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# Optimal dimensioning for the corner combined footings

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**Abstract.** This paper shows optimal dimensioning for the corner combined footings to obtain the most economical contact surface on the soil (optimal area), due to an axial load, moment around of the axis "X" and moment around of the axis "Y" applied to each column. The proposed model considers soil real pressure, i.e., the pressure varies linearly. The classical model is developed by trial and error, i.e., a dimension is proposed, and after, using the equation of the biaxial bending is obtained the stress acting on each vertex of the corner combined footing, which must meet the conditions following: 1) Minimum stress should be equal or greater than zero, because the soil is not withstand tensile. 2) Maximum stress must be equal or less than the allowable capacity that can be capable of withstand the soil. Numerical examples are presented to illustrate the validity of the optimization techniques to obtain the minimum area of corner combined footings under an axial load and moments in two directions applied to each column.

**Keywords:** corners combined footings; optimization techniques; contact surface; more economical dimension; optimal area

## 1. Introduction

Footings are structural elements that transmit column or wall loads to the underlying soil below the structure. Footings are designed to transmit these loads to the soil without exceeding its safe bearing capacity, to prevent excessive settlement of the structure to a tolerable limit, to minimize differential settlement, and to prevent sliding and overturning. The choice of suitable type of footing depends on the depth at which the bearing stratum is localized, the soil condition and the type of superstructure. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems (Bowles 2001, Das *et al.* 2006).

The design of superficial solution is done for the following load cases: 1) the footings subjected to concentric axial load, 2) the footings subjected to axial load and moment in one direction (uniaxial bending), 3) the footings subjected to axial load and moment in two directions (biaxial bending) (Bowles 2001, Das *et al.* 2006, Calabera 2000, Tomlinson 2008, McCormac and Brown

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2013, González-Cuevas and Robles-Fernandez-Villegas 2005).

A combined footing is a long footing supporting two or more columns in (typically two) one row. The combined footing may be rectangular, trapezoidal or T-shaped in plan. Rectangular footing is provided when one of the projections of the footing is restricted or the width of the footing is restricted. Trapezoidal footing or T-shaped is provided when one column load is much more than the other. As a result, both projections of the footing beyond the faces of the columns will be restricted (Kurian 2005, Punmia *et al.* 2007, Varghese 2009).

Construction practice may dictate using only one footing for two or more columns due to:

a) Closeness of column (for example around elevator shafts and escalators).

b) To property line constraint, this may limit the size of footings at boundary. The eccentricity of a column placed on an edge of a footing may be compensated by tying the footing to the interior column.

Conventional method for design of combined footings by rigid method assumes that (Bowles 2001, Das *et al*. 2006, McCormac and Brown 2013, González-Cuevas and Robles-Fernandez-Villegas 2005):

1. The footing or mat is infinitely rigid, and therefore, the deflection of the footing or mat does not influence the pressure distribution.

2. The soil pressure is linearly distributed or the pressure distribution will be uniform, if the centroid of the footing coincides with the resultant of the applied loads acting on foundations.

3. The minimum stress should be equal to or greater than zero, because the soil is not capable of withstand tensile stresses.

4. The maximum stress must be equal or less than the allowable capacity that can withstand the soil.

Optimization of building structures is a prime target for designers and has been investigated by many researchers in the past and its papers are: Optimum design of unstiffened built-up girders (Ha 1993); Shape optimization of RC flexural members (Rath *et al.* 1999); Sensitivity analysis and optimum design curves for the minimum cost design of singly and doubly reinforced concrete beams (Ceranic and Fryer 2000); Optimal design of a welded I-section frame using four conceptually different optimization algorithms (Jarmai *et al.* 2003); New approach to optimization of reinforced concrete beams (Leps and Sejnoha 2003); Cost optimization of singly and doubly reinforced concrete beams with EC2-2001 (Barros *et al.* 2005); Cost optimization of reinforced concrete flat slab buildings (Sahab *et al.* 2005); Multi objective optimization for performance-based design of reinforced concrete frames (Zou *et al.* 2007); Design of optimally reinforced RC beam, column, and wall sections (Aschheim *et al.* 2008); Optimum design of reinforced concrete columns subjected to uniaxial flexural compression (Bordignon and Kripka 2012); A hybrid CSS and PSO algorithm for optimal design of structures (Kaveh and Talatahari 2012); Structural optimization and proposition of pre-sizing parameters for beams in reinforced concrete buildings (Fleith de Medeiros and Kripka 2013); Optimum cost design of RC columns using artificial bee colony algorithm (Ozturk and Durmus 2013); Optimization of a sandwich beam design: analytical and numerical solutions (Awad 2013); Cold-formed steel channel columns optimization with simulated annealing method (Kripka and Chamberlain Pravia 2013); Cost optimization of reinforced high strength concrete T-sections in flexure (Tiliouine and Fedghouche 2014); Optimal design of reinforced concrete plane frames using artificial neural networks (Kao and Yeh 2014); Reliability-based design optimization of structural systems using a hybrid genetic algorithm (Abbasnia *et al.* 2014); Numerical experimentation for the optimal design of reinforced rectangular concrete beams for singly reinforced sections (Luévanos-Rojas 2016a).

The papers for optimal design of reinforced concrete foundations are: flexural strength of square spread footing (Jiang 1983); Closure to "Flexural strength of square spread footing" by Da Hua Jiang (Jiang 1984); Flexural limit design of column footing (Hans 1985); Economic design optimization of foundation (Wang and Kulhawy 2008); Reliability-Based Economic design optimization of spread foundation (Wang 2009); Structural cost of optimized reinforced concrete isolated footing (Al-Ansari 2013); Multi-objective optimization of foundation using global-local gravitational search algorithm (Khajehzadeh *et al.* 2014).

Some papers presenting the equations to obtain the dimension of footings are: A mathematical model for dimensioning of footings rectangular (Luévanos-Rojas 2013); A mathematical model for dimensioning of footings square (Luévanos-Rojas 2012a); A mathematical model for the dimensioning of circular footings (Luévanos-Rojas 2012b); A new mathematical model for dimensioning of the boundary trapezoidal combined footings (Luévanos-Rojas 2015); A mathematical model for the dimensioning of combined footings of rectangular shape (Luévanos-Rojas 2016b).

This paper shows optimal dimensioning for the corner combined footings to obtain the most economical contact surface on the soil (optimal area), due to an axial load, moment around of the axis "X" and moment around of the axis "Y" applied to each column. The proposed model considers soil real pressure, i.e., the pressure varies linearly. The classical model is developed by trial and error, i.e., a dimension is proposed, and after, using the equation of the biaxial bending is obtained the stress acting on each vertex of the corner combined footing, which must meet the conditions following: 1) Minimum stress should be equal or greater than zero, because the soil is not withstand tensile. 2) Maximum stress must be equal or less than the allowable capacity that can be capable of withstand the soil. The paper presents numerical examples for two property lines adjacent to illustrate the validity of the optimization techniques to obtain the minimum area of the corner combined footings under an axial load and moments in two directions applied to each column.

## 2. Formulation of the proposed model

The general equation for any type of footings subjected to bidirectional bending (Luévanos-Rojas 2012a, b, 2013, 2015, 2016b, Gere and Goodno 2009)

$$\sigma = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y y}{I_y} \tag{1}$$

Where: $\sigma$ is the stress exerted by the soil on the footing (soil pressure), $A$ is the contact area of the footing, $P$ is the axial load applied at the center of gravity of the footing, $M_x$ is the moment around the axis "$X$", $M_y$ is the moment around the axis "$Y$", $x$ is the distance in the direction "$X$" measured from the axis "$Y$" to the fiber under study, $y$ is the distance in direction "$Y$" measured from the axis "$X$" to the farthest under study, $I_y$ is the moment of inertia around the axis "$Y$" and $I_x$ is the moment of inertia around the axis "$X$".

Fig. 1 shows a corner combined footing under axial load and moment in two directions (biaxial bending) in each column, the pressure below the footing vary linearly (Luévanos-Rojas 2012a, b, 2013, 2015, 2016b).

Fig. 2 presents the pressure diagram below the corner combined footing, and also the stresses in each vertex are shown.

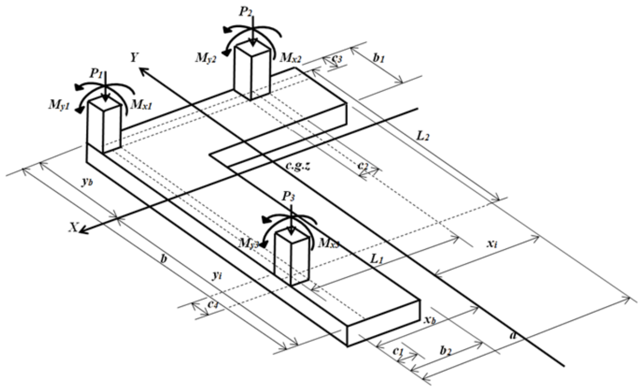

Fig. 1 Corner combined footing

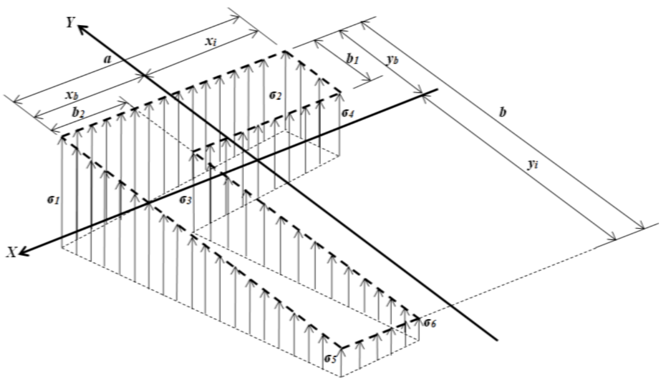

Fig. 2 Diagram of pressure below the footing

From the Eq. (1) the stresses in each vertex of footing are obtained

$$\sigma_1 = \frac{R}{A} + \frac{M_{xT} y_b}{I_x} + \frac{M_{yT} x_b}{I_y} \tag{2}$$

$$\sigma_2 = \frac{R}{A} + \frac{M_{xT} y_b}{I_x} - \frac{M_{yT} x_i}{I_y} \tag{3}$$

$$\sigma_3 = \frac{R}{A} + \frac{M_{xT}(y_b - b_1)}{I_x} + \frac{M_{yT}(x_b - b_2)}{I_y} \tag{4}$$

$$\sigma_4 = \frac{R}{A} + \frac{M_{xT}(y_b - b_1)}{I_x} - \frac{M_{yT} x_i}{I_y} \tag{5}$$

$$\sigma_5 = \frac{R}{A} - \frac{M_{xT} y_i}{I_x} + \frac{M_{yT} x_b}{I_y} \tag{6}$$

$$\sigma_6 = \frac{R}{A} - \frac{M_{xT} y_b}{I_x} + \frac{M_{yT}(x_b - b_2)}{I_y} \tag{7}$$

Where $R$ is resultant force, $M_{xT}$ is resultant moment around the axis "$X$" and $M_{yT}$ is resultant moment around the axis "$Y$" are obtained

$$R = P_1 + P_2 + P_3 \tag{8}$$

$$M_{xT} = M_{x1} + M_{x2} + M_{x3} + P_1\left(y_b - \frac{c_3}{2}\right) + P_2\left(y_b - \frac{c_3}{2}\right) - P_3\left(L_2 + \frac{c_3}{2} - y_b\right) \tag{9}$$

$$M_{yT} = M_{y1} + M_{y2} + M_{y3} + P_1\left(x_b - \frac{c_1}{2}\right) - P_2\left(L_1 + \frac{c_1}{2} - x_b\right) + P_3\left(x_b - \frac{c_1}{2}\right) \tag{10}$$

The geometric properties of section are

$$A = (a - b_2)b_1 + bb_2 \tag{11}$$

$$y_b = \frac{(a - b_2){b_1}^2 + b^2 b_2}{2[(a - b_2)b_1 + bb_2]} \tag{12}$$

$$y_i = \frac{(2b - b_1)(a - b_2)b_1 + b^2 b_2}{2[(a - b_2)b_1 + bb_2]} \tag{13}$$

$$I_x = \frac{a^2 {b_1}^4 + 2ab_1 b_2 (b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2 (b - b_1)^4}{12[(a - b_2)b_1 + bb_2]} \tag{14}$$

$$x_b = \frac{a^2 b_1 + (b - b_1){b_2}^2}{2[(a - b_2)b_1 + bb_2]} \tag{15}$$

$$x_i = \frac{a^2 b_1 + (2a - b_2)(b - b_1)b_2}{2[(a - b_2)b_1 + bb_2]} \tag{16}$$

$$I_y = \frac{b^2 {b_2}^4 + 2bb_1 b_2 (a - b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2 (a - b_2)^4}{12[(a - b_2)b_1 + bb_2]} \tag{17}$$

Geometry conditions are

$$a \geq \frac{c_1}{2} + L_1 + \frac{c_2}{2} \tag{18}$$

$$a = x_b + x_i \tag{19}$$

$$b \geq \frac{c_3}{2} + L_2 + \frac{c_4}{2} \tag{20}$$

$$b = y_b + y_i \tag{21}$$

Substituting the Eqs. (12)-(15) into Eqs. (9)-(10) to obtain the moments in function of "$a$", "$b$", "$b_1$" and "$b_2$", these are

$$M_{xT} = \frac{R\left[(a-b_2){b_1}^2 + b^2 b_2\right]}{2[(a-b_2)b_1 + bb_2]} + M_{x1} + M_{x2} + M_{x3} - \frac{Rc_3}{2} - P_3 L_2 \tag{22}$$

$$M_{yT} = \frac{R\left[a^2 b_1 + (b-b_1){b_2}^2\right]}{2[(a-b_2)b_1 + bb_2]} + M_{y1} + M_{y2} + M_{y3} - \frac{Rc_1}{2} - P_2 L_1 \tag{23}$$

Substituting the Eqs. (11) to (17) into Eqs. (2) to (7) to find the stresses in function of "$a$", "$b$", "$b_1$" and "$b_2$", these are

$$\sigma_1 = \frac{R}{(a-b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a-b_2){b_1}^2 + b^2 b_2\right]}{a^2{b_1}^4 + 2ab_1b_2(b-b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b-b_1)^4} + \frac{6M_{yT}\left[a^2 b_1 + (b-b_1){b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a-b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a-b_2)^4} \tag{24}$$

$$\sigma_2 = \frac{R}{(a-b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a-b_2){b_1}^2 + b^2 b_2\right]}{a^2{b_1}^4 + 2ab_1b_2(b-b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b-b_1)^4} - \frac{6M_{yT}[a^2 b_1 + (2a-b_2)(b-b_1)b_2]}{b^2{b_2}^4 + 2bb_1b_2(a-b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a-b_2)^4} \tag{25}$$

$$\sigma_3 = \frac{R}{(a-b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b-b_1)^2 b_2 - a{b_1}^2\right]}{a^2{b_1}^4 + 2ab_1b_2(b-b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b-b_1)^4} + \frac{6M_{yT}\left[(a-b_2)^2 b_1 - b{b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a-b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a-b_2)^4} \tag{26}$$

$$\sigma_4 = \frac{R}{(a-b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b-b_1)^2 b_2 - a{b_1}^2\right]}{a^2{b_1}^4 + 2ab_1b_2(b-b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b-b_1)^4} - \frac{6M_{yT}[a^2 b_1 + (2a-b_2)(b-b_1)b_2]}{b^2{b_2}^4 + 2bb_1b_2(a-b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a-b_2)^4} \tag{27}$$

$$\sigma_5 = \frac{R}{(a-b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b-b_1)(a-b_2)b_1 + b^2 b_2]}{a^2{b_1}^4 + 2ab_1b_2(b-b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b-b_1)^4} + \frac{6M_{yT}\left[a^2 b_1 + (b-b_1){b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a-b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a-b_2)^4} \tag{28}$$

$$\sigma_6 = \frac{R}{(a-b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b-b_1)(a-b_2)b_1 + b^2 b_2]}{a^2{b_1}^4 + 2ab_1b_2(b-b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b-b_1)^4} + \frac{6M_{yT}\left[(a-b_2)^2 b_1 - b{b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a-b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a-b_2)^4} \tag{29}$$

The stresses generated by soil on contact surface of the combined footing must meet the following conditions: 1) The minimum stress should be equal or greater than zero; 2) The maximum stress must be equal or less than the soil allowable load capacity “$\sigma_{adm}$” (Bowles 2001, Das *et al.* 2006, McCormac and Brown 2013, González-Cuevas and Robles-Fernandez-Villegas 2005).

Now the objective function to minimize the total area of the contact surface “$A_t$” is

$$A_t = (a - b_2)b_1 + bb_2 \tag{30}$$

Constraint functions for dimensioning of the corner combined footings are

$$R = P_1 + P_2 + P_3 \tag{31}$$

$$M_{xT} = \frac{R\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{2[(a - b_2)b_1 + bb_2]} + M_{x1} + M_{x2} + M_{x3} - \frac{Rc_3}{2} - P_3 L_2 \tag{32}$$

$$M_{yT} = \frac{R\left[a^2 b_1 + (b - b_1){b_2}^2\right]}{2[(a - b_2)b_1 + bb_2]} + M_{y1} + M_{y2} + M_{y3} - \frac{Rc_1}{2} - P_2 L_1 \tag{33}$$

$$\begin{aligned}\sigma_1 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{a^2{b_1}^4 + 2ab_1b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} \\ & + \frac{6M_{yT}\left[a^2 b_1 + (b - b_1){b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a - b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a - b_2)^4}\end{aligned} \tag{34}$$

$$\begin{aligned}\sigma_2 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{a^2{b_1}^4 + 2ab_1b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} \\ & - \frac{6M_{yT}[a^2 b_1 + (2a - b_2)(b - b_1)b_2]}{b^2{b_2}^4 + 2bb_1b_2(a - b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a - b_2)^4}\end{aligned} \tag{35}$$

$$\begin{aligned}\sigma_3 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2{b_1}^4 + 2ab_1b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} \\ & + \frac{6M_{yT}\left[(a - b_2)^2 b_1 - b{b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a - b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a - b_2)^4}\end{aligned} \tag{36}$$

$$\begin{aligned}\sigma_4 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2{b_1}^4 + 2ab_1b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} \\ & - \frac{6M_{yT}[a^2 b_1 + (2a - b_2)(b - b_1)b_2]}{b^2{b_2}^4 + 2bb_1b_2(a - b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a - b_2)^4}\end{aligned} \tag{37}$$

$$\begin{aligned}\sigma_5 = {} & \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2{b_1}^4 + 2ab_1b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} \\ & + \frac{6M_{yT}\left[a^2 b_1 + (b - b_1){b_2}^2\right]}{b^2{b_2}^4 + 2bb_1b_2(a - b_2)\left(2a^2 - ab_2 + {b_2}^2\right) + {b_1}^2(a - b_2)^4}\end{aligned} \tag{38}$$

$$\sigma_6 = \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2{b_1}^4 + 2ab_1b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} \tag{39}$$

$$0 \leq \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \leq \sigma_{adm} \tag{40}$$

$$\frac{c_1}{2} + L_1 + \frac{c_2}{2} \leq a \tag{41}$$

$$\frac{c_3}{2} + L_2 + \frac{c_4}{2} \leq b \tag{42}$$

## 3. Numerical problems

Tables present five cases for dimensioning of corner combined footings with two boundaries adjacent; each case varies the soil allowable load capacity of "$\sigma_{adm}$=250, 225, 200, 175, 150 kN/m$^2$", and each case presented four types that are same in all cases. Table 1 presented the four types of corner combined footings; the types vary in the mechanical elements acting on the footing.

Table 1 Mechanical elements acting on the footing

| Type | Loads of the column 1 | | | Loads of the column 2 | | | Loads of the column 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $P_1$ kN | $M_{x1}$ kN-m | $M_{y1}$ kN-m | $P_2$ kN | $M_{x2}$ kN-m | $M_{y2}$ kN-m | $P_3$ kN | $M_{x3}$ kN-m | $M_{y3}$ kN-m |
| 1 | 500 | 150 | 200 | 1000 | 300 | 200 | 900 | 200 | 250 |
| 2 | 500 | -150 | 200 | 1000 | -300 | 200 | 900 | -200 | 250 |
| 3 | 500 | 150 | -200 | 1000 | 300 | -200 | 900 | 200 | -250 |
| 4 | 500 | -150 | -200 | 1000 | -300 | -200 | 900 | -200 | -250 |

Tables 2, 3 and 4 presented the results and it make the following considerations: 1) Dimensions of the three columns are of 40×40 cm in all cases; 2) Soil allowable load capacity varies for each case; 3) Distance between columns is $L_1$=5.00 m, $L_2$=6.00 m.

Tables 2, 3 and 4 show the results using the optimization techniques; the objective function (minimum contact surface) by Eq. (30) is obtained, and constraint functions by Eqs. (31) to (42) are found, and the minimum areas and dimensions for corner combined footings are obtained using the MAPLE-15 software, and it is assumed that dimensions are nonnegative.

This problem assumes that the constant parameters are: $P_1$, $M_{x1}$, $M_{y1}$, $P_2$, $M_{x2}$, $M_{y2}$, $P_3$, $M_{x3}$, $M_{y3}$, $c_1$, $c_2$, $c_3$, $c_4$, $L_1$, $L_2$, $\sigma_{adm}$, $R$, and the decision variables are: $M_{xT}$, $M_{yT}$, $a$, $b$, $b_1$, $b_2$, $\sigma_1$, $\sigma_2$, $\sigma_3$, $\sigma_4$, $\sigma_5$, $\sigma_6$. Table 2 makes the following considerations: $R$=2400 kN, $M_{xT}$ and $M_{yT}$ are not constrained, 5.40 m≤$a$, 6.40 m≤$b$, 0≤$b_1$, 0≤$b_2$, $A_t$ is objective function, $0\leq\sigma_1\leq\sigma_{adm}$, $0\leq\sigma_2\leq\sigma_{adm}$, $0\leq\sigma_3\leq\sigma_{adm}$, $0\leq\sigma_4\leq\sigma_{adm}$, $0\leq\sigma_5\leq\sigma_{adm}$, $0\leq\sigma_6\leq\sigma_{adm}$. Table 3 makes the following considerations: $R$=2400 kN, $M_{xT}$ and $M_{yT}$ are not constrained, 5.40 m≤$a$, 6.40 m≤$b$, 1.00 m≤$b_1$, 1.00 m≤$b_2$, $A_t$ is objective function, $0\leq\sigma_1\leq\sigma_{adm}$, $0\leq\sigma_2\leq\sigma_{adm}$, $0\leq\sigma_3\leq\sigma_{adm}$, $0\leq\sigma_4\leq\sigma_{adm}$, $0\leq\sigma_5\leq\sigma_{adm}$, $0\leq\sigma_6\leq\sigma_{adm}$. Table 4 makes the following considerations: $R$=2400 kN, $M_{xT}$ and $M_{yT}$ are not constrained, 5.40 m=$a$, 6.40 m=$b$, $0 \leq b_1$, $0 \leq b_2$, $A_t$ is objective function, $0 \leq \sigma_1 \leq \sigma_{adm}$, $0 \leq \sigma_2 \leq \sigma_{adm}$, $0 \leq \sigma_3 \leq \sigma_{adm}$, $0 \leq \sigma_4 \leq \sigma_{adm}$, $0 \leq \sigma_5 \leq \sigma_{adm}$, $0 \leq \sigma_6 \leq \sigma_{adm}$.

Tables 2, 3 and 4 are shown in the appendix.

## 4. Results

Table 2 shows the following results (dimensions $a$, $b$, $b_1$ and $b_2$ are assumed nonnegative): 1) The minimum area is the same for the four types in each case; 2) If the soil allowable load capacity decreases, the minimum area increases; 3) The resultant mechanical elements are $R$=2400 kN, $M_{xT}$=0, $M_{yT}$=0 for all cases; 4) The stresses generated by loads in each vertex are the same to $\sigma_{adm}$ in each case. It means that the resultant force is located in the center of gravity of the contact area of the footing with soil, i.e., the total eccentricity of the resultant force "R" in the two directions is zero.

Table 3 presents the following results (dimensions $a$ and $b$ are assumed nonnegative, and $b_1$ and $b_2$ are greater than or equal to one): 1) The minimum area is deferent for the four types in each case; 2) If the soil allowable load capacity decreases, the minimum area increases; 3) The resultant mechanical elements are $R$=2400 kN, $M_{xT}$ and $M_{yT}$ are equal or less than zero for all cases; 4) The stresses generated by loads in each vertex are equal or less than $\sigma_{adm}$ and greater than zero in each case (meets the conditions indicated by the stresses). It means that the resultant force is located in the center of gravity of the contact area of the footing with soil for the case 4 of the type 1 and case 5 of the types 1, 2, 3, because the total moments are $M_{xT}$=0, $M_{yT}$=0 (the stresses generated by loads in each vertex are the same to $\sigma_{adm}$), and for the other cases the resultant force is not located in the center of gravity of the contact area of the footing with soil, and therefore the resultant force "R" has an eccentricity.

Table 4 shows the following results (dimensions $b_1$ and $b_2$ are assumed nonnegative. and $a$=5.40 m and $b$=6.40 m): 1) The minimum area is deferent for the four types in each case; 2) If the soil allowable load capacity decreases, the minimum area increases; 3) The resultant mechanical elements are $R$=2400 kN, $M_{xT}$ and $M_{yT}$ are equal or less than zero for all cases; 4) The stresses generated by loads in each vertex are equal or less than $\sigma_{adm}$ and greater than zero in each case (meets the conditions indicated by the stresses). It means that the resultant force is not located in the center of gravity of the contact area of the footing with soil, and therefore the resultant force "R" has an eccentricity for all cases.

Table 2 shows the optimal area for the dimensions $a \geq 0$, $b \geq 0$, $b_1 \geq 0$ and $b_2 \geq 0$. Table 3 presents the optimal area for the dimensions $a \geq 0$, $b \geq 0$, $b_1 \geq 1$ and $b_2 \geq 1$. Table 4 shows the optimal area for the dimensions $a$=5.40, $b$=6.40, $b_1 \geq 0$ and $b_2 \geq 0$. If the three tables are compared for the same cases and types in function of the optimal area: Table 2 is smaller than Table 3, but Table 3 is smaller than Table 4.

## 5. Conclusions

The foundation is an essential part of a structure that transmits column or wall loads to the underlying soil below the structure.

The mathematical approach suggested in this paper produces results that have a tangible accuracy for all problems, main part of this research to find the more economical dimensions of corner combined footings using the optimization techniques.

The main conclusions are:

1. The most economical dimension is presented if there are fewer restricted dimensions with respect to positive values.

2. The methodology shown in this paper is more accurate and converges more quickly.

3. The classical model will not be practical compared to this methodology, because the classical model is developed proposing the dimensions and then verified to comply with the stresses limits mentioned above.

The model presented in this paper is recommended for the localized columns very close together or if the loads applied are too large in such a way that the isolated footings are overlap between them.

The proposed model presented in this paper for dimensioning of corner combined footings subjected to an axial load and moment in two directions in each column, also it can be applied to others cases: 1) Footings subjected to a concentric axial load in each column, 2) Footings subjected to a axial load and one moment in each column.

Then the proposed model is recommended for dimensioning of corner combined footings with two continuous property line (see Table 2) and also for three and four property lines (see Table 4) subjected to an axial load, moment around of the axis "*X*" and moment around of the axis "*Y*" applied in each column.

The model presented in this paper applies only for dimensioning of corner combined footings assumed than the structural member is rigid and the supporting soil layers elastic, which meet expression of the biaxial bending, i.e., the variation of pressure is linear.

The suggestions for future research are:

1. Design for the corner combined footings assuming these are rigid and the supporting soil layers elastic.

2. Dimensioning and design for the corner combined footings supported on another type of soil by example in totally cohesive soils (clay soils) and totally granular soils (sandy soils), the pressure diagram is not linear and should be treated differently.

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## Appendix

Table 2 Results obtained by software for $b_1 \geq 0$ and $b_2 \geq 0$

| Type | Resultant mechanical elements | | | Optimal area | Dimension of the footing | | | | Stresses generated by loads in each vertex | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R$ kN | $M_{xT}$ kN-m | $M_{yT}$ kN-m | $A_t$ m$^2$ | $a$ m | $b$ m | $b_1$ m | $b_2$ m | $\sigma_1$ kN/m$^2$ | $\sigma_2$ kN/m$^2$ | $\sigma_3$ kN/m$^2$ | $\sigma_4$ kN/m$^2$ | $\sigma_5$ kN/m$^2$ | $\sigma_6$ kN/m$^2$ |
| Case 1 ($\sigma_{adm}$=250.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 9.60 | 6.12 | 9.13 | 0.99 | 0.43 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 |
| 2 | 2400 | 0.00 | 0.00 | 9.60 | 8.11 | 8.99 | 0.54 | 0.62 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 |
| 3 | 2400 | 0.00 | 0.00 | 9.60 | 10.44 | 7.15 | 0.41 | 0.79 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 |
| 4 | 2400 | 0.00 | 0.00 | 9.60 | 9.19 | 10.46 | 0.56 | 0.45 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 | 250.00 |
| Case 2 ($\sigma_{adm}$=225.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 10.67 | 7.34 | 7.15 | 0.71 | 0.85 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 |
| 2 | 2400 | 0.00 | 0.00 | 10.67 | 7.98 | 8.90 | 0.61 | 0.70 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 |
| 3 | 2400 | 0.00 | 0.00 | 10.67 | 9.67 | 7.44 | 0.54 | 0.79 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 |
| 4 | 2400 | 0.00 | 0.00 | 10.67 | 10.25 | 9.26 | 0.48 | 0.65 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 | 225.00 |
| Case 3 ($\sigma_{adm}$=200.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 12.00 | 6.46 | 7.69 | 1.06 | 0.78 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| 2 | 2400 | 0.00 | 0.00 | 12.00 | 7.82 | 8.76 | 0.70 | 0.81 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| 3 | 2400 | 0.00 | 0.00 | 12.00 | 8.21 | 8.53 | 0.86 | 0.64 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| 4 | 2400 | 0.00 | 0.00 | 12.00 | 8.73 | 10.56 | 0.77 | 0.54 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| Case 4 ($\sigma_{adm}$=175.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 13.71 | 6.28 | 7.32 | 1.26 | 0.96 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 |
| 2 | 2400 | 0.00 | 0.00 | 13.71 | 6.88 | 7.37 | 1.06 | 0.77 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 |
| 3 | 2400 | 0.00 | 0.00 | 13.71 | 7.91 | 8.47 | 1.06 | 0.72 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 |
| 4 | 2400 | 0.00 | 0.00 | 13.71 | 10.47 | 8.64 | 0.57 | 0.96 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 |
| Case 5 ($\sigma_{adm}$=150.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 16.00 | 5.72 | 7.04 | 1.77 | 1.12 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |
| 2 | 2400 | 0.00 | 0.00 | 16.00 | 7.37 | 8.20 | 1.00 | 1.20 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |
| 3 | 2400 | 0.00 | 0.00 | 16.00 | 7.40 | 8.51 | 1.41 | 0.78 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |
| 4 | 2400 | 0.00 | 0.00 | 16.00 | 8.70 | 9.73 | 1.00 | 0.84 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |

Table 3 Results obtained by software for $b_1 \geq 1$ and $b_2 \geq 1$

| Type | Resultant mechanical elements | | | Optimal area | Dimension of the footing | | | | Stresses generated by loads in each vertex | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R$ kN | $M_{xT}$ kN-m | $M_{yT}$ kN-m | $A_t$ m$^2$ | $a$ m | $b$ m | $b_1$ m | $b_2$ m | $\sigma_1$ kN/m$^2$ | $\sigma_2$ kN/m$^2$ | $\sigma_3$ kN/m$^2$ | $\sigma_4$ kN/m$^2$ | $\sigma_5$ kN/m$^2$ | $\sigma_6$ kN/m$^2$ |
| Case 1 ($\sigma_{adm}$=250.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -404.93 | -436.54 | 11.44 | 6.04 | 6.40 | 1.00 | 1.00 | 169.13 | 240.54 | 190.41 | 250.00 | 229.65 | 241.47 |
| 2 | 2400 | -862.14 | -665.20 | 12.34 | 6.04 | 7.30 | 1.00 | 1.00 | 131.87 | 236.18 | 162.95 | 250.00 | 232.74 | 250.00 |
| 3 | 2400 | -693.66 | -837.18 | 12.43 | 7.03 | 6.40 | 1.00 | 1.00 | 131.64 | 234.58 | 161.70 | 250.00 | 230.33 | 250.00 |
| 4 | 2400 | -1176.81 | -1114.65 | 13.35 | 7.03 | 7.32 | 1.00 | 1.00 | 101.13 | 232.20 | 137.57 | 250.00 | 231.37 | 250.00 |
| Case 2 ($\sigma_{adm}$=225.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -300.82 | -281.14 | 11.87 | 6.28 | 6.59 | 1.00 | 1.00 | 176.30 | 218.64 | 189.40 | 225.00 | 218.26 | 225.00 |
| 2 | 2400 | -667.06 | -515.91 | 12.91 | 6.31 | 7.60 | 1.00 | 1.00 | 142.00 | 215.61 | 163.05 | 225.00 | 213.33 | 225.00 |
| 3 | 2400 | -565.96 | -647.79 | 12.95 | 7.32 | 6.63 | 1.00 | 1.00 | 140.74 | 213.81 | 161.92 | 225.00 | 215.01 | 225.00 |
| 4 | 2400 | -982.82 | -931.38 | 13.98 | 7.34 | 7.63 | 1.00 | 1.00 | 112.30 | 212.01 | 138.87 | 225.00 | 211.42 | 225.00 |
| Case 3 ($\sigma_{adm}$=200.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -111.86 | -104.62 | 12.48 | 6.58 | 6.90 | 1.00 | 1.00 | 183.73 | 197.96 | 187.94 | 200.00 | 197.84 | 200.00 |
| 2 | 2400 | -439.64 | -340.97 | 13.57 | 6.62 | 7.95 | 1.00 | 1.00 | 150.86 | 194.65 | 162.82 | 200.00 | 193.38 | 200.00 |
| 3 | 2400 | -377.74 | -431.73 | 13.62 | 7.66 | 6.96 | 1.00 | 1.00 | 149.59 | 193.58 | 161.75 | 200.00 | 194.26 | 200.00 |
| 4 | 2400 | -756.01 | -716.86 | 14.71 | 7.70 | 8.00 | 1.00 | 1.00 | 122.28 | 191.41 | 139.85 | 200.00 | 191.03 | 200.00 |
| Case 4 ($\sigma_{adm}$=175.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 13.71 | 6.37 | 7.20 | 1.21 | 1.00 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 | 175.00 |
| 2 | 2400 | -169.55 | -131.93 | 14.37 | 6.99 | 8.38 | 1.00 | 1.00 | 158.20 | 173.25 | 162.10 | 175.00 | 172.85 | 175.00 |
| 3 | 2400 | -153.17 | -174.75 | 14.42 | 8.08 | 7.34 | 1.00 | 1.00 | 156.93 | 172.81 | 161.09 | 175.00 | 173.03 | 175.00 |
| 4 | 2400 | -485.70 | -460.87 | 15.58 | 8.13 | 8.45 | 1.00 | 1.00 | 130.86 | 170.35 | 140.36 | 175.00 | 170.14 | 175.00 |
| Case 5 ($\sigma_{adm}$=150.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | 0.00 | 0.00 | 16.00 | 5.72 | 7.04 | 1.77 | 1.12 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |
| 2 | 2400 | 0.00 | 0.00 | 16.00 | 7.06 | 8.50 | 1.11 | 1.10 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |
| 3 | 2400 | 0.00 | 0.00 | 16.00 | 8.31 | 7.39 | 1.07 | 1.13 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 | 150.00 |
| 4 | 2400 | -155.49 | -147.67 | 16.65 | 8.66 | 8.99 | 1.00 | 1.00 | 137.73 | 148.78 | 140.23 | 150.00 | 148.72 | 150.00 |

Table 4 Results obtained by software for $a$=5.40 and $b$=6.40

| Type | Resultant mechanical elements | | | Optimal area | Dimension of the footing | | | | Stresses generated by loads in each vertex | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R$ kN | $M_{xT}$ kN-m | $M_{yT}$ kN-m | $A_t$ m$^2$ | $a$ m | $b$ m | $b_1$ m | $b_2$ m | $\sigma_1$ kN/m$^2$ | $\sigma_2$ kN/m$^2$ | $\sigma_3$ kN/m$^2$ | $\sigma_4$ kN/m$^2$ | $\sigma_5$ kN/m$^2$ | $\sigma_6$ kN/m$^2$ |
| Case 1 ($\sigma_{adm}$=250.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -562.10 | -502.51 | 11.89 | 5.40 | 6.40 | 1.32 | 0.94 | 146.52 | 231.78 | 179.59 | 250.00 | 235.15 | 250.00 |
| 2 | 2400 | -1073.17 | -830.21 | 13.91 | 5.40 | 6.40 | 1.25 | 1.39 | 81.98 | 224.41 | 144.27 | 250.00 | 213.30 | 250.00 |
| 3 | 2400 | -927.87 | -985.36 | 14.50 | 5.40 | 6.40 | 2.04 | 0.80 | 68.49 | 198.43 | 139.42 | 250.00 | 230.64 | 250.00 |
| 4 | 2400 | -1491.83 | -1344.31 | 16.46 | 5.40 | 6.40 | 2.00 | 1.28 | 21.35 | 191.08 | 120.64 | 250.00 | 209.63 | 250.00 |
| Case 2 ($\sigma_{adm}$=225.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -471.98 | -421.88 | 12.85 | 5.40 | 6.40 | 1.44 | 1.02 | 142.35 | 209.24 | 170.77 | 225.00 | 212.34 | 225.00 |
| 2 | 2400 | -952.61 | -738.92 | 15.05 | 5.40 | 6.40 | 1.35 | 1.54 | 82.17 | 202.04 | 139.21 | 225.00 | 190.92 | 225.00 |
| 3 | 2400 | -817.15 | -861.13 | 15.68 | 5.40 | 6.40 | 2.26 | 0.84 | 70.30 | 176.14 | 135.59 | 225.00 | 208.57 | 225.00 |
| 4 | 2400 | -1358.36 | -1213.51 | 17.84 | 5.40 | 6.40 | 2.23 | 1.39 | 25.65 | 168.33 | 119.05 | 225.00 | 188.27 | 225.00 |
| Case 3 ($\sigma_{adm}$=200.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -366.72 | -327.49 | 13.96 | 5.40 | 6.40 | 1.59 | 1.12 | 138.98 | 187.32 | 161.66 | 200.00 | 190.00 | 200.00 |
| 2 | 2400 | -810.79 | -632.51 | 16.36 | 5.40 | 6.40 | 1.47 | 1.71 | 83.32 | 180.27 | 133.73 | 200.00 | 169.32 | 200.00 |
| 3 | 2400 | -689.05 | -713.66 | 17.01 | 5.40 | 6.40 | 2.53 | 0.86 | 73.43 | 155.09 | 131.40 | 200.00 | 187.02 | 200.00 |
| 4 | 2400 | -1203.77 | -1057.16 | 19.41 | 5.40 | 6.40 | 2.51 | 1.50 | 31.13 | 146.27 | 116.89 | 200.00 | 167.97 | 200.00 |
| Case 4 ($\sigma_{adm}$=175.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -242.80 | -216.32 | 15.24 | 5.40 | 6.40 | 1.77 | 1.23 | 136.65 | 166.25 | 152.12 | 175.00 | 168.27 | 175.00 |
| 2 | 2400 | -643.34 | -507.74 | 17.85 | 5.40 | 6.40 | 1.60 | 1.92 | 85.71 | 159.25 | 127.63 | 175.00 | 148.83 | 175.00 |
| 3 | 2400 | -540.85 | -540.38 | 18.48 | 5.40 | 6.40 | 2.86 | 0.85 | 78.39 | 135.83 | 126.63 | 175.00 | 165.93 | 175.00 |
| 4 | 2400 | -1024.52 | -868.40 | 21.19 | 5.40 | 6.40 | 2.87 | 1.61 | 38.14 | 125.21 | 113.89 | 175.00 | 149.04 | 175.00 |
| Case 5 ($\sigma_{adm}$=150.00 kN/m$^2$) | | | | | | | | | | | | | | |
| 1 | 2400 | -95.82 | -84.93 | 16.73 | 5.40 | 6.40 | 1.99 | 1.36 | 135.64 | 146.37 | 141.96 | 150.00 | 147.30 | 150.00 |
| 2 | 2400 | -446.25 | -360.53 | 19.56 | 5.40 | 6.40 | 1.73 | 2.18 | 89.70 | 139.07 | 120.60 | 150.00 | 130.02 | 150.00 |
| 3 | 2400 | -370.32 | -344.97 | 20.04 | 5.40 | 6.40 | 3.24 | 0.80 | 85.73 | 120.00 | 120.81 | 150.00 | 144.93 | 150.00 |
| 4 | 2400 | -816.14 | -639.89 | 23.19 | 5.40 | 6.40 | 3.34 | 1.68 | 47.20 | 105.87 | 109.61 | 150.00 | 131.72 | 150.00 |