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Transient thermoelastic analysis of carbon/carbon composite multidisc brake using finite element method

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Abstract. In the current paper, a generalization of the results of Zhao *et al.* (2008) on a new design of C/C composite multidisc brake system is presented. The purpose of this paper is to study the effect of thermal sensitivity of Carbon/Carbon (C/C) composite material on the temperature distributions, deformation, and stress during braking. In this regard, a transient temperature–displacement coupled analysis for C/C composite brake discs with frictional heat generation under simulated operating conditions is performed. An axisymmetric model for brake system is used for the finite element analysis according to the theory of energy transformation and transportation. The transient temperature distributions on the friction surfaces, deformation, and stress are obtained. To check the validity, the results are corroborated with other solutions available in the literature, wherever possible. The current study could be used as a guide in the initial design of a high performance multidisc brake system.

Keywords: thermoelastic; C/C composite brake system; finite element method

1. Introduction

Extreme temperature is an important concern in the design and analysis of sliding contacts such as clutches, bearings and brakes. Vehicle braking results in converting large amounts of kinetic energy into heat at the disc interface between brake rotors and stators. Thermal deformation and frictional heating have a significant influence on the contact pressure and temperature on friction surfaces in sliding contact systems and also causes misaligned and often localized contact, which leads to stiction with rise in localized temperature and wear and often observed in brakes and clutch discs as hot spots. Also as the result of localized stick-slipping brake or clutch, in some cases judder and brake squeal occurs (Menday and Rahnejat 2010, Centea *et al.* 2001, Papinniemi *et al.* 2002). Slipping of the contact surfaces results in increasing the temperature and the peak value of temperature is one of the most important factors in the design of sliding system.

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Therefore, it is crucial to investigate temperature variation of the brake discs under the braking condition for developing tribology technology.

Thermal characteristics of discs have been estimated and analyzed in the literature depending on different assumptions and geometries. Talati and Jalalifar (2009) extracted the governing equations for heat transfer on a simple disc and pad assembly and solved the equations using Green's function approach. They estimated the temperature rise of the disc assuming that all of the kinetic energy of a vehicle was converted into thermal energy. The convective part in the thermal cooling was found to have the most significant influence to obstruct overheating the components of the sliding system which causes decrease in friction coefficient. Zhang *et al.* (2011) Simulated transient temperature field during braking in aircraft for carbon/silicon-carbide (C/SiC) composite brake discs adopting a finite element method based on the theory of energy transformation. They investigated the effects of initial velocity, deceleration and friction coefficient on the highest temperature of the brake components. Yevtushenko and Grzes (2016) investigated the effects of sliding velocity and temperature on transferred heat to the disk brake. Choi and Lee (2004) studied the transient thermoelastic analysis of disc brakes in repeated brake applications using the finite element method. They solved the coupled elastic contact and heat conduction problems. Also, they investigated the thermoelastic instability phenomenon on each of the friction surfaces between the contacting bodies. Adamowicz and Grześ (2012) studied the effect of convective heat transfer on the distributions of transient temperature of a real brake system. They found that cooling of the contacted surfaces of the disc during relatively short braking has insignificant effect. However brake release after braking period leads significant decrease in temperature of the disc. Zhao *et al.* (2008) studied the thermomechanical behavior of a carbon–carbon (C/C) composite multidisc clutch under simulated operating conditions and obtained the contact pressure and temperature distributions on the friction surfaces. They were concluded that the peak temperature decreases by increasing clutch disc thickness. They also showed the specific heat and transverse thermal conductivity have a significant effect on the thermomechanics of the clutch contact. Belhocine and Bouchetara (2012a, 2012b) investigated temperature field of solid and ventilated brake disc using three different cast iron materials. They have separately calculated heat transfer coefficient on different locations on surfaces of the disc and implemented a finite element model. Gkinis *et al.* (2018) evaluated characteristics of friction lining for unused and used clutches under operational conditions related to slippage and contact temperatures. They showed increasing the coefficient of friction of lining material results in decreasing the slippage and heat generation. Jiang *et al.* (2012) analyzed the thermal finite element model of SiC/Al discs of a brake system at a speed of 300 km/h within emergency braking with airflow cooling using computational fluid dynamics (CFD) model. They were concluded that higher convection due to airflow cooling reduce the maximum temperature and thermal gradients in the braking, since heat will be dissipated faster from hot parts of the disc. Ghadimi *et al.* (2013) studied thermal analysis of the brake disc mounted on wheel R920K for the locomotive type ER24PC. They found that the fin aerodynamic design on brake has critical role in the thermal stresses removal at the internal surface and insignificant role on the reduction of maximum temperature on contact surface. Bauzin *et al.* (2018) evaluated the heat flux induced by breaking of aircraft wheel. They have identified the Induced heat flux due to friction from experimental results. Belhocine (2017) investigated thermal effects in structure and contact behavior of a disc-pad assembly using finite element approach, and presented temperature distribution, deformation, stress, and contact pressure on the disc-pad and compared mechanical and thermomechanical analysis results. Zhang *et al.* (2018) used a displacement gradient circulation method for the thermal–structural coupling analysis of friction pair during the braking process considering geometric and motion characteristics of brake pads and brake discs. They used a three-dimensional model of transient heat transfer for brake friction pair in the Abaqus software. Alnaqi *et al.* (2015) proposed a scaling methodology to evaluate the thermal performance of a disc brake at a reduced scale and used a two dimensional axisymmetric transient thermal finite element model on the Abaqus software for validation of the results by comparing the full and small scale discs. Belhocine *et al.* (2016) determined disc temperature and examined stress concentration, structural deformation and contact pressure of brake disc and pads during single braking stop event using ANSYS commercial finite-element software. They investigated the effects of using a fixed caliper, different friction coefficients and different speeds of the disc on the stress concentration, structural deformation and contact pressure of the brake disc and pads. Mew *et al.* (2015) studied transient thermal response of a highly porous ventilated brake disc. In this regard, they compared the transient thermal response of a newly developed ventilated brake disc with a porous medium core (wire-woven bulk diamond) and those of a solid brake disc and a conventionally ventilated brake disc with pin fins. Patil *et al.* (2016) presented transient thermal analysis of magnetorheological (MR) brake proposed for an e-bicycle in order to estimate the temperature rise of MR fluid on account of braking maneuver. They also compared the obtained numerical results by the experimental results. Ambekar *et al.* (2017) carried out thermo-structural analysis on the seven different geometric models of disc brakes with carbon fiber reinforced silicon carbide material (C/SiC) and determined steady state temperature variations, transient structural deformations and von mises stresses. Yevtushenko *et al.* (2017) investigated the time-dependent frictional heating of a disc with applied thermal barrier coating (TBC) on its working surface and determined the temperature fields in the coating and the disc during braking taking into account the dependency of thermal properties of materials to temperature. Also several researchers have reported different studies on thermal and thermoelastic behavior of aerospace structures (Lin *et al.* 2018, Ebrahimi and Heidari 2018, Mehar and Panda 2018).

It should be noted that this paper is a generalization of the results of Zhao *et al.* (2008) on a new design of C/C composite multidisc brake system. The purpose of this paper is to study the effect of thermal sensitivity of C/C composite material on the temperature distributions, deformation, and stress during braking. In this regard, thermal analysis of full multidisc brake system is presented using the ANSYS parametric design language code (APDL). In order to assure accuracy of the solution the results are compared with the results of Zhao *et al.* (2008). The effects of friction coefficient on the transient temperature distributions, deformation, and stress of the brake system are investigated.

2. Numerical modeling

Fig. 1 shows the new design of C/C composite multidisc brake system. It consists of multiple rotating discs sandwiched between stationary discs. The rotating discs are driven by the wheel, while stationary discs are restrained by torque tube keys. The material properties, and parameters used in the thermal estimation are depicted in Table 1. The discs material is C/C composite with desirable thermo-physical characteristics.

Brakes are essentially a mechanism to change the energy types. At the initial time of breaking, rotors of brake system rotate with an angular speed ω and vehicle has kinetic energy. Applying the brakes, result in conversion of total kinetic energy into heat at the disc interface between brake rotors and stators. Linear and angular velocity of the vehicle and disc decrease linearly by time:

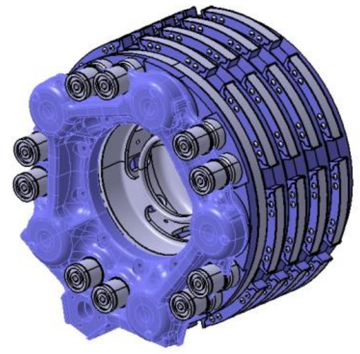


Fig. 1 C/C composite discs assembly

Table 1 Contact status of elements between back plate and rotor

Time (s)	0	2	4	6	8	10	12	14	16	18	20
Contact status	S	C	C	C	C	C	C	C	C	C	C

$$V = V_0(1-\frac{t}{t_0}) \ \rightarrow \ \omega = \omega_0(1-\frac{t}{t_0}), \quad 0 \le t \le t_0 \tag{1}$$

2.1 Heat flux

The generated heat by friction is dissipated through conduction within the bodies being in contact and convection from the free surfaces of the system with the same constant heat transfer coefficient h_{conv} being average value for the analyzed braking process. The thermophysical properties of materials of the rotors and stators are constant. Heat flux on a surface due to convection, is governed by:

$$q = h_{conv}(T - T_{ambient}) \tag{2}$$

where T is the temperature at a point on the surface, and $T_{ambient}$ is the ambient temperature. It is assumed that the thermal resistance on the surfaces of contact is negligible and therefore temperatures of the discs on these surfaces are equal.

2.2 Heat transfer model

The two-dimensional unsteady heat conduction equation of each body for an axisymmetric problem within the cylindrical coordinate system is derived as:

$$\rho c \frac{\partial T}{\partial t} = \frac{k_r}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + k_z \frac{\partial^2 T}{\partial z^2} \tag{3}$$

where ρ, c, kr and kz are the density, specific heat and thermal conductivities in the r and z direction of the material, respectively.

Using Galerkin's approach, a finite element form of unsteady heat Eq. (3) can be written in the following matrix form as

$$[C_T]\{\dot{T}\}+[K_T]\{T\}=\{R\} \tag{4}$$

where $[C_T]$ is the heat capacity matrix, $[K_T]$ the heat conductivity matrix, $\{T\}$ the nodal temperature and $\{R\}$ is the thermal force matrix. The detailed derivations and method for solving of Eq. (4) can be found in the literature (Hsu 2012, Choi and Lee 2003, Cook 2007, Shahzamanian *et al.* 2010).

2.3 Elastic problem

The constitutive equation for the elastic problem with thermal expansions and mechanical loading can be written as:

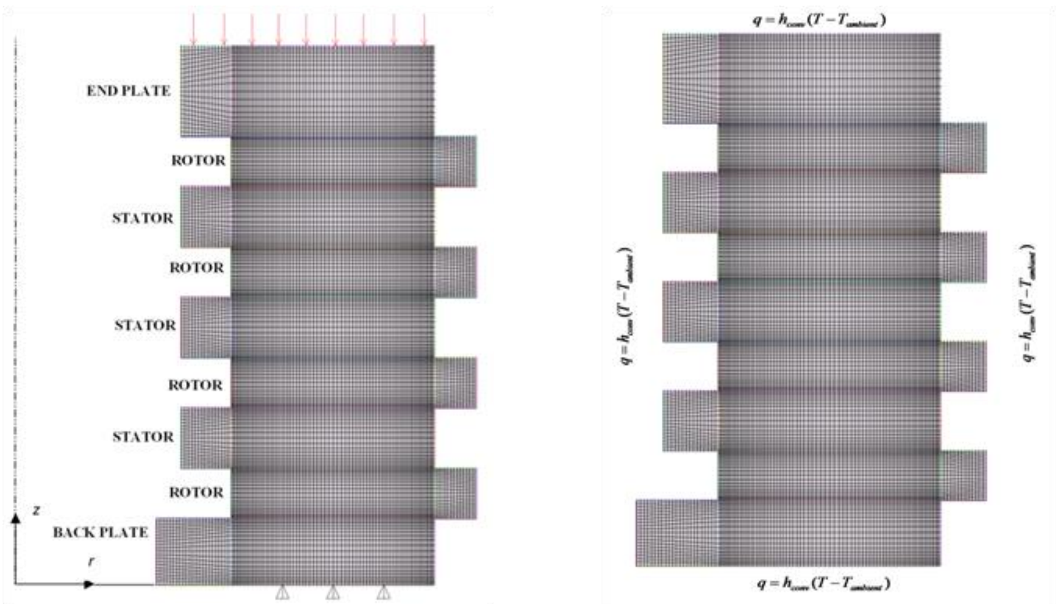
$$\{\sigma\}=[D](\{\varepsilon\}-\{\varepsilon_0\}) \tag{5a}$$

where

$$\{\varepsilon_0\}=\{\alpha\}\Delta T \tag{5b}$$

In the above equation $\{\sigma\}$ is the stress vector, $[D]$ is the elasticity matrix and $\{\alpha\}$is the thermal expansion coefficient vector, respectively.

The detailed derivations and method for solving the thermoelastic finite element equation of equilibrium can be found in the literature (Hsu 2012, Choi and Lee 2003, Cook 2007).



(a) Mechanical boundary condition (b) Thermal boundary condition

Fig. 2 The finite element model of multidisc brake system

2.4 Contact model of brake discs

The intensity of heat flux induced by friction between contacts surfaces that enters the discs are calculated from the formula:

$$\Delta q = \mu VP \tag{6a}$$

where

$$V = r\omega \tag{6b}$$

In the above equation μ is the friction coefficient, P is the contact pressure and ω is the angular sliding velocity.

In the numerical modeling of the contact problem, special attention is needed because the actual contact area between the contacting bodies is not known beforehand. Although the bodies in contact have linear materials, the contact is a nonlinear problem. Therefore iterative solution is necessary for accurate solution of the contact problems (Choi and Lee 2004, Adamowicz and Grześ 2012, Zhao *et al.* 2008).

2.5 Finite element modelling

This section describes the steps of simulation of brake discs by using the parametric design language code of ANSYS 14. In this study, an axisymmetric model for brake system was used for the finite element analysis. The axisymmetric modeling of the problem leads significant reduction in computational time. This method is limited to rotational symmetry shapes. Fig. 2 shows the element mesh of multidisc brake system with boundary conditions. The convective boundary conditions are imposed on all boundaries to consider more realistic heat conditions. It consists of four rotors, three stators, one end plate and one back plate. The hydraulic pressure is applied to the surface of the end plate. The procedure of implementation of the presented formulation in finite element program is described as follows. At the beginning of each time increment, the temperature distribution over the disk brakes have been obtained from heat flux of friction and heat transfer by conduction within disks and convection to ambient. Then the thermoelastic strain and stress have been obtained by the disk thermal and mechanical loading.

Fine finite element mesh has been used near contact surfaces due to high temperature gradients. For contact analysis, ANSYS requires separate identification between contacting elements. The element that makes contact is called “Conta171” and the other is the element to be connected and contacted, or target element, called “Targe169”. For the present C/C brake “Conta171” is used for contact surfaces of end plate, back plate and stators while “Targe169” element is used for contact surfaces of rotors. The geometry of “Conta171” and “Targe169” elements are shown in Figs. 3 and 4. Actually the geometries of these two elements (Conta171 and Targe169) are the same and they are applied to describe the contact status of surfaces.

Also, three types of status are exists for contacts as designated by ANSYS, contact (C), near contact (N) and sticking (S). Contact (C) is the status that all elements of two surfaces are in contact; near contact (N) is the status that some elements are not in contact and are in near contact and sticking (S) is the status that some elements stick together. All contact elements in the results are in contact (C) status with sliding condition. In the Table 1 contact status has been shown between back plate and rotor at the time of operation.

On the other hand, there are different algorithms used for surface-to-surface contact. In this

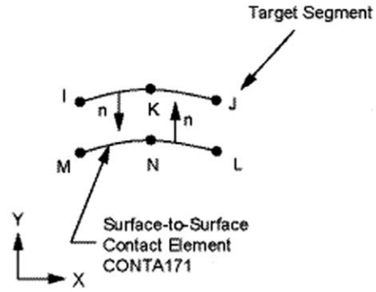


Fig. 3 Geometry of Conta171 and Targe169 elements (ANSYS 14 user's manual)

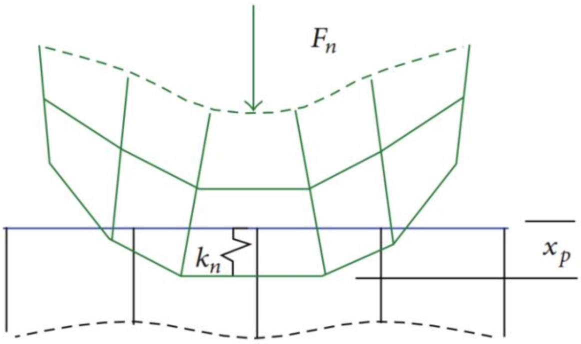


Fig. 4 The contact stiffness between two contact parts (ANSYS 14 user's manual)

paper penalty method is used for contact surfaces. This algorithm used constant "spring" to establish the relationship between the two contact surfaces (Fig. 4). The contact force (pressure) between two contact parts can be written as follows:

$$F_n = k_n x_n \tag{7}$$

where F_n is the contact force, k_n is the contact stiffness, and x_p is the distance between two existing nodes or separate contact parts (penetration or gap).

Also, there is the contact stiffness factor (F_{kn}) in the analysis that controls the extent to which penetration between contact and target surfaces nodes can take place. The usual range for F_{kn} factor is from 0.01 to 1.0. High values of F_{kn} decrease the amount of penetration, but can lead to ill-conditioning of the global stiffness matrix and to convergence difficulties. Lower F_{kn} values give a non-convergent result with excessive penetration. Ideally, F_{kn} should be in the specified range such that it is high enough for small penetration and also low enough to facilitate convergence of the solution. In this study, the contact stiffness value is selected based on accuracy of results of validation problem and appropriate value is taken to be 1.

ANSYS specific "Plane13" element is used for all brake discs. The rectangular shape of this element with 4 nodes has been used in the finite element model. "Plane13" is a four node 2D plane element. It is suitable for problems involving structural field, magnetic, thermal, electrical and piezoelectric. Plane stress, plane strain and axisymmetric models are available for this element. This element was used for steady state and transient analyses (Shahzamanian *et al.* 2010). In this paper, the degrees of freedom of displacement in x, UX, displacement in y, UY and temperature, TEMP, are applied together with axisymmetric behavior. The final model consists of 14400 elements. Fig. 5 presents procedure of the transient thermoelastic analysis in ANSYS.

3. Results and discussion

The results of the transient thermomechanical behavior in the composite multidisc brake system are given for carbon–carbon material (Zhao *et al.* 2008). The axes proposed for distribution of material properties and boundary conditions are shown in Fig. 2 (a) (Zhao *et al.* 2008, Choi *et al.* 2004). The material properties are given in Table 2.

3.1 Validation of finite element code

Before some basic characteristics of thermoelastic behavior of multidisc brake system are discussed, it is convenient to valid the present code with other solutions available in the literature.

The transient temperature obtained at selected time intervals is compared in Fig. 6 with those given by Zhao *et al.* (2008) in the same Operation conditions.

As seen, the present code is very successful in the prediction of the transient temperature and the agreement between the results of the present code and those given by Zhao et al are very good.

3.2 Thermoelastic behavior of C/C composite multidisc brake system

The time history of hydraulic pressure P_h and angular velocity ω assumed for a brake cycle is shown in Fig. 7. In braking process, the hydraulic pressure P_h was assumed to linearly increase to 1 MPa by 1.5 s and then kept constant until 2.5 s. Also, the angular velocity ω was assumed to linearly decrease and finally became zero at 2.5 s. The time step Δt = 0.066 *s* was used in the computations. Fig. 8 illustrates the distributions of temperature on all contact surfaces at selected time intervals of 1 s, 2 s, 3 s and 4 s.

Table 2 Material Properties of Carbon/Carbon composite

Property	Value	Unit
Density	1800	Kg/m^3
Young's Modulus (E_r=E_θ)	50.2 E9	Pa
Young's Modulus (E_z)	5.89 E9	Pa
Poisson's Ratio ($\upsilon_{r\theta}$)	0.3	-
Poisson's Ratio ($\upsilon_{r\theta} = \upsilon_{rz}$)	0.33	-
Shear Modulus ($G_{\theta z} = G_{rz}$)	2.46 E9	Pa
Shear Modulus ($G_{r\theta}$)	1.93E9	Pa
Thermal expansion ($\alpha_r = \alpha_\theta$)	3.1E-07	-
Thermal expansion (α_z)	2.9E-07	-
Friction Coefficient (μ)	0.2	-
Convection Coefficient (h)	100	$W/(m^2K)$
Ambient Temprature	293	K

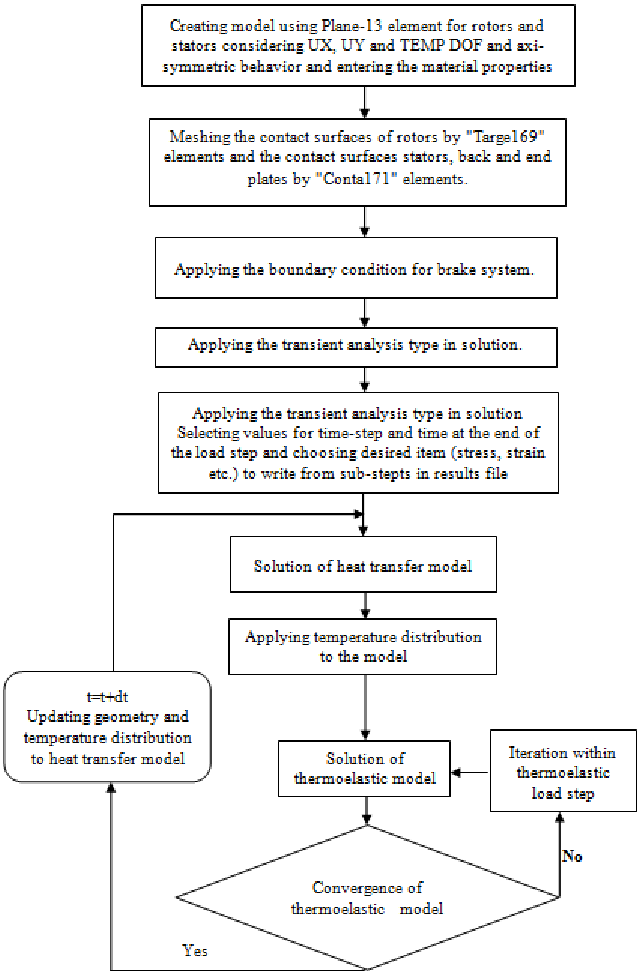
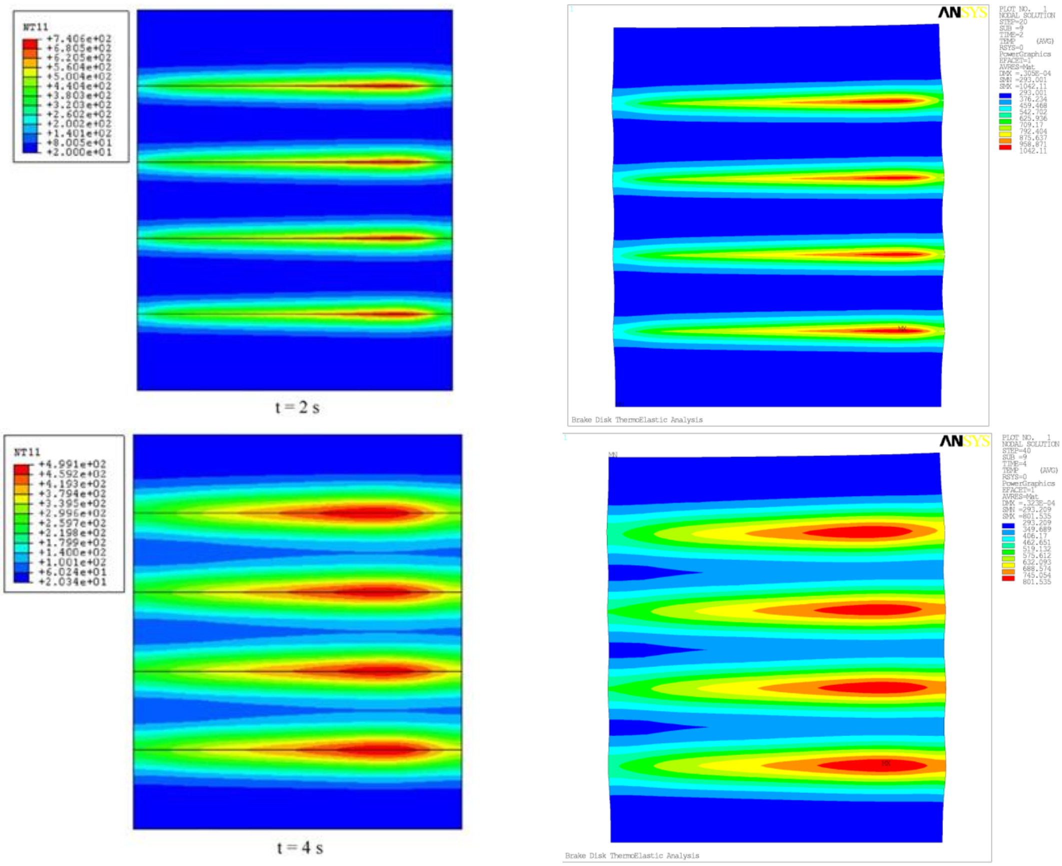


Fig. 5 Procedure of the transient thermoelastic analysis

As seen, at the beginning of the braking action the generated heat due to friction is very high and cause a fast rise of the temperature of the discs and this frictional heat generation decreases gradually because, the speed of brake discs are high in the beginning which leads to large frictional heat flow and reduce with time during the braking action. Therefore at the end of the braking time, the generated heat equals to zero. It can be concluded that the maximum disc surface temperature is obtained almost at the middle of the braking time.

Also, Fig. 8 presents the maximum transient temperature on all the friction surfaces occur near the outer radius of the discs and has non-uniform distribution on the contact surfaces. As the braking step progresses, due to the non-uniform growth of normal pressure on the friction surfaces and non-symmetric boundary conditions, the distribution of temperature of disc brakes becomes non-symmetric. The temperature for surface 8 is lower than that of the others. This is because of less friction surface and low thickness of the back plate.

To obtain a clear view of the thermomechanical behavior of the C/C composite multidisc brake



(a) Zhao *et al.* (2008) (0C) (b) Present Code (0K)

Fig. 6 Comparison of the transient temperature obtained at selected time intervals

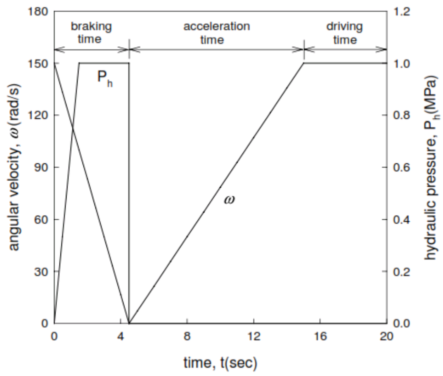
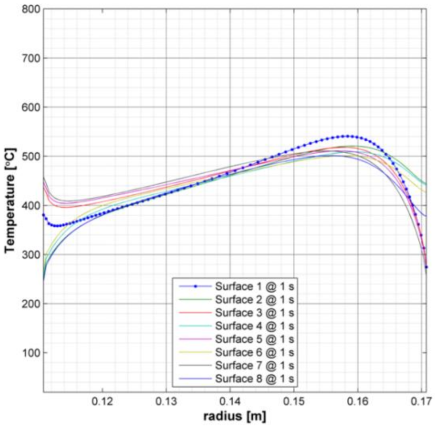
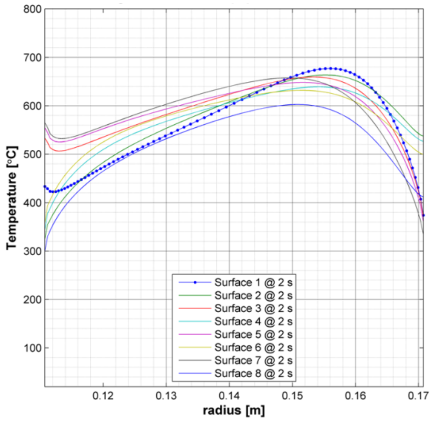


Fig. 7 The operation conditions used in transient thermoelastic analysis

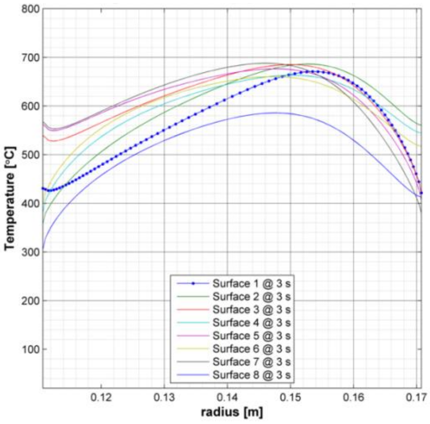
system, the transient evolution of temperature at selected time intervals is shown in Fig. 9. As expected, the temperature gradient is localized in the region of the friction surfaces at the early



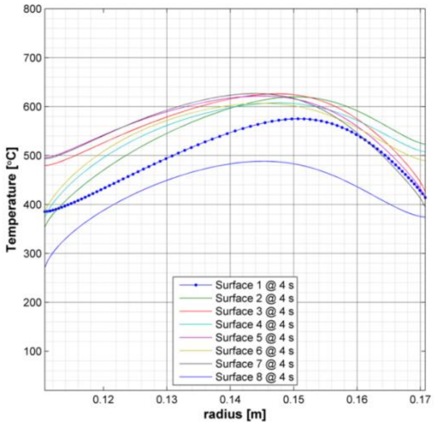
(a) at 1 s



(b) at 2 s

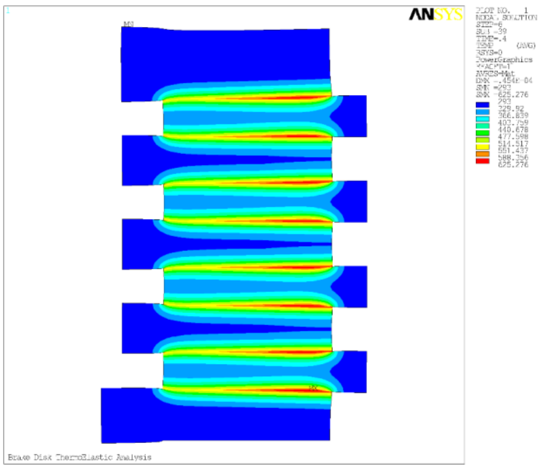


(c) at 3 s

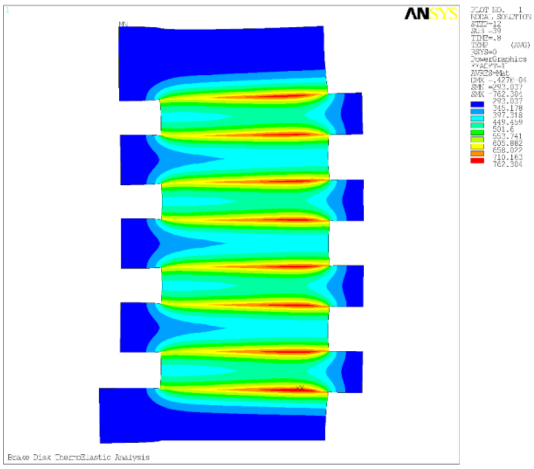


(d) at 4 s

Fig. 8 The distributions of temperature on all contact surfaces at selected time intervals

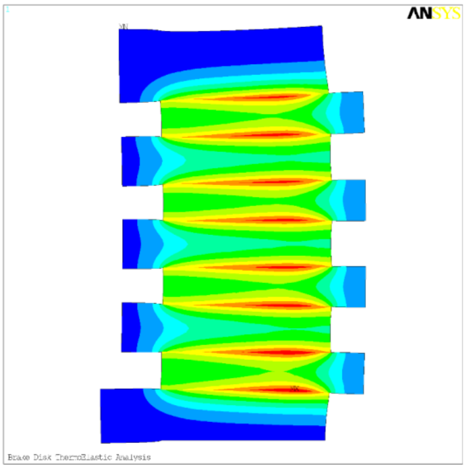


(a) 0.4 s

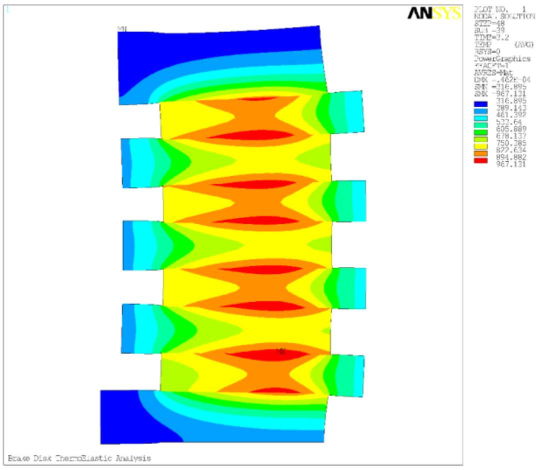


(b) 0.8 s

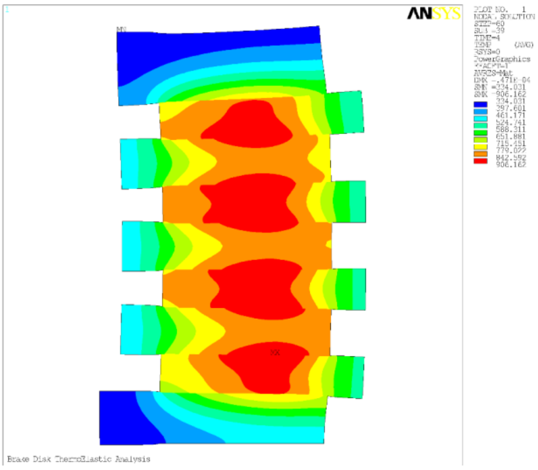
Fig. 9 Temperature contours of multidisc brake system obtained at selected time intervals (^{0}K)



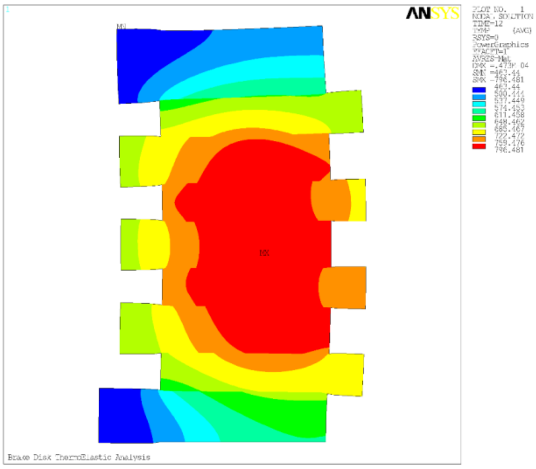
(c) 1.6 s



(d) 3.2 s



(e) 8 s



(f) 12 s

Fig. 9 Continued

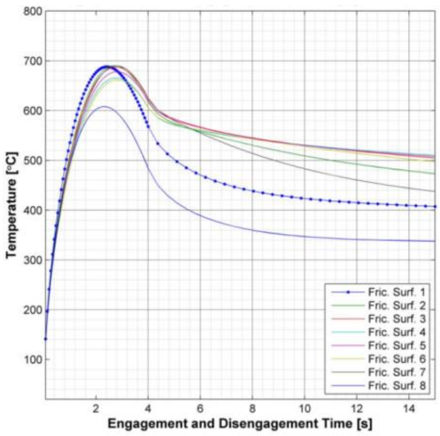


Fig. 10 Maximum temperature in different discs surfaces versus time

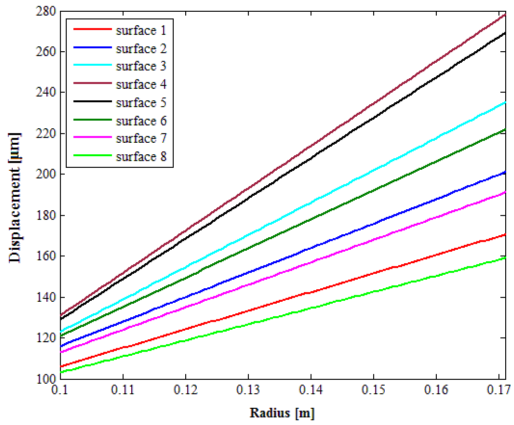


Fig. 11 Maximum displacement in different discs surfaces versus radius

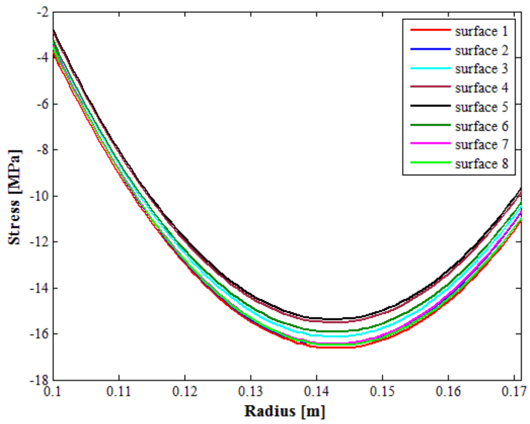


Fig. 12 Maximum stresses in different discs surfaces versus radius

stage of engagement. From Fig. 9, the heat conduction through the interior of the brake system can be clearly observed.

Maximum contact surface temperatures of the discs are illustrated in Fig. 10. The highest temperature of the surfaces occur at about 2.5 s, which is approximately 690 ^{0}C. At the early stage of engagement, the temperature initially increases at a rapid rate, achieving a peak value at approximately 2.5 s. Then, the temperature gradually decreases with time to a stable condition. Maximum displacements and stresses on contact surfaces of the discs are shown in Fig. 11 and Fig. 12, respectively. Maximum deformations occur at the outer radius of the discs. As seen, maximum deformation occurs at about 3.8 s, which is approximately 279 µm. Also maximum stress occurs at about 3.4 s, which is approximately 16 MPa.

4. Conclusions

In the present paper, a transient temperature–displacement coupled analysis for C/C composite brake discs with frictional heat generation under simulated operation conditions has been performed by adopting a axisymmetric finite element method according to the theory of energy transformation. The temperature distributions on the friction surfaces, deformation, and stress were obtained. The current study can assist brake engineers in choosing a suitable analysis method to critically evaluate the contact behavior of the C/C composite brake discs and could be used as a guide in the initial design of a high performance multidisc brake system. Based on the obtained results, the following conclusions can be derived:

- Maximum deformations occur at the outer radius of the discs.
- The temperature of the brake discs increase firstly at a rapid rate and then decreases with time to a stable condition.
- The highest temperature occurs at approximately 2.5 s, which is approximately 690°C.
- Maximum deformation occurs at about 3.8 s, which is approximately 279 μm.
- Maximum stress occurs at about 3.4 s, which is approximately 16 MPa.

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