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Two-temperature thermoelastic surface waves in micropolar thermoelastic media via dual-phase-lag model

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Abstract. This article is concerned with a two-dimensional problem of micropolar generalized thermoelasticity for a half-space whose surface is traction-free and the conductive temperature at the surface of the half-space is known. Theory of two-temperature generalized thermoelasticity with phase lags using the normal mode analysis is used to solve the present problem. The formulas of conductive and mechanical temperatures, displacement, micro-rotation, stresses and couple stresses are obtained. The considered quantities are illustrated graphically and their behaviors are discussed with suitable comparisons. The present results are compared with those obtained according to one temperature theory. It is concluded that both conductive heat wave and thermodynamical heat wave should be separated. The two-temperature theory describes the behavior of particles of elastic body more real than one-temperature theory.

Keywords: thermoelasticity; thermoelastic plane waves; micropolar material; two temperatures theory; heat transfer

1. Introduction

In the theory of uncoupled thermoelasticity the heat equation is independent of both mechanical effects and equation of motion. The uncoupled theory has two defects according to the dependency relation and that the temperature is contained in the dynamic equation as a known function. Biot (1952) has introduced coupled thermoelasticity theory to overcome the first shortcoming of the uncoupled theory in which that mechanical state of a solid has no effect on its temperature. However the coupled theory of Biot (1952) failed to treat the second shortcoming in which heat equation is in a diffusion type (parabolic). This means that the speed of propagation of temperature is infinite, which contradicts physical experiments.

The generalized theories of thermoelasticity have treated the second shortcoming of the uncoupled theory. They have involved hyperbolic type of heat transport equation which predicts infinite speed of propagation of thermal signals. Among the generalized theories the extended thermoelasticity, the temperature rate dependent thermoelasticity and others (Lord and Shulman 1967, Green and Lindsay 1971, Green and Naghdi 1991, 1992, 1993) have been the subject of

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many investigations. Tzou (1995a, b, 1996) has proposed the dual-phase-lag (DPL) model to modify the classical thermoelastic model with two different time translations, one for heat flux and the other for temperature gradient.

Wave propagations in micropolar materials have many applications in different fields of science and technology. The linear theory of micropolar thermoelasticity is developed by many investigators (Nowacki 1966, Tauchert *et al.* 1968, Eringen 1970, 1971) and coupled by the inclusion of heat effect. Boschi and Iesan (1973) have generalized the theory of thermoelasticity of Green and Lindsay (1971) to a homogeneous micropolar continua. Scalia (1990) has established a basic relation of micropolar thermoelasticity that implies in a simple way the reciprocal theorem and another uniqueness result. Kumar and Singh (1996) have discussed two problems in micropolar generalized thermoelastic half-space with stretch. El-Karamany and Ezzat (2004) have presented the boundary integral equation formulation for generalized linear micropolar thermoelasticity. Sherief *et al.* (2005) have solved an axisymmetric half-space problem based on the micropolar thermoelasticity theory. Othman and Singh (2007) have investigated the effect of rotation on displacement, microrotation and temperature distributions in micropolar half-space via five theories of thermoelasticity. El-Karamany and Ezzat (2013) have presented the constitutive laws for three-phase-lag micropolar thermoelasticity theory.

The two-temperature model is presented and developed by many investigators (Gurtin and Williams 1966, Chen and Gurtin 1968, Chen *et al.* 1968, 1969, Nath *et al.* 1998, Allam *et al.* 2002, Quintanilla 2004, Youssef 2006, Abbas and Zenkour 2014, Zenkour and Abouelregal 2014a,b, 2015, Carrera *et al.* 2015, Abouelregal and Zenkour 2016). This paper is concerned with the problem of wave propagations in an isotropic micropolar generalized thermoelastic half-space in which its surface is assumed to be traction-free. The model for linear theory of micropolar generalized two temperatures thermoelasticity (MP2TE) based on dual-phase-lag model is introduced. This model is solved by using the normal mode analysis. The exact formulae of thermodynamic temperature, conductive temperature, displacements, microrotation and stresses are obtained. The distributions of the considered variables are computed numerically and presented graphically, for a specific model. Comparisons are made with the results obtained in case of one-temperature theory (MP1TE). Some comparisons will be shown in figures to estimate the effect of the two-temperature parameter.

2. Mathematical model and basic equations

The concept of micro-continuum, proposed by Eringen (1984), takes into account the microstructure of material while the theory itself remains still in a continuum formulation. Governing equations in a homogeneous isotropic micropolar generalized thermoelastic solid in the absence of body forces, body couples and heat sources are given by Eringen (1970)

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + (\mu + \alpha) \varepsilon_{ij} + \mu \varepsilon_{ji} - \gamma \theta \delta_{ij}, \tag{1}$$

$$m_{ij} = \epsilon \omega_{k,k} \delta_{ij} + (\upsilon + \beta) \omega_{i,j} + (\upsilon - \beta) \omega_{i,j}, \tag{2}$$

$$\rho T_0 S = \rho C_E \theta + \gamma T_0 \varepsilon_{kk}, \tag{3}$$

and the linear equations of balance law are

$$\sigma_{ij,j} = \rho \ddot{u}_i, \tag{4}$$

$$\epsilon_{ijk}\sigma_{jk} + m_{ji\,j} = \rho J \ddot{\omega}_{,i}, \quad i,j,k = 1,2,3. \tag{5}$$

The linearized form of heat conduction is

$$\rho T_0 \dot{S} = -q_{i,i}. \tag{6}$$

The modified classical thermoelasticity model with two temperatures in which Fourier law is replaced by the assumption

$$\left(1 + \tau_q \frac{\partial}{\partial t}\right)\vec{q} = -K\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\nabla\phi. \tag{7}$$

The two-temperature relation is given by (Nowacki 1966)

$$\phi - T = a\phi_{,ii}, \quad a > 0. \tag{8}$$

Now using divergence theorem and Eqs. (3) and (6), from Eq. (8) we obtain (Tzou 1995b)

$$K\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\nabla^2\phi + \left(1 + \tau_q \frac{\partial}{\partial t}\right)(\rho Q) = \left(1 + \tau_q \frac{\partial}{\partial t}\right)\left(\rho C_E \frac{\partial \theta}{\partial t} + \gamma T_0 \frac{\partial \varepsilon_{kk}}{\partial t}\right), \tag{9}$$

Using Eqs. (1) and (2) in Eqs. (4) and (5), we get

$$(\mu + \alpha)\nabla^2\vec{u} + (\lambda + \mu - \alpha)\nabla(\nabla\cdot\vec{u}) + 2\alpha\nabla\times\vec{\omega} - \gamma\nabla\theta = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{10}$$

$$(\upsilon + \beta)\nabla^2\vec{\omega} + (\varepsilon + \upsilon - \beta)\nabla(\nabla\cdot\vec{\omega}) + 2\alpha\nabla\times\vec{u} - 4\alpha\vec{\omega} = \rho J\frac{\partial^2\vec{\omega}}{\partial t^2}. \tag{11}$$

3. Formulation of the problem

Let us consider a half-micropolar generalized thermoelastic space, whose surface is traction-free and subjected to decreasing thermal source with time which affects a narrow band of width $2L$ surrounding the x-axis. Let us consider plane waves in plane such that all particles on a line parallel to z-axis are equally displaced. Therefore, all the field variables are functions of x, y and t only, and independent of the variable z. So, we assume the components of displacement and micro-rotation vectors in the form

$$\vec{u} \equiv (u, v, 0), \quad \vec{\omega} \equiv (0, \omega, 0), \tag{12}$$

and the cubical dilatation is given by

$$e = \varepsilon_{kk} = \nabla\cdot\vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}. \tag{13}$$

Equations of motion and equation of balance of momentum (10) and (11) will be in the form

$$\begin{gathered}(\mu + \alpha)\nabla^2 u + (\lambda + \mu - \alpha)\frac{\partial e}{\partial x} + 2\alpha\frac{\partial \omega}{\partial y} - \gamma\frac{\partial \theta}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \\ (\mu + \alpha)\nabla^2 v + (\lambda + \mu - \alpha)\frac{\partial e}{\partial y} - 2\alpha\frac{\partial \omega}{\partial x} - \gamma\frac{\partial \theta}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \\ (\upsilon + \beta)\nabla^2\omega + 2\alpha\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 4\alpha\omega = \rho J\frac{\partial^2\omega}{\partial t^2}.\end{gathered} \tag{14}$$

The constitutive relations are given by

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial v}{\partial y}-\gamma\theta,$$
$$\sigma_{yy}=(\lambda+2\mu)\frac{\partial v}{\partial y}+\lambda\frac{\partial u}{\partial x}-\gamma\theta, \tag{15}$$
$$\sigma_{zz}=\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\gamma\theta,$$

$$\sigma_{xy}=(\mu+\alpha)\frac{\partial v}{\partial x}+(\mu-\alpha)\frac{\partial u}{\partial y}-2\alpha\omega,$$
$$\sigma_{yx}=(\mu-\alpha)\frac{\partial v}{\partial x}+(\mu+\alpha)\frac{\partial u}{\partial y}+2\alpha\omega, \tag{16}$$

$$m_{zx}=(\upsilon-\beta)\frac{\partial \omega}{\partial x}=(\upsilon-\beta)m_{xz},\quad m_{zy}=(\upsilon-\beta)\frac{\partial \omega}{\partial y}=(\upsilon-\beta)m_{yz}, \tag{17}$$

$$\phi-\theta=a\nabla^2\phi,\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}. \tag{18}$$

Introduce the following boundary conditions for the present application:
(1) Mechanical boundary conditions that the surface of the half-space is traction free

$$\sigma_{yy}(x,0,t)=\sigma_{xy}(x,0,t)=0,\quad m_{yz}(x,0,t)=0. \tag{19}$$

(2) Thermal boundary condition that the conductive temperature at the surface of the half-space is known

$$\phi(x,0,t)=f(x,t), \tag{20}$$

where the function $f(x,t)$, applied on the boundary, is taken as follows

$$f(x,t)=\phi_0 H(L-|x|)\mathrm{e}^{-bt}, \tag{21}$$

where ϕ_0 is constant, $H(\cdot)$ is the Heaviside unit step function $-L<x<L$ and t is a certain value of time. This means that the applied thermomechanical shock acts only on a band of width $2L$ centered around the x-axis on the surface of the half space and zero everywhere else.

4. Solution of the problem

For simplification we use the following dimensionless variables

$$\{x',y'\}=\frac{c_1}{\eta}\{x,y\},\quad \{u',v',t'\}=\frac{c_1^2}{\eta}\{u,v,t\}\quad \{\theta',\phi'\}=\frac{\gamma}{\rho c_1^2}\{\theta-\phi_0,\phi-\phi_0\},$$
$$\sigma'_{ij}=\frac{1}{\mu+\alpha}\sigma_{ij},\quad \omega'=\frac{\alpha}{\mu+\alpha}\omega,\quad c_1^2=\frac{\lambda+2\mu}{\rho},\quad \eta=\frac{K}{\rho C_E},\quad m'_{ij}=\frac{\alpha\eta}{c_1(\mu+\alpha)(\upsilon+\beta)}m_{ij}. \tag{22}$$

Using Eq. (22), we find the dimensionless forms of equations of motion and the equation of balance of momentum (14) and the heat conduction Eq. (9) with $Q = 0$ as follows (suppressing primes for simplicity in the notation)

$$\zeta_1^2 \frac{\partial^2 u}{\partial x^2} + \zeta_2^2 \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial \omega}{\partial y} - \zeta_1^2 \frac{\partial \theta}{\partial x} = \zeta_1^2 \frac{\partial^2 u}{\partial t^2},$$
$$\zeta_1^2 \frac{\partial^2 v}{\partial y^2} + \zeta_2^2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} - 2\frac{\partial \omega}{\partial x} - \zeta_1^2 \frac{\partial \theta}{\partial y} = \zeta_1^2 \frac{\partial^2 v}{\partial t^2}, \tag{23}$$
$$\nabla^2 \omega + g_1 \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) - g_2 \omega = g_3 \frac{\partial^2 \omega}{\partial t^2},$$

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \nabla^2 \phi = \left(1 + \tau_q \frac{\partial}{\partial t}\right)\left(\frac{\partial \theta}{\partial t} + g \frac{\partial \varepsilon_{kk}}{\partial t}\right), \tag{24}$$

where

$$\zeta_1^2 = \frac{\lambda + 2\mu}{\mu + \alpha}, \quad \zeta_2^2 = \zeta_1^2 - 1, \quad g_1 = \frac{2\alpha^2 \eta^2}{c_1^2 (\mu + \alpha)(\upsilon + \beta)},$$
$$g_2 = \frac{4\alpha \eta^2}{c_1^2 (\upsilon + \beta)}, \quad g_3 = \frac{\rho J c_1^2}{\upsilon + \beta}, \quad g = \frac{\eta \gamma^2 T_0}{\rho c_1^2 K}. \tag{25}$$

Also, using the non-dimensional forms (22), the constitutive Eqs. (15)-(18) take the form

$$\sigma_{xx} = \zeta_1^2 \frac{\partial u}{\partial x} + \delta_2 \frac{\partial v}{\partial y} - \zeta_1^2 \theta,$$
$$\sigma_{yy} = \zeta_1^2 \frac{\partial v}{\partial y} + \delta_2 \frac{\partial u}{\partial x} - \zeta_1^2 \theta, \tag{26}$$
$$\sigma_{zz} = \delta_2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \zeta_1^2 \theta,$$

$$\sigma_{xy} = \frac{\partial v}{\partial x} + \delta_3 \frac{\partial u}{\partial y} - 2\omega, \quad \sigma_{yx} = \delta_3 \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + 2\omega, \tag{27}$$

$$m_{zx} = \delta_4 \frac{\partial \omega}{\partial x} = \delta_4 m_{xz}, \quad m_{zy} = \delta_4 \frac{\partial \omega}{\partial y} = \delta_4 m_{yz}, \tag{28}$$

$$\phi - \theta = a_1 \nabla^2 \phi, \tag{29}$$

where

$$\delta_2 = \frac{\lambda}{\mu + \alpha}, \quad \delta_3 = \frac{\mu - \alpha}{\mu + \alpha}, \quad \delta_4 = \frac{\upsilon - \beta}{\upsilon + \beta}, \quad a_1 = \frac{c_1^2}{\eta^2} a. \tag{30}$$

The decomposition on displacement vector is considered in the form

$$\vec{u} = \nabla \Phi + \nabla \times \vec{\Psi}, \quad \vec{\Psi} \equiv (0,0,\Psi). \tag{31}$$

The expression relating the radial displacement $u(x,y,t)$ and the axial displacement $v(x,y,t)$ to the displacement potentials (31) follows as

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y}, \quad v = \frac{\partial \Phi}{\partial y} - \frac{\partial \Psi}{\partial x}. \tag{32}$$

Using Eq. (32), we can simplify Eqs. (23) and (24) as follow

$$\nabla^2 \left(\zeta_1^2 \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y} \right) + 2 \frac{\partial \omega}{\partial y} - \zeta_1^2 \frac{\partial \theta}{\partial x} = \zeta_1^2 \frac{\partial^2}{\partial t^2} \left(\frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y} \right), \tag{33}$$

$$\nabla^2 \left(\zeta_1^2 \frac{\partial \Phi}{\partial y} - \frac{\partial \Psi}{\partial x} \right) - 2 \frac{\partial \omega}{\partial x} - \zeta_1^2 \frac{\partial \theta}{\partial y} = \zeta_1^2 \left(\frac{\partial \Phi}{\partial y} - \frac{\partial \Psi}{\partial x} \right), \tag{34}$$

$$\left(\nabla^2 - g_3 \frac{\partial^2}{\partial t^2} - g_2 \right) \omega = g_1 \nabla^2 \Psi, \tag{35}$$

$$\left(1 + \tau_\theta \frac{\partial}{\partial t} \right) \nabla^2 \Phi = \left(1 + \tau_q \frac{\partial}{\partial t} \right) \left(\frac{\partial \theta}{\partial t} + g \frac{\partial^2}{\partial t^2} \nabla^2 \Phi \right). \tag{36}$$

Eqs. (33) and (34) may be reduced to the following forms (assuming without loss of generality of our problem that all quantities are initially zero)

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} \right) \Phi = \theta, \quad \left(\nabla^2 - \zeta_2^2 \frac{\partial^2}{\partial t^2} \right) \Psi = -2\omega. \tag{37}$$

5. Normal mode analysis

The solution for the considered physical variables can be decomposed in terms of normal modes in the following form

$$\{\theta, \phi, u, v, \omega, \sigma_{ij}, \Phi, \Psi\}(x, y, t) = \{\bar{\theta}, \bar{\phi}, \bar{u}, \bar{v}, \bar{\omega}, \bar{\sigma}_{ij}, \bar{\Phi}, \bar{\Psi}\}(y) e^{(i\xi x + \Omega t)}, \tag{38}$$

where Ω is the complex time constants, $i = \sqrt{-1}$ is the imaginary unit and ξ is the wave number in the x-direction, and $\{\theta, \phi, u, v, \omega, \sigma_{ij}, \Phi, \Psi\}$ are the amplitudes of the represented plane waves to the considered variables.

By using Eq. (38) in Eqs. (33)-(36), we can get the following equations

$$(D^2 - \varepsilon_1)\bar{\Phi} = \bar{\theta}, \tag{39}$$

$$(D^2 - \varepsilon_2)\bar{\Psi} = -2\bar{\omega}, \tag{40}$$

$$(D^2 - \varepsilon_3)\bar{\omega} = g_1 (D^2 - \xi^2)\bar{\Psi}, \tag{41}$$

$$(D^2 - \xi^2)\bar{\phi} = \varepsilon_4 [\bar{\theta} + g(D^2 - \xi^2)\bar{\Phi}], \tag{42}$$

$$\bar{\phi} - \bar{\theta} = a_1 (D^2 - \xi^2), \tag{43}$$

where

$$D = \frac{\mathrm{d}}{\mathrm{d}y}, \quad \varepsilon_1 = \xi^2 + \Omega^2, \quad \varepsilon_2 = \xi^2 + \zeta_2^2\Omega^2,$$
$$\varepsilon_3 = \xi^2 + g_2 + g_3\Omega^2, \quad \varepsilon_4 = \frac{\Omega(1+\tau_\theta\Omega)}{1+\tau_\theta\Omega}. \tag{44}$$

Putting $f(x,y)$ in normal mode form and substituting Eq. (38) into Eqs. (19) and (20) we get the boundary conditions in the form

$$\bar{\sigma}_{yy}(x,0,t) = \bar{\sigma}_{xy}(x,0,t) = \bar{m}_{yz}(x,0,t) = 0, \quad \bar{\phi}(x,0,t) = \bar{f}(\xi,\Omega), \tag{45}$$

where $\bar{f}(\xi,\Omega)$ may be written as follows

$$\bar{f}(\xi,\Omega) = \phi_0 \mathrm{e}^{-[\mathrm{i}\xi x + (\Omega+b)t]}. \tag{46}$$

By eliminating $\bar{\theta}$, $\bar{\phi}$, $\bar{\Phi}$ in Eqs. (39), (42), (43) and $\bar{\omega}$, $\bar{\Psi}$ in Eqs. (40), (41), we get the following equations

$$(D^4 - \eta_1 D^2 + \gamma_1)\{\bar{\theta},\bar{\phi},\bar{\Phi}\} = 0, \tag{47}$$

$$(D^4 - \eta_2 D^2 + \gamma_2)\{\bar{\omega},\bar{\Psi}\} = 0, \tag{48}$$

where

$$\eta_1 = \frac{\xi^2 + \varepsilon_1 + \varepsilon_4[a_1(\varepsilon_1 + g\xi^2) + (a_1\xi^2 + 1)(g+1)]}{a_1\varepsilon_4(g+1)+1}, \quad \eta_2 = \varepsilon_2 + \varepsilon_3 - g_1,$$
$$\gamma_1 = \frac{\xi^2\varepsilon_1 + \varepsilon_4(\varepsilon_1 + g\xi^2)(a_1\xi^2+1)}{a_1\varepsilon_4(g+1)+1}, \quad \gamma_2 = \varepsilon_2\varepsilon_3 - 2g_1\xi^2. \tag{49}$$

The solutions of Eqs. (47) and (48), which are bounded for the values $y \geq 0$, are given as follows

$$\{\bar{\Phi},\bar{\theta},\bar{\phi},\bar{\Psi},\bar{\omega}\} = \sum_{j=1}^{2}\left\{A_j, A_j^1, A_j^2, A_j^3, A_j^4\right\}\mathrm{e}^{-\lambda_j y}, \tag{50}$$

where A_j, A_j^n, $n = 1,2,3,4$ are some parameters depending on ξ and Ω, and λ_i, $i = 1,2$ are the positive roots of the equation

$$\lambda^4 - A_1\lambda^2 + B_1 = 0, \tag{51}$$

whereas λ_i, $i = 3,4$, are the positive roots the equation

$$\lambda^4 - A_2\lambda^2 + B_2 = 0. \tag{52}$$

Substituting Eq. (50) into Eqs. (39), (40) and (43) we get the following useful relations

$$A_j^1(\xi,\Omega) = \left(\lambda_j^2 - \varepsilon_1\right)A_j(\xi,\Omega), \quad j = 1,2, \tag{53}$$

$$A_j^2(\xi,\Omega) = \frac{\lambda_j^2 - \varepsilon_1}{1 - a_1\left(\lambda_j^2 - \xi^2\right)}A_j(\xi,\Omega), \quad j = 1,2, \tag{54}$$

$$A_j^4(\xi,\Omega) = -\frac{1}{2}\left(\lambda_j^2 - \varepsilon_2\right)A_j^3(\xi,\Omega), \quad j = 3,4. \tag{55}$$

Now substituting Eqs. (53)-(55) into Eqs. (40), (41) and (43), we get

$$\bar{\theta} = \sum_{j=1}^{2}\left(\lambda_j^2 - \varepsilon_1\right)A_j \mathrm{e}^{-\lambda_j y}, \tag{56}$$

$$\bar{\phi} = \sum_{j=1}^{2}\frac{\lambda_j^2 - \varepsilon_1}{1 - a_1\left(\lambda_j^2 - \xi^2\right)}A_j \mathrm{e}^{-\lambda_j y}, \tag{57}$$

$$\bar{\omega} = \sum_{j=3}^{4} -\frac{1}{2}\left(\lambda_j^2 - \varepsilon_2\right)A_j^3 \mathrm{e}^{-\lambda_j y}, \tag{58}$$

To obtain the displacement components $\bar{u}$ and $\bar{v}$ substituting Eq. (38) into Eq. (32) and then using Eq. (50), we get

$$\bar{u} = \mathrm{i}\xi\sum_{j=1}^{2}\lambda_j A_j \mathrm{e}^{-\lambda_j y} - \sum_{j=3}^{4}\lambda_j A_j^3 \mathrm{e}^{-\lambda_j y}, \tag{59}$$

$$\bar{v} = -\sum_{j=1}^{2}\lambda_j A_j \mathrm{e}^{-\lambda_j y} - \mathrm{i}\xi\sum_{j=3}^{4}\lambda_j A_j^3 \mathrm{e}^{-\lambda_j y}. \tag{60}$$

The stress and couple stress components can be obtained by using Eq. (38) into Eqs. (26)-(29) and then substituting Eqs. (56)-(58) into resulting equations, we get

$$\bar{\sigma}_{ii} = \sum_{j=1}^{2}\alpha_{ij}A_j \mathrm{e}^{-\lambda_j y} - \sum_{j=3}^{4}\zeta_{ij}A_j^3 \mathrm{e}^{-\lambda_j y}, \quad i = x,y,z, \tag{61}$$

$$\bar{\sigma}_{is} = \sum_{j=1}^{2}\alpha_{isj}A_j \mathrm{e}^{-\lambda_j y} - \sum_{j=3}^{4}\zeta_{ijs}A_j^3 \mathrm{e}^{-\lambda_j y}, \quad i,s = x,y, \tag{62}$$

$$\bar{m}_{xz} = -\frac{\mathrm{i}\xi}{2}\sum_{j=3}^{4}\left(\lambda_j^2 - \varepsilon_2\right)A_j^3 \mathrm{e}^{-\lambda_j y}, \tag{63}$$

$$\bar{m}_{yz} = -\frac{1}{2}\sum_{j=3}^{4}\lambda_j\left(\lambda_j^2 - \varepsilon_2\right)A_j^3 \mathrm{e}^{-\lambda_j y}, \tag{64}$$

where

$$\alpha_{xj} = (\delta_2 - \beta_1^2)\lambda_j^2 + \beta_1^2\varepsilon_1 - \xi^2\delta_1, \quad \zeta_{xj} = \mathrm{i}\xi\lambda_j(\delta_2 - \delta_1), \tag{65}$$

$$\alpha_{yj}=\beta_1^2\lambda_j^2-\xi^2\delta_2+\beta_1^2\left(\varepsilon_1-\lambda_j^2\right),\quad \zeta_{yj}=-\zeta_{xj}, \tag{66}$$

$$\alpha_{zj}=-\xi^2\delta_2+\beta_1^2\left(\varepsilon_1-\lambda_j^2\right),\quad \zeta_{zj}=-\mathrm{i}\xi\lambda_j\delta_2, \tag{67}$$

$$\alpha_{xyj}=-2\mathrm{i}\xi\lambda_j^2,\quad \zeta_{xyj}=\xi^2-\varepsilon_2+\lambda_j^2(\delta_3+1), \tag{68}$$

$$\alpha_{yxj}=-\mathrm{i}\xi\lambda_j(\delta_3+1),\quad \zeta_{yxj}=\varepsilon_2+\xi^2\delta_3. \tag{69}$$

Now applying the thermal boundary conditions (45), we get

$$\sum_{j=1}^{2}\alpha_{yj}A_j+\sum_{j=3}^{4}\zeta_{yj}A_j^3=0, \tag{70}$$

$$\sum_{j=1}^{2}\alpha_{yxj}A_j+\sum_{j=3}^{4}\zeta_{yxj}A_j^3=0, \tag{71}$$

$$\frac{1}{2}\sum_{j=3}^{4}E_jA_j^3=0, \tag{72}$$

$$\sum_{j=1}^{2}E_jA_j=\bar{f}, \tag{73}$$

where

$$E_j=\lambda_j\left(\lambda_j^2-\varepsilon_2\right),\quad j=3,4,\quad E_j=\frac{\lambda_j^2-\varepsilon_1}{1-a_1\left(\lambda_j^2-\xi^2\right)},\quad j=1,2. \tag{74}$$

Eqs. (70)-(73) can be solved for the unknowns A_j and A_j^3. These solutions are

$$A_1=\frac{\Delta_1\bar{f}}{\Delta},\quad A_2=\frac{\Delta_4\bar{f}}{\Delta},\quad A_3^3=\frac{\Delta_3\bar{f}}{\Delta},\quad A_4^3=\frac{\Delta_4\bar{f}}{\Delta}, \tag{75}$$

where

$$\Delta_1=-E_3\left(\alpha_{y2}\zeta_{yx4}-\alpha_{yx2}\zeta_{y4}\right)-E_4\left(\alpha_{yx2}\zeta_{y3}-\alpha_{y2}\zeta_{yx3}\right), \tag{76}$$

$$\Delta_2=E_3\left(\alpha_{y1}\zeta_{yx4}-\alpha_{yx1}\zeta_{y4}\right)+E_4\left(\alpha_{yx1}\zeta_{y3}-\alpha_{y1}\zeta_{yx3}\right), \tag{77}$$

$$\Delta_3=E_4\left(\alpha_{y2}\zeta_{yx1}-\alpha_{y1}\zeta_{yx2}\right),\quad \Delta_4=E_3\left(\alpha_{y2}\zeta_{yx1}-\alpha_{y1}\zeta_{yx2}\right), \tag{78}$$

$$\Delta=E_1\Delta_1+E_2\Delta_2. \tag{79}$$

Thus, Eq. (75) constitute the complete solution of the thermal boundary condition problem.

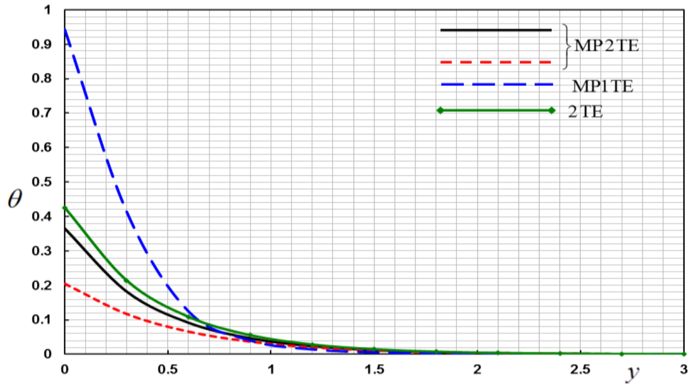

Fig. 1 Variations of dynamical temperature θ with distance y for three different theories

We can get two particular cases for the present problem. For example, if we neglect the effect of micropolarity, the analytical expressions for displacement component and force stresses may be obtained in theory of thermoelasticity with two temperatures (2TE). In addition, as $a_1 \to 0$ and $\phi \to 0$ and neglecting the effect of micropolarity, the classical theory (one-temperature generalized thermoelasticity theory 1TT) is recovered.

6. Numerical results and discussions

The magnesium crystal is chosen for the purposes of evaluations following Eringen (1970). The values of physical constants of magnesium, are given at reference temperature $T_0 = 23\ ^\circ\mathrm{C}$ as

$$\lambda = 9.4 \times 10^{11}\ \mathrm{dyne/cm^2}, \quad \mu = 4.5 \times 10^{11}\ \mathrm{dyne/cm^2},$$
$$\alpha = 0.5 \times 10^{11}\ \mathrm{dyne/cm^2}, \quad \upsilon + \beta = 0.779 \times 10^{-4}\ \mathrm{dyne/cm^2},$$
$$K = 0.6 \times 10^{-2}\ \mathrm{cal/(cm\ s\ ^\circ C)}, \quad C_E = 0.23\ \mathrm{cal/(gm\ ^\circ C)},$$
$$\rho = 1.74\ \mathrm{gm/cm^3}, \quad J = 0.2 \times 10^{-15}\ \mathrm{cm^2}, \quad \gamma = 4.834 \times 10^4\ \mathrm{dyne}.$$

We have $\Omega = \Omega_0 + \mathrm{i}\Omega_1$ then $\mathrm{e}^{\Omega t} = \mathrm{e}^{\Omega_0 t}(\cos\Omega_1 t + \mathrm{i}\sin\Omega_1 t)$, so for small values of time we can take Ω is real (i.e., $\Omega = \Omega_0$), in numerical calculations, the other constants of the problem is taken as follows

$$\Omega_0 = 2, \quad \xi = 2, \quad L = 2, \quad \phi_0 = 1, \quad b = 1.$$

The computations are carried out at the non-dimensional time $t = 0.2$, in the plane $x = 0.1$ and in the range $0 \le y \le 3$, and we consider the real part of the amplitudes of the field quantities which are represented on the vertical axis.

The conductive and the dynamical temperature ϕ and θ, the thermal stresses σ_{xx}, σ_{yy} and σ_{yx}, the displacement components u and v, the micro-rotation ω and the components of the couple stress m_{yz} are represented graphically at different positions of y and time t. The figures show that two-temperature parameter a_1 has significant effect on all the fields. The waves reach the steady state depending on the value of the temperature discrepancy a_1. So, according to the results of this work, it is important to distinguish between the dynamical temperature and the conductive temperature.

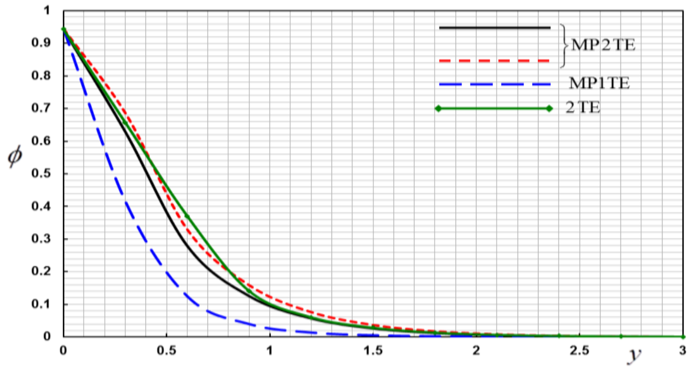

Fig. 2 Variations of conductive temperature ϕ with distance y for three different theories

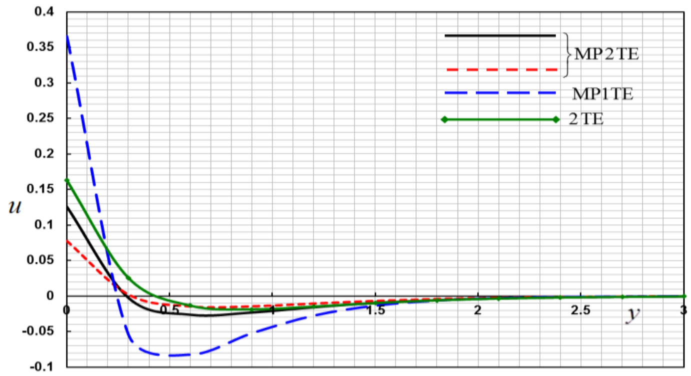

Fig. 3 Variations of displacement u with distance y for three different theories

Fig. 1 exhibits the variation of dynamical temperature θ in the context of the three theories (MP2TE) and (MP1TE), in which we observe that a significant difference in the dynamical temperature is noticed for the value of the non-dimensional two-temperature parameter a_1 where the case of $a_1 = 0$ indicates the old case, one type temperature (MP1TE) and the case of $a_1 = 0.2, 0.4$ indicates the new case, two-temperature (MP2TE). In MP2TE, the mechanical temperature represents the temperature comes from the mechanical process between the particles and the layers of the elastic solid, so it seems to be less than the corresponding one in the theory MP1TE, which is the total temperature. In fact this figure indicates to the amount of thermodynamic process occurred inside the investigated material. If we neglect the effect of micropolarity, that is when $\alpha = \beta = \upsilon = \varepsilon = J = 0$, we get theory of thermoelasticity with two temperatures (2TE). It is observed that the dynamical temperature θ decreases as the axial distance y increases to move in the direction of wave propagation. The dynamical temperature of MP2TE model may be differing than those of MP1TE theory.

Fig. 2 describes the variation of conductive temperature ϕ in the context of the two theories MP1TE and MP2TE. The conductive temperature in MP2TE records values higher than those values recorded in MP1TE, and this can be explained that is the sum of the two temperatures; the mechanical one and the temperature coming from the second gradient of the total temperature ϕ_{ii}. We can observe that there exists a slight difference between the three lines in Fig. 2, and this is due to the choice of the two temperatures discrepancy coefficient a_1. For a large value of a_1, the

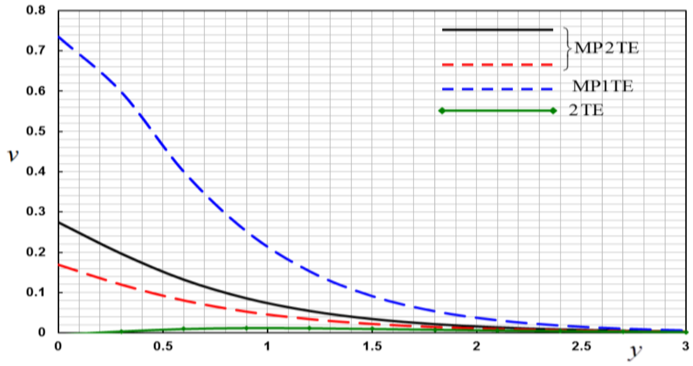

Fig. 4 Variations of displacement v with distance y for three different theories

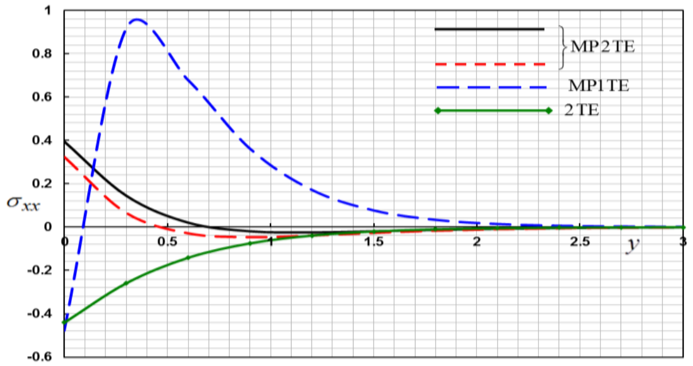

Fig. 5 Variations of stress component σ_{xx} with distance y for three different theories

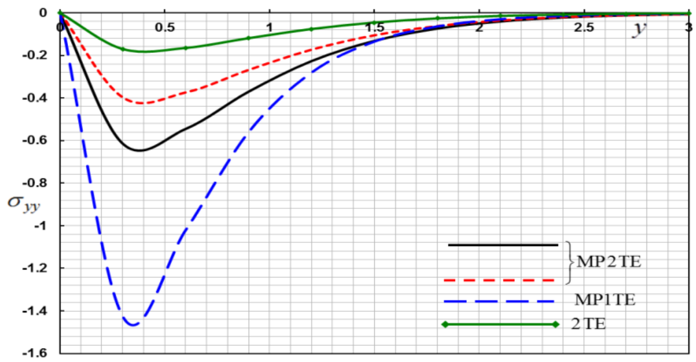

Fig. 6 Variations of stress σ_{yy} with distance y for three different theories

difference will be significant. The conductive temperature ϕ starts with its maximum value at the origin (due to the presence of the thermal boundary) and decreases until attaining zero beyond the thermal wavefront for the generalized theory, whereas it is continuous everywhere else for the coupled theory.

Figs. 3 and 4 exhibit the space variation of normal and transverse displacements u and v in which we observe that a significant difference in the dynamical temperature is noticed for different value of the non-dimensional two-temperature parameter. It is evident that the values of transverse

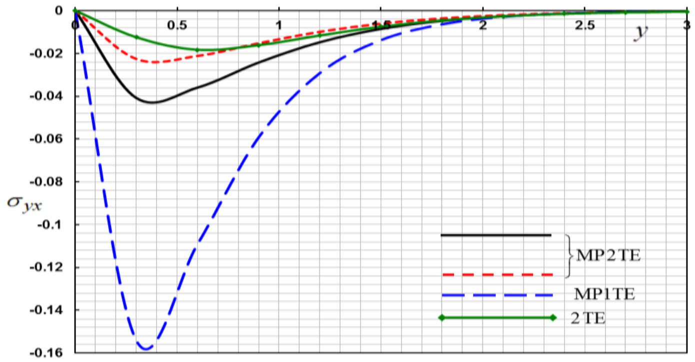

Fig. 7 Variations of stress σ_{yx} with distance y for three different theories

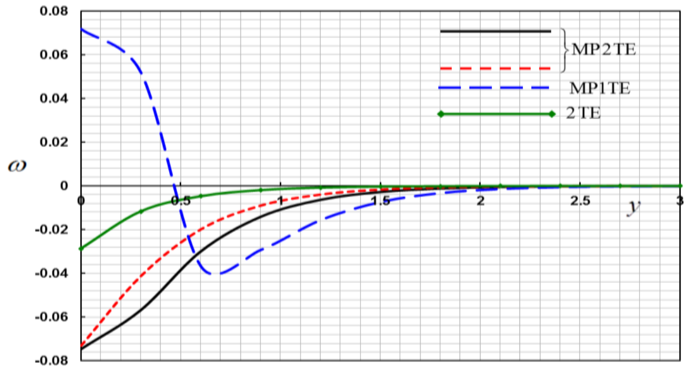

Fig. 8 Variations of micro-rotation ω with distance y for three different theories

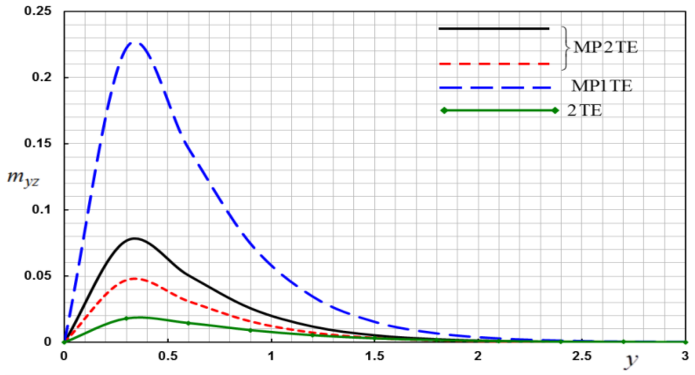

Fig. 9 Variations of tangtial couple stresses m_{yz} with distance y for three different theories

displacement v recorded in MP2TE are less than those values recorded in MP1TE, and the same behavior is noticed for the normal displacement u.

Figs. 5-7 describe the variations of normal and shearing stresses σ_{xx}, σ_{yy} and σ_{yx}, and we observe that, the two-temperature influence still decreases the values of normal and shearing stresses. From Figs. 5 to 7, we found that, the two-temperature parameter has significant effects on all the fields. The waves reach the steady state depending on the value of the temperature discrepancy a_1 The behavior of variations are similar in nature in entire range with signifficant difference in their magnitude of variation.

Figs. 8 and 9 exhibit the variation of micro-rotation ω and tangtial couple stresses m_{yz} under the three theories MP2TE, MP1TE and 2TE in which we observe that a significant difference in the values is noticed for different value of the non-dimensional two-temperature parameter a_1. It is noticed that the value of tangential couple stress for MP1TE theory are large in comparison to MP2TE and MP2TE, theories and the values are small for the rest of the range. The tangential couple stress starts with a zero value at the origin (according to the boundary condition).

7. Conclusions

This work constructed a new model of two-temperature generalized micropolar thermoelasticity for thermoelastic medium. When we distinguish between the two temperatures, the first comes from the mechanical process and the second comes from the thermal process. We obtain that the values of most relevant variables record values less than that recorded in the conventional model of micropolar thermoelasticity (MP1TE). Indeed, this can be considered as an expected result according to the main dependence of these variables on the thermodynamic temperature and the decreasing inherent in this temperature. We think that, these results may be the nearer to the correct values, especially in the cases in which the small values of time in the elastic solids are considered. These facts indicate that the two temperatures discrepancy coefficient a_1 has well pronounced effect on predominantly elastic motions and only marginal effect on predominantly thermal disturbances.

The generalized theory with phase lags is perhaps a more natural candidate for its identification as thermoelasticity than the usual theory. We may conclude that this theory (of heat conduction) is a good model to explain the heat conduction for several kinds of solids and fluids. According to the present study, the theory of two-temperature generalized thermoelasticity describes the behavior of the particles of elastic solid more real than that of one-temperature generalized thermoelasticity theory. It is appropriate to separate both the conductive heat wave and the thermodynamical heat wave.

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EC

Nomenclature

a	Two-temperature parameter (temperature discrepancy)
C_E	Specific heat at constant deformation
$e = \mathrm{div}\,\vec{u}$	Volumetric strain
J	Micro-inertia
K	Thermal conductivity
m_{ij}	Components of couple stress tensor
t	Time
T_0	Environment temperature
$\vec{u}$	Displacement vector
Q	Heat source
S_{ij}	Kronecker’s delta function
ε_{ij}	Components of micropolar strain tensor
ϵ_{ijk}	Permutation symbol
ω_i	Component of microrotation vector
ϕ	Conductive temperature
ϕ_0	Reference conductive temperature
γ	Thermal elastic coupling tensor
$\lambda,\ \mu,\ \alpha,\ \beta,\ \upsilon,\ \varepsilon$	Material constants
∇^2	Laplacian

ρ	Material density
σ_{ij}	Stress components
τ_q	Phase-lag of gradient of temperature
τ_θ	Phase-lag of heat flux
$\theta = T - T_0$	Thermodynamical temperature