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Locating cracks in RC structures using mode shape-based indices and proposed modifications

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Abstract. This study presents the application of two indices for the locating of cracks in Reinforced Concrete (RC) structures, as well as the development of their modified forms to overcome limitations. The first index is based on mode shape curvature and the second index is based on the fourth derivative of the mode shape. In order to confirm the indices' effectiveness, both eigenvalues coupled with nonlinear static analyses were carried out and the eigenvectors for two different damage locations and intensities of load were obtained from the finite element model of RC beams. The values of the damage-locating indices derived using both indices were then compared. Generally, the mode shape curvature-based index suffered from insensitivity when attempting to detect the damage location; this also applied to the mode shape fourth derivative-based index at lower modes. However, at higher modes, the mode shape fourth derivative-based index gave an acceptable indication of the damage location. Both the indices showed inconsistencies and anomalies at the supports. This study proposed modification to both indices to overcome identified flaws. The results proved that modified forms exhibited better sensitivity for identifying the damage location. In addition, anomalies at the supports were eliminated.

Keywords: crack identification; damage locating indices; mode shape curvature; mode shape fourth derivatives

1. Introduction

The development of damage detection techniques based on modal parameters has attracted significant attention with regards to civil engineering applications. Damage inspection of structures is important in order to come up with a planned asset management strategy for maintenance activities. Numerous research projects have been done in the field of damage detection and a variety of methods has been developed and proposed. These methods are mainly based on the relationship between the dynamic characteristics and the damage parameters, such as the depth of a crack and its location. Cracks on main structural elements can be a major cause of concern since they can lead to structural failure. Thus, early crack detection is crucial in order to avoid sudden failure, especially when there is the likelihood of overloading on the structure. In

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general, cracks can cause a reduction in stiffness and correspondingly cause a change in dynamic parameters such as mode shape and its derivatives. Therefore, it is possible to detect the damage location by measuring the change in the mode shape derivatives. Mode shape curvatures are more sensitive to damage. The concept of curvature mode shape was introduced by Pandey *et al*. (1991). Using a finite element model of a simply supported beam with a reduction in Modulus of Elasticity E value of 50% at one third of the span, it was demonstrated that the modal curvature was a much more sensitive damage indicator than the Modal Assurance Criteria (MAC) or Co-ordinate Modal Assurance Criteria (COMAC). This approach was expanded upon by Ratcliffe (1997) using both analytical and experimental results of the curvature of a damaged beam without the need of *a priori* knowledge of the undamaged state. The proposed method applies the Laplace operator on the discrete mode shape. The presence of severe damage was detectable in the form of a jump in the Laplace. Stubbs *et al*. (1995) developed a damage index method to locate the damage which utilised the characteristics of the mode shape curvature for a beam as the main variable in the derived damage localisation algorithm based on the relative differences in modal strain energy before and after damage. Dutta and Talukdar (2004) carried out eigenvalue analysis using the Lanczos algorithm in an adaptive h-version finite element environment in order to control the discretisation error for accurate evaluation of the modal parameters. It was found that there was better localisation of damage when considering curvature of mode shapes. Choi *et al*. (2008) proposed modifications to two existing algorithms for global non-destructive evaluation. These were not effective in locating multiple damage scenarios and were also unable to quantify the severity of damage. The mode shape data from an experimental modal analysis were used for calculating the damage index. This utilised the change in modal strain energy between the undamaged and the damaged timber beam model. The modified damage index by Choi *et al*. (2008) normalised the mode shape curvature, and the hybrid algorithm combined the modified damage index and change in the flexibility algorithm, which reflected the changes of natural frequency and mode shape. Fayyadh and Abdul Razak (2011) and Fayyadh and Abdul Razak (2012) studied the effects of support conditions, especially differential support conditions, on modal parameters and found that mode shape 1 had the highest sensitivity to bearing deteriorations, while mode 3 could be used to identify whether the shift in modal parameters is caused by elastic bearing or stiffness of the structural element. Fayyadh and Abdul Razak (2011) investigated the possibility of using mode shape deviation to detect damage location and found that mode shapes for the damage cases deviated from that of the undamaged cases and the deviation was towards the damage location. Pholdee and Bureerat (2016) conducted comparative analyses of the performance of various meta heuristics for use in structural damage detection based on changes in modal data. Three truss structures were used to pose several test problems, and meta-heuristics were used to solve the test problems. The study concluded that the Teaching-Learning-Based-Optimisation (TLBO) method was outstanding for a large-scale problem, while differential evaluation was the overall best method. Fayyadh and Abdul Razak (2011) developed a new damage algorithm called the Stiffness Reduction Index to detect damage locations in steel structures. This achieved greater accuracy than the COMAC algorithm. Abdul Razak and Fayyadh (2013) studied the effects of RC elements' composite actions in terms of interactions within concrete in tension and compression or interactions between reinforcement steel and concrete, and concluded that the first bending mode could be a good indicator for bond action between reinforcement steel and concrete. They also noted that Mode 2 was governed by the amount of steel reinforcement while Modes 3 and above could be indicators for concrete softening. Frans *et al*. (2017) conducted comparative studies of mode shape curvature and damage location vector methods for damage detection in structures and concluded that for plane truss structures, the damage location vector method could predict the damaged component, while the mode shape curvature method could only predict the nodes in the vicinity of the damaged members. Fayyadh and Abdul Razak (2011) proposed a weighting method for modal parameters using the area under the curve as an indicator for the energy required to from the mode shape and subsequently indicate individual modes weight for averaging purposes. Fayyadh and Abdul Razak (2013) reviewed the use of natural frequencies and mode shape for damage detection in RC structures and concluded that more work was required in the area of damage detection and source identification in RC structures. Shokrani *et al*. (2018) introduced a new method of using mode shape for damage localisation under varying environmental conditions. The method comprised three stages, which all pertained to training, validation and diagnostics. The proposed method was applied on numerical case studies and found to be effective, however further investigations was deemed required for further validation.

From previous studies, it was apparent that some of the indices used to locate damage on reinforced concrete structural elements were of low accuracy in detecting damage locations. The objective of this study is thus to propose modifications to two of the indices to improve their accuracy in detecting damage locations. The first is called the Curvature Damage Factor and is based on the change in curvature of mode shape. The second is called the Local Stiffness Index and is based on the fourth derivative of mode shape.

2. Case study and finite element modelling

The case study presented herein was of a simply supported beam design according to ACI-318-08 with a span of 2.2 m. Its dimensions were 150 mm wide and 250 mm deep. The beam was reinforced with two of 12 mm diameter high yield bars as the main longitudinal reinforcement and 8 mm shear links of 100 mm spacing. The details of the beam are shown in Fig. 1. The steel reinforcement yield stress was 535 MPa, rapture stress was 665 MPa and elasticity modulus was 180 GPa. The concrete compressive strength was 38 MPa, tensile stress was 3.8 MPa and elasticity modulus was 36 GPa. A finite element model of the beam was created using 20-node brick elements to represent the concrete while the reinforcement was modelled with 2-node embedded bar elements inside 3-D brick elements, as shown in Fig. 2. Concrete was modelled using the smeared crack model. For concrete, the linear behaviour was modelled as isotropic material with certain compressive strength value, modulus of elasticity, Poisson's ratio and mass density. For nonlinear behaviour, it was modelled using linear stress cut-off, linear tension softening as well as ultimate strain-based and constant shear retention models. Reinforcement steel bars were represented as fully bonded reinforcement, as the objective of this study was to investigate the accuracy of modal analysis-based damage locating indices and to avoid impact of reinforcement-concrete bond issues on the study objective. For steel nonlinearity, the Von-Mises plasticity criteria was used with the work hardening rule to present the actual steel stress-strain curves. Initially, eigenvalue analysis of the undamaged beam was performed, and the mode shape vectors were obtained. Subsequently, non-linear static analysis was carried out for two different applied load conditions; namely, up to 50% and 70% of the ultimate load in order to induce damage in the beam. For the case when the load was applied at 0.5 L, the ultimate load was 45 kN, such that 50% and 70% were 23 kN and 32.2 kN, respectively. Correspondingly, when the applied load was at 0.25 L, the ultimate load was 56 kN, which gave the respective 50% and 70% values of 28 kN and

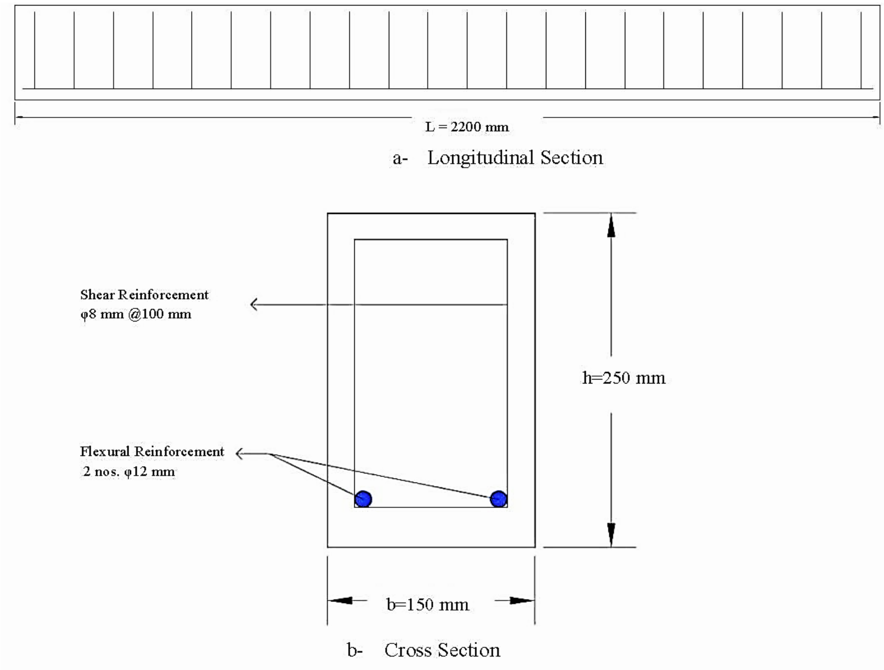


Fig. 1 Reinforced concrete beam dimensions and details

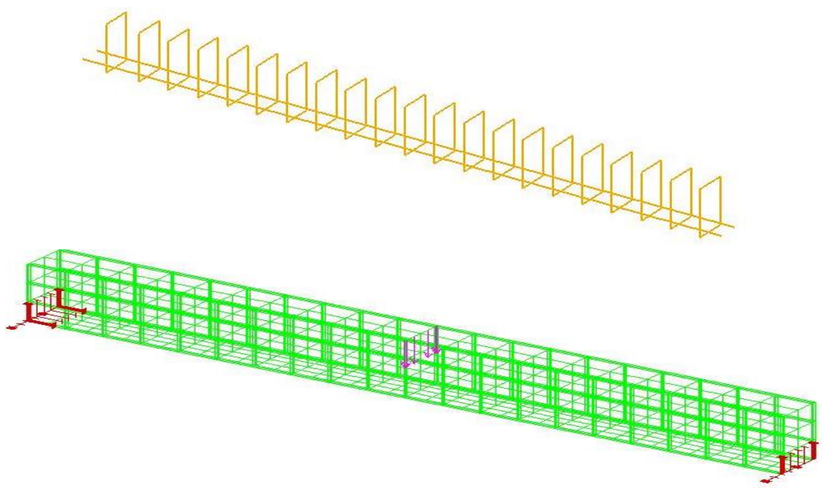
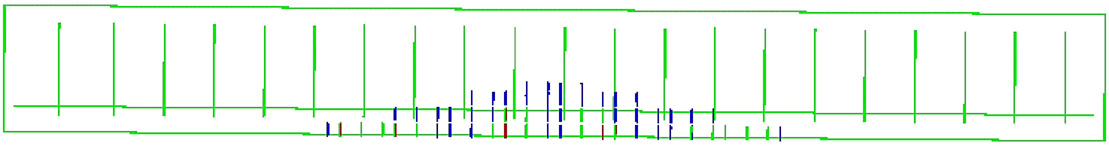
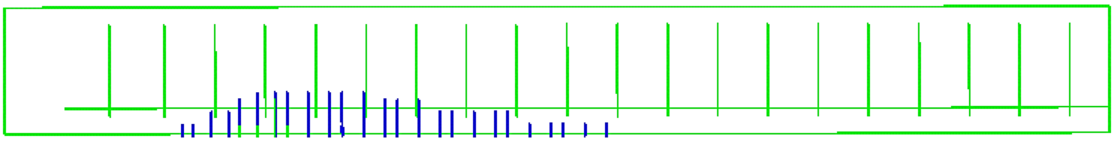


Fig. 2 Finite element model of reinforced concrete beam (top: reinforced bars modelling; bottom: concrete beam modelling) – *Orange colour is reinforcement elements, green colour is concrete brick elements, red colour is support restrains, and burble colour is applied load*

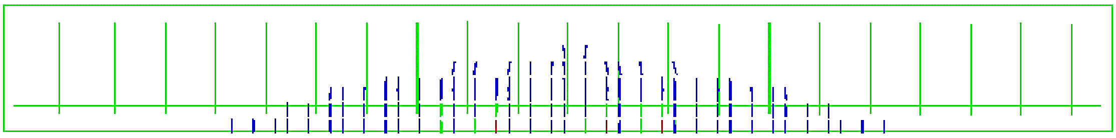
39.2 kN. After each of the applied load conditions, the load was released, and eigenvalue analysis was again carried out to obtain the mode shape vectors for the damaged beam. Different locations of damage in the beam were achieved by applying the concentrated point load at mid-span and quarter-span points along the beam. Fig. 3 shows the crack pattern in the RC beam after application of the damaged load.



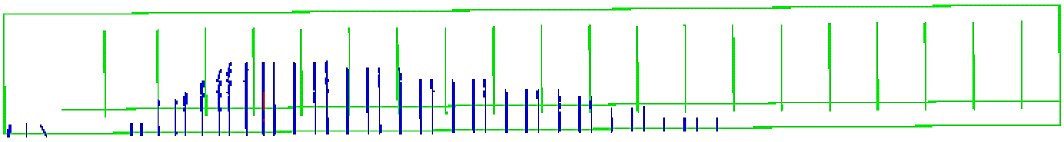
(a) under 50% (23 kN) at 0.5 L



(b) under 50% (28 kN) at 0.25 L



(c) under 70% (32.2 kN) at 0.5 L



(d) under 70% (39.2 kN) at 0.25 L

Fig. 3 Crack pattern of RC beam under different damage load located at different distance from left support (Cracks are shown in blue and red)

3. Damage detection indices

The results obtained from the finite element analysis were subsequently utilised to verify and compare the sensitivity and accuracy and to detect and locate the damage areas, respectively, in this study. The eigenvectors were substituted into the equations for the damage indices, namely the Curvature Damage Factor (CDF) and Local Stiffness Indicator (LSI), and also the corresponding proposed modified indices based on the two mentioned above.

3.1 Curvature damage factor (CDF)

Proposed by Wahab and De Roeck (1999), the mode shape curvature at each point was computed from central difference approximation using mode displacement as given in Eq. (1) below:

$$Ci = (y_{i+1} - 2y_i + y_{i-1})/h^2 \tag{1}$$

where C is the curvature, i is node number, y is the eigenvector at the ith node, and h is the distance between each sequenced node.

Subsequently, the change in curvature between two sets of mode vectors, i.e., the control and damaged cases, is as shown in Eq. (2):

$$\text{CDF} = \frac{1}{N}\sum_{j=1}^{N}|C_{ci} - C_{di}| \qquad (2)$$

where *CDF* is the Curvature Damage Factor, *N* is the total number of modes, '*c*' indicates the control case when no load was applied and '*d*' indicates the damage case when the damage load was applied and released and *C* is the curvature at the ith node.

Fig. 4 and Fig. 5 show the results of CDF according to finite element modelling results with 50% of the ultimate load (UL) applied at 550 mm and 1100 mm from the left support. Correspondingly, the values of CDF with 70% of the ultimate load applied to the beam at the same locations are illustrated in Figs. 6 and 7. The results were summarised for the first six bending modes.

The results show that the CDF correlated well when the damage location was at 550 mm from the left support for different degrees of damage, as is apparent in Fig. 5 and Fig. 7. This matches up to the crack pattern shown in Figs. 3(b) and 3(d). However, when the damage was at mid-span, it was less sensitive. Furthermore, values of CDF in all the cases considered returned high values at the supports, which is an anomaly and indicates a flaw in the index. Thus, CDF in its original form was rather unreliable near the supports.

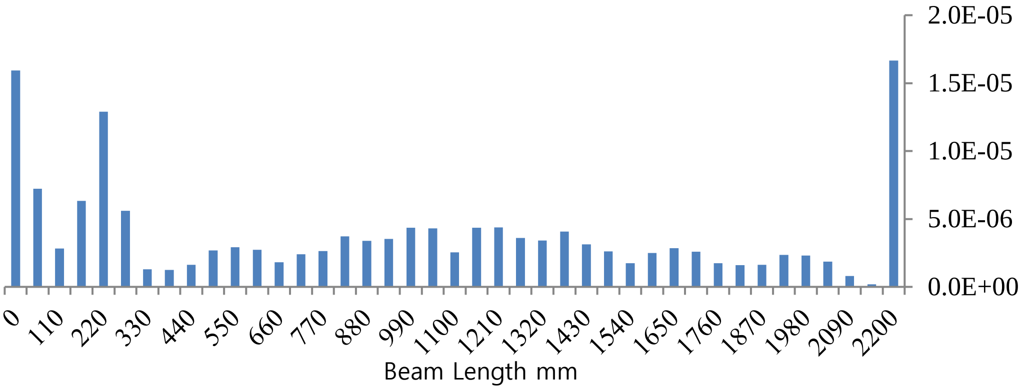


Fig. 4 CDF for beam under 50% UL (23 kN) located 1100mm from left support

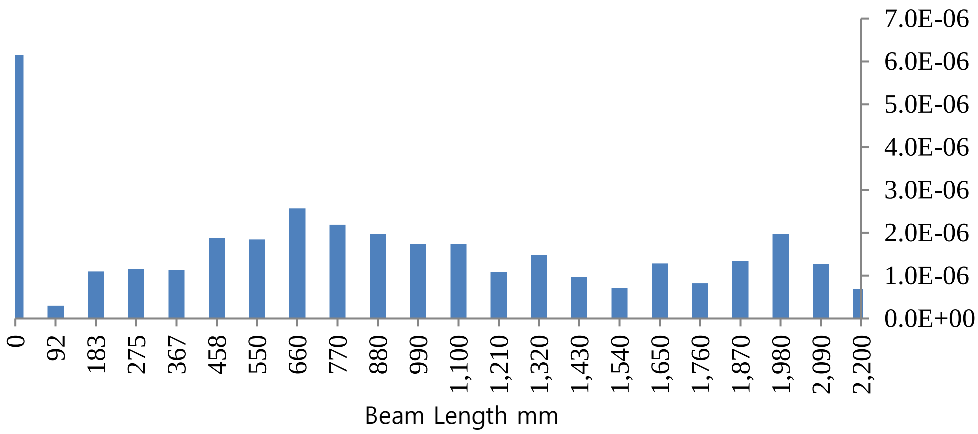


Fig. 5 CDF for beam under 50% UL (28 kN) located 550 mm from left support

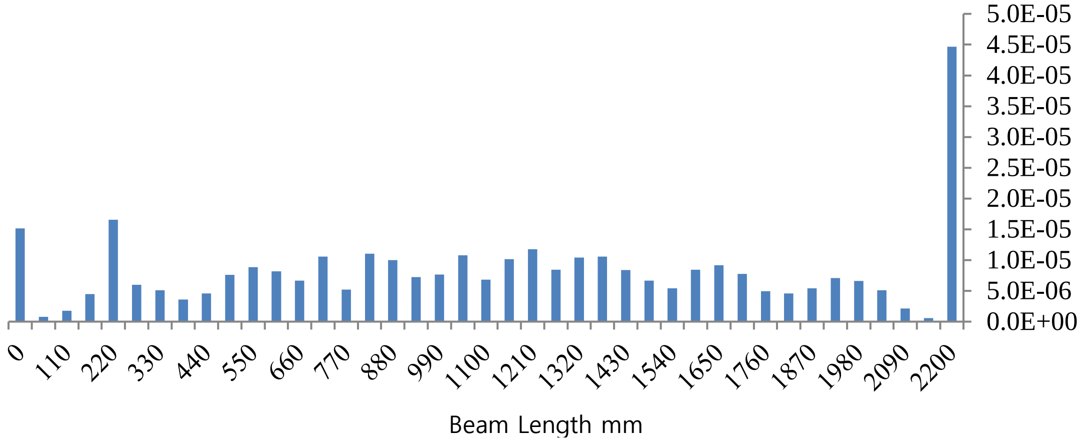


Fig. 6 CDF for beam under 70% UL (32.2 kN) located 1100 mm from left support

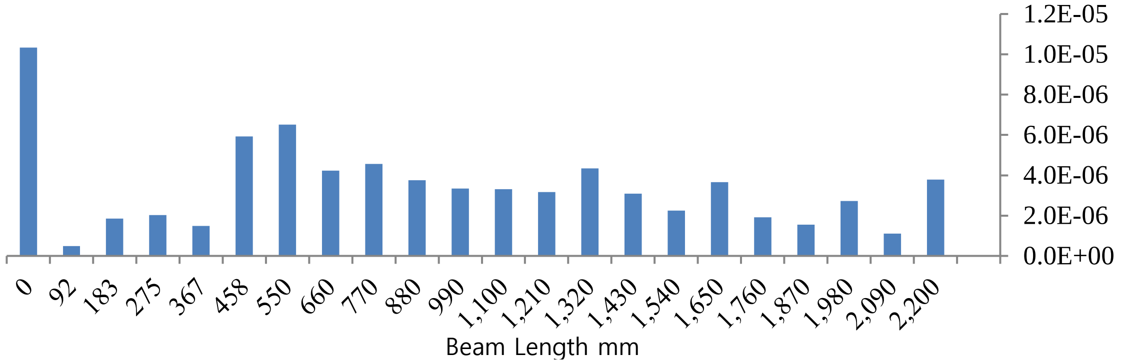


Fig. 7 CDF for beam under 70% UL (39.2 kN) located 550 mm from left support

3.2 Local Stiffness Indicator (LSI)

Proposed by Ismail and Abdul Razak (2006), this is based on the equation for free vibration of the Euler beam as shown below:

$$\frac{d^4y}{dx^2} - \lambda^4 y = 0 \tag{3}$$

Re-arranging the above equation in the following form:

$$\lambda^4 = \left|\frac{y^4}{y}\right| \tag{4}$$

In addition, applying the fourth order-centred finite difference,

$$y^4 = (y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2})/h^4 \tag{5}$$

where y^4 is the fourth derivative.

Thus, the Local Stiffness Indicator is defined as:

$$LSI = \lambda^4 \tag{6}$$

The solutions to Eqs. (3) and (6), when the node is located at the supports, require special consideration. In this case, for a simply supported beam, there is an angle of rotation at the support. This implies that the line of elastic deformation, which has the shape of the jth mode shape, will pass through the support node to the next span and have the same angle of slope. Thus, the elastic deformation line will have the same shape but opposite curvature. Fig. 8 shows this assumption for the support node case. Subsequently, $y_{i-1} = - y_{i+1}$ and $y_{i-2} = - y_{i+2}$. The same assumption can be used at the other support.

The eigenvectors from the finite element model were extracted and substituted into the above equations to determine the LSI at each node. The occurrence of damage in the beam was predicted to cause a change in the LSI value at the damage location as compared to the undamaged beam where the values should remain constant throughout its length. Figs. 9 and 10 depict the values of LSI for the finite element RC beam model with 50% of ultimate load applied at two different positions, i.e., 550 mm and 1100 mm from the left support. Figs. 11 and 12 show the value of LSI for the finite element RC beam model with 70% of ultimate load applied at two different positions, i.e., 550 mm and 1100 mm from the left support.

Apparently, the LSI is a less sensitive damage indicator compared to the CDF for damage location in the regions considered. However, the sensitivity is better at higher modes. In this study, this could be observed when the damage was located at 0.25 L by eliminating the support anomalies for modes 1, 4, 5 and 6. This also reflected its ability to detect damage, as shown in Figs. 10 and 12. For regions at the support, the values appeared as anomalies. Thus, this is the major drawback of the LSI indicator in its original form.

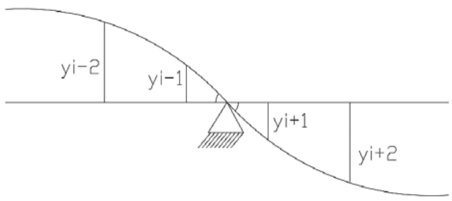


Fig. 8 Special consideration at support

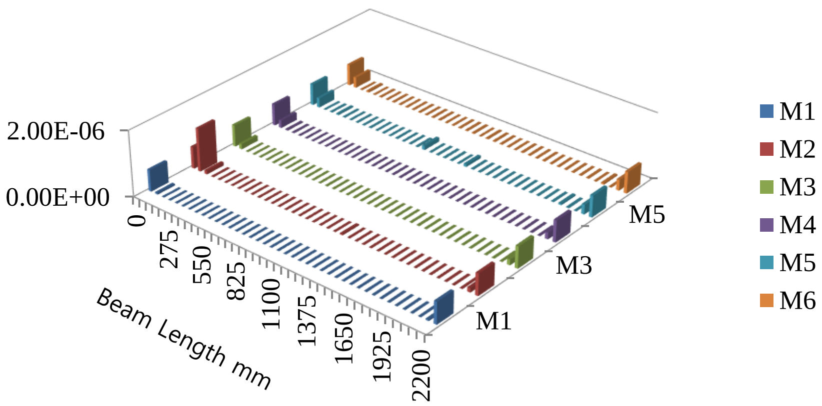


Fig. 9 LSI for beam under 50% UL (23kN) located at 1100 mm from left support, for mode shapes M1 to M6

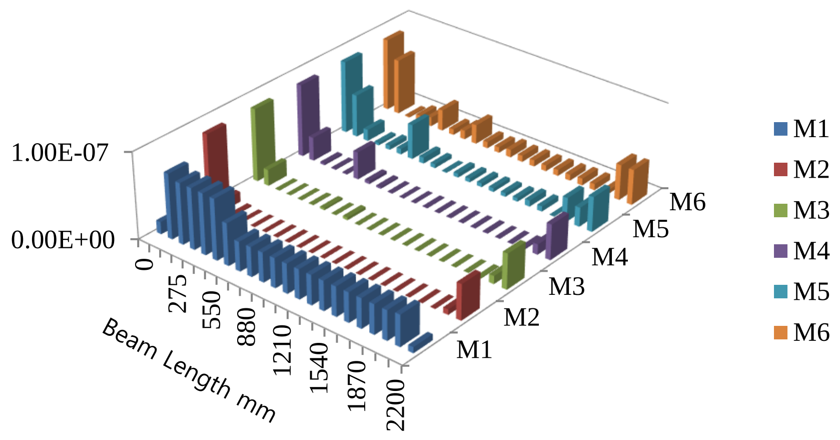


Fig. 10 LSI for beam under 50% UL (28 kN) located at 550 mm from left support, for mode shapes M1 to M6

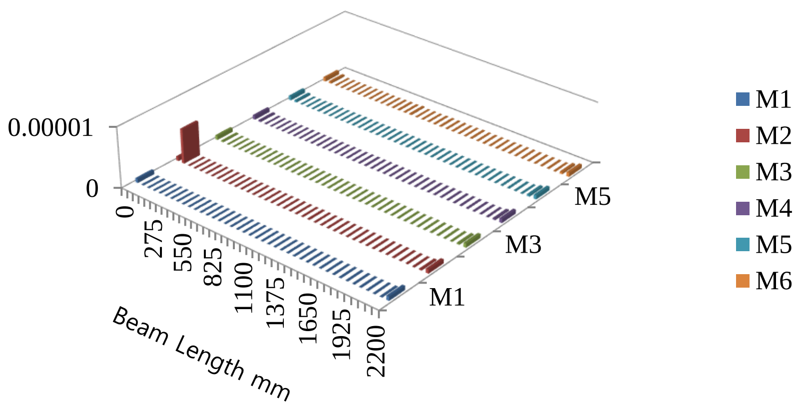


Fig. 11 LSI for beam under 70% UL (32.2 kN) located at 1100 mm from left support, for mode shapes M1 to M6

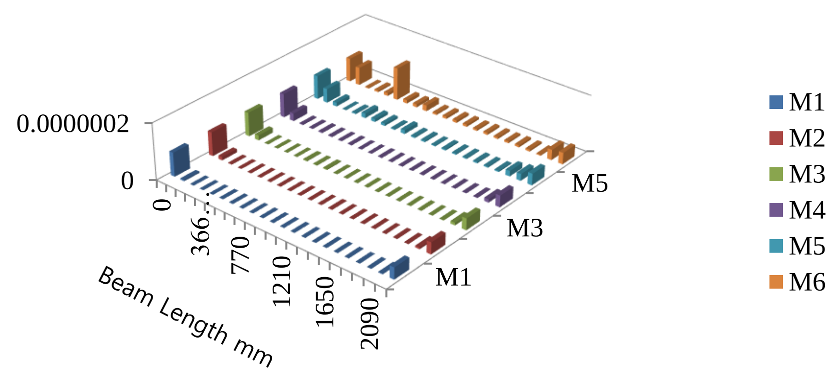


Fig. 12 LSI for beam under 70% UL (39.2 kN) located at 550 mm from left support, for mode shapes M1 to M6

4. Modified damage detection indices

4.1 Modified curvature damage factor (MCDF)

The Curvature Damage Factor (CDF) proposed by Wahab and De Roeck (1999) accounts for all available mode shapes through the summation of the mode shape curvatures. The values of the mode shape curvatures are dependent on the shapes of each individual mode. Instead of reflecting the changes in the curvature due to damage, summation of non-normalised mode shape curvature will distort the damage index in favour of higher modes, which results in false damage identifications. To overcome this problem, in order to use the index for detecting damage location (which is a local phenomenon), it was proposed to calculate the change in curvature at each node for each respective mode considered and compare the values between damage and control cases similar to that proposed by the authors for steel beams (Fayyadh and Razak, 2011).

The mode shape curvature at each point is given in Eq. (1) above. However according to the proposed change above, the equation for the Modified Curvature Damage Factor (MCDF) is given as below:

$$\text{MCDFij} = \left| \frac{C_{di} - C_{ci}}{C_{ci}} \right| 100\% \tag{7}$$

where *MCDF* is the Modified Curvature Damage Factor, '*i*' is node number and '*j*' is the mode shape number. *C* is the curvature at each node, with '*i*' for both '*c*' being the control case where no initial damage load is applied and '*d*' being the damage case when the initial damage load is applied and released. Figs. 13 and 14 show the values of MCDF for the finite element results of the RC beam model with 50% of the ultimate load located at two different locations. Values of MCDF with 70% of the ultimate load are illustrated in Figs. 15 and 16 for applied loads at two different positions, i.e., 550 mm and 1100 mm from left support.

The results showed that the modified MCDF was more sensitive for damage location compared to the CDF. It had the ability to indicate the damage for different levels of damage severity and locations along the beam length. Modes 1 3, 4 and 6 were most sensitive when damage was at 550 mm from the support, as shown in Figs. 14 and 16. When the damage was at mid-span, mode 2

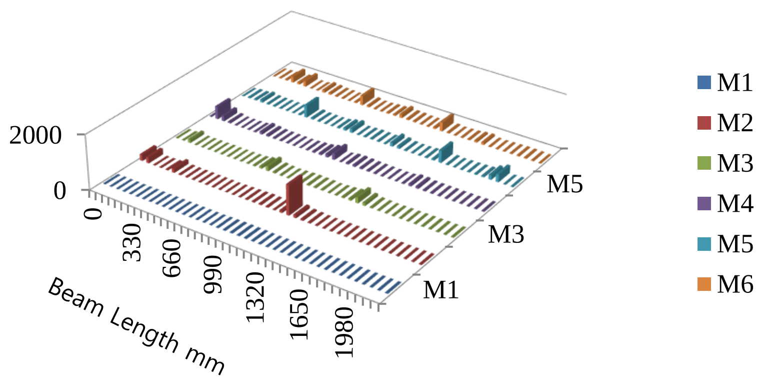


Fig. 13 MCDF for beam under 50% UL (23kN) located at 1100 mm from left support, for mode shapes M1 to M6

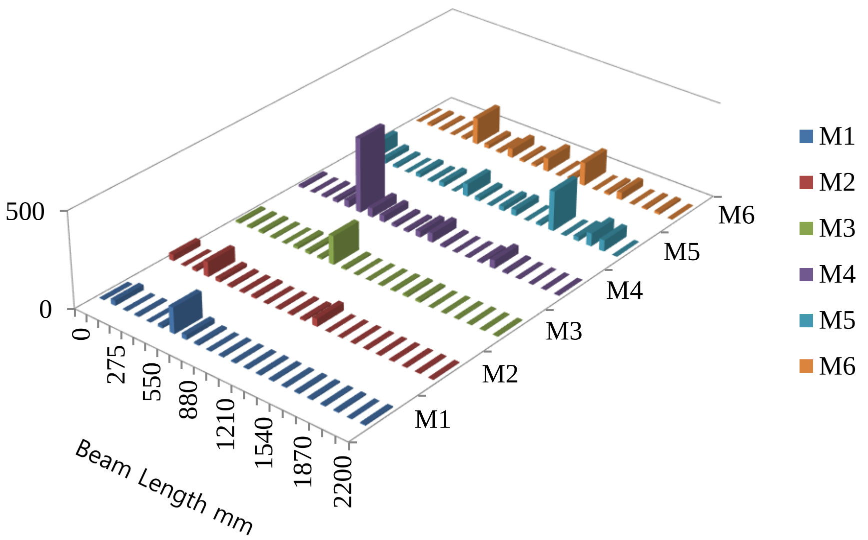


Fig. 14 MCDF for beam under 50% UL (238kN) located at 550 mm from left support, for mode shapes M1 to M6

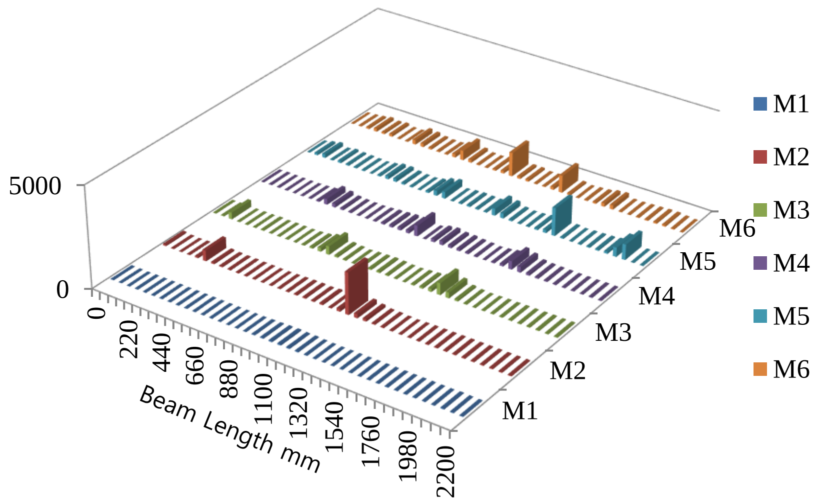


Fig. 15 MCDF for beam under 70% UL (32.2 8kN) located at 1100 mm from left support, for mode shapes M1 to M6

gave the best result, as shown in Figs. 13 and 15. Similarly, mode 1 possessed good sensitivity to locate the damage but was not apparent in the same plots because the values of MCDF for mode 1 were small compared to mode 2, i.e., the higher value mode. To highlight this, Figs. 17 and 18 show MCDF magnified plot for mode 1 when the load is located at 0.5 L. In contrast, the lower modes appeared to have improved sensitivity as damage indicators by utilising the modified form, as indicated by modes 1 and 3 in Figs. 14 and 16 and modes 1 and 2 in Figs. 13, 15, 17 and 18.

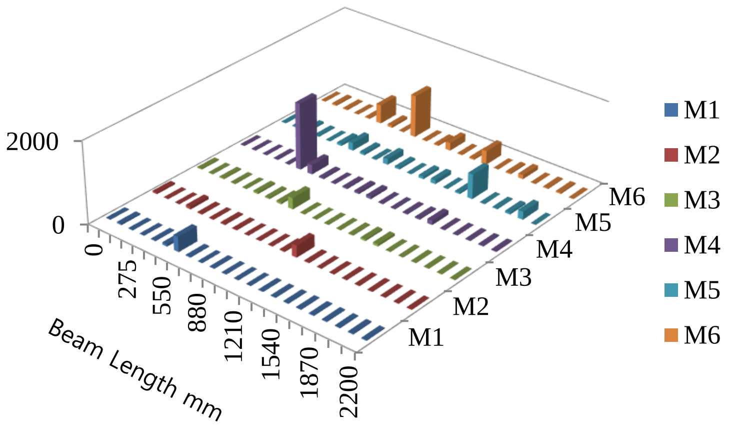


Fig. 16 MCDF for beam under 70% UL (39.2 8kN) located at 550 mm from left support, for mode shapes M1 to M6

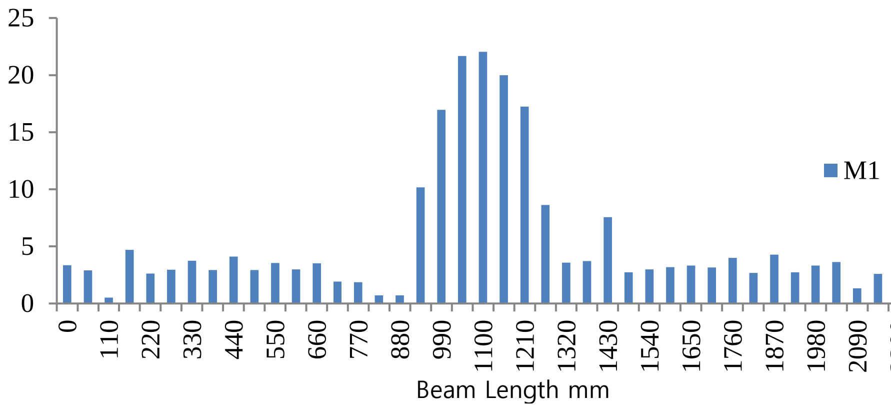


Fig. 17 MCDF for Mode 1 of beam under 50% UL (23 8kN) located at 1100 mm from left support

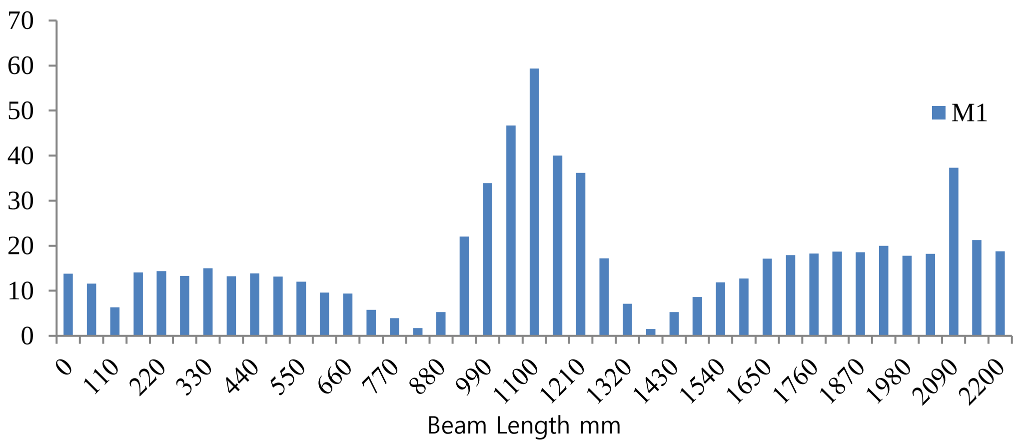


Fig. 18 MCDF for Mode 1 of beam under 70%UL (32.2 kN) located at 1100 mm from left support

4.2 Modified local stiffness indicator (MLSI)

The Local Stiffness Indicator (LSI) developed by Ismail and Abdul Razak (2006) was developed to have indicators for cases when data for the initial state of the structure before damage was unavailable. However, it was concluded that there exist anomalies due to boundary conditions presumably due to the free vibration equation of the Euler beam used for simply supported cases. Furthermore, the LSI was based on the fourth derivative of mode shape and any anomaly would be amplified depending on the degree of the derivative and thus made significant at the support. For cases when the data is available and to overcome the anomalies problem, the modified form is given by Eq. (8), and expressed as a ratio of the LSI for the damage and control cases. In this modified form, if there are anomalies due to boundary conditions at the supports, they will be eliminated by dividing the damage over control.

$$\mathrm{MLSI}_{ij} = \frac{LSI_d}{LSI_C} = \left| \frac{y_d^4 * y_c}{y_c^4 * y_d} \right| \tag{8}$$

where MLSI is the modified indicator, '*i*' is the node number, '*j*' is the mode shape number and y^4 is the fourth derivative of mode shape. The subscript '*d*' is the damage case and the subscript '*c*' is the control case.

Figs. 19 and 20 show the results of MLSI for the finite element results of the RC beam model with 50% of the ultimate load applied at 550 mm and 1100 mm from the left support. Correspondingly, the values of MLSI with 70% of ultimate load applied to the beam at the same locations are depicted in Figs. 21 and 22.

The results showed that the modified index was more sensitive for damage location detection compared to the LSI in its original form. In addition, the peak values at the supports were absent, thus eliminating the anomalies which were obvious when using the unmodified version of the LSI. Although all the modes managed to sense the damaged location at the two levels of severity, in particular, modes 2, 4 and 5 gave the best indication when the damage location was at 1100 mm from the support, as shown in Figs. 19 and 21. Similarly, modes 3 and 6 were most sensitive when the damage was at 550 mm from the support, as shown in Figs. 20 and 22.

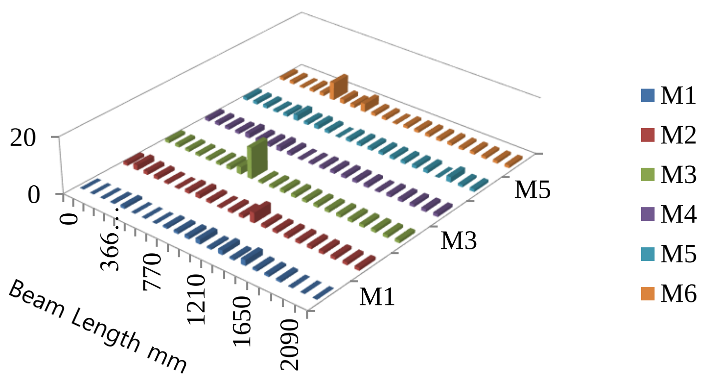


Fig. 20 MLSI for beam under 50% UL (28 KN) located at 550 mm from left support, for mode shapes M1 to M6

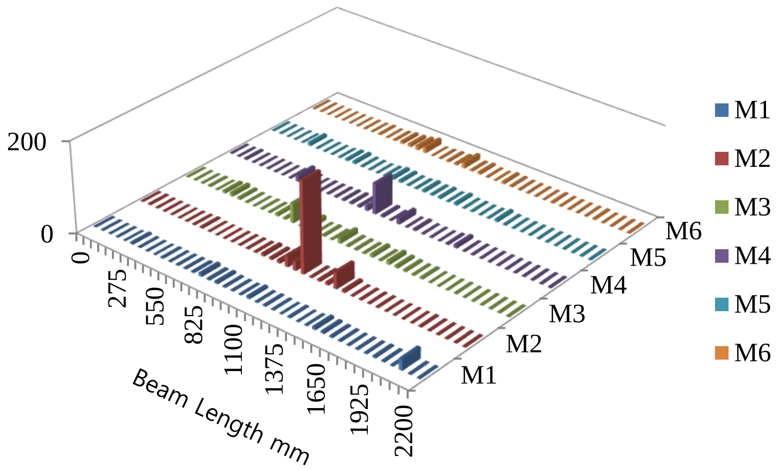


Fig. 21 MLSI for beam under 70% UL (32.2 KN) located at 1100 mm from left support, for mode shapes M1 to M6

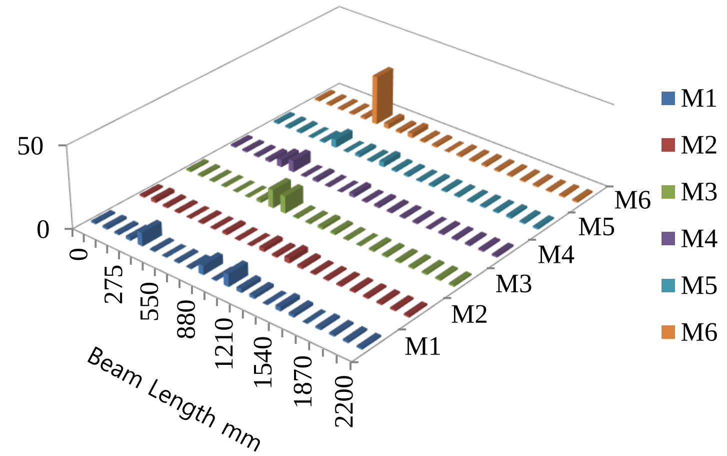


Fig. 22 MLSI for beam under 70% UL (39.2 KN) located at 550 mm from left support, for mode shapes M1 to M6

To verify the accuracy of the approximate solution for calculating the curvature as well as fourth derivatives using the Laplace operator, as shown in Eqs. (1) and (5), a curve fitting software (Table Curve 2D v 5.01) was utilised. This had the ability to estimate the first four derivatives of any set of data. One of the methods used to estimate the second and fourth derivatives was the Fourier estimation method. For comparison, the results obtained using the Fourier estimation were in good agreement as shown in Figs. 23 and 24, for the case of MCDF under 50% UL at mid span as well as MLSI under the same conditions. For MCDF, it was obvious that mode 2 had the best sensitivity, while the f or MLSI modes 2, 4 and 5 gave the best indication. These conclusions were consistent with the results calculated by using the Laplace operator.

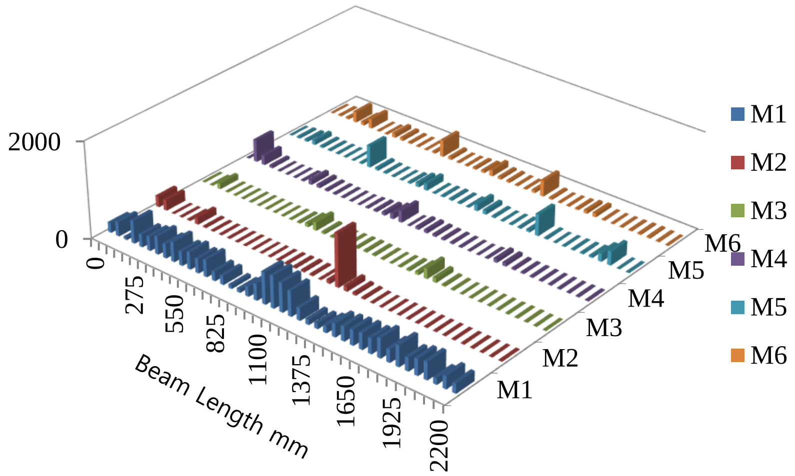


Fig. 23 MCDF for beam under 50% UL (23 KN) located at 1100 mm by using Fourier Estimation, for mode shapes M1 to M6

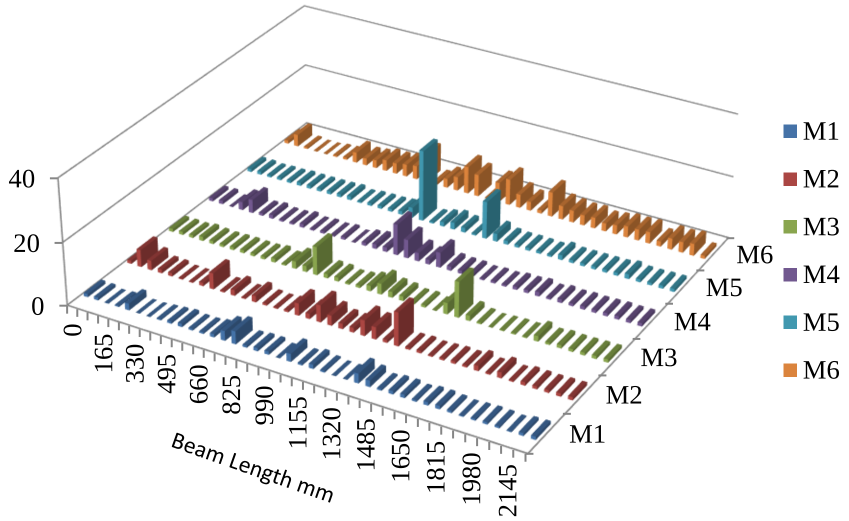


Fig. 24 MLSI for beam under 50% UL (23 KN) located at 1100 mm by using Fourier Estimation, for mode shapes M1 to M6

For the MCDF index based on curvature of mode shape, mode 1 had the ability to locate the damage for different damage cases. This was due to the fact that mode 1 possessed curvature values, i.e., no node or zero curvature, along the length of beam. (See Fig. 25). The distribution of curvature values along the length of beam had an effect on the damage localisation index. If there was a node or zero curvature at a particular point, sensitivity to detect the damage was less at that point in reference. (See Figs. 26 and 27).

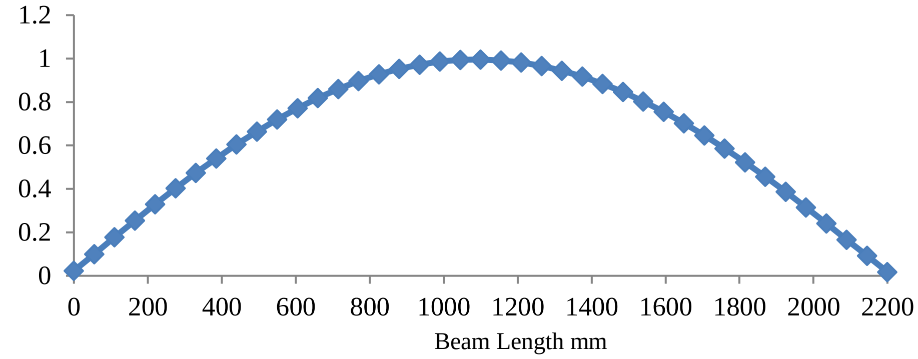


Fig. 25 Curvature distribution along beam length for mode 1

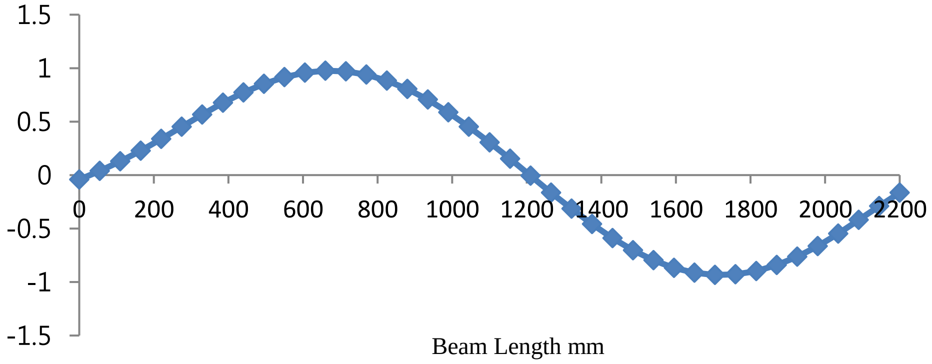


Fig. 26 Curvature distribution along beam length for mode 2

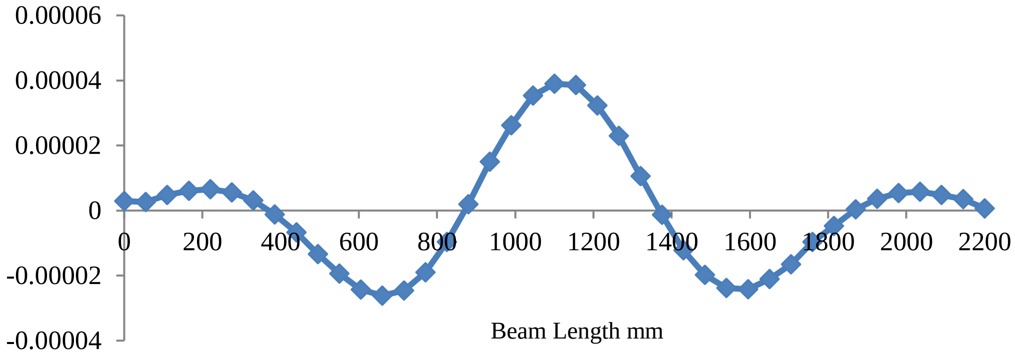


Fig. 27 Curvature distribution along beam length for modes 3

5. Conclusions

From this study, the following conclusions can be drawn:

• Both change in curvature and the fourth derivative of mode shape are satisfactory indicators for locating of singular cracks in the beam.

• The proposed modified forms of the CDF and LSI improve damage detection sensitivity and also overcome the problem of anomalies at the supports. This conclusion is based on a numerical study conducted using finite element analysis.

• Lower modes show improved ability to detect the damage location.

• For the MCDF index based on curvature of mode shape, mode 1 has the ability to locate the damage for different damage cases. This is due to the fact that mode 1 possesses curvature values, i.e., no node or zero curvature, along the length of beam.

• The distribution of curvature values along the length of beam has an effect on the damage localisation index. If there is a node or zero curvature at a particular point, sensitivity to detect the damage is less at the point in reference.

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