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Modified harmony search and its application to cost minimization of RC columns

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Abstract. This paper presents a variant of the Harmony Search Algorithm (HS) and its application to discrete optimization. The main proposed modifications regarding original HS are related to stopping criterion and reinitialization of population, called Harmony Memory. In order to investigate the efficiency of the algorithm, it was applied for obtaining optimal sections of reinforced concrete columns subjected to uniaxial flexural compression. To minimize the cost of the section, the amount and diameters of the reinforcement bars and the dimensions of the columns cross sections were considered as design variables. The obtained results were compared to those generated by other optimization methods. Since, to the examples, Harmony Search reached the same results achieved by Simulated Annealing, some additional analysis are presented in order to compare these methods regarding success rate and number of iterations to reach the optimum.

Keywords: heuristics; modified harmony search; simulated annealing; columns; reinforced concrete

1. Introduction

Structural analysis and design usually involve both highly complex procedures and a great number of variables. As a consequence, the solution has to be found iteratively while initial values are set to the variables based mainly on designer's sensitivity and experience. Also, the number of analysis steps is remarkably increased if optimum values are to be found among all possible alternatives. To mathematically describe the physical response of a structure, extreme function values can be found by using optimization techniques.

The great development of structural optimization took place in the early 60's, when programming techniques were used in the minimization of structures weight. From then on, a great diversity of general techniques has been developed and adapted to structural optimization. However, one of the reasons normally attributed to the little application of the optimization techniques to real structural engineering problems consists of the complexity of the mathematic model generated, normally described by non-linear behavior functions and producing a non-convex space of solutions (several points of optimum), problems for which the resolution by

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traditional mathematical programming methods have proved to be little efficient. For the resolution of these kind of problems the heuristic methods have played an important role, since they involve only values of functions in the process, regardless if there is unimodality or even continuity in their derivatives. Despite the great emphasis in the development of global optimization methods, researchers are even far from the attainment of a method that can be applied with the same efficiency to any class of problems.

The Harmony Search Algorithm is a metaheuristic proposed by Geem, Kim *et al*. (2001), which makes an analogy to musical improvisation of jazz. Since the method was first developed and published in 2001, it has been successfully applied to various research areas (Yoo, Kim *et al*. 2014).

This work presents a variant of Harmony Search Algorithm, illustrating its application with the cost optimization of reinforced concrete columns subjected to uniaxial flexural compression. Amongst the articles that involve the optimization of reinforced concrete columns, stand out the studies by Zielinski, Long *et al*. (1995), who presented a procedure for the optimization of reinforced concrete columns; Argolo (2000), who developed an optimization study of reinforced concrete sections subjected to uniaxial flexion using Genetic Algorithms; Camp, Pezeshk *et al*. (2003), who minimized the cost of frames and short columns using Genetic Algorithms; Rodrigues Júnior (2005), who proposed a formulation for the optimal design of reinforced concrete columns of tall buildings; Martínez-Martín (2008), who compared several optimization algorithms that allow to obtain the design of reinforced concrete rectangular columns with hollow sections for road and railway viaducts of different heights and spans; Bordignon and Kripka (2012), who minimized the cost of rectangular concrete columns subjected to uniaxial flexural by Simulated Annealing, and Medeiros and Kripka (2014), who minimized the environmental costs of reinforced concrete columns by Harmony Search.

The next sections of this paper present, a description of the optimization method, the developed formulation, some examples, a comparison between Harmony Search and Simulated Annealing performance and the conclusions.

2. Harmony Search Optimization Algorithm

The Harmony Search Algorithm, or simply HS, makes an analogy to musical improvisation of jazz, where musicians try to find, through repeated attempts, the perfect harmony (best solution to a problem). Iterations are called improvisations or practice. Variables correspond to musical instruments. Values for variables are the sounds of instruments. Each solution is called harmony, and the calculation of the objective function is called aesthetic estimation. The method can be sumarized in five steps:

Step 1 - Initialization of problem and algorithm parameters: definition of the objective function, the constraints and parameters of the algorithm. Main parameters are Harmony Memory Size (HMS), Harmony Memory Considering Rate (HMCR), Pitch Adjusting Rate (PAR) and Maximum Improvisation (MI).

Step 2 - Initialization of Harmony Memory: definition of first Harmony Memory (initial group of solutions). Harmony Memory (HM) is represented by a matrix of Eq. (1). Each line corresponds to a solution vector. The matrix has a number of rows equal to HMS and number of columns equal to the number of variables of the problem (N). Harmonies are generated randomly between a lower and upper range.

$$HM = \begin{bmatrix} x_1^1 & \cdots & x_N^1 \\ \vdots & \ddots & \vdots \\ x_1^{HMS} & \cdots & x_N^{HMS} \end{bmatrix} \tag{1}$$

Step 3 - Improvise a new harmony: from the initial solution, a new harmony is generated. This step is performed by using the parameters PAR and HMCR. For each variable of the new solution, a random number between 0 and 1 is generated. This number is compared to the value of HMCR (Harmony Memory Considering Rate). If the random number is lesser (probability equal to HMCR), the value of the respective variable in the new solution vector is retrieved from Harmony Memory existing. If the random number is greater (probability equal to 1-HMCR), a new value for the variable is generated, Eq. (2)

$$x_i^{'} \longleftarrow \begin{Bmatrix} x_i^{'} \in \{x_i^1, x_i^2, \dots, x_i^{HMS}\}\ with\ probability\ HMCR \\ x_i^{'} \in X_i\ with\ probability\ (1 - HMCR) \end{Bmatrix} \tag{2}$$

The choice of this new value can be done in two different ways, according to Eq. (3). Again, a random number between 0 and 1 is generated and compared to the parameter PAR. If the number is less than the rate (probability equal to PAR), Harmony Memory is considered, but with little adjustment, according to Eq. (4). This adjustment is defined by *bw* (maximum variation of tone) and a random number. If this is greater than PAR (probability equal to 1-PAR), the new value for the variable is randomly generated within the interval of possible solutions

$$Adjustment\ of\ the\ note\ x_i^{'} \longleftarrow \begin{Bmatrix} YES, with\ probability\ PAR \\ NOT, with\ probability\ \ (1 - PAR) \end{Bmatrix} \tag{3}$$

$$x_i^{'} \longleftarrow x_i^{'} \pm random\ number * bw \tag{4}$$

Step 4 - Update of Harmony Memory: At each new harmony improvised, it is checked whether this is better than the worst harmony of Harmony Memory (HM), relative the objective function. If confirmed this condition, the new harmony replaces the worst harmony of HM.

Step 5 - Check the stopping criterion: usually, the maximum number of improvisations MI. If it is not achieved, the algorithm returns to the third step.

Regarding the original work of Geem, Kim *et al*. (2001), several improvements and variations of the method have been proposed by other authors. An extensive study regarding these variations can be found, e.g., in Ingram and Zhang (2009), and in Fourie, Green *et al*. (2013).

Mahadavi, Fesanghary *et al*. (2007), for example, refined the method by developing the Harmony Improved Search Algorithm (IHS). It was suggested in IHS the dynamic variation of parameters PAR and *bw*, according to the number of iterations, between minimum and maximum limits for each factor. PAR increases linearly, while the parameter *bw* decreases exponentially.

The algorithm proposed and incorporated into present work considers the inclusion of the variable parameters of Mahadavi, Fesanghary and Damangir (2007). In addition to IHS, our algorithm considers the following modifications regarding original HS:

- Initialization of Harmony Memory: instead of generating all initial solutions randomly, one (or more) predefined solution can be included in the Harmony Memory. It can accounts for the designer’s knowledge regarding the specific problem;

-Harmony Memory reinitialization: to avoid premature convergence to local minimum, the Harmony Memory is restarted when all solutions achieve the same value. Only the best current solution is kept in this new HM. This procedure is similar to *reanealling*, adopted in Simulated Annealing Method;

-Stopping criterion as an additional criterion to avoid unnecessary calculations, the algorithm developed in this work can terminate the search when the best solution found does not varies after successive restarts, being NR the considered number of restarts.

3. Formulation for minimizing the cost of reinforced concrete columns

Considering a rectangular cross section, the objective of optimum design is to obtain a configuration that is capable of producing internal forces and moments (N_{rd} and M_{rd}) equal or higher than the applied external loadings (N_{sd} and M_{sd}), with minimal cost.

Moreover, the Brazilian standard NBR 6118/07 (2007) establishes some dimensional constraints for the columns. The minimum area of the section (A_{Cmin}) can not be less than 360 cm². The height *h*, the largest cross-sectional dimension, must be limited to five times the width *b*. The Brazilian standard also provides that cross-sectional area of the columns can not have dimensions less than 19 cm. Dimensions between 12 cm and 19 cm are allowed, but the loads must be multiplied by an additional coefficient.

In each corner of the rectangular sections, there should be at least one steel bar. The space between the bars can not be smaller than 2 cm, than the largest diameter of the bars, and than 1.2 times the maximum diameter of aggregate. The maximum spacing between bars is 40 cm or twice the section’s dimension *b*. The diameters adopted for the steel bars should not be less than 10 mm, nor more than 1/8 of the smallest cross-sectional dimension. In addition, there are minimum and maximum areas of reinforcements that must be respected according to NBR 6118/07 (2007). A detailed procedure regarding strength verification of reinforced concrete columns and standard requirements can be seen in Medeiros and Kripka (2014).

The formulation of the optimization problem starts out from the knowledge of some input parameters, previously defined and which basically represent the stresses acting on the element and the materials characteristics and costs. These design parameters do not change during the optimization process and are defined as: N_{sd} - axial force; M_{sd} - bending moment in relation to the axis *x*; *c* - cover depth; *fyk* - characteristic strength of steel; E_s - elasticity modulus of steel; C_c - unit cost of concrete; C_s - unit cost of steel; C_f - unit cost of formwork.

The design variables (x_i) are the values that represent the cross section dimensions, characteristic strength of concrete (*fck*) and the steel bar diameters as identified in Fig. 1, where x_1 and x_2 represent, respectively, the width (*b*) and the height (*h*) of the cross section; x_3 is the diameter of the four corner bars; x_4 represents the number of bars in the two layers parallel to x_1; x_5 is the diameter of the bars in the two layers parallel to x_1; x_6 represents the number of layers with two bars parallel to x_2; and x_7 is the diameter of the bars in the layers parallel to x_2. The strength *fck* is represented by the variable x_8.

In this study, all variables were considered as discrete, with the dimensions of the cross section varying in steps of one or five centimeters, this last to reduce the wastes if a timber formwork is used. The diameters of the reinforcement bars were limited to those available in commercial stores.

The cost function to be minimized in the optimization process considers the total cost of materials (concrete and steel) and formwork, and can be expressed by Eq. (5)

$$f(x) = x_1.x_2.C_c + \gamma_s.\pi.(x_3{}^2 + 0.5.x_4.x_5{}^2 + 0.5.x_6.x_7{}^2).C_S + 2.(x_1 + x_2).C_f \qquad (5)$$

The first term of the function represents the cost of concrete per unit volume (C_c), while the

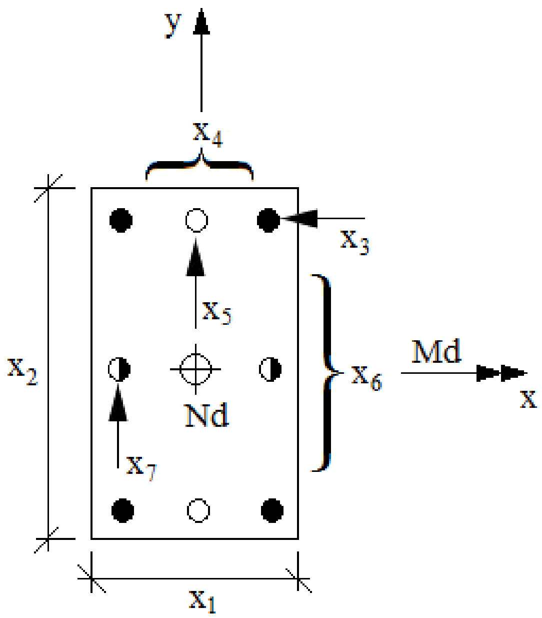

Fig. 1 Design variables

second represents the cost of the longitudinal reinforcement per unit mass (C_s), being γ_s the specific weight of steel. The last term represents the cost of formwork per unit area (C_f). All costs provide a relative value per unit length of the optimized element.

In the process of minimizing the cost function, all constraints imposed to the problem must be respected. Basically, the constraints are related to the strength criteria and construction requirements, as previously mentioned.

All design variables must satisfy the prescriptions of the Brazilian standard NBR 6118/07 (2007) with reference to the limitations of size, spacing, and steel ratio. Relatively to side constraints, the variables x_1 and x_2 may belong to the intervals contained in Eqs. (6)-(7) (units in cm)

$$x_1 \in [12; \dots;\ 200] \tag{6}$$

$$x_2 \in [12; \dots;\ 1000] \tag{7}$$

Upper limits were determined to be large enough to not interfere on the results of the optimization process. For the same reason, the variables x_4 and x_6 (number of steel bars in the two layers parallel to x_1 and the number of layers with two bars parallel to x_2, respectively) can take only integer values between 0 and 18. Also, x_3, x_5, and x_7 are variables that represent the longitudinal steel bars, restricted to the following diameters (units in mm)

$$x_3, x_5, x_7 \in [10; 12.5; 16; 20; 22; 25] \tag{8}$$

The variable x_8, related to characteristic strength of concrete (fck), can assume discrete values shown in Eq. (9). The units are exposed in MPa.

$$x_8 \in [20; 25; 30; 35; 40; 45; 50] \tag{9}$$

In the sequence, the inequality constraints of the problem are presented in normalized equations.

$$g_1 = 1 - \frac{N_{rd}}{N_{sd}} \leq 0 \tag{10}$$

$$g_2 = 1 - \frac{M_{rd}}{M_{sd}} \leq 0 \tag{11}$$

$$g_3 = 1 - \frac{5.x_1}{x_2} \leq 0 \tag{12}$$

$$g_4 = 1 - \frac{e}{e_{mín}} \leq 0 \tag{13}$$

$$g_5 = 1 - \frac{e_{máx}}{e} \leq 0 \tag{14}$$

$$g_6 = 1 - \frac{\rho}{\rho_{mín}} \leq 0 \tag{15}$$

$$g_7 = 1 - \frac{\rho_{máx}}{\rho} \leq 0 \tag{16}$$

$$g_8 = 1 - \frac{x_1.x_2}{Ac_{min}} \leq 0 \tag{17}$$

$$g_9 = 1 - \frac{x_1}{8.x_3} \leq 0 \tag{18}$$

$$g_{10} = 1 - \frac{x_1}{8.x_5} \leq 0 \tag{19}$$

$$g_{11} = 1 - \frac{x_1}{8.x_7} \leq 0 \tag{20}$$

The constraints described by Eqs. (10)-(11) determine, respectively, that internal resistant force N_{rd} and bending moment M_{rd} should be greater or equal than acting forces and moments N_{sd} and M_{sd}. Eq. (12) limits the maximum height of the section to five times the size of the base. Minimum and maximum spacing *e* to steel bars constrained by Eqs. (13)-(14), while Eqs. (15)-(16) define minimum and maximum reinforcement rates ρ. Eq. (17) ensures sections with concrete area larger than A_C minimum. Finally, Eqs. (18)-(20) guarantee that the diameter of the steel bars are smaller than 1/8 of the base length *b*.

Regarding the constraints, a penalty function technique was adopted, in which constrained problems are transformed into unconstrained ones by adding to the function *f*(*x*) a penalty function *P*(*x*), which considers a multiplying factor *r* applied to all the constraints that are not satisfied. Thus, the penalized function *F*(*x*) can be written as

$$F(x) = f(x) + P(x) \tag{21}$$

Being

$$P(x) = \sum r.g(x) \tag{22}$$

4. Examples

The formulation described was implemented using the Fortran programming language, and numerical simulations were performed in order to test the efficiency of the proposed procedure.

Regarding HS, the following parameters were adopted:

bw Maximum = 2.0
bw Minimum = 1.0
PAR Maximum = 0.5
PAR Minimum = 0.3
Harmony Memory Size (HMS) = 50
Harmony Memory Considering Rate (HMCR) = 0.9
Pitch Adjusting Rate (PAR) = 0.45
Maximum Variation of Tone (bw) = 1.5
Maximum Improvisation (MI) = 250,000
Number of restarts (NR) = 5

This first example was taken from Argolo (2000), who optimized columns sections according to the Brazilian standard NBR-6118/80. From a column section initially dimensioned by iteration abacuses (fixed section of 30×70 cm), the author proposed a new configuration using Genetic Algorithms (GA). The acting force and moment, along with the costs of the materials (in Brazilian Reais, R\$) used in the example are: N_{sd}=2,142.86 kN, M_{sd}=375 kN.m, C_c=125.00 R\$/m³, C_s=1.27 R\$/kg and C_f=16.49 R\$/m². It was adopted *fck*=25 MPa and *fyk*=500 MPa.

Fig. 2(a) and Fig. 2(b) show sections obtained by Argolo (2000) by iteration abacuses and

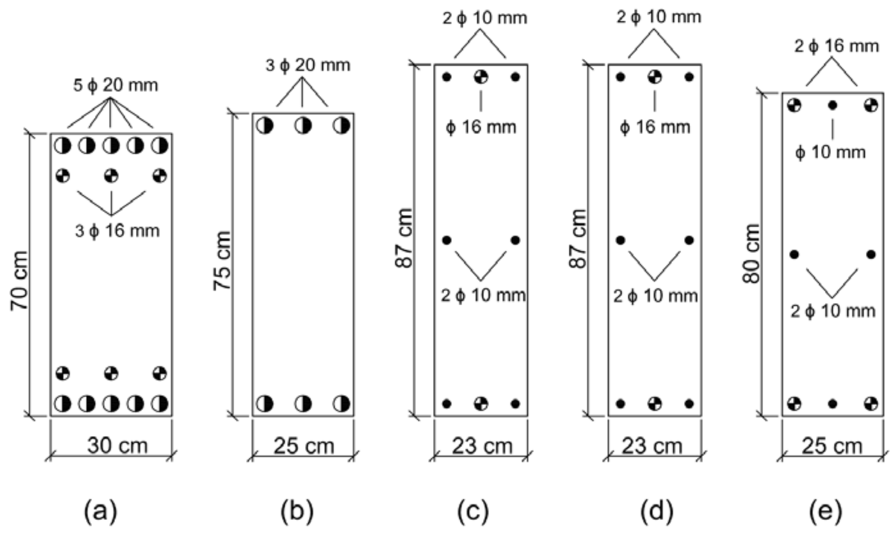

Fig. 2 Detailing of sections for example I

Table 1 Optimal solutions costs for example I (R$/m)

Method	Concrete	Steel	Formwork	Total Cost	Variation
Iteration Abacuses (a)	26.25	43.35	32.98	102.58	-
Genetic Algorithm (b)	23.44	18.79	32.98	75.21	-26.68%
Simulated Annealing (c)	25.01	8.70	36.28	70.00	-31.76%
Harmony Search (d)	25.01	8.70	36.28	70.00	-31.76%
Harmony Search (e)	25.00	11.15	34.63	70.78	-31.00%

Genetic Algorithms, respectively. This author detailed the dimensions of the concrete sections as multiple of 5 cm. Fig. 2(c) shows the best result obtained by Bordignon and Kripka (2012) by Simulated Annealing. The authors discretized sections in 1 cm and used the Brazilian standard NBR-6118/07 (2007) in analyzes. Fig. 2(d) and Fig. 2(e) refer to solutions obtained in present work, by the usage of Harmony Search, for sections multiple of 1 cm and 5 cm, respectively.

Table 1 shows the optimal cost obtained by each method employed. In the last column it is indicated the percentual variation in cost, regarding the original solution proposed by iteration abacuses. It can be observed that SA and HS led to the same results, with an additional economy in relation to GA (6.93%) and to Abacus (31.76%). In addition, optimum sections generated by Argolo (2000) violate the constraint related to minimum spacing between longitudinal bars.

The solution obtained by Harmony Search, considering multiple dimensions of 5 cm, causes an increase of 1.11% on the cost, but the solution is still economical compared to that obtained by Iterations Abacuses (31%) and Genetic Algorithms (5.89%).

For this example, some tests of the Harmony Search Algorithm were performed in order to study the dependence of the algorithm regarding the initial solution. In some trials, all variables initially assumed its maximum value (upper limit). In others, they initially assumed its minimum value (lower limit). Results confirmed the conversion to the optimal solution in both situations (Fig. 3). The graph illustrates the cost function variation (not penalized) according to the number of iterations. These results were obtained with an average of ten trials in sequence.

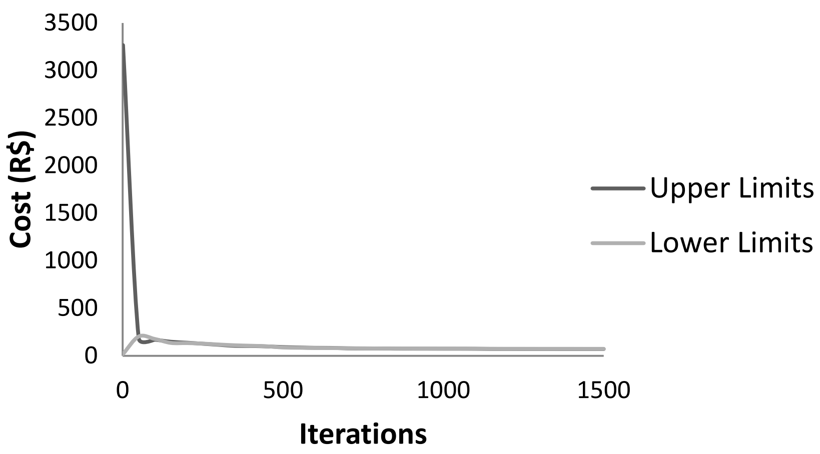

Fig. 3 Convergence for optimal from different initial solutions

Another example was taken from Zielinski, Long, and Troitsky (1995), who studied a case of uniaxial flexural compression according to the Canadian standard CSA CAN3-A23.3-M84. The costs of the materials used were: C_c=110.00 \$/m³, C_s=2.10 \$/kg and C_f=27.00 \$/m² along with the acting force N_{sd}=1780 kN and moment M_{sd}=362 kN.m. It was adopted *fck*=30 MPa and *fyk*=500 MPa.

The optimum design, based on Mathematical Programming (MP) and using the Powell method suggested by the authors, corresponds to a rectangular cross section of 31.96×59.36 cm and a steel section of 25.8 cm². This section has been simplified assuming the practical dimensions of 35×60 cm (Fig. 4(a)). Argolo (2000) compared these results to those obtained from the implementation of the Genetic Algorithm method, following the same Canadian standard, and adopting multiple sections of 5 cm. The section optimized by this method assumed values of 30×65 cm in cross section (Fig. 4(b)), resulting in a reduction of 6.04% in the final cost of the section when compared to the optimal result obtained by Zielinski, Long, and Troitsky (1995), and 13.87% in relation to the practical result suggested by the same authors. Camp, Pezeshk, and Hansson (2003) employed the American standard ACI 318-89 to optimize the same column, also using GA. An optimal section of 30.48 cm×63.5 cm was obtained (steel area of 20.26 cm²), when working with continuous variables. A cost reduction of 6.45% was achieved in relation to the optimal section obtained by Zielinski, Long, and Troitsky (1995), and 14.24% in relation to practical section proposed by the same authors.

The optimal section generated by the Harmony Search (Fig. 4(d)), following the criteria prescribed by the Brazilian standard NBR 6118/07 (2007), and adopting sections dimensions multiple of 1 cm, showed a decrease in cost of 24.23% when compared to the optimal section of Zielinski, Long *et al*. (1995), 30.54% over the practical result suggested by the same authors, 19.35% in comparison to the section optimized by Argolo (2000), and 19.01% in relation to the section by Camp, Pezeshk *et al*. (2003). The same results were obtained by Bordignon and Kripka (2012) with Simulated Annealing, employing the Brazilian standard NBR 6118/07 (2007), according to section of Fig. 4(c). The same figure shows another section obtained by the Harmony Search, but considering dimensions of the concrete section discretized in 5 cm. In this case, the cost was also effectively reduced compared to solutions obtained by Mathematical Programming (28.78%) and Genetic Algorithms (17.31%), considering only the practical sections. Regarding the solution proposed by Harmony Search, the increase obtained was only 2.54% to section dimensions discretized in 1 cm.

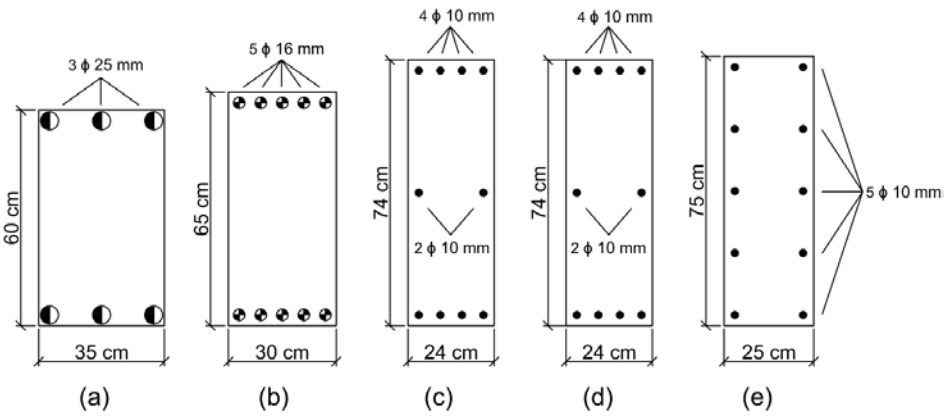

Fig. 4 Detailing of sections to example II

Table 2 Optimal solutions costs for example II (R$/m)

Method	Concrete	Steel	Formwork	Total Cost	Variation
Mathematical Optimization[1]	20.87	42.53	49.31	112.71	-
Mathematical Optimization (a)	23.10	48.55	51.30	122.95	9.08%
Genetic Algorithm[2]	21.29	33.40	50.75	105.44	-6.45%
Genetic Algorithm (b)	21.45	33.15	51.30	105.90	-6.04%
Simulated Annealing (c)	19.54	12.94	52.92	85.40	-24.23%
Harmony Search (d)	19.54	12.94	52.92	85.40	-24.23%
Harmony Search (e)	20.63	12.94	54.00	87.57	-22.31%

1) Results obtained by continuous variables [Zielinski, Long, and Troitsky (1995)]
2) Results obtained by continuous variables [Camp, Pezeshk, and Hansson (2003)]

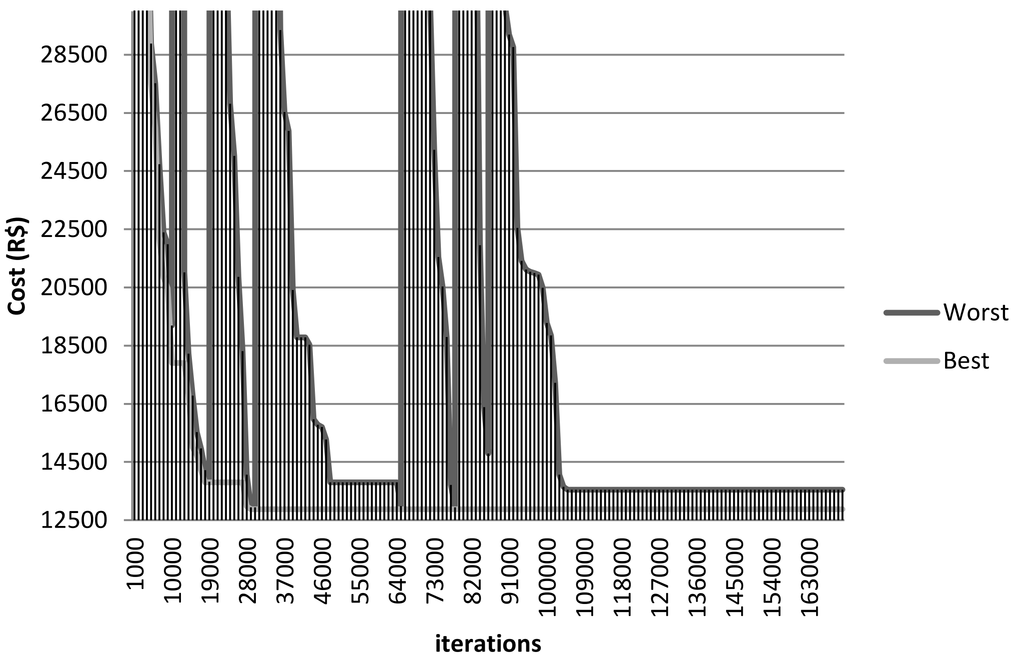

Fig. 5 Iteration history to example of Kripka *et al*. (2015)

Again, the sections obtained by Zielinski, Long, and Troitsky (1995) and Argolo (2000) did not consider some constructive provisions required by the Brazilian standard NBR 6118/07 (2007) for reinforced concrete columns. Table 2 shows the optimal cost obtained by each method employed. The last column indicates the percentual variation in cost, regarding the original solution proposed by Zielinski, Long, and Troitsky (1995), who employed Mathematical Programming.

Fig. 5 illustrates the process of reinitialization of the Harmony Memory to another structure analyzed by the authors (Kripka *et al*. 2015). It can be noticed that, when the worst and best solution achieve the same value, just the best result is maintained in HM. In general, the worst result corresponds to an unfeasible solution, which was penalized as previously described.

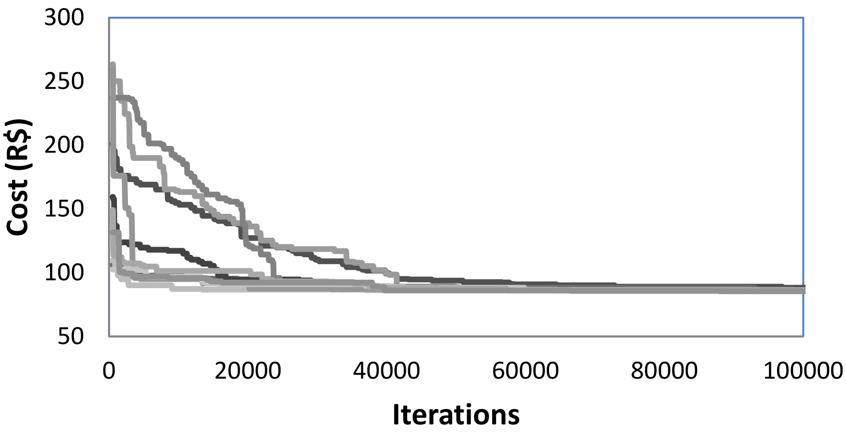

Fig. 6 Optimal cost versus number of iterations (Simulated Annealing)

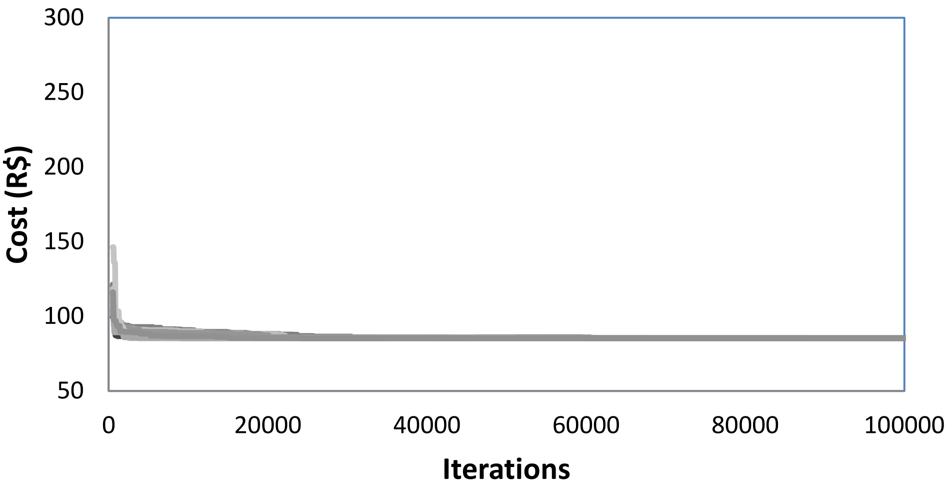

Fig. 7 Optimal cost versus number of iterations (Harmony Search)

5. Comparison between Harmony Search and simulated annealing

In previous examples, it was verified that Harmony Search reached the same results achieved by Bordignon and Kripka (2012) by the usage of Simulated Annealing. This item presents a more detailed comparison between these optimization methods, using software developed in this paper and the computer program of Bordignon and Kripka (2012).

Comparing the number of iterations required for convergence, Harmony Search showed more satisfactory results. In Fig. 6 and Fig. 7 the relation between cost variation and number of iterations for the second example is shown, considering ten runs to each method. The results for each attempt by Harmony Search and Simulated Annealing are presented, respectively. Upper limits were the initial solution of variables. It can be seen that optimal cost decreases rapidly to HS, when compared to SA.

Table 3 Comparison between SA and HS algorithms (upper limits)

Method	Number of Iterations	Best Solution	Worst Solution	Average Solution	Number of Repetitions of Best Solution
Simulated Annealing	156,400[1]	85.40	86.05	85.55	1
	250,000[2]	85.40	85.47	85.44	4
Harmony Search	156,400[1]	85.40	85.47	85.42	8
	250,000[2]	85.40	85.47	85.42	8

1) Average number of iterations for HS
2) Maximum number of iterations for SA and HS

Table 4 Comparison between SA and HS algorithms (lower limits)

Method	Number of Iterations	Best Solution	Worst Solution	Average Solution	Number of Repetitions of Best Solution
Simulated Annealing	184,950[1]	85.40	85.64	85.47	3
	250,000[2]	85.40	85.47	85.45	3
Harmony Search	184,950[1]	85.40	85.47	85.41	9
	250,000[2]	85.40	85.47	85.41	9

1) Average number of iterations for HS
2) Maximum number of iterations for SA and HS

A higher convergence rate using the Harmony Search was also observed, which achieved 85%, for the example considered, against 35% obtained by Simulated Annealing to the maximum number of iterations (MI). Combined with a faster convergence, this makes the Harmony Search competitive with Simulated Annealing. The results for this example, starting the solution from upper and lower limits of the variables, are presented in Table 3 and Table 4, respectively. The number of iterations used for the comparison is related to the average number of iterations for Harmony Search and the maximum number of iterations for both methods, according to the legend of tables.

6. Conclusions

This paper dealt with the problem of optimization of rectangular reinforced concrete columns subjected to uniaxial flexural compression, following the requirements of the Brazilian standard NBR 6118/07 (2007), and using the Harmony Search Method.

Modifications were proposed in the original Harmony Search Algorithm developed by Geem *et al*. (2001), which proved to be valid to obtain the optimum solutions with fewer iterations and higher convergence rate. Aiming to test the proposed formulation and the Harmony Search Algorithm developed, three examples were performed.

For the examples analyzed, Harmony Search led to a better solution than the Genetic Algorithms, Mathematical Programming and the practical dimensioning performed with the aid of iteration abacuses. Regarding Simulated Annealing, the results obtained were the same, but HS outperformed SA in relation to the convergence rate (85% versus 25% for the example analyzed), as well as the number of iterations required to achieve the optimal solution, showing up Harmony Search quite competitive with the Simulated Annealing.

Acknowledgements

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