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Computational methodology to determine the strength of reinforced concrete joint

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Abstract. Seismic performance of structures depends on the force flow mechanism inside the structure. Discontinuity regions, like beam-column joints, are often affected during earthquake event due to the complex and discontinuous load paths. The evaluation of shear strength and identification of failure mode of the joint region are helpful to (i) define the strength hierarchy of the beam-column sub-assemblage, (ii) quantify the influence of different parameters on the behaviour of beam-column joint and, (iii) develop suitable and adequate strengthening scheme for the joints, if required, to obtain the desired strength hierarchy. In view of this, it is very important to estimate the joint shear strength and identify the failure modes of the joint region as it is the most critical part in any beam-column sub-assemblage. One of the most effective models is softened strut and tie model which was developed by incorporating force equilibrium, strain compatibility and constitutive laws of cracked reinforced concrete. In this study, softened strut and tie model, which incorporates force equilibrium equations, compatibility conditions and material constitutive relation of the cracked concrete, are used to simulate the shear strength behaviour and to identify failure mechanisms of the beam-column joints. The observations of the present study will be helpful to arrive at the design strategy of the joints to ensure the desired failure mechanism and strength hierarchy to achieve sustainability of structural systems under seismic loading.

Keywords: beam-column joints; strength hierarchy; failure mechanism; softened strut and tie model; joint shear strength

1. Introduction

Most of the existing reinforced concrete structures were designed only for gravity loads, even in the regions prone to high seismicity. They do perform well in the conventional cases under gravity loading but the question on their performance under seismic event arises. These structures are prone to devastating damages during any moderate or major earthquake attack. Many regions in India, New Zealand, Japan, southern parts of America, Italy, Iran, etc. fall under high seismicity. Every year, these regions often experience moderate to major earthquakes. It is truly said that the

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earthquake does not kill people but buildings does. Hence, it is important to improve the performance of the existing structures to take care of any catastrophic failures. There are even places in many countries, believed to be non seismic zone that has had devastating earthquakes. Some of the places in India have records of high earthquakes but were recognized to be placed in mild seismic zone. A three phase description is followed for design of structures for seismic loads, (i) during mild earthquakes, structure should have sufficient lateral stiffness to control the inter storey drift thereby reducing the damages to the non-structural elements, (ii) during a moderate earthquake, the non-structural components can be allowed to damage while the structural elements should have sufficient strength to remain in their elastic range, (iii) during strong earthquakes, the structural elements can go to plastic stage and large deformations can occur with permissible damages to the structural elements but in any of these cases, the structure should not collapse. The important parameter for any structural element subjected to earthquake load is their energy dissipation characteristics at critical locations. A good structure is that which dissipates more energy so that the structural components are prevented from damages. Shear failures and anchorage failures are wrong types of failure because they dissipate very less energy. These kinds of failures are often cited at the beam-column joints (as typically shown in Fig. 1) which make them, the most vulnerable component to dictate the strength of the structure.

Current structural codes are recently developing a new kind of design procedure based on the performance of the structures under seismic loading. The primary objective of this Performance Based Seismic Design (PBSD) is human life safety. Now, it has transformed significantly from its literal meaning. The specific objective of the PBSD is not only the human life safety but also the socio-economic loss. The requirements of these objectives are, (a) accurate prediction of demands of the earthquake loading such as loads, deformations and the deterioration in strength and stiffness of the structures, (b) design of components to meet these demands including the unexpected situations, (c) quick repair of the structure even after a major earthquake. To accomplish these requirements, there are considerable amount of demands in the component level like beams, columns, joints, etc. Non-linear analyses are being employed to predict these demands in order to design the components for their specific performances. The performance based design procedures require inelastic response of the structures to determine their level of deformations during such earthquake loading. Failure of any structure is inherited to the failure of the joints present in them rather than the other components because of their complex behaviour during any seismic or cyclic loads.

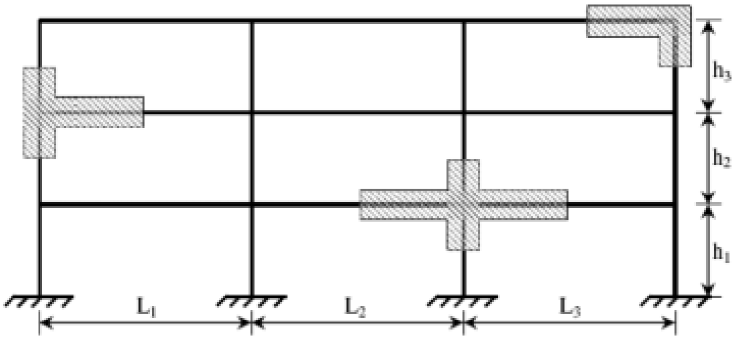


Fig. 1 Critical regions in RC structures in earthquake

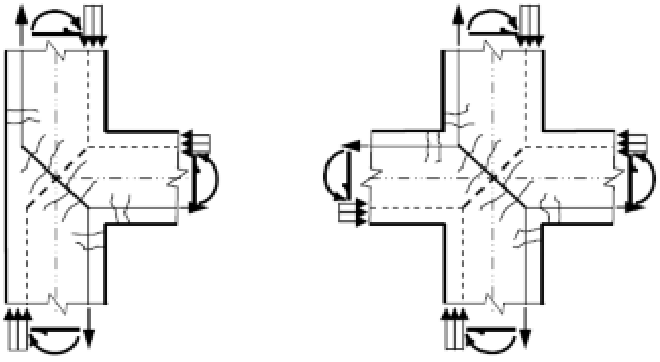


Fig. 2 Failure pattern in exterior and interior joint

Beam-column joint is a critical region in a whole structure because of its brittle shear failure during earthquake loading. This is of a major concern to the performance based design concepts. The ultimate capacity of these joints depend on many factors in which some of them are the material behaviour of steel and concrete, bond between concrete and steel, aspect ratio of the joint, reinforcement arrangement in the side joint, shear force transferred to the joints and the energy dissipation capacity. Since the performance of the crucial components critically depends on the load transfer mechanism in the disturbed region (shown in Fig. 2), it is important to evaluate the behaviour of the joint (as also emphasized in Fisher and Sezen (2011), Unal and Burak (2012), Wang *et al*. (2012), Park and Mosalam (2012), Zhou and Zhang (2012), Wong and Kuang (2014), Pauletta *et al*. (2015)) region (i) to define the strength hierarchy of the beam-column sub-assemblage, (ii) to quantify the influence of different parameters on it and, (iii) to develop suitable strengthening procedures for the joints (Sasmal *et al*. 2011a, b, Tsonos 2014, Vecchio *et al*. 2015), if required, to obtain the desired strength hierarchy.

In view of this, it is important to estimate the joint shear strength and to identify the failure modes of the joint.

2. Various types of analytical models for shear strength evaluation

2.1 Empirical models

Empirical models were developed by many researchers by extracting some of the parameters that affect the joint shear strength. The basic assumption is that all the parameters are independent of each other.

Sarsam and Phipps model

Sarsam and Phipps (1985) suggested an empirical model for predicting the shear capacity of the beam-column joints, as given in Eqs. (1) to (3).

$$V_{cd} = 5.08(f_{cu}\rho_c)^{0.33}\left(\frac{d_c}{d_b}\right)^{1.33} b_c d_c \sqrt{1 + 0.29\frac{N}{A_g}} \tag{1}$$

$$V_{sd} = 0.87 f_{yv} A_{js} \tag{2}$$

$$V_{ud} = V_{cd} + V_{sd} \tag{3}$$

where, f_{cu} is the concrete cube strength (MPa) and ρ_c is the column reinforcement ratio, $\rho_c = \frac{A_{so}}{b_c d_c}$, where A_{so} is the area of the layer of steel reinforcement farthest from the maximum compression region in the column (mm^2), d_c and b_c are the depth and width of column, d_b is the depth of beam, N is the column axial load (in N), A_g is the gross area of the column, V_{sd} is the shear strength offered by steel. V_{cd} is shear strength offered by concrete, A_{js} is the area of the transverse reinforcement (mm^2) crossing the diagonal plane from corner to corner of the joint between the beam compression and tension reinforcements, f_{yv} is the tensile strength of the transverse reinforcement (MPa) and V_{ud} is total shear strength.

Scott *et al*. model

Scott *et al*. (1994) suggested a model based only on a single diagonal strut mechanism without the horizontal and vertical mechanisms. The formulae suggested are

$$v_{crsh} = \frac{2\sqrt{f_{cu}}}{\left(\frac{z_{col}}{z_{bm}} + \frac{z_{bm}}{z_{col}}\right)} \tag{4}$$

$$V_{crsh} = v_{crsh} b_c d_c \tag{5}$$

where, v_{crsh} is crushing strength of diagonal strut in the joint, f_{cu} is the concrete cube strength, $\frac{z_{bm}}{z_{col}}$ is the slope of the diagonal strut to the horizontal in which, z_{col} is the distance between the two centers of outer column reinforcement bars, z_{bm} is determined by section analysis, and d_c and b_c are the depth and width of column,.

Vollum model

This model was proposed by Vollum (1998) for exterior joints with or without the transverse reinforcements. Vollum and Newman (1999) concluded that a realistic strut and tie model is difficult to construct due to its complexity. The difficulties lie in the determination of nodal sizes, column bar forces and the width of strut. The suggested equations of this model are

$$V_c = 0.642\beta[1 + 0.552(2 - h_b/h_c)] b_c h_c \sqrt{f_c'} \tag{6}$$

$$\begin{aligned} V_j &= max\left[\left(V_c - \alpha b_c h_c \sqrt{f_c'}\right) + A_{sj} f_y V_c\right] \\ &\le 0.97 b_c h_c \sqrt{f_c'}[1 + 0.552(2 - h_b/h_c)] \\ &\le 1.33 b_c h_c \sqrt{f_c'} \end{aligned} \tag{7}$$

Eq. (6) is for joints without transverse reinforcement and Eq. (7) is for joints with transverse reinforcements. β depends on the anchorage detail, β=1.0 for anchorage type A (connections with beam using L shaped bars), β=0.9 for anchorage type C (connections with beam sing U shaped bars), α represents the effect of column axial load and concrete strength and is taken as 0.2, h_c and h_b are the depths of column and beam respectively, b_c is the width of column, f_c' is the concrete cylinder strength and f_y is the yield stress of reinforcement, A_{sj} is the area of the stirrups within the top two thirds of the beam in the joint depth below the main beam reinforcement.

Bakir and Boduroğlu model

Bakir and Boduroğlu (2002) proposed a model based on the regression analysis of available data reported by various researchers. In this model, the percentage of beam reinforcement and the aspect ratio of the joint are considered. The total joint shear strength is a sum of the strength contributed by the concrete and the steel

$$V_c = \frac{0.71\beta\gamma\rho_s^{0.4289}}{\left(\frac{h_b}{h_c}\right)^{0.61}}\left(\frac{b_c + b_b}{2}\right)h_c\sqrt{f_c'} \tag{8}$$

$$V_s = \alpha A_{sje} f_y \tag{9}$$

The parameter β represents the anchorage detail where, β=1.0 for anchorage type A, β=0.85 for anchorage type C, γ= 1.37 for inclined bars in the joint and γ= 1.0 for others, α= 0.664 for joints with less transverse reinforcement, 0.6 for medium reinforcements and 0.37 for higher reinforcements, A_{sje} represents the transverse reinforcement area in the joint, ρ_s is the percentage of tension steel in the beam, b_c and b_b are the widths of column and beam respectively, h_c and h_b are the depths of column and beam respectively, f_c' is the concrete cylinder strength and f_y is the yield stress of reinforcement.

Hegger *et al*. model

Hegger *et al*. (2003) developed a model which considers the column reinforcement ratio and joint aspect ratio in calculating the joint shear strength. The suggested relations for the model are given in Eqs. (10) to (12), as,

$$V_j = V_c + V_s \tag{10}$$

$$V_s = \alpha_2 A_{sj,eff} f_y \tag{11}$$

$$V_c = \alpha_1 ABC b_f h_c \tag{12}$$

where, α_1 represents the anchorage detail, α_1=0.95 for type A, α_1=0.85 for type C, A, B and C are the parameters representing the effect of aspect ratio, the effect of column reinforcement ratio and the ratio of concrete strength respectively, α_2 is the efficiency of transverse reinforcements, $A_{sj,eff}$ is the effective area of transverse reinforcements in the joint.

2.3 Comparison of empirical models

The shear strengths of the beam-column joints predicted by the empirical models are compared to understand the accuracy of the models (presented in Table 1). The specimen details reported in Fujii and Morita (1991) (specimens K-B1, B4) is used to compare the models. The models which directly predict the shear strength of the joints are used to compare the results. The comparison is shown in the Table 1.

It is to state that the empirical models as presented in Table 1 are primarily developed to predict the shear strength of the joints under monotonic loads whereas the experimental investigation was carried out under cyclic loading. From the above table, it is observed that the Vollum model predicts the shear strength of the joints with better accuracy but it overestimates the shear strength

Table 1 Comparison of empirical models

Specimen	V_{jh_test} (kN)	Sarsam & Phipps		Scott *et al.*		Bakir & Boduroğlu		Hegger *et al.*		Vollum	
		Vjh (kN)	% diff	Vjh (kN)	% diff	Vjh (kN)	% diff	Vjh (kN)	% diff	Vjh (kN)	% diff
Fujii & Morita (K-B1)	246	170.02	-30.9	261.63	6.3	194.55	-20.9	240.63	-2.2	251.33	2.2
Fujii & Morita (K-B4)	287	226.73	-21.0	261.63	-8.8	218.67	-23.8	263.45	-8.2	296.08	3.1

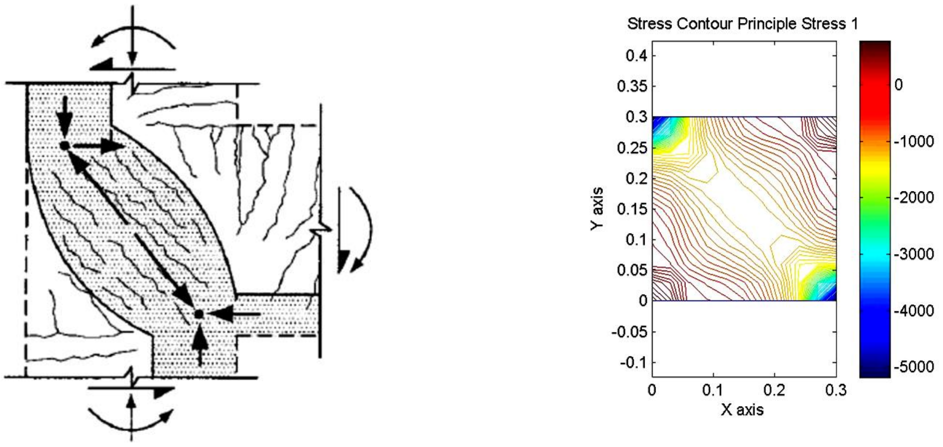
of the beam-column joints. Hegger et al. model (2003) and Scott et al. model (1994) are also observed to have good accuracy in predicting the joint shear strength.

2.4 Analytical models

Softened strut and tie model

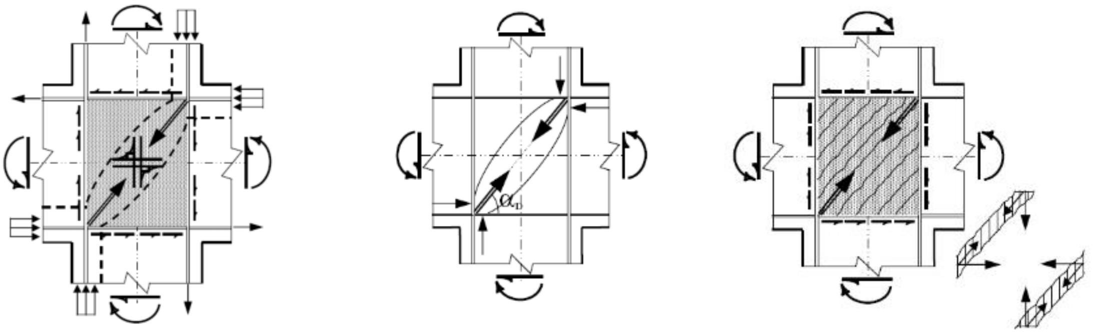
Selection of the mechanism of the model is based on the load transfer mechanism within the joint. Usually, the strut-tie models are considered to satisfy only the equilibrium conditions. The satisfaction of other conditions like compatibility and constitutive relations can be achieved only through selection of a proper strut-tie and mechanisms based on force flow. Fig. 3(a) shows how a strut is formed in an exterior joint from the load conditions, the joint experiences during an earthquake.

The principal stresses in the joint panel can be visualized through a linear finite element simulation model of a square joint sample (as shown in Fig. 3(b)). This figure can be used to validate the assumption that the principal direction of compression lies with the direction of diagonal strut, but is valid to some extent of the aspect ratio of the joint. Selection of the



(a) (b)

Fig. 3 (a) Diagonal strut mechanism, (b) Force transfer obtained from finite element modelling



(a) (b) (c)

Fig. 4 Force transfer mechanism inside the joint (a) development of shear, (b) strut mechanism, (c) strut and tie mechanism

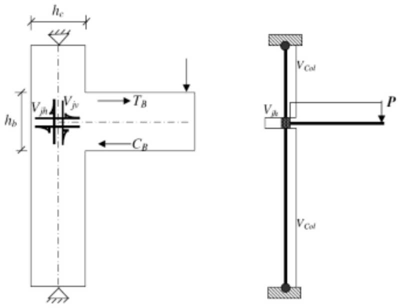


Fig. 5 Development of joint shear in a beam-column sub-assemblage

mechanism of the model is based on the load transfer mechanism within the joint. Usually, the strut-tie models are considered to satisfy only the equilibrium conditions. The satisfaction of other conditions like compatibility and constitutive relations can be achieved only through selection of a proper strut-tie and mechanisms based on force flow. Fig. 4(a) to (c) depict the free body diagram of the forces act on the beam column joint under seismic event.

From Fig. 5, it is clear that the summation of force in the horizontal direction gives

$$V_{jh} = T_B - V_{col} \tag{13}$$

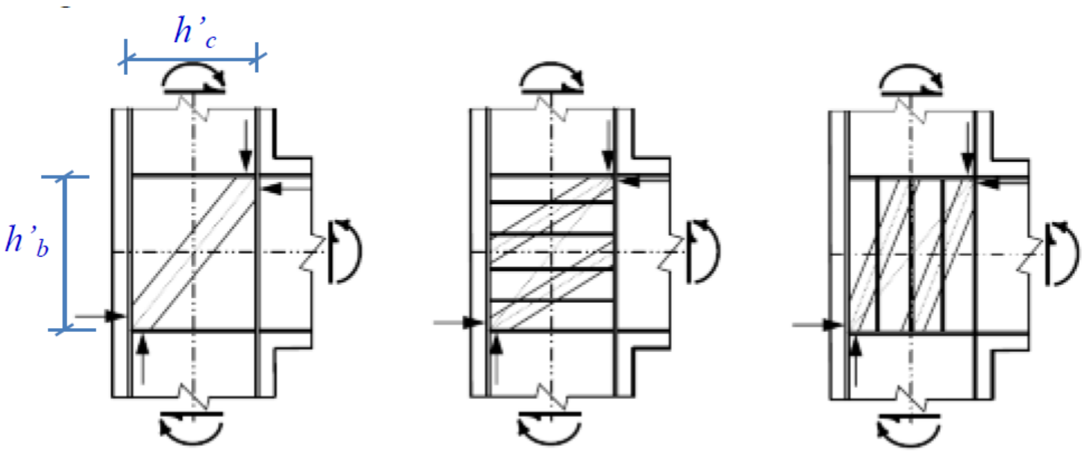
where, V_{jh} is the horizontal shear force in the joint, V_{col} is the shear force produced by the column action and T_B is the tensile force created by the beam reinforcement bars.

The vertical joint shear force can be approximated from the geometry of the joint as

$$V_{jv} \approx \left(\frac{h_b'}{h_c'}\right) \times V_{jh} \tag{14}$$

where, h'_b and h'_c are the lever arms of forces in the beam and column respectively (as shown in Fig. 6).

The strut-tie model consists of three mechanisms namely, (i) diagonal, (ii) horizontal and (iii) vertical mechanisms. The diagonal mechanism of the joint is simulated by a compression strut,



(a) (b) (c)

Fig. 6 (a) Diagonal mechanism (b) Horizontal mechanism (c) Vertical mechanism

placed diagonally, which makes an angle θ with the horizontal. The θ is defined from the geometry as

$$\theta = \tan^{-1}\left(\frac{h_b''}{h_c''}\right) \tag{15}$$

where, h_b'' is the lever arm between the extreme reinforcement bars in the beam; and h_c'' is the distance between the centroid of extreme longitudinal reinforcement to the centroid of the extended reinforcement with a 90 degree hook from the beam. The orientation of the diagonal strut between the centroids of extreme reinforcements rather than centroids of compression and tension zones is to simulate the joint core with sufficient accuracy. The diagonal, horizontal and vertical mechanisms are shown in Fig. 6.

Equilibrium

Fig. 7 shows the strut and tie model for a beam-column joint. The joint forces must be in equilibrium; this can be achieved by summation of the joint forces and equating it to zero. So, the horizontal shear force resisted by the joint will be

$$V_{str,h} = F_D \cos\theta + F_H + F_V \cot\theta \tag{16}$$

where, F_D is the diagonal compressive force on the strut, F_H is the tensile force acting on the horizontal ties and F_V is the tensile force acting on the vertical steel. The joint vertical shear force can also be determined in a similar way as

$$V_{str,v} = F_D \sin\theta + F_H \tan\theta + F_V \tag{17}$$

It can be noted from Eqs. (16) and (17) that the ratio is always preserved to be $\tan\theta$. The three mechanisms in the joint create three load paths, as given in Fig. 6. The forces acting on the joint should be distributed to these load paths. Since, the structure is statically indeterminate; the forces are apportioned by considering either horizontal or vertical mechanism at a time along with the diagonal mechanism. In the absence of the vertical ties, the vertical mechanism does not occur. So, the horizontal and diagonal mechanisms resist the external forces.

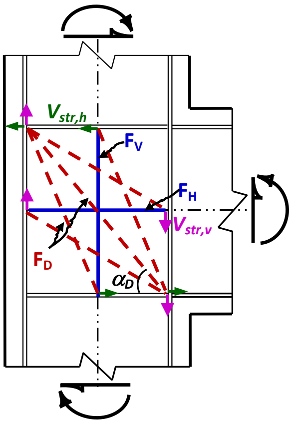


Fig. 7 Free body diagram of the strut and tie model in the joint subjected to external loads

Schäfer (1996) and Jennewein and Schäfer (1992) suggested equations for calculating the horizontal and vertical forces and their proportional constants for statically indeterminate strut-tie structure. With that, the horizontal fraction of force F_H is assumed as

$$\begin{gathered} F_H = \gamma_h \times V_{str,h} \\ \gamma_h = \frac{2\tan\theta - 1}{3} \\ for\ 0 \le \gamma_h \le 1 \end{gathered} \tag{18}$$

where, γ_h is the proportioning constant. Eq. (18) is an interpolation equation between two cases, that the entire horizontal shear is in horizontal direction (F_H=$V_{str,h}$) and that the entire horizontal shear is transferred by diagonal compression strut (F_H=0). In a very similar way, the diagonal and vertical mechanisms can also be derived.

If R_d, R_v, R_h are the ratios of the joint shears resisted by the diagonal, vertical and horizontal mechanisms, the same can be calculated using the proportioning constants, as

$$R_d = \frac{(1-\gamma_h)(1-\gamma_v)}{(1-\gamma_h\gamma_v)} \tag{19}$$

$$R_v = \frac{(1-\gamma_h)(\gamma_v)}{(1-\gamma_h\gamma_v)} \tag{20}$$

$$R_h = \frac{(\gamma_h)(1-\gamma_v)}{(1-\gamma_h\gamma_v)} \tag{21}$$

Constitutive laws

The shear strength of the beam-column joint is believed to be governed by the softening effect of the concrete. This so called compression softening is quantified and proposed by Vecchio and Collins (1993) and Zhang and Hsu (1998). The understanding of shear problems is enhanced by Collins *et al.* (1996). Generally, the cracked concrete exhibits lesser strength than the uniaxially loaded concrete under compression. In this model, principal direction of compressive stresses is assumed to be coincidental with the diagonal strut. This assumption can be improved. The ascending region of softened stress-strain relationship, as proposed by Zhang and Hsu (1998) is given by

$$\sigma_d = \zeta f_c' \left[2\left(\frac{\varepsilon_d}{\zeta\varepsilon_0}\right) - \left(\frac{\varepsilon_d}{\zeta\varepsilon_0}\right)^2\right] \quad \text{for} \left(\frac{\varepsilon_d}{\zeta\varepsilon_0}\right) \le 1 \tag{22}$$

$$\zeta = \frac{5.8}{\sqrt{f_c'}} \frac{1}{\sqrt{1+400\varepsilon_r}} \le \frac{0.9}{\sqrt{1+400\varepsilon_r}} \tag{23}$$

where, ζ is the softening coefficient which is considered equal for both stress and strains, σ_d is the average principal stress in the concrete along diagonal direction, ε_d and ε_r are the principal strains in diagonal, perpendicular to diagonal directions and ε_0 is the concrete cylinder strain corresponding to f_c'. ε_0 can be approximated as in Foster and Gilbert (1996)

$$\varepsilon_0 = -0.002 - 0.001\left(\frac{f_c' - 20}{80}\right) \text{ for } 20 \le f_c' \le 100 \text{ MPa} \tag{24}$$

The shear strength of the joint is reached whenever the compressive stresses and strains of the diagonal strut satisfy the following formulae

$$\sigma_d = \zeta \times f_c' \tag{25}$$

$$\varepsilon_d = \zeta \times \varepsilon_0 \tag{26}$$

Compatibility

The strain in the principal direction of tension can be calculated from the compatibility condition suggested by Hsu (1993). The compatibility equation relates the vertical, horizontal and the magnitude of principal compressive strains to get principal tension strains

$$\varepsilon_r = \varepsilon_h + (\varepsilon_h - \varepsilon_d)\, cot^2\theta \tag{27}$$

$$\varepsilon_r = \varepsilon_v + (\varepsilon_v - \varepsilon_d)\, tan^2\theta \tag{28}$$

where, ε_r is the principal strain in tension direction, ε_d is the principal strain in compression direction, ε_h and ε_v are the horizontal and vertical strains in the joint.

3. Development of computer programs in MATLAB

Using the softened strut-tie model proposed by Hwang and Lee (1999) and based on the flowchart (shown in Fig. 8), computer program is developed in this study for evaluation of joint shear strength of exterior beam column sub-assemblages for seismic resistance. It is to mention that the confinement effect on the constitutive model of concrete in the joint is considered in the present study, as it is found to be more reasonable. Further, unlike Hwang and Lee (1999), the post peak behavior was also studied in the present work. Sasmal *et al*. (2012, 2013) developed the methodology to develop the strength hierarchy of the structural components after obtaining the strength of the joints. The present study will further be useful to characterise the strength based on type and geometrical parameters of the joints. These are validated by comparing the results of this study with those of the tests and the analytical studies reported in the literature. For easy reference, the flowchart is explained briefly.

Step1: The algorithm is an iterative procedure with selection of initial value for the horizontal joint shear strength $V_{str,h}$ (hereinafter briefly termed as V_{jh}).

Step2: Calculations are based on the force equilibrium equations to calculate the forces in different directions in the joint

Step3: Check the strain state from the constitutive model and ensuring the compatibility

Step4: Check the failure of any of the 5 types of failure modes are considered, as (i) concrete strut reaches its strength while horizontal and vertical ties are in elastic range (Type E), (ii) if yielding of horizontal tie occurs, joint shear beyond the yielding of horizontal ties is resisted by reduced mechanism i.e., diagonal and vertical mechanisms (Type YH), (iii) if yielding of vertical tie occurs, joint shear beyond yielding of vertical ties is resisted by reduced mechanism i.e., diagonal and horizontal mechanisms (Type YV), (iv) yielding of horizontal tie occurs and strut arrives at its capacity after second yielding of vertical ties (Type YHV) and (v) yielding of vertical tie occurs and strut arrives at its capacity after second yielding of horizontal ties (Type YVH).

Step5: An iterative procedure to calculate the stress and strain softening. In case of exceeding tolerance, the iteration start with a new value of horizontal joint shear strength $V_{str,h}$ in Step1.

3.1 Numerical Investigations

Using the programme developed in the present study based on the softened strut-tie models proposed by Hwang and Lee (2002), influence of different parameters on the joint shear strength is studied. All the parameters are varied in a range to observe how the variables affect the shear strength of the joints. The basic parameters considered in this study are, (i) Concrete cylinder strength f_c', (ii) Diagonal strut angle θ and (iii) Area of vertical and horizontal steel in the joint core, A_{tv} and A_{th}. The last two parameters A_{tv} and A_{th} are used to define the shear strength envelope of the section such that, their implications on the shear strength of the joints can be investigated. The patterns of the parameter effects are studied along with their failure mode in order to understand their complete behaviour.

3.2 Validation of models and computer program

MATLAB code developed based on the softened strut-tie model is used to calculate the joint shear strengths of 63 specimens described in the various literatures and to compare the joint shear strength value obtained by Hwang and Lee (1999). The specimens encompass a wide range of geometric, loading, reinforcement detailing and material properties. These test data were used to evaluate the exterior joint shear strength and to validate the model for its accuracy and repeatability in order to study the insights in the behaviour of joints (Vishnu Pradeesh 2014). It has been observed that the results obtained using the model and the computer program are reasonably corroborated with the experimental results pertaining to the strengths of shear dominant joints. For brevity, only few results are presented in Table 2. From the table 2, it can be observed that the model has a good accuracy in predicting the joint shear strength test results.

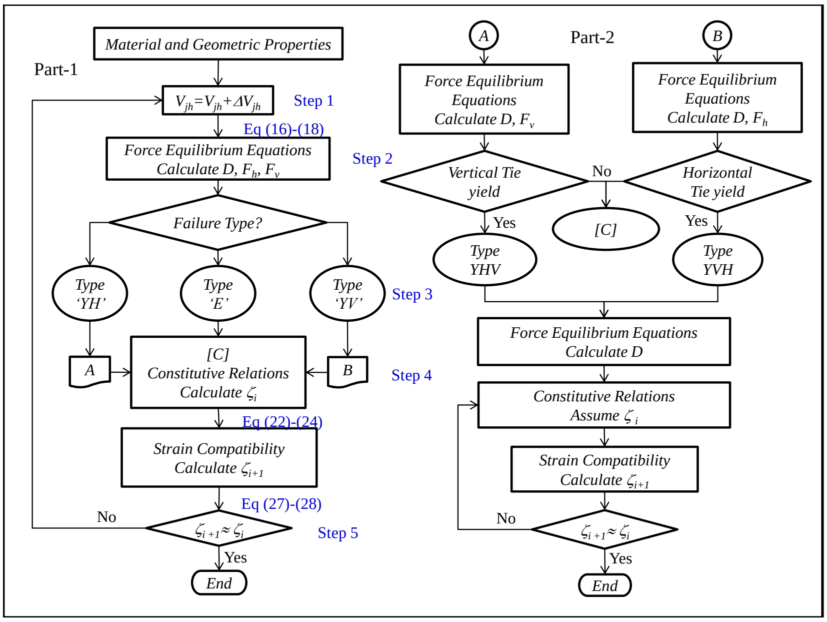


Fig. 8 Schematic flow chart of algorithm

Table 2 Experimental validation for exterior joints

Authors	Specimens	f_c' (MPa)	f_{yh} (MPa)	f_{yv} (MPa)	$N/A_g f_c$'	$V_{j,h,test}$ (kN)	θ	A_{str} (cm^2)	A_{th} (mm^2)	A_{tv} (mm^2)	Type	$V_{jh,H}$ (kN)	Type	$V_{j,h,S}$ (kN)	% Diff	$V_{j,h,test}$ /$V_{j,h,S}$
											Hwang result		Present study			
Megget (1974)	Unit A	22.1	317	365	0.07	576	53	389	1330	774	E	419	E	410	2.15	1.40
Blakeley *et al.* (1975)	Exterior	48.0	297	289	0.00	1104	56	1176	5680	4914	E	2398	E	2433	1.46	0.45
Lee *et al.* (1977)	2	29.0	389	0	0.11	194	42	194	426	0	YV	261	YV	257	1.53	0.75
	5	24.8	389	0	0.00	206	42	142	426	0	YV	172	YV	165	4.07	1.25
	6	24.8	273	0	0.00	208	42	142	126	0	YVH	155	YVH	130	15.97	1.60
Paulay and Scarpas (1981)	Unit 1	22.6	326	296	0.05	754	55	616	1356	1256	E	631	E	640	1.43	1.18
	Unit 2	22.5	326	296	0.15	990	55	788	942	1256	YH	711	YH	732	2.95	1.35
	Unit 3	26.9	316	296	0.05	753	55	616	628	1256	YH	634	YH	651	2.68	1.16
Park and Milburn (1983)	Unit 3	38.2	321	485	0.10	606	49	415	471	628	YH	643	YH	648	0.78	0.94

3.3 Investigations on the role of key parameters

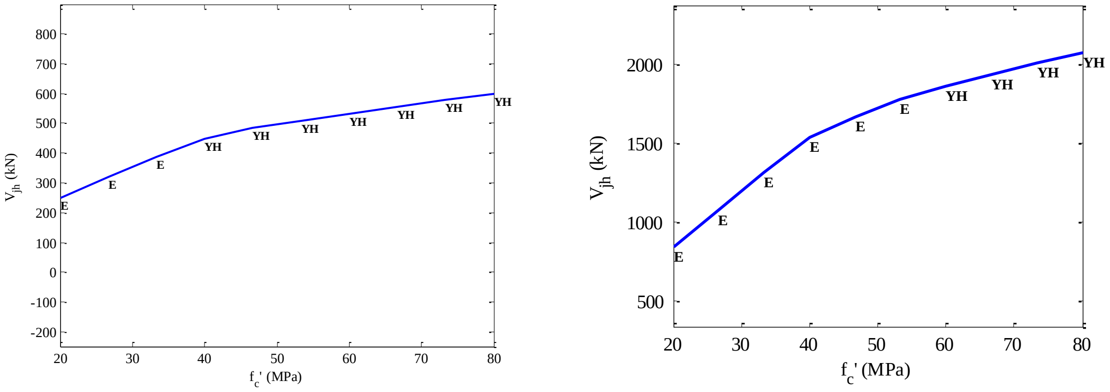
Using the softened strut-tie model, a computer program is developed without considering the confinement effect. Using the program, influence of different parameters on the joint shear strength is studied. All the parameters are varied in a range to observe how the variables affect the shear strength of the joints. The external joints are analyzed. The basic parameters studied in this study are, (*i*) concrete cylinder strength f_c', (*ii*) diagonal strut angle θ, (*iii*) area of horizontal and vertical steel in the joint core, A_{th} and A_{tv}. Further, it was attempted to study the role of horizontal steel (A_{th}) on shear strength envelop of the beam column joint region. The observations of the present study along with their failure mode developed would help in understanding their complete behavior of the beam column joint under lateral loading.

3.4 Influence of concrete cylinder strength on shear strength

The basic strength of concrete comes from its compressive strength since it is weak in tension. The concrete compressive strength is a very important parameter in the determination of joint shear strength. It is related to both the strut and truss mechanisms which are formed when the joint starts resisting the external forces transferred to it. Fig. 9 shows that the variation of shear strength of joints with the compressive strength of concrete. These specimens are selected due to the failure mode encountered during the increment in the concrete strength. The cylinder compressive strength is varied between 20 MPa and 80 MPa. It is well understood that the increase in compressive strength will increase the shear strength of the joint. This is evident from the Fig. 9 that with the increase in compressive strength of concrete, the diagonal strut becomes capable of taking more loads before its failure stress. It is valid for both exterior and interior joints. But, the rate of change in shear strength with compressive strength of concrete is found to be monotonic. A steep increase in joint shear strength with the increase in concrete compressive strength is observed when the reinforcement is in elastic stage whereas the rate of change decreases when steel yielding governs the failure of the joint.

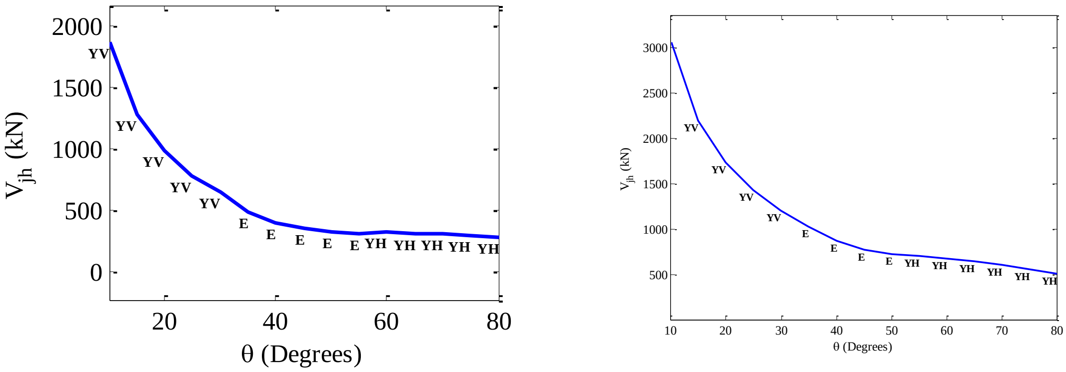
3.5 Influence of diagonal strut angle on shear strength

In the truss and strut-tie analogy, the angle of the diagonal strut formed in the joint plays a key role in determining the shear capacity of the joint (V_{jh}). The angle of strut has a considerable effect



(a) (b)

Fig. 9 Variation of shear strength with concrete compressive strength for (a) exterior joints, (b) interior joints



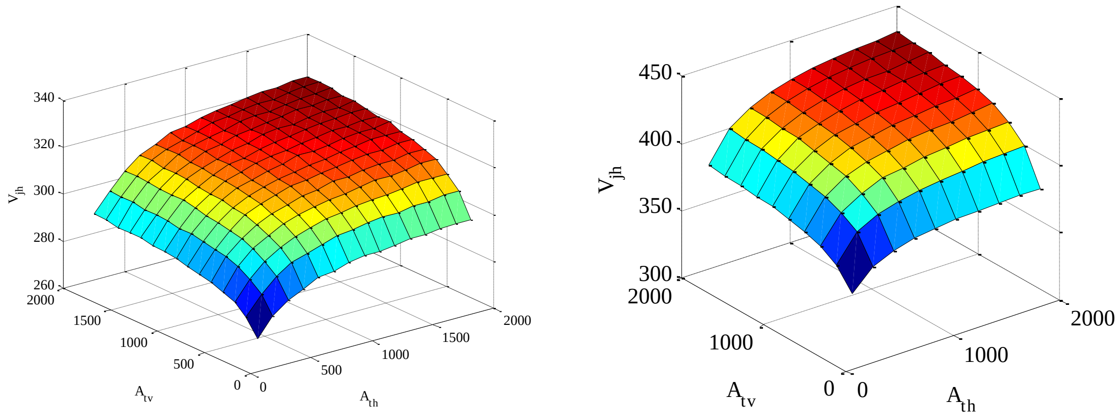
(a) (b)

Fig. 10 Variation of shear strength with strut angle for (a) exterior joints, (b) interior joints

on the failure mode of the joint. Change in the strength with respect to angle of inclination for the joint is given in Fig. 10. The angle of strut is changed from 10 degrees to 80 degrees. With these angles, the shear capacities of the joints are plotted against them. It is observed that the strength of the joint keeps descending as the angle of strut increases from 10 to 80 degrees. The associated failure mode is also indicated. The descending portion of the Fig. 10 indicates that the larger the beam depth, lower is the shear capacity of the joints, which proves that the stronger beams have lower joint shear strength than the weaker beams. This is due to the fact that steep struts provide less resistance towards the horizontal forces. The change in failure mode can be understood from the mechanism which is happening inside the joints. When the angle is very small, the strut transfers the forces to the vertical ties and a reduced mechanism is followed. When the stresses in the vertical ties reach their yielding stress, the joint fails by the yielding of the vertical tie bars. A similar kind of mechanism happens when the angle of strut is larger. The strut transfers a larger portion of the load to the horizontal tie. The middle portion of the curve shows that the strut takes more load than the ties which makes the failure of diagonal strut when the ties are within their elastic limit.

3.6 Influence of area of horizontal and vertical reinforcement in the joint

The failure mode and the shear strength of the beam-column joints are largely controlled by the area of the horizontal and vertical ties. The shear strength envelope of the exterior and interior joints is given in the Fig. 11. The areas of the horizontal (A_{th}) and vertical (A_{tv}) steel tie bars are varied from 250 mm^2 and 2000 mm^2. The variation in the shear capacity of the joint is obtained due to the changes in areas of horizontal and vertical tie bars. The shear strength envelope explains the amount of reinforcement required in the joint to achieve target failure strength and the failure mode. It is evident that, as the areas of the horizontal and vertical tie are increased, the shear capacity of the joint gets saturated after some level. This level of saturation is for a particular experimental specimen. The shear strength envelope shifts from the initial position, if the other parameters are changed, but the phenomenon of saturation is true for all the cases. This implies that for a given vertical steel area, the increase in horizontal steel area, after some threshold has no significant improvement in the shear capacity of the joint and vice versa. A distinct interesting



(a) (b)

Fig. 11 Shear strength envelope for (a) exterior joints (b) interior joints

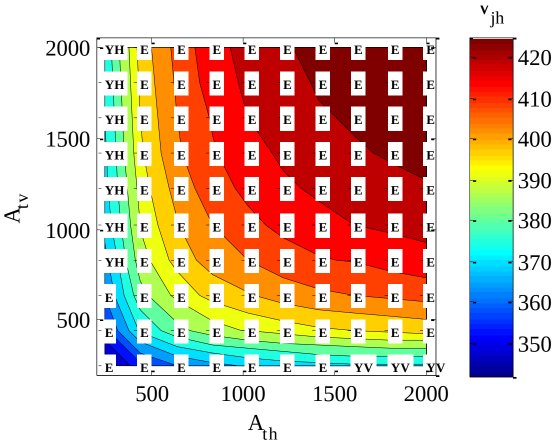


Fig. 12 Failure mode pattern with shear strength contour

pattern of the failure modes can be observed from Fig. 12 consisting of the shear strength envelope contours. The diagonal strut failure dominates when both the steel areas are large. The horizontal or vertical steel yielding occurs when the respective steel area is less.

4. Conclusions

The present study focuses on the evaluation of the available models (both empirical and analytical) for joint shear strength evaluation of beam-column sub-assemblages. In the present study, soften strut and tie model as proposed by Huang and Lee (2002) is suitably improved and used to determine the shear strength of RC beam column joint subjected to seismic type loading. It is found from the present study that the contribution from both concrete strut and steel ties plays key role in evaluation of the joint shear strength. It is found that increase in concrete compressive strength changes the failure mode of the joint. Shear strength of the joint reduces with increase in diagonal strut angle. Participation of core concrete is significant for diagonal strut angles in the range 35 to 55 degrees. Steel reinforcement enhances joint shear strength upto a saturation limit after which, the joint failure mode is determined by core concrete failure. Failure modes with various compressive strength, strut angle and steel in joint region obtained from the present study would help in zoning of various failure mechanisms which can be a tool in achieving desired strength hierarchy of the beam-column sub-assemblages for seismic upgradation. A failure envelope of shear strength is developed for RC beam-column joints using the MATLAB program. It is found that exterior joints are extremely vulnerable, though degradation of strength is faster in interior joint. This can be used as useful tool to assess the influence of amount of horizontal and vertical reinforcement in joint with given geometry and material properties on the failure mode of the joint. This computer aided procedure for estimating the joint shear strength would help the designers to achieve desired failure mechanism in the design strategy.

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