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Simulating large scale structural members by using Buckingham theorem: Case study

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Abstract. Scaling and similitude large scale structural member to small scale model is considered the most important matter for the experimental tests because of the difficulty in controlling, lack of capacities and expenses, furthermore that most of MSc and PhD students suffering from choosing the suitable specimen before starting their experimental study. The current study adopts to take large scale slab with opening as a case study of structural member where the slab is squared with central squared opening, the boundary condition is fixed from all sides, the load represents by four concentrated force in four corners of opening, as well as, the study adopts Buckingham theorem which has been used for scaling, all the parameters of the problem have been formed in dimensionless groups, the main groups have been connected by a relations, those relations are represented by force, maximum stress and maximum displacement. Finite element method by ANSYS R18.1 has been used for analyzing and forming relations for the large scale member. Prediction analysis has been computed for three small scale models by depending on the formed relations of the large scale member. It is found that Buckingham theorem is considered suitable way for creating relations among the parameters for any structural problem then making similitude and scaling the large scale members to small scale members. Finally, verification between the prediction and theoretical results has been done, it is observed that the maximum deviation between them is not more than 2.4%.

Keywords: Buckingham theorem; dimensional analysis; large scale structure; scaling

1. Introduction

The scaling is called isotropic or uniform scaling, if all dimensions of structure have been decreased or increased linearly by multiplying them by a suitable factor, this is can be done in drawing or maps without any limitations, but in engineering structures, scaling should be studied well to create same effectiveness between prototype system and model system (Kline 2011). Modeling the real or actual structure (prototype) is very important in many cases where that is reducing the cost, saving time and easy to control. In many cases, an experimental work has been done for a scale model instead of the prototype, the scale model has to has similar behavior of the actual prototype, so it is important to find useful scale factor to get similar effect, take into consideration that the accuracy between a scale model and an actual structure depends on the number of parameters and variables between them. Too many varies articles have been submitted for comparing the experimental study with theoretical study but they all depend on finite element method for comparing the results between the model and prototype, the aims of all the studies is

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getting an actual overview about the behavior of actual structure, for examples, (Kossakowski 2017) has used non-destructive testing method, in this approach, combination of field measurements with a numerical static analysis of the structure has been submitted.

Small-and large-scale analysis for behavior of rock-soil has been submitted by (Moradi *et al*. 2019), in this approach only finite element represented by ABAQUS software has used for confirmation between the model and prototype. Some of authors depended on previous models, (Kossakowski and Uzarska 2019) has used Barcelona model which is an extending of Drucker and Prager model of concrete elastic-rigid-plastic materials.

Numerical simulation of bridge piers has submitted by (Chiou *et al*. 2019), the study totally depends on finite element method for simulating the large scale member. Other trials used the optimization methods by forming an objective function and constraints, then some processes will be in charge of producing new model, other trials used similarity equations and dimensionless analysis for simulating the prototype as in (Altunışık *et al*. 2018) who submitted a study of the similarity and scaling laws between the supposed model and the prototype, factors for modeling the prototype were used and they were constant for the cases. as well as, it is observed that nonlocal elasticity theory can be used for very small structure, the effect of size in the mechanical properties is taken into consideration, this theory can be used not only on local strain but also nonlocal stains (Li 2023)

Other trials used the dimensionless analysis by depending on Buckingham's pi theorem, the non-dimensional parameters can be described by using Buckingham's pi theorem where it is used for linking the physical variables with the fundamental dimensions. This method is used for different problems of various sciences, the present study adopts this approach for finding the suitable factor for scaling the actual structure (Russo *et al*. 2017) and (Tanimoto 2017)

2. Finite element modeling

The methodology of present study will be according to the following points:

• Specify the parameters of problem then formed in dimensionless groups.

• Produce a relation among the main groups, the groups can be equated by a factors because they are represented a pure scalar quantity (Laghaar 1971)

• Finding the factors of previous point by substituting the information of large scale slab with opening, the study requires analyzing the slab by a Finite Element program (Shehadeh *et al*. 2015)

• Apply the previous factors in small scale slab with opening for getting the prediction results then make a comparison with analyzed results

Schematic figure (Fig. 1) clarify the previous points:

3. Buckingham theorem

This section will be divided in two parts, the first will clarify the application of the theorem while the second will explain the implementation of Buckingham theorem of the present case study.

3.1 Applications Buckingham theorem

Buckingham theorem has very wide use in many fields, like civil engineering, mechanical

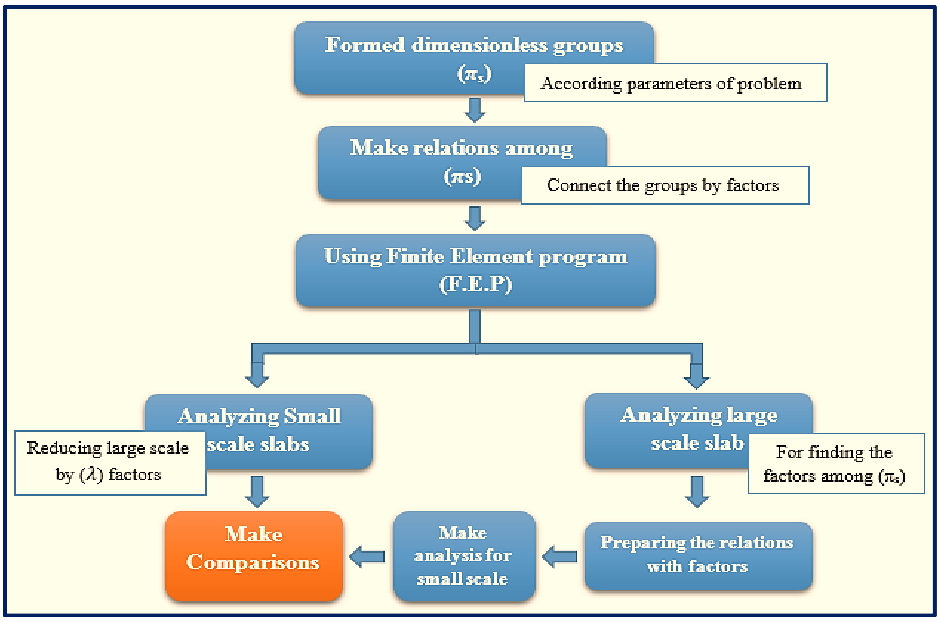

Fig. 1 Methodology of work

engineering, environment engineering, and even botany and social science, because it is a general method and produces a perfect result. The most useful applications of this theorem are in fields where large-scale modeling is difficult and expensive, like most structural members in civil engineering and some machines in mechanical engineering. The following points clarify some of these uses:

a. Thermal engineering, many articles have been submitted in this field, like one on estimating heat release due to a phase change of high-pressure condensing steam (Salmani and Mahpeykar 2019). On the other hand, an investigation about the roughness effects has been undertaken in another article (Salmani *et al*. 2022), while the effect of estimated wetness terms has been studied in another article (Salmani *et al*. 2019).

b. Geotechnical engineering, a study has been done for a pipe line crossing a soil; the pipe line has been represented as a beam under an elastic foundation according to the Winkler formula; the modeling of the pipe using Buckingham theorem was used by (Bhattacharya *et al*. 2021).

c. Structural engineering (Phatak and Dhonde 2003) submitted a study about finding the permissible torsional stress of a reinforced concrete beam. The Buckingham theorem has been used to predict the general equation of torsional stress.

3.2 Implemintation of Buckingham theorem

According to the Buckingham theorem, the parameters of any problem have to be formed in dimensionless groups, the present study will include to make dimensionless groups for a case study of slab with opening, the slab is squared with central squared opening, the boundary condition is fixed from all sides, the load represents by four concentrated force in four corners of opening as shown in Fig. 2.

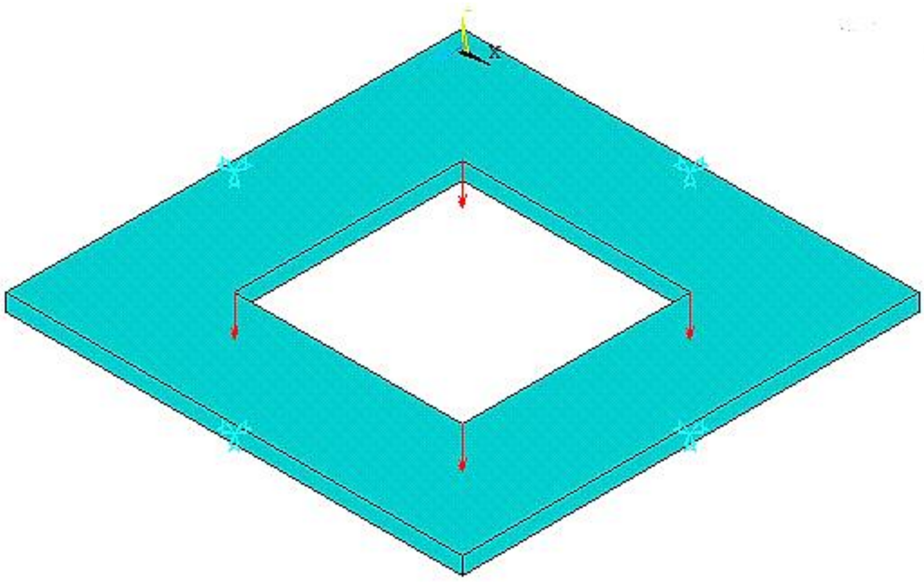

Fig. 2 General shape of present problem

Table1 Primary parameters and dimensions

Parameter	σ	δ	E	h	I	ρ	a	$a_\circ$	F
Primary dimension	$ML^{-1}T^{-2}$	L	$ML^{-1}T^{-2}$	L	L^4	ML^{-3}	L	L	MLT^{-2}

3.2.1 Determine dimensionless groups

The parameters of the present case study are:

σ: *maximum stress*	δ: *maximum displacement*	E: *modulus of elasticity*
h: *thickness of slab*	I: *second moment of inertia*	ρ: *density*
a: *dimension of slab*	$a_\circ$: *dimension of opening*	F: *concentrated force*

According to Buckingham theorem, the dimensionless groups can be determined by following points:

• Number of involved parameters in present case study are (n = 9)

• The primary parameters (r) in present problem are (M, L, T) which represents mass, length and time. The primary dimensions of the previous parameters will be according to Table 1.

• Number of groups $(\pi - groups)$ will be equal to (n – r = 9 – 3 = 6)

• Deriving $\pi - groups$ using the repeating parameters (h, ρ, E)

- Group π_1 (use variable σ)

$\pi_1 = \sigma.\left(h^a.\rho^b.E^c\right) = (ML^{-1}T^{-2}).((L)^a.(ML^{-3})^b.(ML^{-1}T^{-2})^c)$

The powers (a, b, c) should be equal to zero for getting dimensionless values, So:

Primary parameters	Powers equal to zero	Results
L	-1 + a – 3b – c = 0	a = 0
M	1 + b + c = 0	b = 0
T	-2 – 2c = 0	c = -1

$\pi_1 = \frac{\sigma}{E}$

- Group π_2 (use variable δ)

$\pi_2 = \delta.\left(h^a.\rho^b.E^c\right) = (L).((L)^a.(ML^{-3})^b.(ML^{-1}T^{-2})^c)$

The powers (a, b, c) should be equal to zero for getting dimensionless values, So:

Primary parameters	Powers equal to zero	Results
L	1 + a – 3b – c = 0	a = -1
M	b + c = 0	b = 0
T	-2c = 0	c = 0

$\pi_2 = \frac{\delta}{h}$

- Group π_3 (use variable I)

$\pi_3 = I.\left(h^a.\rho^b.E^c\right) = (L^4).((L)^a.(ML^{-3})^b.(ML^{-1}T^{-2})^c)$

The powers (a, b, c) should be equal to zero for getting dimensionless values, So:

Primary parameters	Powers equal to zero	Results
L	4 + a – 3b – c = 0	a = -4
M	b + c = 0	b = 0
T	-2c = 0	c = 0

$\pi_3 = \frac{I}{h^4}$

- Group π_4 (use variable a)

$\pi_4 = a.\left(h^a.\rho^b.E^c\right) = (L).((L)^a.(ML^{-3})^b.(ML^{-1}T^{-2})^c)$

The powers (a, b, c) should be equal to zero for getting dimensionless values, So:

Primary parameters	Powers equal to zero	Results
L	1 + a – 3b – c = 0	a = -1
M	b + c = 0	b = 0
T	-2c = 0	c = 0

- Group π_5 (use variable $a_\circ$)

$\pi_5 = a_\circ.\left(h^a.\rho^b.E^c\right) = (L).((L)^a.(ML^{-3})^b.(ML^{-1}T^{-2})^c)$

The powers (a, b, c) should be equal to zero for getting dimensionless values, So:

Primary parameters	Powers equal to zero	Results
L	1 + a – 3b – c = 0	a = -1
M	b + c = 0	b = 0
T	-2c = 0	c = 0

$\pi_5 = \frac{a_\circ}{h}$

- Group π_6 (use variable F)

$\pi_6 = F.\left(h^a.\rho^b.E^c\right) = (MLT^{-2}).((L)^a.(ML^{-3})^b.(ML^{-1}T^{-2})^c)$

The powers (a, b, c) should be equal to zero for getting dimensionless values, So:

Primary parameters	Powers equal to zero	Results
L	1 + a – 3b – c = 0	a = -2
M	1 + b + c = 0	b = 0
T	-2 - 2c = 0	c = -1

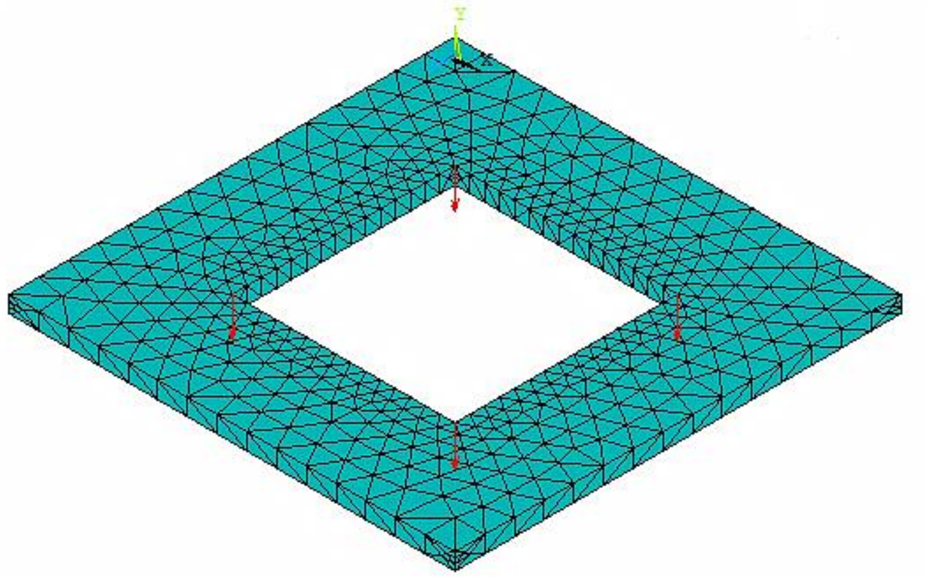

Fig. 3 Meshing the prototype slab with opening

$$\pi_6 = \frac{F}{h^2.E}$$

The relation of groups can be written:

$$\pi = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6),\ \pi = f(\frac{\sigma}{E}, \frac{\delta}{h}, \frac{I}{h^4}, \frac{a}{h}, \frac{a_\circ}{h}, \frac{F}{h^2.E})$$

3.2.2 Finding relation among the groups

It clear that all the groups are dimensionless values-scalar quantities- and they can be connected by each other's by a relation and constant (Shehadeh *et al*. 2015). The relation will be according to requirements, the required relations for present problem are:

1- Produce relation between the force and maximum stress

$\pi_1 = C_1.\pi_4.\pi_5.\pi_6$ or $\pi_1 = C_1.\frac{\pi_4.\pi_5}{\pi_6}$ or any other form

2- Produce relation between the force and maximum displacement

$\pi_2 = C_2.\pi_4.\pi_5.\pi_6$ or $\pi_2 = C_2.\frac{\pi_4.\pi_6}{\pi_5}$ or any other form

3.2.3 Modeling and analyzing of large scale slab with opening

ANSYS $R_{18.1}$ has been used for building the prototype slab with opening. Concrete 65 element is the materials specified in present problem, this element is initially assumed to be an isotropic material. The cracks in three orthogonal dimensions, crushing and failure by plasticity can be occurred in this element. Free meshing with tetrahedral element has been adopted as shown in Fig. 3 (Cho and Wood 1997).

The materials properties and dimensions of the prototype are:

a: $4\ m$ $a_\circ$: $2\ m$ h: $0.15\ m$ μ: 0.18

F: $15000\ N$ E: $2.48E + 10\ \frac{N}{m^2}$ ρ: $2400\ \frac{kg}{m^3}$

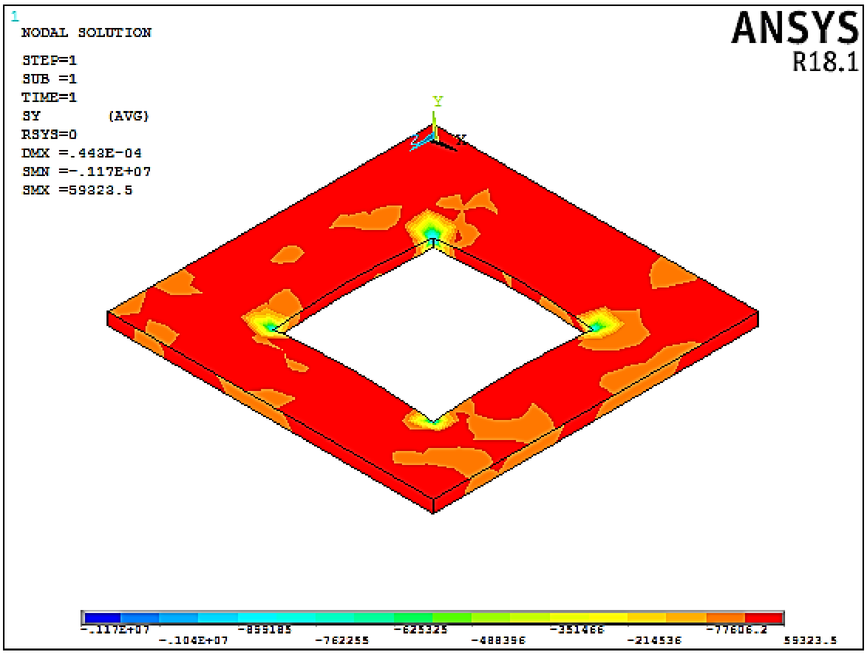

Fig. 4 Maximum stress of prototype

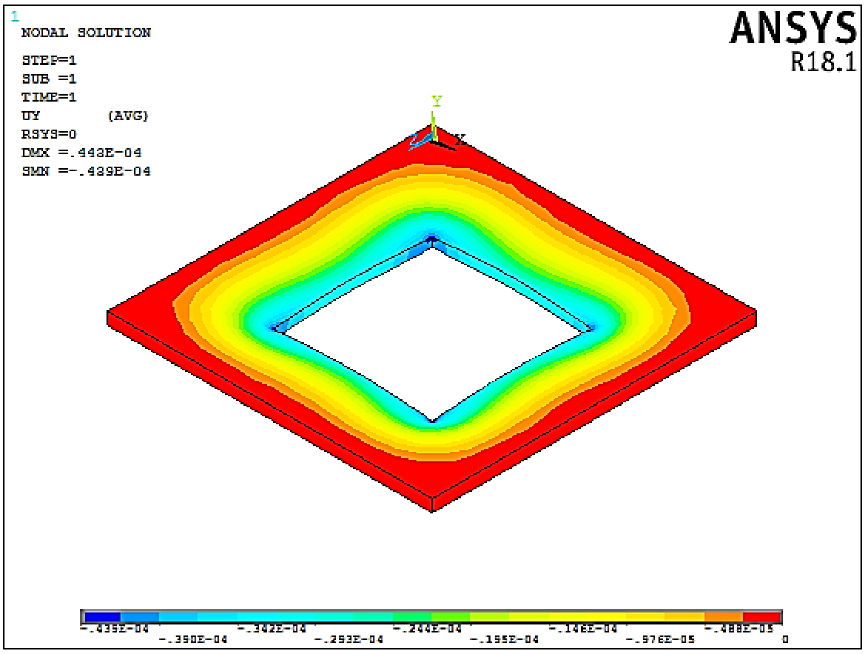

Fig. 5 Maximum displacement of prototype

The results of Finite Element analyzing of max stress and displacement has been obtained as shown in Figs. 4 and 5.

Table 2 different values of forces versus maximum stresses and displacements

Force (N)	Maximum stress (N/m^2)	Maximum displacement (m)
15000	59323.5	4.43E-05
20000	79098	5.90E-05
25000	98872.5	7.38E-05
30000	118647	8.85E-05

3.2.4 Driving the equation constant of the large scale slab with opening

The relation between force and stress will be according to following form:

$$\pi_1 = C_1.\pi_4.\pi_5.\pi_6 \quad (1)$$

$$\frac{\sigma}{E} = C_1.\frac{a}{h}.\frac{a_\circ}{h}.\frac{F}{h^2.E} \quad (2)$$

$$\sigma = C_1.\frac{F.a}{h^2.a_\circ} \quad (3)$$

The relation between force and displacement will be according to following form:

$$\pi_2 = C_2.\frac{\pi_4.\pi_6}{\pi_5} \quad (4)$$

$$\frac{\delta}{h} = C_2.\frac{\frac{a}{h}.\frac{F}{h^2.E}}{\frac{a_\circ}{h}} \quad (5)$$

$$\delta = C_2.\frac{F.a}{E.h.a_\circ} \quad (6)$$

Different values of forces haven been applied on the prototype in order to specify the constants of Eqs. (1)-(6). Table 2 views different values of forces with versus of maximum stresses and maximum displacements.

The constants (C_1&C_2) have been found by substituting the average of forces, maximum stresses and maximum displacements:

$C_1 = 0.00025,\qquad C_2 = 5.517283$

4. Results and discussion

This section will include three parts to view the results and to discuss them, as they shown below:

4.1 Prediction analyzing of small scale slab with opening

The aim of this article is similitude and scaling large scale structural member to small scale member, three small scale models will be checked by reducing the dimensions and the external forces by a factor (λ), the materials properties and restraints will not be changed (Kim and Choi 2016). Table 3 shows the details of the models.

Table 2 different values of forces versus maximum stresses and displacements

Model name	λ	$F\ (N)$	$a(m)$	$a_{\circ}(m)$	$h(m)$
$Model_1$	0.75	λF	3	1.5	0.1125
$Model_2$	0.5	λF	2	1	0.075
$Model_3$	0.25	λF	1	0.5	0.0375

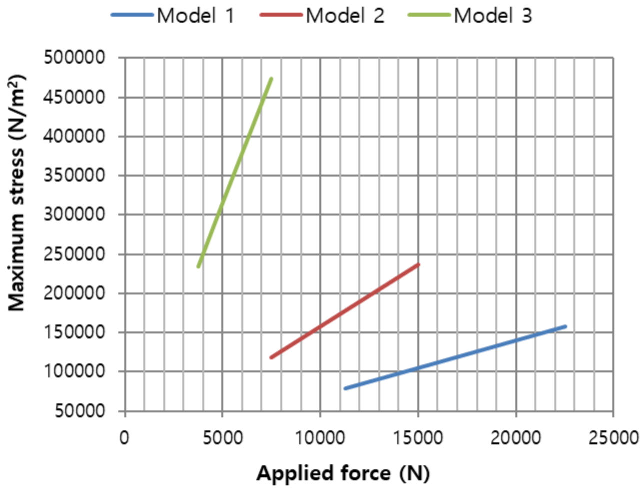

Fig. 6 Predicted maximum stress for models

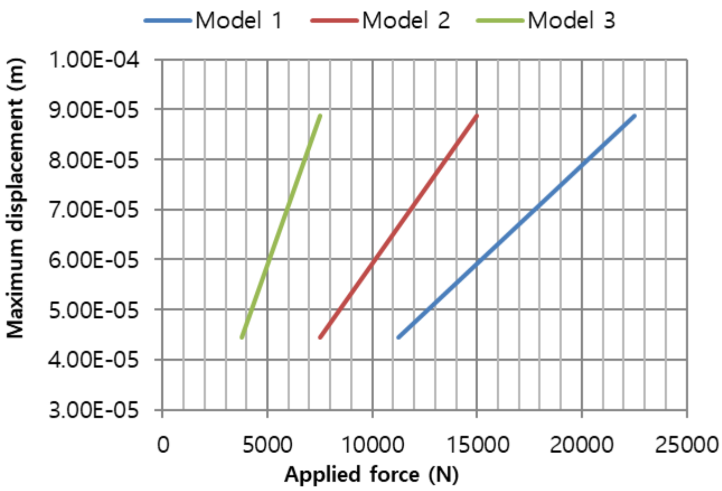

Fig. 7 Predicted maximum displacement for models

The following two steps show the procedure for finding the predicted analysis:

• Substitute the details of each model in Eqs. (1)-(6), the materials properties and restraints will be the same of the prototype.

• Use the same constants (C_1,C_2) of the prototype because of the required similarity between the large scale model and small scale model.

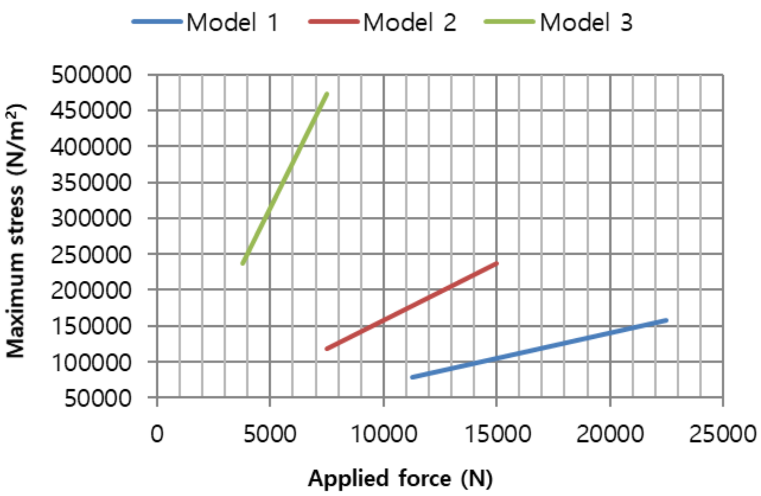

Fig. 8 Theoretical maximum stress for models

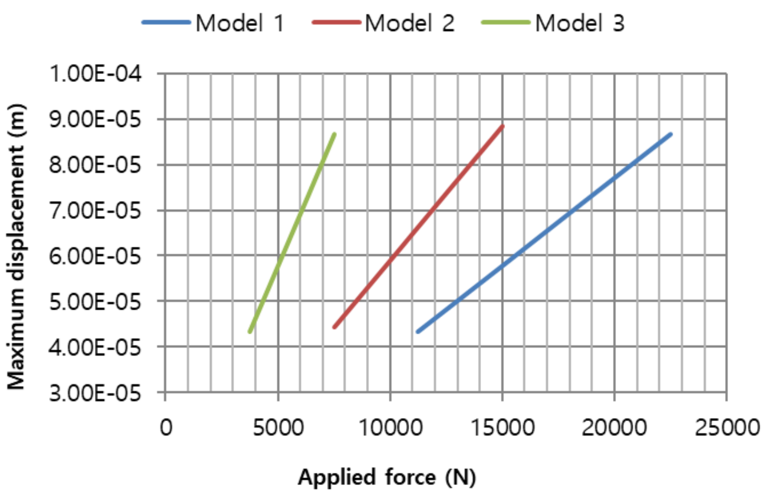

Fig. 9 Theoretical maximum displacement for models

Figs. 6 and 7 clarify the relation between the applied force with maximum stress and maximum displacement.

4.2 Comparison the predicated analysis with theoretical analysis

The same theoretical analyzing in item (3.3) has been repeated for the models 1, 2, and 3 in order to make comparison with the predicted analyzing, Figs. 8-9 show the theoretical maximum stress and displacement respectively.

The prediction and theoretical curves have been merged in one figure in order to clarify the comparison between them, Figs. 10 and 11 show predication and theoretical of maximum stress and displacement for the model (1), (2) and (3).

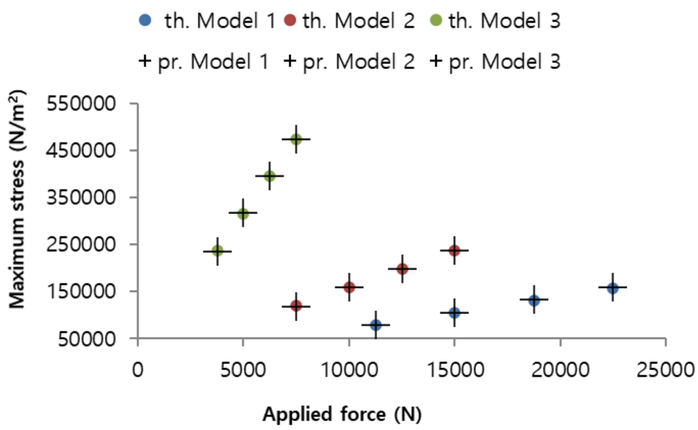

Fig. 10 Prediction and theoretical maximum stress for models

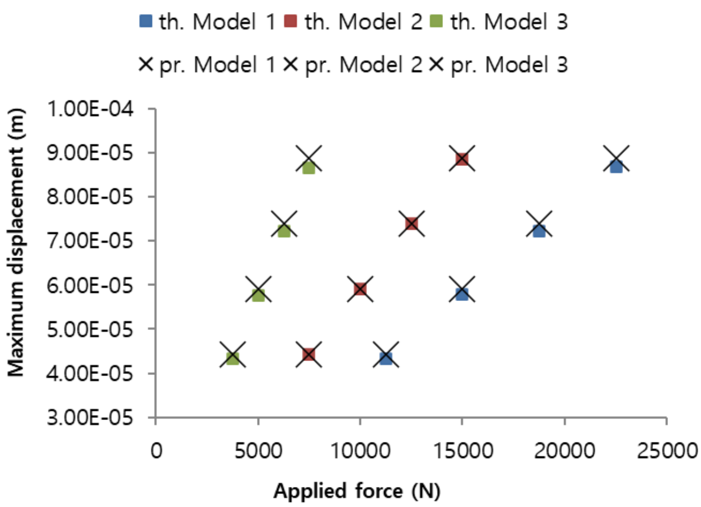

Fig. 11 Prediction and theoretical maximum displacement for models

4.3 Discussion

It is clear from Figs. 10 and 11 that there is semi exact fitting between the prediction and the theoretical results, the reason of the small deviation among those charts is probably of Finite Element method where the number of elements in prototype is differ than the number of elements in models. In other hand, the kind of element may change the results, so it is suggested to make same study with other type of element, the suggested element has to has more factors and properties for getting near to the real or field element, an experimental study is recommended in order to make comparison. In general, it seems that the large scale structural members behave like of small scale members where engineering similarity can be used by reducing the dimensions and the external forces by multiplying with a factor (λ) as in model 1, 2, and 3. The following equation is used to find the percentage diffraction between prediction and theoretical of maximum stress and displacement.

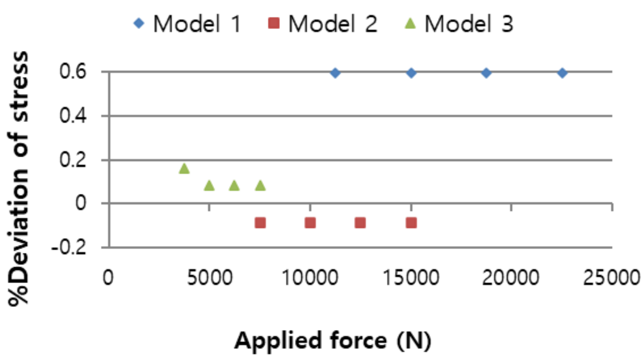

Fig. 12 Percentage deviation of stress between prediction and theoretical results

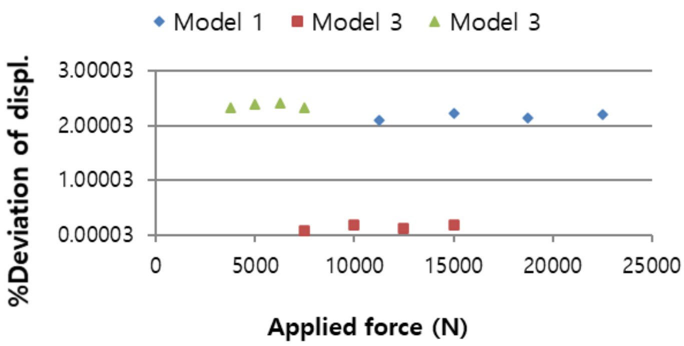

Fig. 13 Percentage deviation of displacement between prediction and theoretical results

$$\%\ \text{Deviation} = \frac{\text{prediction} - \text{theoretical}}{\text{prediction}} * 100 \quad (7)$$

The percentage deviation between the prediction and theoretical results has been calculated, Figs. 12 and 13 show the maximum deviation in stress and displacement respectively, it is observed that the maximum deviation in stress is (0.59%) while the maximum deviation in displacement is (2.4%).

5. Conclusions

The main conclusions that have been obtained from a prototype that produced by using Buckingham theorem and the theoretical results are:

• Buckingham theorem is considered suitable way for creating relations among the parameters for any structural problem then making similitude and scaling the large scale members to small scale members.

• Maximum deviation between prediction and theoretical results of the structural members is very small and it is not exceeding (2.4%), so engineering similarity by reducing the dimensions and forces by a factor might be considered suitable way for structural members.

• Other element in Finite element method has to be checked to take into consideration all other effects, an experimental work is recommended.

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