Advances in Aircraft and Spacecraft Science, *Vol. 10, No. 3 (2023) 245-256*
https://doi.org/10.12989/aas.2023.10.3.245

Aerodynamic design and optimization of a multi-stage axial flow turbine using a one-dimensional method

Xinyang Yin[1a], Hanqiong Wang[1b], Jinguang Yang[*1], Yan Liu[1c],
Yang Zhao[2d] and Jinhu Yang[3e]

[1]*School of Energy and Power, Dalian University of Technology, Dalian, 116000, China*
[2]*Shenyang Blower Works Group Corporation, Shenyang, 110000, China*
[3]*Qingdao Institute of Aeronautical Technology, Qingdao, 266000, China*

(Received March 6, 2023, Revised May 10, 2023, Accepted June 5, 2023)

Abstract. In order to improve aerodynamic performance of multi-stage axial flow turbines used in aircraft engines, a one-dimensional aerodynamic design and optimization framework is constructed. In the method, flow path is generated by solving mass continuation and energy conservation with loss computed by the Craig & Cox model; Also real gas properties has been taken into consideration. To obtain an optimal result, a multi-objective genetic algorithm is used to optimize the efficiencies and determine values of various design variables; Final design can be selected from obtained Pareto optimal solution sets. A three-stage axial turbine is used to verify the effectiveness of the developed optimization framework, and designs are checked by three-dimensional CFD simulation. Results show that the aerodynamic performance of the optimized turbine has been significantly improved at design point, with the total-to-total efficiency increased by 1.17% and the total-to-static efficiency increased by 1.48%. As for the off-design performance, the optimized one is improved at all working points except those at small mass flow.

Keywords: aerodynamic design and optimization; axial flow turbine; multi-stage; one-dimensional design

1. Introduction

It is well known that aeroengine, as the heart of an aircraft, determines its flight capability in a large amount, and axial flow turbine is one of the key components of an aircraft engine, so its aerodynamic performance is vital to achieve the design target of an airplane. It has been an everlasting task for engine designers to improve aerodynamic performance of axial flow turbines.

Many researches have been done in the axial flow turbine aerodynamic optimization filed, among them, the most popular method is to combine three-dimensional (3D) computational fluid dynamics (CFD) simulation with optimization algorithms, such as the work of Li *et al.* (2017), Tang *et al.* (2016), Aponte *et al.* (2020), Zhang *et al.* (2022) etc. Nonetheless, due to the limitation

∗Corresponding author, Professor, E-mail: jinguang_yang@dlut.edu.cn
[a]Master Student, E-mail: y4260521@163.com
[b]Master Student, E-mail: wang13081920704@163.com
[c]Professor, E-mail: yanliu@dlut.edu.cn
[d]Senior Engineer, E-mail: zhaoyang86@shengu.com.cn
[e]Associate Professor, E-mail: yangjinhu@iet.cn

http://www.techno-press.org/?journal=aas&subpage=7 ISSN: 2287-528X (Print), 2287-5271 (Online)

of computing resources, such methods usually suffer a long turn-around time and a high computing cost. For multi-stage axial flow turbines optimization problem, one-dimensional (1D, sometimes it is termed as "mean-line") based optimization will be more productive and effective than its 3D counterpart.

Rao *et al.* (1980) expressed the design problem of axial flow turbines as a nonlinear mathematical programming problem, and their 1D based optimization have achieved a fundamental success in improving isentropic efficiency. Based on constructal theory, Feng *et al.* (2022) optimized turbine performance by optimizing 17 geometric parameters, thermal parameters, and flow parameters. Nishi *et al.* (2022) combined a 1D design method with design of experiment, response surface method, as well as optimization method to establish a design and optimization framework for flow path of axial flow turbine. Besides that, Agromayor *et al.* (2019) proposed a mean-line model based optimization method for the preliminary design of multi-stage axial turbines, and a 1D analysis module with influence of exhaust diffusers being considered was incorporated in his codes. Other researchers (Li *et al.* 2023, Ghisu *et al.* 2006, Zhdanov *et al.* 2013) have contributed to the development of 1D design optimization of axial flow turbines either.

From the above literature review, it is found that although 1D optimization is frequently used for axial flow turbine aerodynamic design, its effectiveness is not well validated, since there are many steps involved for a turbine aerodynamic design process, and 1D design is only the first step, many factors in the following steps will affect the design performance. In the current research, a complete design flow of an axial flow turbine is being conducted, including 1D design and optimization, blade design and 3D CFD simulation. A parametric turbine blade is generated by an optimization database enhanced by machine learning technique, so that there is little manual intervention involved, which makes a consistent comparison of different 1D designs by 3D CFD results possible. The design process is detailed and a 1D based optimization framework is built. A three-stage turbine is taken as an example to demonstrate the effectiveness of the proposed method.

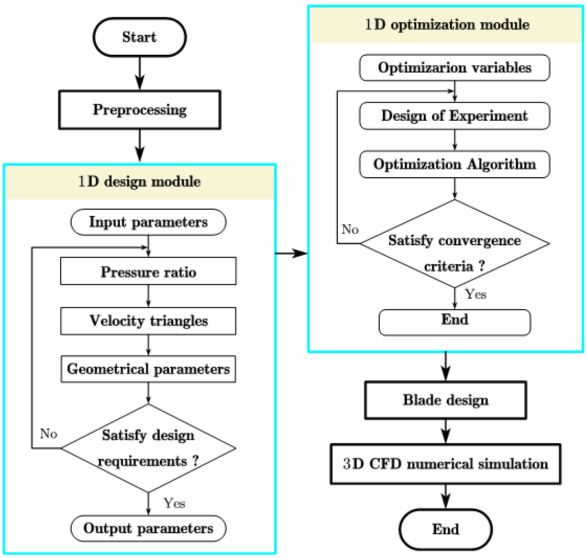


Fig. 1 Flow chart of 1D design optimization and verification

2. One-dimensional based axial flow turbine design and optimization process

The flow chart of 1D based axial turbine design and optimization used in the present study is shown in Fig. 1. The core of the design process is the 1D design and optimization, which determines the streamwise work distribution, the flow path and the velocity triangles, and is detailed in the section 2.1. Then, the 1D turbine design is geometrically realized by a parametric blade modeling code (Li *et al.* 2022) for further 3D CFD verification. In the process of blade modeling, only the parameters related to 1D optimization are changed, and the other parameters remain unchanged to exclude the influence of blade modeling. Finally, 3D CFD is used to check the performance of the optimized design. The whole process can be run without human intervention and a consistent performance check by CFD can be obtained, which makes validation of the 1D based optimization framework easier.

2.1 Axial flow turbine 1D design method

One can write the extended form of the Hamilton's Principle with the notations used in the present study as.

The 1D design program used in the present study is an inhouse code named AXTD (Jiang 2019), which conducts design calculation of axial flow turbines at mean line position. Different flow path options are available in the code, such as constant inner diameter, constant mean diameter, constant outer diameter, and user specified diameter etc. Input data of AXTD includes inlet geometric data, working condition, design target, stage non-dimensional parameters (degree of reactions, flow coefficient etc.). The calculation process can be introduced by taking a stator for example. Parameters at stator inlet can be solved iteratively by the following equations.

$$\dot{m} = \rho_1 V_1 A_1 \tag{1}$$

$$h_{t1} = h_1 + \frac{V_1^2}{2} \tag{2}$$

For each stage, the stage total-to-static pressure ratio is specified, so is the flow angle at each blade row, so an isentropic enthalpy at stator outlet can be obtained by

$$h_{2\mathrm{is}} = h_{t1} + (1 - \Omega) h_{4is} \tag{3}$$

where Ω is the degree of reaction of the stage. Then the actual static pressure p_2 can be obtained from isentropic relations. Other parameters at outlet are iterated with an assumed total pressure loss coefficient $\overline{\omega}$. Primary equations are listed in Eqs. (4)-(8).

$$\overline{\omega} = (p_{t1} - p_{t2})/(p_{t2} - p_2) \tag{4}$$

$$h_{t2} = h_{t1} \tag{5}$$

$$h_2 = f(p_2, p_{t2}, h_{t2}) \tag{6}$$

$$h_2 = h_{\mathrm{t}2} - \frac{V_2^2}{2} \tag{7}$$

$$V_{x2} = V_2 \cos \alpha_2 \tag{8}$$

The other state variables, such as t_2, p_2, ρ_2 etc., can be calculated from equations of state and thermodynamic properties. Then a loss model can be called to compute a new loss coefficient, and iteration can be done until converged. In the present application, the Craig&Cox loss model (Craig *et al.* 1970) is adopted, and both total-to-total and total-to-static efficiency can be obtained. Finally geometric parameters can be computed from given flow path pattern.

Working medium in axial turbine (gas in this case) usually is not behaved as perfect gas, in this case a look-up table method is used to calculate its thermal and transport properties. Along the radial direction, the simple radial equilibrium equation is solved, so that twisted blade can be designed.

AXTD has been validated in a number of engineering-oriented turbine design cases (Zhang *et al.* 2021). In the present study, a 1D design without optimization is generated using AXTD as the baseline design. Then optimization is conducted, its results will be compared to the baseline one to demonstrate the validity of the optimization framework.

2.2 Optimization algorithm

The selection of optimization algorithm directly dictates final optimized results. According to the literature review, popular optimization methods in turbine aerodynamic optimization include gradient-based optimization algorithm, tabu search method and genetic algorithm etc. In this study, the Multi-Objective Genetic Algorithm II (MOGA-II) (Poles 2003) is selected after some numerical experiments. It uses a more smart and efficient multi-search elitism, which is able to preserve excellent (Pareto or non-dominated) solutions without converging prematurely to a local optimum, and it is more appropriate for multi-objective optimization in the present application. Moreover, elitism strategy enhances convergence and robustness of the algorithm.

MOGA-II handles constraints by applying penalty policy. The general idea behind penalty functions is to transform a constrained optimization problem into an unconstrained problem by adding a value to - or subtracting the value from - the objective functions based on the amount of constraint violation present in a solution.

2.3 Optimization frame of 1D design

There are dozens of input data for multi-stage axial turbine 1D design, and their values for a particular design depend on designers' experience. In order to achieve better performance, an optimization strategy is integrated into the current design process, which carefully selected design variables are optimized, and proper constraints are imposed. The 1D optimization flow chart is showed in the 1D optimization module in Fig. 1.

The optimization problem of axial flow turbine design can be formulated as

$$
\begin{array}{ll}
\text{Max.} & \eta_{\text{tt}} \\
\text{Max.} & \eta_{\text{ts}} \\
 & \eta_{\text{tt}} - \eta_{\text{ts}} \leq \Delta\eta \\
 & \Omega_{\text{hub},i} > 0, i = 1,2,\cdots,n \\
\text{s.t.} & \Omega_{\text{tip},i} < 1, i = 1,2,\cdots,n \\
 & \delta_i \leq \Delta\delta, i = 1,2,\cdots,N \\
 & h_{\text{b},i} < h_{\text{b},i+1}\ i = 1,2,\cdots,N
\end{array}
\tag{9}
$$

where *N* and n are number of blades and stages respectively. The objective functions are

Table 1 Design variables for 1D optimization of multi-stage axial flow turbines

Symbol	Variable
H_{in}	Inlet blade height /m
D_{in}	Inlet diameter /m
Ω_i	Stage degree of reaction
π_i	Stage pressure ratio
$\beta_{\text{geo,s},i}$	Stator outlet geometric angle /°
$\beta_{\text{geo,r},i}$	Rotor outlet geometric angle /°
$c_{\text{s},i}$	Stator axial chord length /m
$c_{\text{r},i}$	Rotor axial chord length /m

Table 2 Outlet variables of the 1D optimization of axial flow turbines

Symbol	Parameter
η_{ts}	Total-to-static efficiency
η_{tt}	Total-to-total efficiency
H_i	Blade height
H_{max}	Maximum blade height
δ_i	Flaring angle
$\Omega_{\text{hub},i}$	Minimum degree of reaction at hub
$\Omega_{\text{tip},i}$	Maximum degree of reaction at tip

maximization of total-to-total and total-to-static efficiencies, which are defined as

$$\eta_{\text{tt}} = \frac{h_{\text{t,in}} - h_{\text{t,out}}}{h_{\text{t,in}} - h_{\text{t,out,is}}} \tag{10}$$

$$\eta_{\text{ts}} = \frac{h_{\text{t,in}} - h_{\text{t,out}}}{h_{\text{t,in}} - h_{\text{out,is}}} \tag{11}$$

where $h_{\text{t,in}}$, $h_{\text{t,out}}$, $h_{\text{t,out,is}}$ and $h_{\text{out,is}}$ are the total enthalpy of the inlet and outlet, the total isentropic enthalpy and static enthalpy of the outlet, respectively.

Proper constraints are imposed to ensure the soundness of the optimized results. Here degree of reactions at hub and tip, the flare angle, and the blade height are constrained. It is worth to say that constraint on the two efficiencies is beneficial to the optimization, and this can be explained as follows. The η_{tt} characterizes the loss, the higher the value, the smaller the turbine loss. The η_{ts} represents the work conversion ability. The higher the value, the higher the work output. The difference constraint that η_{tt} and η_{ts} can balance turbine loss and work output.

Selected design variables are listed in Table 1, and available output variables for objective function and constraints definition are shown in Table 2. When the optimization is converged, a Pareto solution sets can be obtained, and an optimal case can be selected from the set.

3. Results and analysis

Here a three-stage turbine used in an aeroengine is taken as an example to validate the

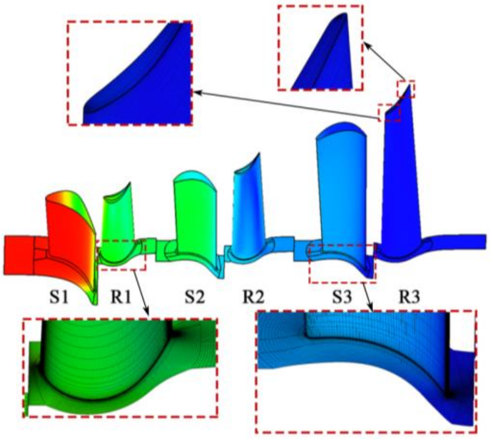


Fig. 2 The structured grid used in the 3D simulation for the optimized case

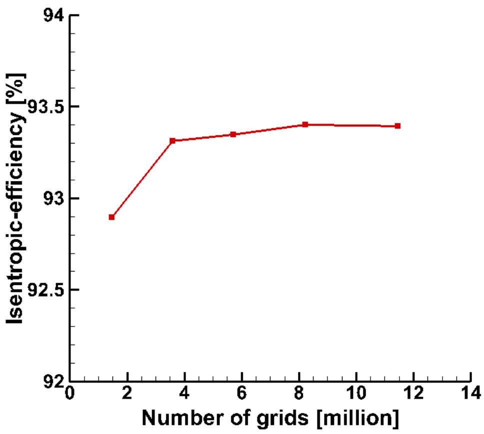


Fig. 3 Grid number in relation to efficiency

developed optimization framework. In this case, 24 design variables are selected for optimization at the aerodynamic design point. For the three-stage turbine case, the constraint bounds are set as: $\Delta\delta = 30, \Delta\eta = 0.02$.

Either baseline or optimal designs are checked by 3D CFD simulation. Here the commercial software NUMECA (2009) is used, and AutoGrid5 is adopted to generate the grids. The calculation domain and some details of the structured grid are shown in Fig. 2. As for the computational setup, the Spalart-Allmaras turbulence model is used to capture turbulence, and total pressure and temperature are set at inlet, and static pressure are given at one outlet radial position, values for other positions are obtained by solving simple radial equilibrium equation. The global conservation option on the mixing plane is used. Blade surfaces and endwalls are assumed to no-slip and adiabatic wall. Computational study had been done by the authors' group extensively, e.g., (Zhang *et al.* 2021) and (Jiang 2019). The calculation settings in this paper are consistent with previous studies. To guarantee grid independence, 5 groups of grids are evaluated. The corresponding isentropic efficiency is illustrated in Fig. 3. When the grid number is greater than 4 million, the efficiency hardly changed, so a grid number of 5.7 million is selected for the

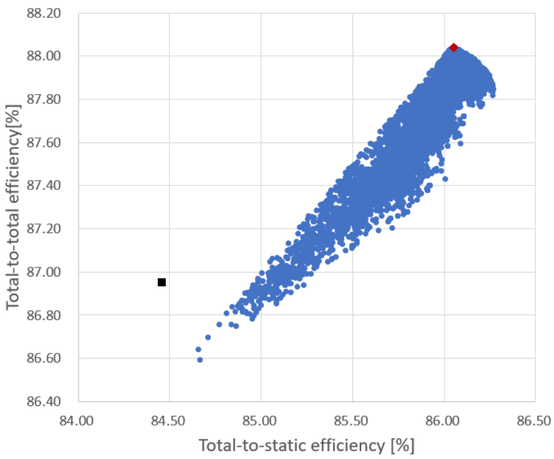


Fig. 4 An effective one-dimension aerodynamic optimization of a multi-stage axial turbine (the red mark represents the solution selected for the 3D calculation, and the black mark represents the baseline design)

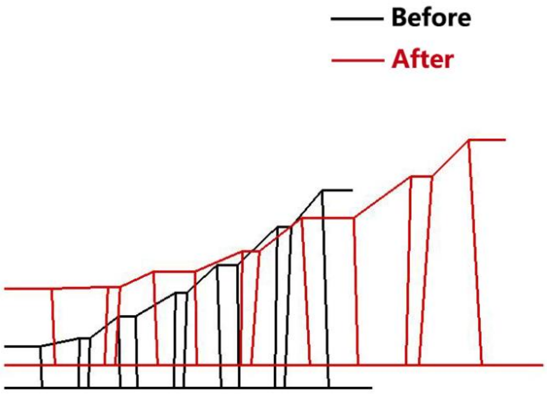


Fig. 5 Comparison of meridional flow path before and after optimization

following CFD simulation.

In this case, 24 design variables and 12 output variables are selected for optimization, and design is optimized at aerodynamic design point. Fig. 4 plots all of the evaluated designs during the 1D optimization. The baseline design is denoted as a black square in the figure, and the selected optimized design is shown as a red circle.

3.1 Comparison of 1D results

The meridional flow path of the baseline and optimized design are compared in Fig. 5. It is evident that after optimization the mean line radius is increased and the length is longer. Some geometric parameters are compared in Table 4, where values are actually ratios of corresponding variables for optimized and baseline designs. It can be seen that blade height of all stages are increased, most obviously for the first stage. Other geometric parameters are also increase proportionally, as does the flow area. Table 5 compares 1D design aerodynamic parameters before

Table 3 Comparison of the result of the CFD calculation before and after optimization

	Baseline design	Optimized design	Relative difference
Total-to-total efficiency	87.31%	88.33%	+1.17%
Total-to-static efficiency	84.98%	86.24%	+1.48%

Table 4 comparison of 1D geometric parameters before and after optimization (non-dimensional value is used and defined as value of optimized design over that of the baseline design)

Parameter	S1	R1	S2	R2	S3	R3
Inner diameter			1.0476			
Blade height	1.852	1.572	1.307	1.196	1.195	1.174
Axial chord length	1.355	1.319	1.180	1.572	1.351	1.408
Blade number	0.872	1.253	1.056	1.384	0.856	1.083

Table 5 Comparison of 1D aerodynamic parameters before and after optimization

	Flow coefficient		Pressure drop ratio		Total-to-total efficiency	
	before	after	before	after	before	after
The 1st stage	1.924	1.782	1.982	2.121	0.877	0.876
The 2nd stage	1.640	1.528	1.951	1.987	0.808	0.862
The 3rd stage	1.415	1.296	1.848	1.750	0.868	0.890

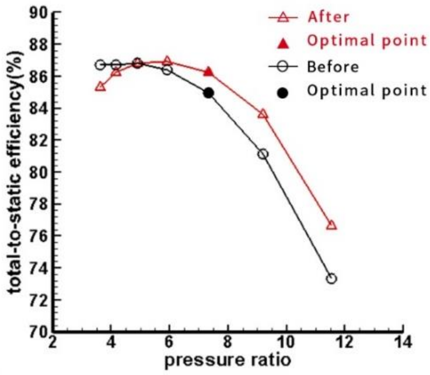


Fig. 6 Comparison of overall performance curves under variable outlet conditions

and after optimization. After optimization, flow coefficients of all stages are decreased and efficiencies increased, respectively, and pressure ratio of the first stage is increased.

3.2 Comparison of overall performance obtained by 3D CFD

Overall performance at the design point calculated by 3D CFD are compared in Table 3. It is obvious that both the total-to-total and total-to-static efficiencies are increased by 1.17% and 1.48%, respectively, after optimization.

Off-design performance for the baseline and optimized turbine are also calculated at the design speed. During the CFD calculation, the inlet boundary conditions are fixed, and the outlet pressure is changed. The result is shown in Fig. 6. It can be found that the higher pressure ratio π is, the

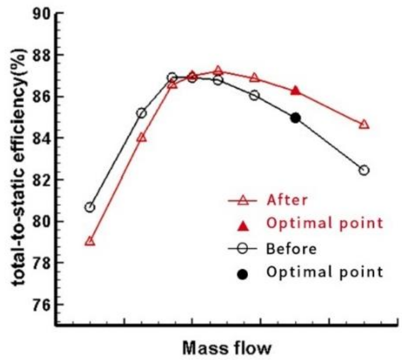
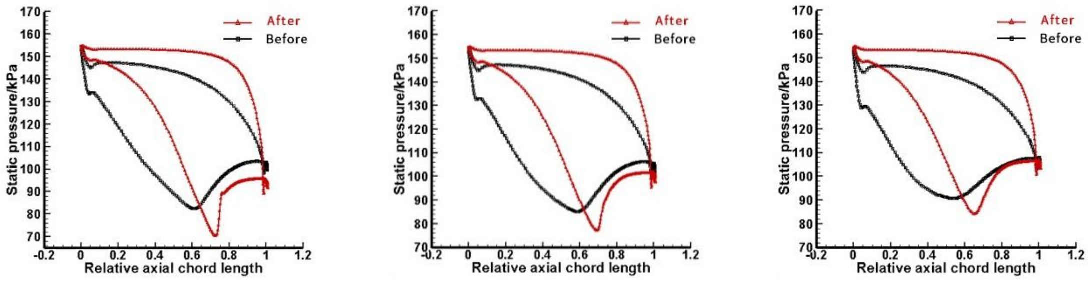


Fig. 7 Comparison of overall performance curves under variable flow conditions



(a) (b) (c)

Fig. 8 Comparison of the static pressure distribution of the 1st stage stator blade before and after optimization at three sections. (a) 10% span; (b) 50% span; (c) 90% span

more efficiency gain. Similarly, comparison is made for fixed outlet boundary condition where as varied inlet boundary conditions, which is shown in Fig. 7. It can be seen that the efficiency of the optimized turbine is improved for the mass flow rate near or larger than the design value. However, the efficiency of the optimized turbine shows a decreasing trend at lower flow rate, which is consistent with Fig. 6.

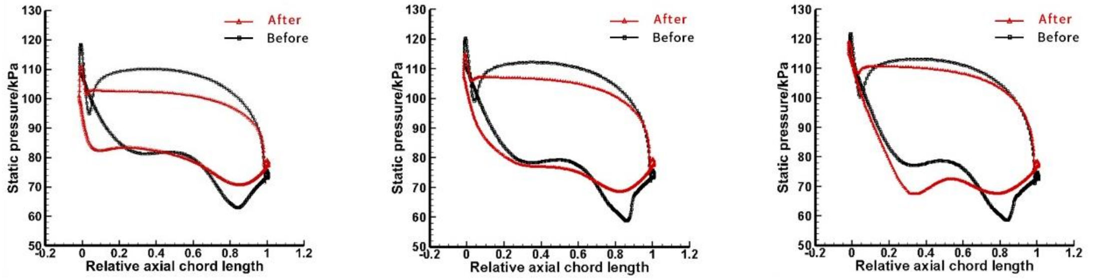
3.3 Comparison of flow field

Figs. 8 and 9 compare static pressure distribution of the 1st stage stator and rotor at different blade heights at design conditions. For the 1st stage stator, after optimization, loading is moved rearward. But for the first-stage rotor, a different tendency is happened.

Fig. 10 compare the streamlines at different span position at the design flow rate before and after optimization, correspondingly. Fig. 11 shown relative Mach number contours. It is easily found that the flow is accelerated in stators, but decelerated in rotors, which is consistent with the pressure distribution.

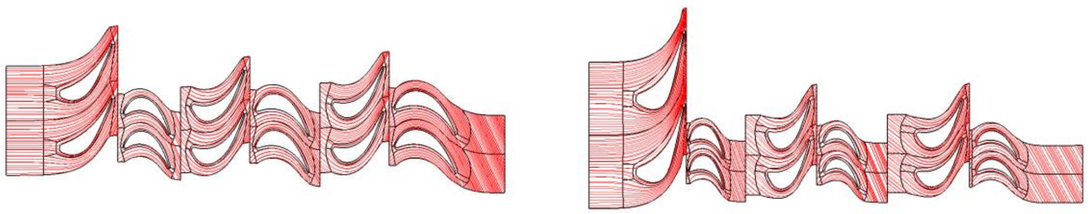
4. Conclusions

A 1D optimization framework of axial flow turbines is constructed and validated in the present

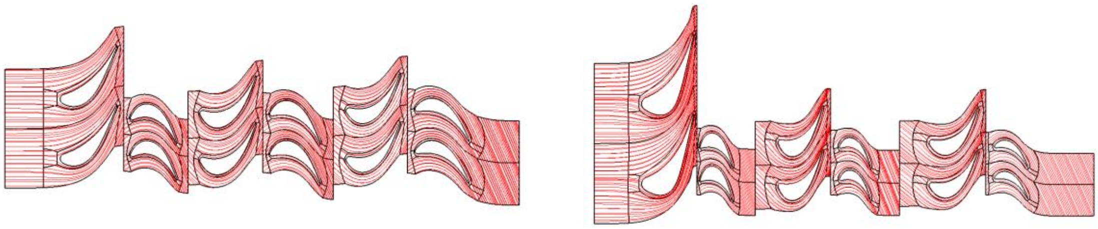


(a) (b) (c)

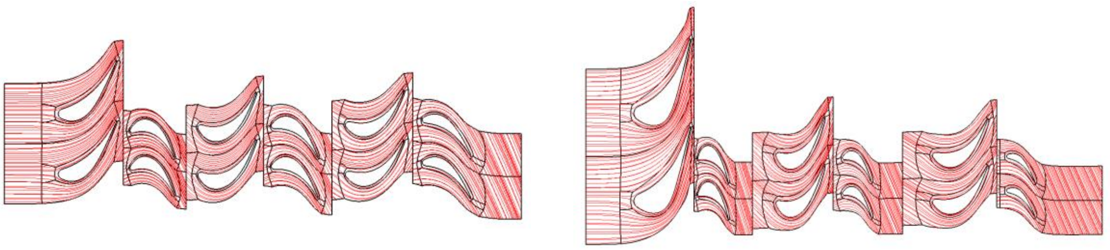
Fig. 9 Comparison of the static pressure distribution of the 1st stage rotor blade before and after optimization at three sections. (a) 10% span; (b) 50% span; (c) 90% span



(a)



(b)



(c)

Fig. 10 Relative velocity streamlines at three sections of the turbine design before (left) and after (right) optimization. (a) 10% span; (b) 50% span; (c) 90% span

study, which combines a 1D design program and a multi-objective genetic algorithm. A complete turbine design process, including blade design and CFD simulation, can be proceeded automatically, and the effectiveness of the 1D optimization can be verified by 3D CFD results. A 3-stage turbine is taken as an example to demonstrate the correctness of the development method. Through the present investigation, the following conclusions can be drawn:

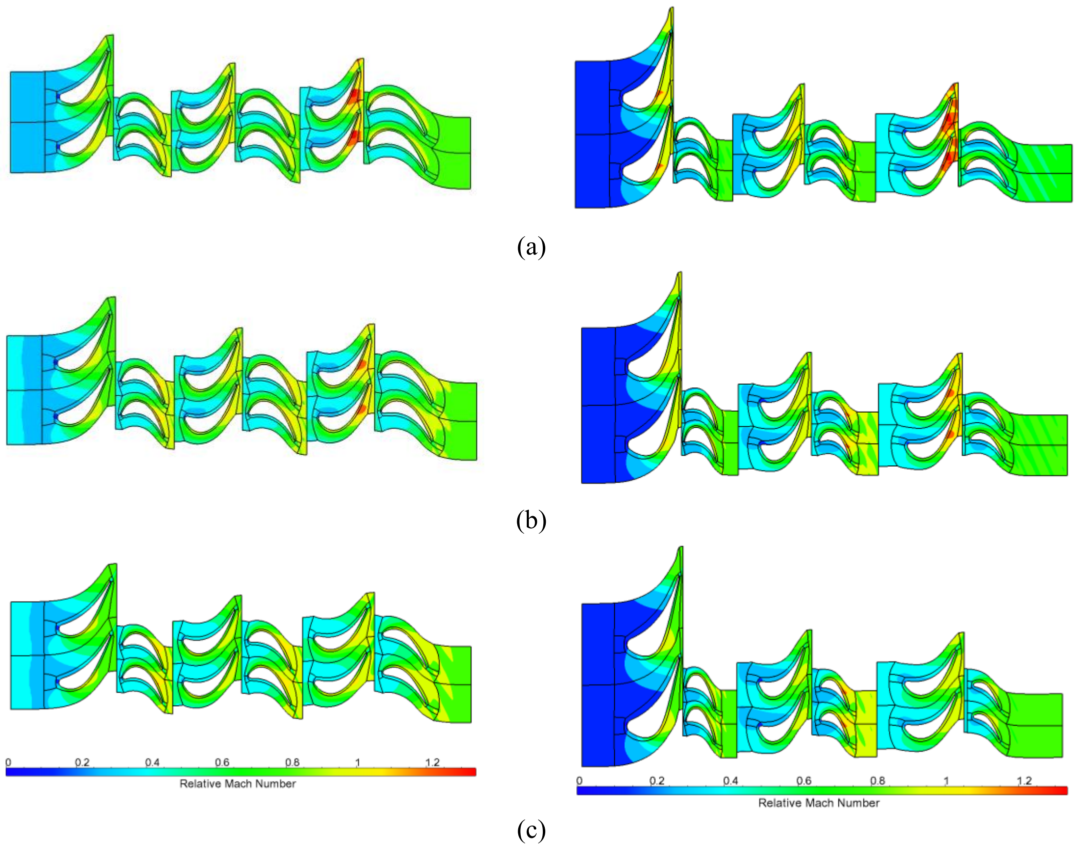


Fig. 11 Relative Mach number at three sections of the turbine design before (left) and after (right) optimization. (a) 10% span; (b) 50% span; (c) 90% span

(1) The optimization problem is carefully formulated. The objective function involves both total-to-total and total-to-static efficiencies, and constraints covers geometric and aerodynamic design criteria.
(2) A blade design program, augmented by a database and machine learning technique, is used in the design process, which can eliminate the human intervention and avoid introducing additional influencing factors, so a 1D design can be checked by 3D CFD results, making the results more reliable.
(3) A 3-stage axial flow turbine is optimized with an increase in total-to-total efficiency by 1.17%, and total-to-static efficiency by 1.48%. As for the off-design performance at the design speed, the performance is improved for mass flow rate equal or larger than the design value, but deteriorated towards lower mass flow rate.

Acknowledgments

The 1st author owes thanks to Mr. Li Zhi for his help on using the blade design program, and the 6th author would like to acknowledge the support by the Taishan Scholars Program.

References

Agromayor, R. and Nord, L.O. (2019), "Preliminary design and optimization of axial turbines accounting for diffuser performance", *Int. J. Turbomach. Propuls. Power*, **4**(3), 32. https://doi.org/10.3390/ijtpp4030032.

Aponte, R.D., Teran, L.A., Grande, J.F., Coronado, J.J., Ladino, J.A., Larrahondo, F.J. and Rodríguez, S.A. (2020), "Minimizing erosive wear through a CFD multi-objective optimization methodology for different operating points of a Francis turbine", *Renew. Energy*, **145**, 2217-2232. https://doi.org/10.1016/j.renene.2019.07.116.

Craig, H.R.M. and Cox, H.J.A. (1970), "Performance estimation of axial flow turbines", *Proc. Inst. Mech. Eng.*, **185**(1), 407-424.

Feng, H., Chen, L., Tang, W. and Ge, Y. (2022), "Optimal design of a dual-pressure steam turbine for rankine cycle based on constructal theory", *Energi.*, **15**(13), 4854. https://doi.org/10.3390/en15134854.

Ghisu, T., Parks, G.T., Jarrett, J.P. and Clarkson, P.J. (2006), "Multi-objective optimisation of aero-engine compressors", *Third International Conference 'The Future of Gas-Turbine Technology'*, European Turbine Network (ETN).

Jiang, X. (2019), "Design and performance analysis of axial turbine with real working fluid", Dalian University of Technology, China.

Li, X., Huo, Y., Xu, H., Liu, Y., Li, G., Niu, X., Zhu, X. and Li, Y. (2023), "Aerodynamic design and optimization of power turbine for marine gas turbine", *Propuls. Technol.*, **44**(02), 143-151.

Li, Z. and Zheng, X. (2017), "Review of design optimization methods for turbomachinery aerodynamics", *Progr. Aerosp. Sci.*, **93**, 1-23.

Li, Z., Liu, Y., Yang, J.G. and Wang, H.Q. (2022), "A data-based policy model for intelligent turbine cascade design", *J. Appl. Fluid Mech.*, **15**(4), 1179-1192. https://doi.org/10.47176/JAFM.15.04.33515.

Nishi, Y., Mori, N., Yamada, N. and Inagaki, T. (2022), "Study on the design method for axial flow runner that combines design of experiments, response surface method, and optimization method to one-dimensional design method", *Renew. Energy*, **185**, 96-110. https://doi.org/10.1016/j.renene.2021.12.009.

Numeca Int. (2009), Numeca User Manuals, NUMECA International, Belgium.

Poles S. (2003), "MOGA-II an improved multi-objective genetic algorithm", Esteco technical Report, 6.

Rao, S.S. and Gupta, R.S. (1980), "Optimum design of axial flow gas turbine stage-Part I: Formulation and analysis of optimization problem", *J. Eng. Power*, **102**(4), 782-789. https://doi.org/10.1115/1.3230341.

Rao, S.S. and Gupta, R.S. (1980), "Optimum design of axial flow gas turbine stage-Part II: Solution of the optimization problem and numerical results", *J. Eng. Power*, **102**(4), 790-797. https://doi.org/10.1115/1.3230342.

Tang, X., Luo, L., Wen, F., Zhou, X. and Wang, S. (2016), "3-D Aerodynamic optimization design of air-cooled turbine with transonic blade", *Turbine Technol.*, **58**(06), 405-409.

Zhang, C. and Janeway, M. (2022), "Optimization of turbine blade aerodynamic designs using CFD and neural network models", *Int. J. Turbomach. Propuls. Power*, **7**(3), 20. https://doi.org/10.3390/ijtpp7030020.

Zhang, M., Liu, Y., Yang, J., Jiang, X., Shao, W. and Wang, X. (2021), "Design and analysis of a multi-stage axial turbine for gasified coal-water power system", *Int. J. Energy Res.*, **45**(5), 7352-7365. https://doi.org/10.1002/er.6319.

Zhdanov, I., Staudacher, S. and Falaleev, S. (2013), "An advanced usage of mean-line loss systems for axial turbine design optimization", *Proceedings of the ASME Turbo Expo 2013: Turbine Technical Conference and Exposition. Volume 6A: Turbomachinery. San Antonio*, Texas, USA. June.

AP