

Numerical experimentation for the optimal design for reinforced concrete rectangular combined footings

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Abstract. This paper shows an optimal design for reinforced concrete rectangular combined footings based on a criterion of minimum cost. The classical design method for reinforced concrete rectangular combined footings is: First, a dimension is proposed that should comply with the allowable stresses (Minimum stress should be equal or greater than zero, and maximum stress must be equal or less than the allowable capacity withstand by the soil); subsequently, the effective depth is obtained due to the maximum moment and this effective depth is checked against the bending shear and the punching shear until, it complies with these conditions, and then the steel reinforcement is obtained, but this is not guaranteed that obtained cost is a minimum cost. A numerical experimentation shows the model capability to estimate the minimum cost design of the materials used for a rectangular combined footing that supports two columns under an axial load and moments in two directions at each column in accordance to the building code requirements for structural concrete and commentary (ACI 318S-14). Numerical experimentation is developed by modifying the values of the rectangular combined footing to from “d” (Effective depth), “b” (Short dimension), “a” (Greater dimension), “ ρ_{P1} ” (Ratio of reinforcement steel under column 1), “ ρ_{P2} ” (Ratio of reinforcement steel under column 2), “ ρ_{yLB} ” (Ratio of longitudinal reinforcement steel in the bottom), “ ρ_{yLT} ” (Ratio of longitudinal reinforcement steel at the top). Results show that the optimal design is more economical and more precise with respect to the classical design. Therefore, the optimal design presented in this paper should be used to obtain the minimum cost design for reinforced concrete rectangular combined footings.

Keywords: optimal design; reinforced concrete rectangular combined footings; minimum cost design; moments; bending shear; punching shear

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1. Introduction

The foundation is the part of the structure which transmits the loads to the soil. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems (Das *et al.* 2006, Ha 1993, Luévanos-Rojas *et al.* 2017).

The footings sizes are mostly governed by the axial load and moments, allowable soil pressure, unit weight of concrete, soil unit weight, and the depth of the footing base below the final grade (Al-Ansari 2013, Luévanos-Rojas *et al.* 2017).

The design of superficial solution is done for the following load cases: 1) the footings subjected to concentric axial load, 2) the footings subjected to axial load and moment in one direction (uniaxial bending), 3) the footings subjected to axial load and moment in two directions (biaxial bending) (Bowles 2001, Das *et al.* 2006, Calabera 2000, Tomlinson 2008, McCormac and Brown 2013, González-Cuevas and Robles-Fernandez-Villegas 2005).

A combined footing is a long footing supporting two or more columns in (typically two) one row. The combined footing may be rectangular, trapezoidal or T-shaped in plan. Rectangular footing is provided when one of the projections of the footing is restricted or the width of the footing is restricted. Trapezoidal footing or T-shaped is provided when one column load is much more than the other. As a result, both projections of the footing beyond the faces of the columns will be restricted (Kurian 2005, Punmia *et al.* 2007, Varghese 2009).

Construction practice may dictate using only one footing for two or more columns due to:

- a) Closeness of column (for example around elevator shafts and escalators).
- b) To property line constraint, this may limit the size of footings at boundary. The eccentricity of a column placed on an edge of a footing may be compensated by tying the footing to the interior column.

Conventional method for design of combined footings by rigid method assumes that (Bowles 2001, Das *et al.* 2006, McCormac and Brown 2013, González-Cuevas and Robles-Fernandez-Villegas 2005):

1. The footing or mat is infinitely rigid, and therefore, the deflection of the footing or mat does not influence the pressure distribution.
2. The soil pressure is linearly distributed or the pressure distribution will be uniform, if the centroid of the footing coincides with the resultant of the applied loads acting on foundations.
3. The minimum stress should be equal to or greater than zero, because the soil is not capable of withstand tensile stresses.
4. The maximum stress must be equal or less than the allowable capacity that can withstand the soil.

Optimization of building structures is a prime target for designers and has been investigated by many researchers in the past and its papers are: Optimum design of unstiffened built-up girders (Ha 1993); Shape optimization of RC flexural members (Rath *et al.* 1999); Sensitivity analysis and optimum design curves for the minimum cost design of singly and doubly reinforced concrete beams (Ceranic and Fryer 2000); Optimal design of a welded I-section frame using four conceptually different optimization algorithms (Jarmai *et al.* 2003); New approach to optimization of reinforced concrete beams (Leps and Sejnoha 2003); Cost optimization of singly and doubly reinforced concrete beams with EC2-2001 (Barros *et al.* 2005); Cost optimization of reinforced concrete flat slab buildings (Sahab *et al.* 2005); Multi objective optimization for performance-based design of reinforced concrete frames (Zou *et al.* 2007); Design of optimally

reinforced RC beam, column, and wall sections (Aschheim *et al.* 2008); Optimum design of reinforced concrete columns subjected to uniaxial flexural compression (Bordignon and Kripka 2012); A hybrid CSS and PSO algorithm for optimal design of structures (Kaveh and Talatahari 2012); Structural optimization and proposition of pre-sizing parameters for beams in reinforced concrete buildings (Fleith de Medeiros and Kripka 2013); Constructability optimal design of reinforced concrete retaining walls using a multi-objective genetic algorithm (Kaveh *et al.* 2013); Optimization of modal load pattern for pushover analysis of building structures (Shayanfar *et al.* 2013); Analysis and optimal design of fiber-reinforced composite structures: sail against the wind (Nascimbene 2013); Optimum cost design of RC columns using artificial bee colony algorithm (Ozturk and Durmus 2013); Optimization of a sandwich beam design: analytical and numerical solutions (Awad 2013); Cold-formed steel channel columns optimization with simulated annealing method (Kripka and Chamberlain Pravia 2013); Optimal design of reinforced concrete beams: A review (Rahmanian *et al.* 2014); Optimal design of reinforced concrete plane frames using artificial neural networks (Kao and Yeh 2014a); Cost optimization of reinforced high strength concrete T-sections in flexure (Tiliouine and Fedghouche 2014); Optimal design of plane frame structures using artificial neural networks and ratio variables (Kao and Yeh 2014b); Reliability-based design optimization of structural systems using a hybrid genetic algorithm (Abbasnia *et al.* 2014); The comparative analysis of optimal designed web expanded beams via improved harmony search method (Erdal 2015); Seismic performance and optimal design of framed underground structures with lead-rubber bearings (Chen *et al.* 2016); Nonlinear analysis based optimal design of double-layer grids using enhanced colliding bodies optimization method (Kaveh and Mahdavi 2016a); Numerical experimentation for the optimal design of reinforced rectangular concrete beams for singly reinforced sections (Luévanos-Rojas 2016a); Optimal design of truss structures using a new optimization algorithm based on global sensitivity analysis (Kaveh and Mahdavi 2016b); Probability analysis of optimal design for fatigue crack of aluminium plate repaired with bonded composite patch (Errouane *et al.* 2017).

The main contributions for optimal design of reinforced concrete foundations are: Jiang (1983) investigated the influence of non-uniform soil pressure under the footing slab upon its carrying capacity of the flexural strength of square spread footing. Jiang (1984) closed the paper to thank Gesund for his valuable comment on the upper bound solution of the square spread footing. Hans (1985) studied Flexural collapse loads of eccentrically loaded, individual column footings using the Yield Line Theory; it was found that the cantilever failure mechanism recommended by the ACI Building Code does not give the lowest upper bound on the loads, and Governing equations were derived for mechanisms that led to flexural collapse loads as low as one-half that predicted by the cantilever mechanism for some column/footing combinations. Wang and Kulhawy (2008) developed a design approach that explicitly considers the construction economics and results in a foundation that has the minimum construction cost, and this design approach is expressed as an optimization process, in which the objective is to minimize construction cost, with the design parameters and design requirements as the optimization variables and constraints, respectively. Al-Ansari (2013) presented an analytical model to estimate the cost of an optimized design of reinforced concrete isolated footing base on structural safety. Flexural and optimized formulas for square and rectangular footing are derived base on ACI building code of design, material cost, and optimization. Khajehzadeh *et al.* (2014) introduced a novel optimization technique based on gravitational search algorithm (GSA) for numerical optimization and multi-objective optimization of the foundation, and in the proposed method is applied a chaotic time-varying system into the position updating equation to increase the global exploration ability and accurate local exploitation

of the original algorithm. López-Chavarría *et al.* (2017a) shown optimal dimensioning for the corner combined footings to obtain the most economical contact surface on the soil (optimal area), due to an axial load, moment around of the axis “X” and moment around of the axis “Y” applied to each column; The proposed model considers soil real pressure, i.e., the pressure varies linearly. Luévanos-Rojas *et al.* (2017) presented an optimal design for reinforced concrete rectangular footings using the new model, also a numerical experimentation is shown to show the model capability to estimate the minimum cost design of the materials used for a rectangular footing that supports an axial load and moments in two directions in accordance to the building code requirements for structural concrete and commentary (ACI 318-13). López-Chavarría *et al.* (2017b) shown a mathematical model for dimensioning of square footings using optimization techniques (general case), i.e., the column is localized anywhere of the footing to obtain the most economical contact surface on the soil, when the load that must support said structural member is applied (axial load and moments in two directions).

Some papers that present the equations to obtain the design of footings are: Design of isolated footings of rectangular form using a new model (Luévanos-Rojas *et al.* 2013); Design of isolated footings of circular form using a new model (Luévanos-Rojas 2014a); Design of boundary combined footings of rectangular shape using a new model (Luévanos-Rojas 2014b); Design of boundary combined footings of trapezoidal form using a new model (Luévanos-Rojas 2015); A comparative study for the design of rectangular and circular isolated footings using new models (Luévanos-Rojas 2016b); A new model for the design of rectangular combined footings of boundary with two opposite sides restricted (Luévanos-Rojas 2016c); A new mathematical model for design of square isolated footings for general case (López-Chavarría *et al.* 2017c). These papers present only the design equations and numerical examples of the footings, but the optimal design is not shown.

This paper shows an optimal design for reinforced concrete rectangular combined footings based on a criterion of minimum cost due to an axial load, moment around of the axis “X” and moment around of the axis “Y” applied to each column. The proposed model considers soil real pressure, i.e., the pressure varies linearly. The classical model is developed by trial and error, i.e., a dimension is proposed, and after, using the equation of the biaxial bending is obtained the stress acting on each vertex of the rectangular combined footing, which must meet the conditions following: 1) Minimum stress should be equal or greater than zero, because the soil is not withstand tensile. 2) Maximum stress must be equal or less than the allowable capacity that can be capable of withstand the soil. The paper presents a numerical example for a property line adjacent to illustrate the validity of the optimization techniques to obtain the optimal design due to the minimum cost of the reinforced concrete rectangular combined footings under an axial load and moments in two directions applied to each column.

2. Methodology

According to Building Code Requirements for Structural Concrete and Commentary (ACI 318S-14 2014), the critical sections are: 1) the maximum moment is located in face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall; 2) bending shear is presented at a distance “ d ” (distance from extreme compression fiber to centroid of longitudinal tension reinforcement) shall be measured from face of column, pedestal, or wall for footings supporting a column, pedestal, or wall; 3) punching shear is localized so that its perimeter “ b_o ” is a

minimum but need not approach closer than “ $d/2$ ” to: (a) Edges or corners of columns, concentrated loads, or reaction areas; and (b) Changes in slab thickness such as edges of capitals, drop panels, or shear caps.

2.1 Equations for the dimensioning of rectangular combined footings

Fig. 1 shows a combined footing supporting two rectangular columns of different dimensions (a boundary column and other inner column) subject to axial load and moments in two directions (bidirectional bending) each column.

The general equation for any type of footings subjected to bidirectional bending (Luévanos-Rojas *et al.* 2013, 2017, Luévanos-Rojas 2014a, b, c, 2015, 2016b, c, López-Chavarría *et al.* 2017a, b, c, Gere and Goodno 2009)

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} \quad (1)$$

where: σ is the stress exerted by the soil on the footing (soil pressure), A is the contact area of the footing, P is the axial load applied at the center of gravity of the footing, M_x is the moment around the axis “ X ”, M_y is the moment around the axis “ Y ”, x is the distance in the direction “ X ” measured from the axis “ Y ” to the fiber under study taking into account the direction of the axis, y is the distance in direction “ Y ” measured from the axis “ X ” to the fiber under study considering the direction of the axis, I_y is the moment of inertia around the axis “ Y ” and I_x is the moment of inertia around the axis “ X ”. The moments in the clockwise direction are positive.

The general equation of the bidirectional bending is transformed as follows (Luévanos-Rojas 2014b)

$$\sigma = \frac{R}{ab} + \frac{6Ry_c}{ba^2} + \frac{6Rx_c}{ab^2} \leq \sigma_{adm} \quad (2)$$

where: σ_{adm} is the capacity of available allowable load of the soil, R is the resultant force of the loads, y_c is the distance from the center of the contact area of the footing in the direction “ Y ” to the resultant, x_c is the distance from the center of the contact area of the footing in the direction “ X ” to the resultant.

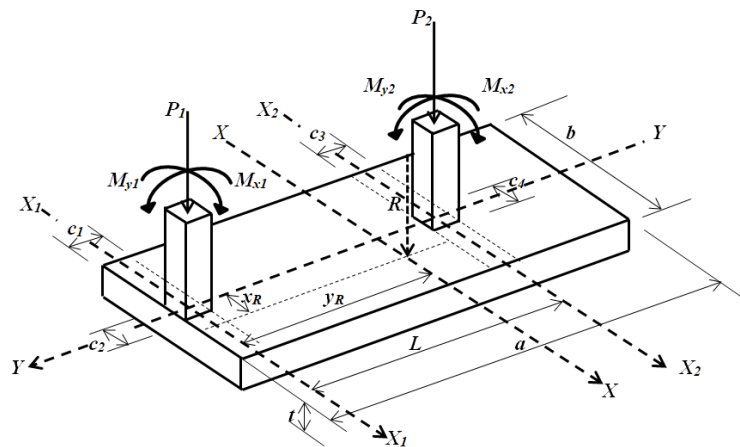


Fig. 1 Boundary combined footing of rectangular shape

Now the sum of moments around the axis “ X_I ” is obtained to find “ y_R ” and the resultant force is made to coincide with the gravity center of the area of the footing with the position of the resultant force in the direction “ Y ”, therefore there is not moment around the axis “ X ” and the value of “ y_c ” is zero, “ $x_R = x_c$ ” is the sum of moments around the axis “ Y ” divided by the resultant, thus the values of “ y_R ” and “ x_R ” are (Luévanos-Rojas 2014b)

$$x_R = \frac{M_{y1} + M_{y2}}{R} \quad (3)$$

$$y_R = \frac{P_2 L - M_{x1} - M_{x2}}{R} \quad (4)$$

Now, the resultant force is made to coincide with the gravity center of the area of the footing with the position of the resultant force in the direction “ Y ”. Thus the value of “ a ” is (Luévanos-Rojas 2014b)

$$a = 2 \left(\frac{c_1}{2} + y_R \right) \quad (5)$$

Substituting the Eq. (4) into Eq. (5) is obtained

$$a = 2 \left(\frac{c_1}{2} + \frac{P_2 L - M_{x1} - M_{x2}}{R} \right) \quad (6)$$

Now, substituting “ $y_c = 0$ ”, and the Eqs. (3) and (6) into Eq. (2) is obtained (Luévanos-Rojas 2014b)

$$b = \frac{R + \sqrt{R^2 + 24 \sigma_{adm} a (M_{y1} + M_{y2})}}{2 \sigma_{adm} a} \quad (7)$$

Note: the values of b and a must be the minimum values.

The capacity of available allowable load of the soil “ σ_{adm} ” is (Luévanos-Rojas 2014b)

$$\sigma_{adm} = q_a - \gamma_{ppz} - \gamma_{pps} \quad (8)$$

where: q_a is the allowable load capacity of the soil, γ_{ppz} is the self-weight of the footing in square meter, γ_{pps} is the self-weight of soil fill in square meter.

Note: if in the combinations are included the wind and/or the earthquake, the allowable load capacity of the soil can be increased by 33% (ACI 318S-14 2014).

Also the Eq. (19) could be presented (Luévanos-Rojas 2014b)

$$\sigma_{adm} = q_a - \gamma_c (d + r) - \gamma_g (H - d - r) \quad (9)$$

where: γ_c is concrete density = 24 kN/m^3 , γ_g is soil density, d is the footing effective depth, r is the footing coating and H is the depth of the footing base below the final grade.

2.2 Equations for the design of rectangular combined footings

2.2.1 Equations for the moments

Critical sections for moments are presented in sections $a_1'-a_1'$, $a_2'-a_2'$, $b'-b'$, $c'-c'$, $d'-d'$ and $e'-e'$ (see Fig. 2).

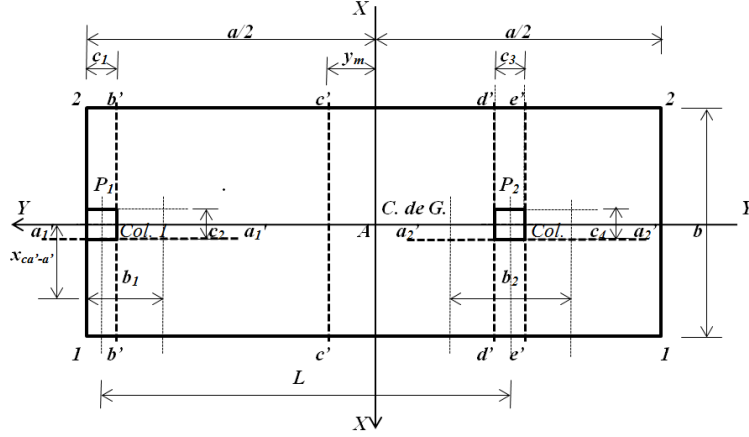


Fig. 2 Critical sections for moments

Moment “ M_{a1} ” acting around the axis $a_1'-a_1'$ is (Luévanos-Rojas 2014b)

$$M_{a_1'} = \frac{(b-c_2)^2 [P_{u1}b^2 + 2M_{uy1}(2b+c_2)]}{8b^3} \quad (10)$$

Moment “ M_{a2} ” acting around the axis $a_2'-a_2'$ is (Luévanos-Rojas 2014b)

$$M_{a_2'} = \frac{(b-c_4)^2 [P_{u2}b^2 + 2M_{uy2}(2b+c_4)]}{8b^3} \quad (11)$$

where: P_{u1} and P_{u2} are loads factored acting on the footing; M_{uy1} and M_{uy2} are moments factored acting on the footing.

Moment “ M_b ” acting around the axis $b'-b'$ is (Luévanos-Rojas 2014b)

$$M_{b'} = \frac{(P_{u1}a - R_u c_1)c_1}{2a} \quad (12)$$

where: R_u is the resultant force of the loads factored acting on the footing

Moment “ M_c ” acting around the axis $c'-c'$ is (Luévanos-Rojas 2014b)

$$M_{c'} = \frac{P_{u1}(P_{u1}a - R_u c_1)}{2R_u} \quad (13)$$

Moment “ M_d ” acting around the axis $d'-d'$ is (Luévanos-Rojas, 2014b)

$$M_{d'} = P_{u1} \left(L - \frac{c_3}{2} \right) - \frac{R_u}{2a} \left(L + \frac{c_1 - c_3}{2} \right)^2 \quad (14)$$

Moment “ M_e ” acting around the axis $e'-e'$ is (Luévanos-Rojas 2014b)

$$M_{e'} = P_{u1} \left(L + \frac{c_3}{2} \right) + \frac{P_{u2}c_3}{2} - \frac{R_u}{2a} \left(L + \frac{c_1 + c_3}{2} \right)^2 \quad (15)$$

2.2.2 Equations for the bending shear

Critical sections for bending shear are obtained at a distance “ d ” to from face of the column with the

footing are presented in sections $f_1'-f_1'$, $f_2'-f_2'$, $g'-g'$, $h'-h'$ and $i'-i'$ (see Fig. 3).

Bending shear " $V_{ff_1'}$ " acting on the axis $f_1'-f_1'$ is (Luévanos-Rojas 2014b)

$$V_{ff_1'} = \frac{P_{u1}(b-c_2-2d)}{2b} + \frac{3M_{uy1}[b^2-(c_2+2d)^2]}{2b^3} \quad (16)$$

Bending shear " $V_{ff_2'}$ " acting on the axis $f_2'-f_2'$ is (Luévanos-Rojas 2014b)

$$V_{ff_2'} = \frac{P_{u2}(b-c_4-2d)}{2b} + \frac{3M_{uy2}[b^2-(c_4+2d)^2]}{2b^3} \quad (17)$$

Bending shear " $V_{fg'}$ " acting on the axis $g'-g'$ is (Luévanos-Rojas 2014b)

$$V_{fg'} = P_{u1} - \frac{R_u(c_1+d)}{a} \quad (18)$$

Bending shear " $V_{fh'}$ " acting on the axis $h'-h'$ is (Luévanos-Rojas 2014b)

$$V_{fh'} = P_{u1} - \frac{R_u}{a} \left(L + \frac{c_1-c_3}{2} - d \right) \quad (19)$$

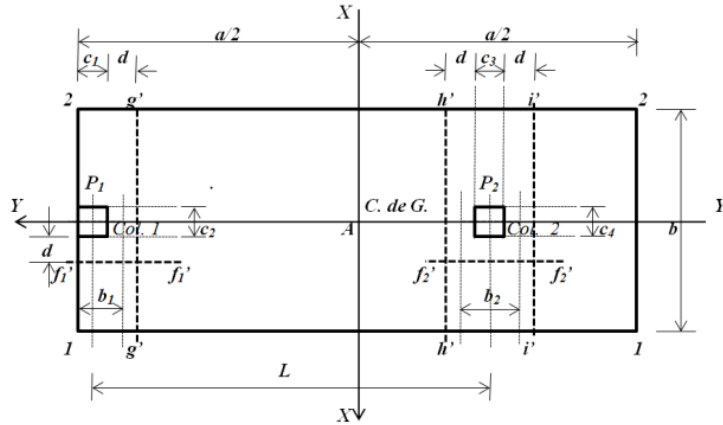


Fig. 3 Critical sections for bending shear

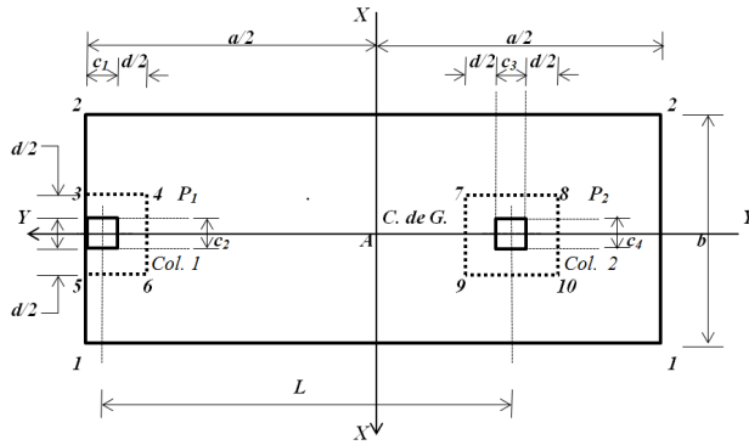


Fig. 4 Critical sections for punching shear

Bending shear “ V_{fi} ” acting on the axis $i'-i'$ is (Luévanos-Rojas 2014b)

$$V_{fi} = R_u - \frac{R_u}{a} \left(L + \frac{c_1 + c_3}{2} + d \right) \quad (20)$$

2.2.3 Equations for the punching shear

Critical section for the punching shear appears at a distance “ $d/2$ ” to from face of the column with the footing in the two directions in section formed by points 3, 4, 5 and 6 for boundary column, and points 7, 8, 9 and 10 for inner column (see Fig. 4).

Punching shear for boundary column “ V_{p1} ” acting on the footing is the force “ P_{u1} ” which acting on column 1 less the pressure volume of the area formed by the points 3, 4, 5 and 6 (Luévanos-Rojas 2014b)

$$V_{p1} = P_{u1} - \frac{R_u(c_1 + d/2)(c_2 + d)}{ab} \quad (21)$$

Punching shear for inner column “ V_{p2} ” acting on the footing is the force “ P_{u2} ” which acting on column 2 less the pressure volume of the area formed by the points 7, 8, 9 and 10 (Luévanos-Rojas 2014b)

$$V_{p2} = P_{u2} - \frac{R_u(c_3 + d)(c_4 + d)}{ab} \quad (22)$$

2.3 Equations by American concrete institute

Equations for moment in both axes are considered at the face of the column are (ACI 318S-14 2014, Luévanos-Rojas 2016a)

$$M_u = \phi_f b_w d^2 \rho f_y \left(1 - \frac{0.59 \rho f_y}{f'_c} \right) \quad (23)$$

$$\rho = \frac{A_s}{b_w d} \quad (24)$$

$$\rho_b = \frac{0.85 \beta_1 f'_c}{f_y} \left(\frac{600}{600 + f_y} \right) \quad (25)$$

$$0.65 \leq \beta_1 = \left(1.05 - \frac{f'_c}{140} \right) \leq 0.85 \quad (26)$$

$$\rho_{\max} = 0.75 \rho_b \quad (27)$$

$$\rho_{\min} = \begin{cases} \frac{0.25 \sqrt{f'_c}}{f_y} \\ \frac{1.4}{f_y} \end{cases} \quad (28)$$

$$A_{st} = 0.0018 b_w t \quad (29)$$

where: M_u is the factored maximum moment, ϕ_f is the strength reduction factor by bending and its value is 0.90, b_w is width of analysis in structural member, ρ is ratio of A_s to bd , β_1 is the factor relating depth of

equivalent rectangular compressive stress block to neutral axis depth, f_y is the specified yield strength of reinforcement of steel, f'_c is the specified compressive strength of concrete at 28 days, A_{st} is the area of reinforcement steel by temperature, t is the total thickness of the footing.

Required strength U to resist factored loads or related internal moments and forces is (ACI 318-14 2014)

$$U = 1.2D + 1.6L \quad (30)$$

where: D are the dead loads, or related internal moments and forces, L are the live loads, or related internal moments and forces.

Equation for the bending shear (unidirectional shear force) is considered at a distance “ d ” to from of column face is (ACI 318-14 2014)

$$\phi_v V_{cf} = 0.17\phi_v \sqrt{f'_c} b_w d \quad (31)$$

where: V_{cf} is bending shear resisting by concrete; ϕ_v is the strength reduction factor by shear is 0.85.

Equations for the punching shear (shear force bidirectional) appears at a distance “ $d/2$ ” to from of column face on the footing in the two directions are shown (ACI 318-14 2014)

$$\phi_v V_{cp} = 0.17\phi_v \left(1 + \frac{2}{\beta_c}\right) \sqrt{f'_c} b_0 d \quad (32a)$$

$$\phi_v V_{cp} = 0.083\phi_v \left(\frac{\alpha_s d}{b_0} + 2\right) \sqrt{f'_c} b_0 d \quad (32b)$$

$$\phi_v V_{cp} = 0.33\phi_v \sqrt{f'_c} b_0 d \quad (32c)$$

where: V_{cp} is punching shear resisting, β_c is the ratio of long side to short side of the column, b_0 is the perimeter of the critical section, α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns. $\phi_v V_{cp}$ must be the largest value of Eqs. (32(a))-(32(c)). For boundary column $b_0 = 2c_1 + c_2 + 2d$, and for inner column $b_0 = 2c_3 + 2c_4 + 4d$.

2.4 Objective function to minimize the cost

A cost function is defined as the total cost “ C_t ” which is equal to cost of flexural reinforcement more the cost of concrete. These costs involve material costs and fabrication costs, respectively. The cost of the rectangular footing is

$$C_t = V_c C_c + V_s \gamma_s C_s \quad (33)$$

where: C_c is cost of concrete for 1 m^3 of ready mix reinforced concrete in dollars, C_s is cost of reinforcement steel for 1 kN of steel in dollars, V_s is volume of reinforcement steel, V_c is volume of concrete and γ_s is steel density = 76.94 kN/m^3 .

The volumes for rectangular footings are

$$V_s = (A_{syLT} + A_{syLB})a + (A_{sxTT} + A_{sP1} + A_{sP2} + A_{sxBT})b \quad (34)$$

$$V_c = abt - (A_{syLT} + A_{syLB})a - (A_{sxTT} + A_{sP1} + A_{sP2} + A_{sxBT})b \quad (35)$$

where: t is the total thickness of the footing, A_{syLT} is the area of longitudinal reinforcement steel at the top (direction of axis “Y”), A_{syLB} is the area of longitudinal reinforcement steel in the bottom (direction of axis “Y”), A_{sxTT} is the area of reinforcement steel at the top with a width a (direction of axis “X”), A_{sP1} is the

area of reinforcement steel at the bottom of the column 1 with a width b_1 (direction of axis “X”), A_{sP2} is the area of reinforcement steel at the bottom of the column 2 with a width b_2 (direction of axis “X”), A_{sxBT} is the area of reinforcement steel at the bottom of the surplus b_1 and b_2 with a width $a - b_1 - b_2$ (direction of axis “X”).

Substituting Eqs. (34) and (35) into Eq. (33) is obtained

$$C_t = C_c [abt - (A_{syLT} + A_{syLB})a - (A_{sxTT} + A_{sP1} + A_{sP2} + A_{sxBT})b] + \gamma_s C_s [(A_{syLT} + A_{syLB})a + (A_{sxTT} + A_{sP1} + A_{sP2} + A_{sxBT})b] \quad (36)$$

Substituting $\alpha = \gamma_s C_s / C_c \rightarrow \gamma_s C_s = \alpha C_c$ into Eq. (36) is presented

$$C_t = C_c \{ ab(d+r) - [(A_{syLT} + A_{syLB})a + (A_{sxTT} + A_{sP1} + A_{sP2} + A_{sxBT})b] (1 - \alpha) \} \quad (37)$$

2.5 Constraint functions

Equations for the dimensioning of rectangular combined footings are

$$a \geq 2 \left(\frac{c_1}{2} + \frac{P_2 L - M_{x1} - M_{x2}}{R} \right) \quad (38)$$

$$b \geq \frac{R + \sqrt{R^2 + 24[q_a - \gamma_c(d+r) - \gamma_g(H-d-r)]a(M_{y1} + M_{y2})}}{2[q_a - \gamma_c(d+r) - \gamma_g(H-d-r)]a} \quad (39)$$

Equations for the design of rectangular combined footings are

$$\frac{(b-c_2)^2 [P_{u1} b^2 + 2M_{uy1}(2b+c_2)]}{8b^3 \phi_f f_y} \leq d A_{sP1} \left(1 - \frac{0.59 A_{sP1} f_y}{b_1 d f'_c} \right) \quad (40)$$

$$\frac{(b-c_4)^2 [P_{u2} b^2 + 2M_{uy2}(2b+c_4)]}{8b^3 \phi_f f_y} \leq d A_{sP2} \left(1 - \frac{0.59 A_{sP2} f_y}{b_2 d f'_c} \right) \quad (41)$$

$$\frac{(P_{u1} a - R_u c_1) c_1}{2a \phi_f f_y} \leq d A_{syLB} \left(1 - \frac{0.59 A_{syLB} f_y}{b d f'_c} \right) \quad (42)$$

$$\frac{P_{u1} (P_{u1} a - R_u c_1)}{2R_u \phi_f f_y} \leq d A_{syLT} \left(1 - \frac{0.59 A_{syLT} f_y}{b d f'_c} \right) \quad (43)$$

$$\frac{2P_{u1} a \left(L - \frac{c_3}{2} \right) - R_u \left(L + \frac{c_1 - c_3}{2} \right)^2}{2a \phi_f f_y} \leq d A_{syLB} \left(1 - \frac{0.59 A_{syLB} f_y}{b d f'_c} \right) \quad (44)$$

$$\frac{2P_{u1} a \left(L + \frac{c_3}{2} \right) + P_{u2} a c_3 - R_u \left(L + \frac{c_1 + c_3}{2} \right)^2}{2a \phi_f f_y} \leq d A_{syLB} \left(1 - \frac{0.59 A_{syLB} f_y}{b d f'_c} \right) \quad (45)$$

$$\frac{P_{u1} (b - c_2 - 2d)}{2b} + \frac{3M_{uy1} [b^2 - (c_2 + 2d)^2]}{2b^3} \leq 0.17 \phi_v \sqrt{f'_c} b_1 d \quad (46)$$

$$\frac{P_{u2} (b - c_4 - 2d)}{2b} + \frac{3M_{uy2} [b^2 - (c_4 + 2d)^2]}{2b^3} \leq 0.17 \phi_v \sqrt{f'_c} b_2 d \quad (47)$$

$$\frac{P_{u1}a - R_u(c_1 + d)}{a} \leq 0.17\phi_v \sqrt{f'_c} bd \quad (48)$$

$$\frac{P_{u1}a - R_u\left(L + \frac{c_1 - c_3}{2} - d\right)}{a} \leq 0.17\phi_v \sqrt{f'_c} bd \quad (49)$$

$$\frac{R_u\left(a - L - \frac{c_1 + c_3}{2} - d\right)}{a} \leq 0.17\phi_v \sqrt{f'_c} bd \quad (50)$$

$$\frac{P_{u1}ab - R_u(c_1 + d/2)(c_2 + d)}{ab} \leq \begin{cases} 0.17\phi_v \left(1 + \frac{2}{\beta_c}\right) \sqrt{f'_c} (2c_1 + c_2 + 2d)d \\ 0.083\phi_v \left(\frac{\alpha_s d}{b_0} + 2\right) \sqrt{f'_c} (2c_1 + c_2 + 2d)d \\ 0.33\phi_v \sqrt{f'_c} (2c_1 + c_2 + 2d)d \end{cases} \quad (51)$$

$$\frac{P_{u2}ab - R_u(c_3 + d)(c_4 + d)}{ab} \leq \begin{cases} 0.17\phi_v \left(1 + \frac{2}{\beta_c}\right) \sqrt{f'_c} [2(c_3 + c_4 + 2d)]d \\ 0.083\phi_v \left(\frac{\alpha_s d}{b_0} + 2\right) \sqrt{f'_c} [2(c_3 + c_4 + 2d)]d \\ 0.33\phi_v \sqrt{f'_c} [2(c_3 + c_4 + 2d)]d \end{cases} \quad (52)$$

$$\rho_{P1}, \rho_{P2}, \rho_{xTT}, \rho_{xBT} \leq 0.75 \left[\frac{0.85\beta_1 f'_c}{f_y} \left(\frac{600}{600 + f_y} \right) \right] \quad (53)$$

$$\rho_{P1}, \rho_{P2}, \rho_{yLB}, \rho_{yLT} \geq \begin{cases} \frac{0.25\sqrt{f'_c}}{f_y} \\ \frac{1.4}{f_y} \end{cases} \quad (54)$$

$$A_{sP1} = \rho_{P1} b_1 d \quad (55)$$

$$A_{sP2} = \rho_{P2} b_2 d \quad (56)$$

$$A_{syLB} = \rho_{yLB} b d \quad (57)$$

$$A_{syLT} = \rho_{yLT} b d \quad (58)$$

$$A_{sxTB} = 0.0018(a - b_1 - b_2)d \quad (59)$$

$$A_{sxTT} = 0.0018ad \quad (60)$$

where: $b_1 = c_1 + d/2$, and $b_2 = c_3 + d$.

3. Numerical experimentation

Design of a reinforced concrete rectangular combined footing supporting two square columns with a boundary column, and another inner column (see Fig. 1), and the basic information following is: $c_1 = 40 \times 40$ cm; $c_2 = 40 \times 40$ cm; $L = 6.00$ m; $H = 1.5$ m; $M_{Dx1} = 140$ kN-m; $M_{Lx1} = 100$ kN-m; $M_{Dy1} = 120$ kN-m; $M_{Ly1} = 80$ kN-m; $P_{D1} = 700$ kN; $P_{L1} = 500$ kN; $M_{Dx2} = 280$ kN-m; $M_{Lx2} = 200$ kN-m; $M_{Dy2} = 240$ kN-m; $M_{Ly2} = 160$ kN-m; $P_{D2} = 1400$ kN; $P_{L2} = 1000$ kN; $f'_c = 21$

MPa; $f_y = 420$ MPa; $q_a = 220$ kN/m²; $\gamma_c = 24$ kN/m³; $\gamma_g = 15$ kN/m³. It is assumed that $r = 8$ cm, and the ratio of reinforcement steel cost to concrete cost is: $\alpha = 90$.

Loads and moments acting on the rectangular combined footing due to columns are: $P_1 = 1200$ kN, $M_{x1} = 240$ kN-m, $M_{y1} = 200$ kN-m, $P_2 = 2400$ kN, $M_{x2} = 480$ kN-m, $M_{y2} = 400$ kN-m, $R = 3600$ kN.

Loads and moments acting on the rectangular combined footing due to columns by Eq. (30) are factored: $P_{u1} = 1640$ kN, $M_{ux1} = 328$ kN-m, $M_{uy1} = 272$ kN-m, $P_{u2} = 3280$ kN, $M_{ux2} = 656$ kN-m, $M_{uy2} = 544$ kN-m, $R_u = 4920$ kN.

Substituting corresponding values into Eq. (37) to obtain the objective function and also into Eqs. (38) to (60) to find the constraints, these are:

Minimize

$$C_t = C_c \{ ab(d + 0.08) + 89[(A_{syLT} + A_{syLB})a + (A_{sxTT} + A_{sP1} + A_{sP2} + A_{sxTB})b] \} \quad (61)$$

Subject to

For the dimensioning

$$a \geq 8.00 \quad (62)$$

$$b \geq \frac{3600 + \sqrt{12960000 + 2833632a - 129600ad}}{393.56a - 18ad} \quad (63)$$

For the design

$$\frac{(5b-2)^2(1025b^2+680b+136)}{b^3} \leq 47250000dA_{sP1} \left[1 - \frac{11.8A_{sP1}}{(0.4+0.5d)d} \right] \quad (64)$$

$$\frac{(5b-2)^2(1025b^2+680b+136)}{b^3} \leq 23625000dA_{sP2} \left[1 - \frac{11.8A_{sP2}}{(0.4+d)d} \right] \quad (65)$$

$$\frac{5a-6}{a} \leq \frac{236250}{41} dA_{syLB} \left(1 - \frac{11.8A_{syLB}}{bd} \right) \quad (66)$$

$$5a - 6 \leq \frac{283500}{41} dA_{syLT} \left(1 - \frac{11.8A_{syLT}}{bd} \right) \quad (67)$$

$$\frac{29a-270}{a} \leq \frac{47250}{41} dA_{syLB} \left(1 - \frac{11.8A_{syLB}}{bd} \right) \quad (68)$$

$$\frac{55a-512}{a} \leq \frac{78750}{41} dA_{syLB} \left(1 - \frac{11.8A_{syLB}}{bd} \right) \quad (69)$$

$$\frac{5125b^3+250b^2(2-41d)-408(5d+1)^2}{b^3} \leq \frac{1445\sqrt{21}(5d+4)d}{16} \quad (70)$$

$$\frac{5125b^3+250b^2(2-41d)-408(5d+1)^2}{b^3} \leq \frac{1445\sqrt{21}(5d+2)d}{16} \quad (71)$$

$$\frac{5a-3(5d+2)}{a} \leq \frac{289\sqrt{21}bd}{656} \quad (72)$$

$$\frac{a+3(d-6)}{a} \leq \frac{289\sqrt{21}bd}{3280} \quad (73)$$

$$\frac{5a-5d-32}{a} \leq \frac{289\sqrt{21}bd}{1968} \quad (74)$$

$$\frac{50ab-3(5d+2)(5d+4)}{ab} \leq \begin{cases} \frac{867\sqrt{21}(5d+3)d}{164} \\ \frac{1411\sqrt{21}(85d+6)d}{1640} \\ \frac{561\sqrt{21}(5d+3)d}{164} \end{cases} \quad (75)$$

$$\frac{50ab-3(5d+2)^2}{ab} \leq \begin{cases} \frac{867\sqrt{21}(5d+2)d}{164} \\ \frac{1411\sqrt{21}(15d+1)d}{410} \\ \frac{561\sqrt{21}(5d+2)d}{164} \end{cases} \quad (76)$$

$$\rho_{P1}, \rho_{P2}, \rho_{yLB}, \rho_{yLT} \leq 0.0159375 \quad (77)$$

$$\rho_{P1}, \rho_{P2}, \rho_{yLB}, \rho_{yLT} \geq 0.0033333 \quad (78)$$

$$A_{sP1} = \rho_{P1}(0.4 + 0.5d)d \quad (79)$$

$$A_{sP2} = \rho_{P2}(0.4 + d)d \quad (80)$$

$$A_{syLB} = \rho_{yLB}bd \quad (81)$$

$$A_{syLT} = \rho_{yLT}bd \quad (82)$$

$$A_{sxTB} = 0.0018(a - 0.8 - 1.5d)d \quad (83)$$

$$A_{sxTT} = 0.0018ad \quad (84)$$

Assume all variables nonnegative

Tables 1-7 show the results using the optimization techniques for the design of a reinforced concrete rectangular combined footing; the objective function (minimum cost) by Eq. (61) is obtained, and constraint functions by Eqs. (62) to (84) are found, and the minimum cost for the design of the rectangular combined footing is obtained using the MAPLE-15 software, and it is assumed that the dimensions (a, b, d), the ratios of reinforcement steel (ρ_{P1} , ρ_{P2} , ρ_{yLB} , ρ_{yLT}), and the areas of reinforcement steel (A_{sP1} , A_{sP2} , A_{syLB} , A_{syLT} , A_{sxTB} , A_{sxTT}) are nonnegative.

Also results are verified by the classical design method using the Eqs. (6) to (32(a)-32(c)).

Table 1 shows, when the effective depth “d” of the rectangular combined footing varies, taking into account the values of 79.52, 80.00, 90.00, 100.00, 110.00 and 120.00 cm.

Table 2 presents, when the short dimension “b” of the rectangular combined footing changes, taking into account the values of b = 270.00, 280.00, 282.34, 290.00, 300.00 and 310.00 cm.

Table 3 shows, when the greater dimension “a” of the rectangular combined footing modifies, taking into account the values of a = 870.00, 890.00, 910.00, 910.56, 920.00 and 940.00 cm.

Table 1 Effective depth “d” of the rectangular combined footing is changed

a	b	d	ρ_{P1}	A_{sP1}	ρ_{P2}	A_{sP2}	ρ_{yLB}	A_{syLB}	ρ_{yLT}	A_{syLT}	A_{sxTB}	A_{sxTT}	C_t
cm	cm	cm		cm ²		cm ²		cm ²		cm ²	cm ²	cm ²	(\$)
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
898.74	285.28	80.00	0.00333	21.33	0.00333	32.00	0.00333	76.07	0.00333	76.07	100.65	129.45	41.94C _c
800.00	314.37	90.00	0.00333	25.50	0.00333	39.00	0.00333	94.31	0.00333	94.31	94.77	129.60	46.16C _c
800.00	315.58	100.00	0.00333	30.00	0.00333	46.67	0.00333	105.19	0.00333	105.19	102.60	144.00	51.32C _c
800.00	316.80	110.00	0.00333	34.83	0.00333	55.00	0.00333	116.16	0.00333	116.16	109.89	158.40	56.55C _c
800.00	318.04	120.00	0.00333	40.00	0.00333	64.00	0.00333	127.21	0.00333	127.21	116.64	172.80	61.82C _c

Table 2 Short dimension “b” of the rectangular combined footing is modified

a	b	d	ρ_{P1}	A_{sP1}	ρ_{P2}	A_{sP2}	ρ_{yLB}	A_{syLB}	ρ_{yLT}	A_{syLT}	A_{sxTB}	A_{sxTT}	C_t
cm	cm	cm		cm ²		cm ²		cm ²		cm ²	cm ²	cm ²	(\$)
962.61	270.00	77.40	0.00333	20.30	0.00333	30.29	0.00333	69.66	0.00395	82.57	106.79	134.11	42.24C _c
920.02	280.00	79.12	0.00333	20.98	0.00333	31.42	0.00333	73.85	0.00344	76.20	102.74	131.03	41.86C _c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
880.92	290.00	80.76	0.00333	21.64	0.00333	32.51	0.00333	78.07	0.00333	78.07	98.82	128.06	42.17C _c
844.90	300.00	82.32	0.00333	22.27	0.00333	33.56	0.00333	82.32	0.00333	82.32	95.04	125.19	42.64C _c
811.62	310.00	83.80	0.00333	22.88	0.00333	34.58	0.00333	86.59	0.00333	86.59	91.40	122.42	43.09C _c

Table 3 Greater dimension “a” of the rectangular combined footing is changed

a	b	d	ρ_{P1}	A_{sP1}	ρ_{P2}	A_{sP2}	ρ_{yLB}	A_{syLB}	ρ_{yLT}	A_{syLT}	A_{sxTB}	A_{sxTT}	C_t
cm	cm	cm		cm ²		cm ²		cm ²		cm ²	cm ²	cm ²	(\$)
870.00	292.95	81.23	0.00333	21.83	0.00333	32.82	0.00333	79.32	0.00333	79.32	97.69	127.21	42.31C _c
890.00	287.60	80.38	0.00333	21.48	0.00333	32.25	0.00333	77.06	0.00333	77.06	99.75	128.77	42.05C _c
910.00	282.48	79.54	0.00333	21.15	0.00333	31.69	0.00333	74.89	0.00333	74.89	101.75	130.29	41.80C _c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
920.00	280.00	79.13	0.00333	20.98	0.00333	31.42	0.00333	73.85	0.00344	76.20	102.73	131.03	41.86C _c
940.00	275.20	78.31	0.00333	20.66	0.00333	30.88	0.00333	71.83	0.00367	79.15	104.66	132.50	42.03C _c

Table 4 presents, when the ratios of reinforcement steel “ ρ_{P1} ” of the rectangular combined footing changes, taking into account the values of $\rho_{P1} = 0.00333, 0.00500, 0.01000, 0.01250, 0.01500$ and 0.01594 .

Table 5 shows, when the ratios of reinforcement steel “ ρ_{P2} ” of the rectangular combined footing changes, taking into account the values of $\rho_{P2} = 0.00333, 0.00500, 0.01000, 0.01250, 0.01500$ and 0.01594 .

Table 6 presents, when the ratios of reinforcement steel “ ρ_{yLB} ” of the rectangular combined footing changes, taking into account the values of $\rho_{yLB} = 0.00333, 0.00500, 0.01000, 0.01250, 0.01500$ and 0.01594 .

Table 7 shows, when the ratios of reinforcement steel “ ρ_{yLT} ” of the rectangular combined footing changes, taking into account the values of $\rho_{yLT} = 0.00333, 0.00500, 0.01000, 0.01250, 0.01500$ and 0.01594 .

This problem assumes that the constant parameters are: $P_1, M_{x1}, M_{y1}, P_2, M_{x2}, M_{y2}, c_1, c_2, c_3, c_4, L, \sigma_{adm}, R, P_{u1}, M_{ux1}, M_{uy1}, P_{u2}, M_{ux2}, M_{uy2}, R_u, q_a, \gamma_c, \gamma_s, f'_c, f_y, \alpha, r, H$, and the decision variables are: $a, b, d, \rho_{P1}, \rho_{P2}, \rho_{yLB}, \rho_{yLT}, A_{sP1}, A_{sP2}, A_{syLB}, A_{syLT}, A_{sxTB}, A_{sxTT}$.

Table 4 Ratios of reinforcement steel “ ρ_{P1} ” of the rectangular combined footing is modified

a	b	d	ρ_{P1}	A_{sP1} cm ²	ρ_{P2}	A_{sP2} cm ²	ρ_{yLB}	A_{syLB} cm ²	ρ_{yLT}	A_{syLT} cm ²	A_{sxTB} cm ²	A_{sxTT} cm ²	C_t (\$)
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
910.56	282.34	79.52	0.00500	31.71	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	42.06C _c
910.56	282.34	79.52	0.01000	63.42	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	42.85C _c
910.56	282.34	79.52	0.01250	79.28	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	43.25C _c
910.56	282.34	79.52	0.01500	95.13	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	43.65C _c
910.56	282.34	79.52	0.01594	101.08	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	43.80C _c

Table 5 Ratios of reinforcement steel “ ρ_{P2} ” of the rectangular combined footing is changed

a	b	d	ρ_{P1}	A_{sP1} cm ²	ρ_{P2}	A_{sP2} cm ²	ρ_{yLB}	A_{syLB} cm ²	ρ_{yLT}	A_{syLT} cm ²	A_{sxTB} cm ²	A_{sxTT} cm ²	C_t (\$)
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
910.56	282.34	79.52	0.00333	21.14	0.00500	47.52	0.00333	74.84	0.00333	74.84	101.81	130.33	42.19C _c
910.56	282.34	79.52	0.00333	21.14	0.01000	95.03	0.00333	74.84	0.00333	74.84	101.81	130.33	43.38C _c
910.56	282.34	79.52	0.00333	21.14	0.01250	118.79	0.00333	74.84	0.00333	74.84	101.81	130.33	43.98C _c
910.56	282.34	79.52	0.00333	21.14	0.01500	142.55	0.00333	74.84	0.00333	74.84	101.81	130.33	44.58C _c
910.56	282.34	79.52	0.00333	21.14	0.01594	151.46	0.00333	74.84	0.00333	74.84	101.81	130.33	44.80C _c

Table 6 Ratios of reinforcement steel " ρ_{yLB} " of the rectangular combined footing is modified

a	b	d	ρ_{P1}	A_{sP1} cm ²	ρ_{P2}	A_{sP2} cm ²	ρ_{yLB}	A_{syLB} cm ²	ρ_{yLT}	A_{syLT} cm ²	A_{sxTB} cm ²	A_{sxTT} cm ²	C_t (\$)
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00500	112.25	0.00333	74.84	101.81	130.33	44.82C _c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.01000	224.51	0.00333	74.84	101.81	130.33	53.92C _c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.01250	280.64	0.00333	74.84	101.81	130.33	58.47C _c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.01500	336.76	0.00333	74.84	101.81	130.33	63.02C _c
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.01594	357.81	0.00333	74.84	101.81	130.33	64.72C _c

Table 7 Ratios of reinforcement steel " ρ_{yLT} " of the rectangular combined footing is changed

a	b	d	ρ_{P1}	A_{sP1} cm ²	ρ_{P2}	A_{sP2} cm ²	ρ_{yLB}	A_{syLB} cm ²	ρ_{yLT}	A_{syLT} cm ²	A_{sxTB} cm ²	A_{sxTT} cm ²	C_t (\$)
910.56	282.34	79.52	0.00333	21.14	0.00333	31.68	0.00333	74.84	0.00333	74.84	101.81	130.33	41.79C_c
985.35	265.00	76.50	0.00333	19.95	0.00333	29.71	0.00333	67.57	0.00500	101.36	101.87	135.68	43.82C _c
985.35	265.00	76.50	0.00333	19.95	0.00333	29.71	0.00333	67.57	0.01000	202.73	101.87	135.68	52.71C _c
985.35	265.00	76.50	0.00333	19.95	0.00333	29.71	0.00333	67.57	0.01250	253.41	101.87	135.68	57.17C _c
985.35	265.00	76.50	0.00333	19.95	0.00333	29.71	0.00333	67.57	0.01500	304.09	101.87	135.68	61.60C _c
985.35	265.00	76.50	0.00333	19.95	0.00333	29.71	0.00333	67.57	0.01594	323.10	101.87	135.68	63.26C _c

4. Results

Table 1 shows the numerical experimentation changing the effective depth "d". When the value of "d" is increased, the values of "b", " A_{sP1} ", " A_{sP2} ", " A_{syLB} " and " A_{syLT} " are increased; the value of "a" is reduced until the minimum value of 8.00 m, and to from of this value is constant; the value of " A_{sxTB} " is reduced until the value of 94.77 cm², and to from of this value is increased; the value of " A_{sxTT} " is reduced until the value of 129.45 cm², and to from of this value is increased; the values of " ρ_{P1} ", " ρ_{P2} ", " ρ_{yLB} " and " ρ_{yLT} " are constant and equals to 0.00333, and the total cost is increased.

Table 2 presents the numerical experimentation modifying the short dimension "b". When the value of "b" is increased, the values of "d", " A_{sP1} ", " A_{sP2} " and " A_{syLB} " are increased; the values of "a", " A_{sxTB} " and " A_{sxTT} " are reduced; the value of " A_{syLT} " is reduced until the value of 74.84 cm², and to from of this value is increased; the value of " ρ_{yLT} " is reduced until the value of 0.00333, and to from of this value is constant; the values of " ρ_{P1} ", " ρ_{P2} " and " ρ_{yLB} " are constant and equals to 0.00333, and the total cost is reduced until the value of 41.79C_c, and to from of this value is increased.

Table 3 shows the numerical experimentation changing the greater dimension "a". When the value of "a" is increased, the values of "b", "d", " A_{sP1} ", " A_{sP2} " and " A_{syLB} " are reduced; the value

of " A_{syLT} " is reduced until the value of 74.84 cm^2 , and to from of this value is increased; the values of " A_{sxtB} " and " A_{sxtT} " are increased; the value of " ρ_{yLT} " is constant and equals to 0.00333, and to from of this value is increased; the values of " ρ_{P1} ", " ρ_{P2} " and " ρ_{yLB} " are constant and equals to 0.00333, and the total cost is reduced until the value of $41.79C_c$, and to from of this value is increased.

Table 4 presents the numerical experimentation modifying the ratios of reinforcement steel " ρ_{P1} ". When the value of " ρ_{P1} " is increased, the value of " A_{sP1} " is increased; the values of " a ", " d ", " ρ_{P2} ", " ρ_{yLB} ", " ρ_{yLT} ", " A_{sP2} ", " A_{syLB} ", " A_{syLT} ", " A_{sxtB} " and " A_{sxtT} " are constant, and the total cost is increased.

Table 5 presents the numerical experimentation changing the ratios of reinforcement steel " ρ_{P2} ". When the value of " ρ_{P2} " is increased, the value of " A_{sP2} " is increased; the values of " a ", " d ", " ρ_{P1} ", " ρ_{yLB} ", " ρ_{yLT} ", " A_{sP1} ", " A_{syLB} ", " A_{syLT} ", " A_{sxtB} " and " A_{sxtT} " are constant, and the total cost is increased.

Table 6 presents the numerical experimentation modifying the ratios of reinforcement steel " ρ_{yLB} ". When the value of " ρ_{yLB} " is increased, the value of " A_{syLB} " is increased; the values of " a ", " d ", " ρ_{P1} ", " ρ_{P2} ", " ρ_{yLT} ", " A_{sP2} ", " A_{syLT} ", " A_{sxtB} " and " A_{sxtT} " are constant, and the total cost is increased.

Table 7 presents the numerical experimentation changing the ratios of reinforcement steel " ρ_{yLT} ". When the value of " ρ_{yLT} " is increased, the value of " A_{syLT} " is increased; the value of " a " is increased until the value of 985.35 cm , and to from of this value is constant; the value of " d " is reduced until the value of 76.50 cm , and to from of this value is constant; the value of " A_{sP1} " is reduced until the value of 19.95 cm^2 , and to from of this value is constant; the value of " A_{sP2} " is reduced until the value of 29.71 cm^2 , and to from of this value is constant; the value of " A_{syLB} " is reduced until the value of 67.57 cm^2 , and to from of this value is constant; the value of " A_{sxtB} " is increased until the value of 101.87 cm^2 , and to from of this value is constant; the value of " A_{sxtT} " is increased until the value of 135.68 cm^2 , and to from of this value is constant; the values of " ρ_{P1} ", " ρ_{P2} ", " ρ_{yLB} " are constant, and the total cost is increased.

5. Conclusions

The foundation is an essential part of a structure that transmits column or wall loads to the underlying soil below the structure.

This paper shows an optimal design for reinforced concrete rectangular combined footings based on a criterion of minimum cost due to an axial load, moment around of the axis "X" and moment around of the axis "Y" applied to each column.

The proposed model assumes that the constant parameters are: $P_1, M_{x1}, M_{y1}, P_2, M_{x2}, M_{y2}, c_1, c_2, c_3, c_4, L, \sigma_{adm}, R, P_{u1}, M_{ux1}, M_{uy1}, P_{u2}, M_{ux2}, M_{uy2}, R_u, q_a, \gamma_c, \gamma_s, f'_c, f_y, \alpha, r, H$, and the decision variables are: $a, b, d, \rho_{P1}, \rho_{P2}, \rho_{yLB}, \rho_{yLT}, A_{sP1}, A_{sP2}, A_{syLB}, A_{syLT}, A_{sxtB}, A_{sxtT}$.

Numerical experimentation takes into account a of the decision variables as a constant parameter to observe the precise of the model, these constant parameters are: $a, b, d, \rho_{P1}, \rho_{P2}, \rho_{yLB}, \rho_{yLT}$.

The main conclusions are:

1.- The most economical cost for design a reinforced concrete rectangular combined footing is presented if there are not restricted with respect to the decision variables.

2.- The methodology shown in this paper is more accurate and converges more quickly.

3.- The classical model cannot be compared to this methodology, because the classical model is not guaranteed that obtained cost is the most economical cost.

The proposed model presented in this paper for optimal design of reinforced concrete rectangular combined footings subjected to an axial load and moment in two directions in each column, also it can be applied to others cases: 1) Footings subjected to a concentric axial load in each column, 2) Footings subjected to a axial load and one moment in each column.

The model presented in this paper applies only for design of reinforced concrete rectangular combined footings assumed than the structural member is rigid and the supporting soil layers elastic, which meet expression of the biaxial bending, i.e., the variation of pressure is linear.

The suggestions for future research are:

1.- Optimal design for reinforced concrete trapezoidal combined footings assuming these are rigid and the supporting soil layers elastic.

2.- Optimal design for reinforced concrete "T" combined footings assuming these are rigid and the supporting soil layers elastic.

3.- Optimal design for reinforced concrete rectangular combined footings supported on another type of soil by example in totally cohesive soils (clay soils) and totally granular soils (sandy soils), the pressure diagram is not linear and should be treated differently.

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