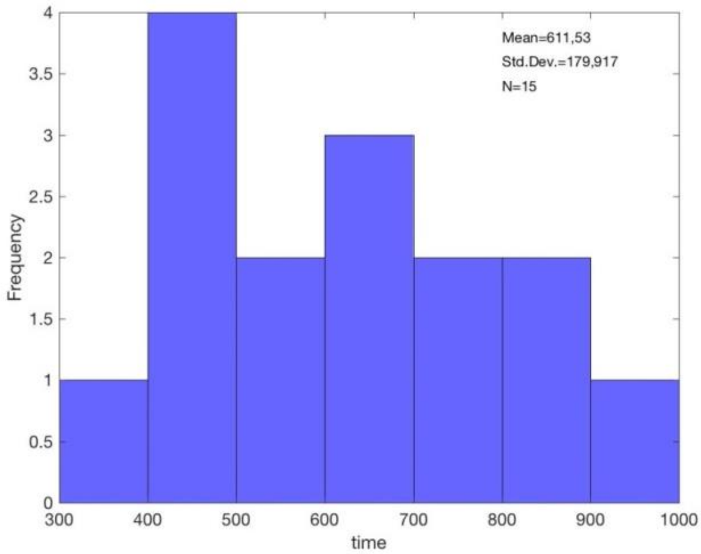



Fig. 3(a) Time Histogram of SA Optimization Algorithm

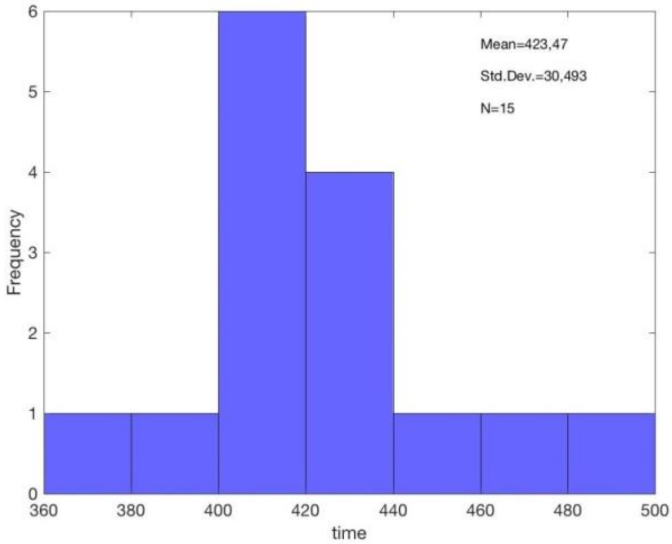



Fig. 3(b) Time Histogram of GA Optimization Algorithm

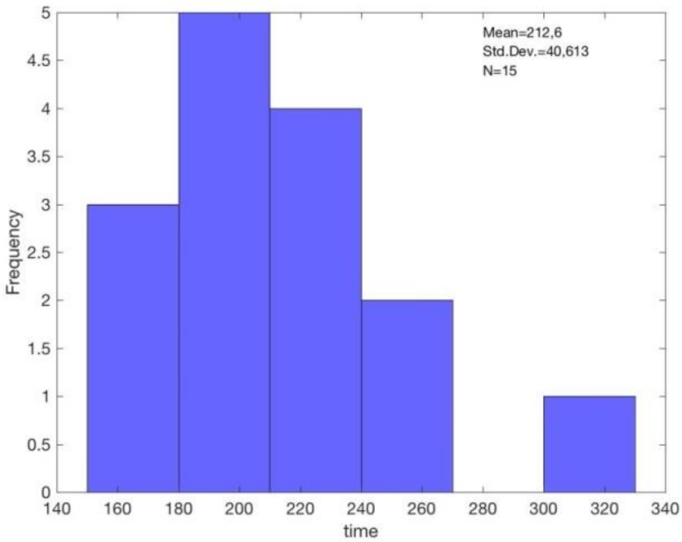


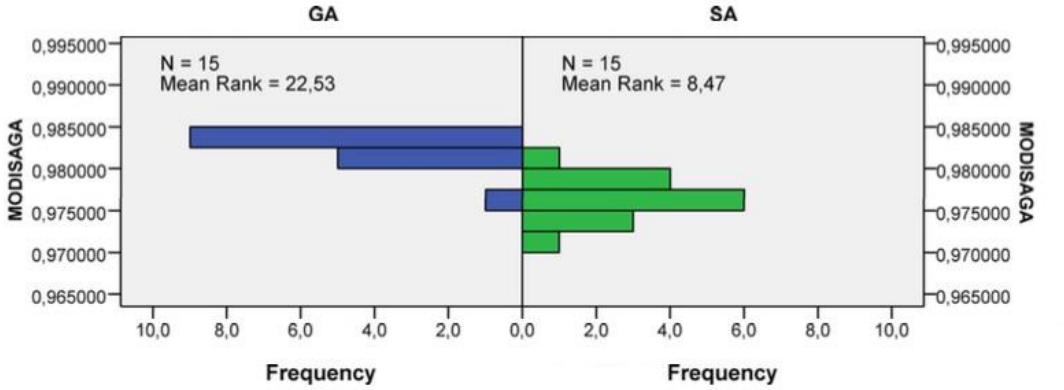

Fig. 3(c) Time Histogram of PSO Algorithm

Fig. 3 Time Histograms of SA, GA and PSO

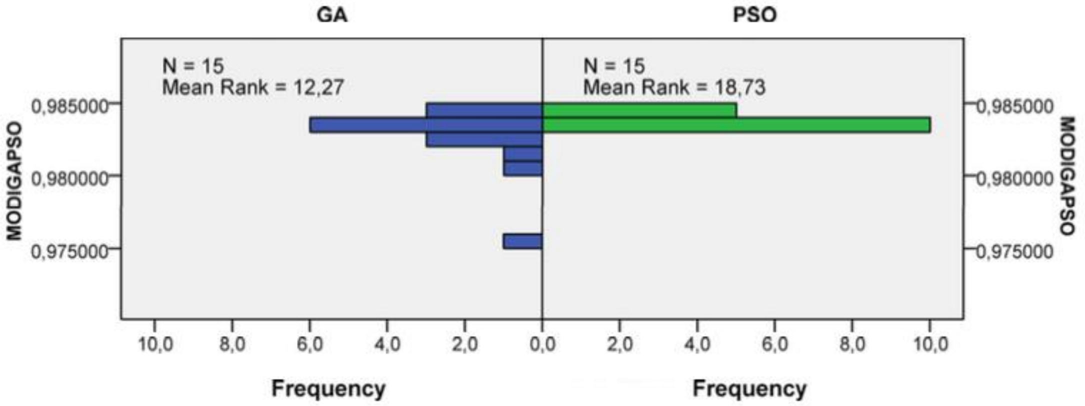
### *5.1.2 Non-parametric Analysis*

A non-parametric analysis was accomplished to determine whether there is a significant difference between the MODI data sets. The typical non-parametric analysis utilized herein is the rank-based Mann-Whitney U Test that is commonly performed for independent variables showing non-normal characteristics. Although the MODI of SA is evaluated as approximately normally distributed according to the normality analysis, the data set is employed in non-parametric analysis for comparison. A comparative analysis was performed for different algorithm pairs such as SA-GA, GA-PSO, and SA-PSO

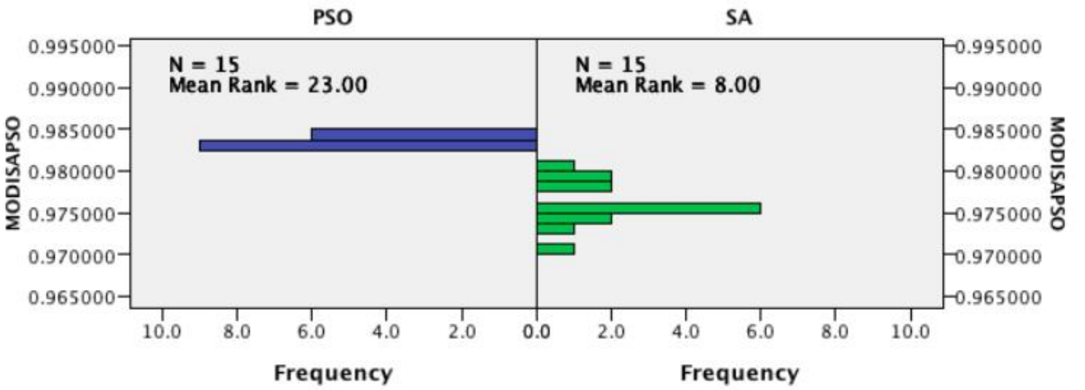
Having performed the Mann-Whitney U Test, we obtained rank-based comparison graphs as shown in Fig. 4(a), (b), (c). The results given in Table 8 show that there is significant difference between the data sets of SA-GA, GA-PSO, and SA-PSO optimization algorithms.




(a) Mann-Whitney U Test: SA-GA Comparison




(b) Mann-Whitney U Test: GA-PSO Comparison

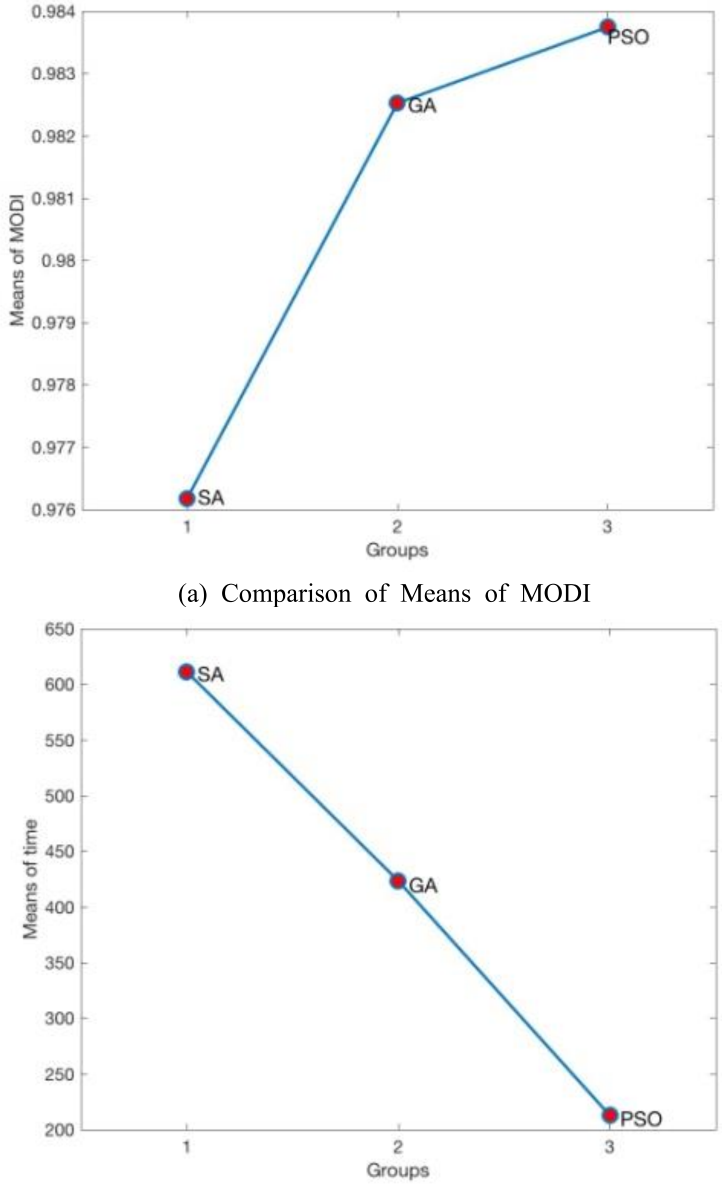



(c) Mann-Whitney U Test: SA-PSO Comparison

Fig. 4 Mann-Whitney U Tests of SA-GA, GA-PSO and SA-PSO

Table 8 Mann-Whitney U Test results for the comparison of MODI data sets

| MODI | SA | GA | PSO |
|---|---|---|---|
| SA | 0 | - | - |
| GA | - | 0 | - |
| PSO | - | - | 0 |




(a) Comparison of Means of MODI



(b) Comparison of Means of Time

Fig. 5 Comparisons of Means of MODI and Time

### *5.1.3 Parametric Analysis for Time data*

Since the time data of SA, GA, and PSO are approximately normally distributed, a parametric analysis was performed to explore the differences between the groups. The parametric analysis shows that the mean of time data is less for PSO algorithm, and also the average MODI is relatively high compared to SA and GA as illustrated in Fig. 5(a) and (b). The parametric analysis revealed that there is a significant difference between the time data of the three groups, and this analysis can be used to determine the optimum algorithm to be selected.

## 5. Conclusions

In this work a benchmark study is carried out among the extensively used heuristic search algorithms i.e., simulated annealing (SA), genetic algorithm (GA) and particle swarm optimization (PSO) in order to observe their performance on a cost function formed with qualitatively and quantitatively dissimilar parts. Since the performance may be the cost function type dependent, the investigation is restricted to a Multi objective optimal design of laminated composite plates with respect to buckling load and non-dimensional fundamental frequencies. In order to avoid fluctuations among the results, each algorithm is run multiple times and the statistical evaluation is presented. The preliminary outputs point out that particle swarm optimization (PSO) proves to be more superior to the remaining two from both computational time and accuracy point of view, on the condition that the previously given settings are applied to the algorithm parameters. In the statistical evaluation scheme, normality analysis and Non-parametric analysis were carried out. Normality tests of all three algorithms were done according to different criteria such as Shapiro-Wilk's and Kolmogorov-Smirnov on the basis of MODI and time values. The normality analysis shows that MODI data of SA, time data of SA, GA, and PSO can be accepted as approximately normally distributed data, however, the MODI data of GA and PSO are not normally distributed.

A comparative analysis was performed for different algorithm pairs such as SA-GA, GA-PSO, and SA-PSO. It was shown that there is significant difference between the data sets of SA-GA, GA-PSO, and SA-PSO optimization algorithms. It is important to note that similar statistical evaluation methodology can be applied to other optimization algorithms on different types of structural problems.

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*CC*

## Nomenclature

| | |
|---|---|
| *Nx* | *load in x direction* |
| $I_0, I_1, I_2$ | *moment of inertias* |
| *Ny.* | *load in y direction* |
| *M* | *mass matrix* |

| | |
|---|---|
| $Ncr$ | *critical load* |
| $\omega$ | *natural frequency* |
| $D_{ij}$ | *bending stiffness* |
| $\{\Theta\}$ | *vibration mode shape* |
| $w$ | *transverse deflection* |
| $\theta$ | *orientation angles* |
| $\lambda_b$ | *minimum critical load* |
| $\lambda$ | *weighted ratios for critical load* |
| $u_0$ | *mid-plane displacements in x directions* |
| $\xi$ | *weighted ratios for natural freq.* |
| $v_0$ | *mid-plane displacements in y directions* |
| $X_i$ | *swarm vector* |
| $w_0$ | *mid-plane displacements in z directions* |
| $V_i$ | *velocity vector* |
| $\phi_x$ | *the rotations of transverse normal about x- axes* |
| $B_i$ | *best position vector* |
| $\phi_y$ | *the rotations of transverse normal about y- axes* |
| $w_p$ | *inertia weight* |
| $A_{ij}$ | *laminate stiffness* |
| $c_1$, $c_2$ | *positive acceleration constants* |
| $B_{ij}$ | *laminate stiffness* |
| $r_1$, $r_2$ | *random numbers* |
| $W$ | *virtual work* |
| $T_j$ | *temperature parameter* |
| $K$ | *linear stiffness matrix* |
| $f_t$ | *current cost* |
| $E$ | *elasticity modulus* |
| $f_h$ | *highest cost* |
| $\rho$ | *density* |
| $\upsilon$ | *Poisson's ratio* |