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FE modeling for geometrically nonlinear analysis of laminated plates using a new plate theory

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Abstract. The aim of the present work is to study the nonlinear behavior of the laminated composite plates under transverse sinusoidal loading using a new inverse trigonometric shear deformation theory, where geometric nonlinearity in the Von-Karman sense is taken into account. In the present theory, in-plane displacements use an inverse trigonometric shape function to account the effect of transverse shear deformation. The theory satisfies the traction free boundary conditions and violates the need of shear correction factor. The governing equations of equilibrium and boundary conditions associated with present theory are obtained by using the principle of minimum potential energy. These governing equations are solved by eight nodded serendipity element having five degree of freedom per node. A square laminated composite plate is considered for the geometrically linear and nonlinear formulation. The numerical results are obtained for central deflections, in-plane stresses and transverse shear stresses. Finite element Codes are developed using MATLAB. The present results are compared with previously published results. It is concluded that the geometrically linear and nonlinear response of laminated composite plates predicted by using the present inverse trigonometric shape function is in excellent agreement with previously published results.

Keywords: laminated composite plates; geometrically nonlinear; finite element method; new trigonometric shear deformation theory

1. Introduction

One of the most significant characteristics of a laminated composite plate is high stiffness to weight ratio, which is particularly vital for various mechanical, aerospace, locomotive industries and several other engineering fields. By ordering layers and fiber direction in laminated plate, essential strength and stiffness properties can be attained.Three dimensional (3D) elasticity solutions for bidirectional bending of laminated composite and sandwich plates are presented by Pagano (1970) and Zenkour (2007). The 3D elasticity solutions are computationally difficult and led the development of approximate theories for the analysis of plate structures.The classical laminate plate theory (CLPT), which is an extension of the classical plate theory (CPT) developed by Kirchhoff (1850), neglects the effects of out-of-plane strains.The CLPT which ignores the effect of transverse shear deformation and under-calculates deflections, becomes insufficient for

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the analysis of laminated composite plates. In the first order shear deformation theory (FSDT) developed by Mindlin (1951) a shear correction factor is multiplied with the shear modulus to correct the unlikely deviation of the shear strains and stresses through the plate thickness.The calculation of shear correction factor becomes difficult as it depends on loadings types, boundary conditions, geometric and material parameters. One more limitation of FSDT is that, it does not satisfy traction free boundary conditions at top and bottom surfaces of the plate. A large number of higher order shear deformation theories (HSDTs) have been suggested to overcome the limitations of CLPT and FSDT. Sayyad and Ghugal (2015, 2017a) have recently reviewed all higher order shear deformation theories for laminated composite plates.Kulkarni *et al.* (2018) reviewed articles of last fifteen years related to structural analysis of composite plates including evolution of all plate theories with their outcome.Sayyad and Ghugal (2017b) reviewed various literature available for the bending, free vibration and buckling analysis of laminated composite and sandwich beams. Also they presented the displacement field of various displacement based equivalent single layer and layerwise theories. One of the well-known higher order theories is developed by Reddy (1984) which is further used by many researchers for the various problems of beams, plates and shells. The finite element analysis of laminated anisotropic plate is presented by Panda and Natarajan (1979), where the stresses are computed instead of their resultants.The volume integration of the stiffness matrix is calculated by Gauss Legendre method of integration.The finite element method (FEM) is used by Phan and Reddy (1985) to study the effects of shear deformation, bending–extension coupling and anisotropy on the response of laminated composite plates.Yin and Ruan (985) have developed an analytical solution for geometrically linear and nonlinear problems of thin plate. Whitney and Pagano (1970) and Senthilnathan *et al.* (1987) presented nonlinear analysis of anisotropic plates. Ren and Hinton (1986) extended the theory of Reddy to develop two finite elements for bending analysis of laminated composite plates. The deformation of antisymmetric angle ply laminated plate under transverse loading has been studied by Ren (1987). In his analysis author have used double Fourier series and obtained closed-form solution for simply supported boundary conditions. Pandya and Kant (1988) developed C^0 isoparametric finite element formulation to estimate interlaminar stresses.The impact analysis for nonlinear deflection of plate using FEM has been presented by Kant and Mallikarjuna (1990). The HSDT is proposed by Kant and Kommineni (1992), accounting for parabolic distribution of the transverse shear strains with inclusion of von Karman higher order terms for linear and nonlinear analysis. Savithri and Varadan (1993) presented the geometric nonlinear analysis of laminated plates using displacement based higher order theory and Galerkin method.The Hybrid-Trefftz finite element model is developed by Qinm (1995) for the nonlinear analysis, in order to simplify by detaching the coupling between in-plane and out-of-plane displacements.The dynamic response of layered plate is studied by Makhecha *et al.* (2001) using C^0 type serendipity element. Kant and Swaminathan (2002) presented the static analysis of simply supported laminated composite and sandwich plates based on higher order refined shear deformation theories. Sayyad and Ghugal (2013a,b,2014a,b,c,2017b) published series of research papers on application of trigonometric shear deformation theory for the bending, buckling and free vibration analysis of isotropic, transversely isotropic, laminated composite and sandwich rectangular plates. Carrera (1999b, 2005) studied the effect of transverse normal strain on static and dynamic responses of multilayered plates. Carrera (2002, 2003 and 2011) developed well-known Carrera Unified Formulation (CUF) for beams, plates and shells theories. The mixed FEM has been proposed by Urthaler and Reddy (2008) to solve plate bending, by involving the bending moments at the discrete points. FEM models based on least square method and weak-form Galerkin method for plates using FSDT and HSDT are studied by Reddy *et* *al.* (2010) treating displacements and stress resultants as the unknown field variables. The coupled analysis of plate by using C^0 finite element formulation is proposed by Hari *et al.* (2011). Somireddy and Rajagopal. (2015) conducted parametric study on geometric nonlinear bending analysis of laminated composite plates using C^0 and C^1 finite elements. Pingulkar and Sureshathis (2016) carried out vibration analysis of cantilever glass fiber and carbon fiber reinforced polymer composites by using ANSYS. Naik and Sayyad (2018) developed a new fifth order shear and normal deformation theory for the bending analysis of laminated plates subjected to transverse loads. Li and Zhao (2015) presented nonlinear bending analysis of laminated plate and numerical load-deflection curves obtained by using a mixed Galerkin perturbation technique.Zuo *et al.*(2015) proposed a wavelet finite element method using two dimensional B-spline wavelet interval for the static and free vibration analysis of laminated composite plates. The effect of modulus ratios and aspect ratios on central deflection and stresses of plate is presented by Reddy (2012). A four node higher order element is formulated by Grover *et al.* (2013) based on a refined shear plate theory with an extension of the in-plane cubic displacements. A new nonpolynomial shear-deformation theory (NPSDTs) is presented by Goswami and Becker (2013) corresponding inverse parabolic and secant function. Suganyadevi and Singh (2013) presented mixed finite element method based on the functional analysis method in combination with the Gateaux differential approach. Nam *et al.* (2017) proposed a polygonal finite element formulation for vibration analysis of laminated composite plates. In the present paper, the geometrically linear and nonlinear analysis of a laminated composite plates subjected to the transverse sinusoidal loading is investigated for the simply supported boundaries using a new inverse trigonometric shear deformation theory. Symbolic computation comparable to Roque (2014) is used in MATLAB coding based on finite element methods for analysis of linear and Von Karman's nonlinearity. The numerical results for assumed displacement fields are presented to compare the deflection response and induced stresses for various lamination schemes.

2. The present theory

2.1 The geometrical configuration

The geometry of plate under consideration is shown in Fig.1, in which a, b, and h indicate plate length, width and thickness respectively. A symmetric and asymmetric laminated plate composed of N laminate, where N=2, 3, 4 etc., with displacement parallels to the Cartesian Coordinates axes x-1, y-2 and z-3 is considered. All layers are perfectly bonded and made up of orthotropic

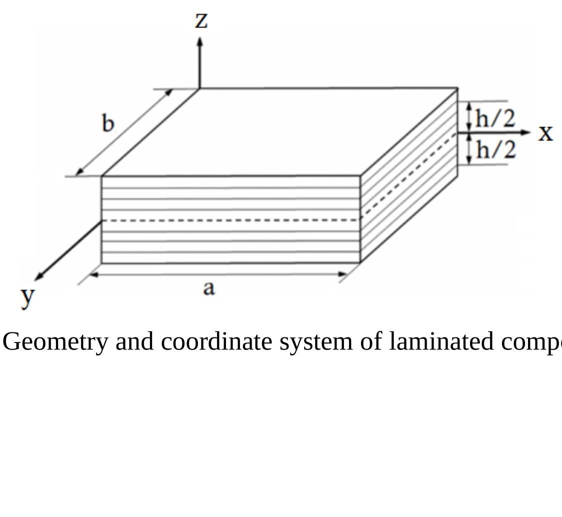

Fig. 1 Geometry and coordinate system of laminated composite plate

elastic material. The plate is exposed to transverse load q (x, y) on the top surface of the plate i.e., z = h/2.

2.2 The displacement field

The displacement field is the start point for the theoretical formulation of the present theory. In this study, the following displacement fields for a plate are assumed for the development of the present shear deformation theory.

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\phi(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\psi(x,y) \\ w(x,y) &= w_0(x,y) \end{aligned} \tag{1}$$

where, the present kinematic function $f(z) = \cot^{-1}\left(\frac{h}{z}\right) - \left(\frac{16}{15}\right)\left(\frac{z}{h}\right)^3$ represents the variation of the displacement within the element, and u, v and w are displacements at general points of the plate with u_0, v_0 and w_0 are displacements components at a point on the midplane of the plate. The ϕ and ψ are rotation about y and x axis respectively. The simple field variables for each discrete point in this formulation are u_0, v_0, w_0, ϕ and ψ. The condition of shear stresses is satisfied with the given function at the top and bottom surface of laminated composite plate.

2.3 Strain-displacement relationship

The strain displacement relation is expressed by assuming small displacements and moderate rotations. Considering geometric nonlinearity the von-Karman's nonlinear strain–displacement relations are considered as follows

$$\varepsilon = \{\varepsilon_l\} + \{\varepsilon_{nl}\} \tag{2}$$

$$\begin{aligned} \varepsilon_x &= \left\{\frac{\partial u_0}{\partial x}\right\} + \left\{\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right\} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{d\phi}{dx} \\ \varepsilon_y &= \left\{\frac{\partial v_0}{\partial y}\right\} + \left\{\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right\} - z\frac{\partial^2 w_0}{\partial y^2} + f(z)\frac{d\psi}{dx} \\ \gamma_{xy} &= \left\{\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\} + \left\{\left(\frac{\partial w_0}{\partial x}\right)\left(\frac{\partial w_0}{\partial y}\right)\right\} - 2z\frac{\partial^2 w_0}{\partial x \partial y} + f(z)\left(\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial x}\right) \\ \gamma_{zx} &= \frac{d}{dz}f(z)\phi, \quad \gamma_{yz} = \frac{d}{dz}f(z)\psi \end{aligned} \tag{3}$$

where, $\varepsilon = (\varepsilon_x, \varepsilon_y, \varepsilon_z)^T$ and $\gamma = (\gamma_{xy}, \gamma_{xz}, \gamma_{yz})^T$ are normal strain vector and shear strain vector respectively. In general, $\varepsilon = (\varepsilon_l, \varepsilon_{nl})$ are called linear and nonlinear strain vectors, including normal and shear strains.

2.4 Constitutive relations

The constitutive relations, for planar orthotropic laminated composite plate material, for each lamina of laminate in terms of principal material directions can be written as:

$$\{\sigma_i\}^k = [Q_{ij}]^k \{\varepsilon_i\}^k$$
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k_{l,nl} \tag{4}$$

where, Q_{ij} are plane stress reduced elastic constants of lamina, $\{\sigma_i\}^k$ is the stress vector and $\{\varepsilon_i\}^k$ is the strain vector. For orthotropic layer, the elastic constants are written as

$$Q_{11}^k = \frac{E_1^k}{1-\mu_{12}\mu_{21}}, Q_{22}^k = \frac{E_2^k}{1-\mu_{12}\mu_{21}}, Q_{12}^k = \frac{\mu_{12}E_2^k}{1-\mu_{12}\mu_{21}}$$
$$Q_{66}^k = G_{12}^k, Q_{44}^k = G_{23}^k, Q_{55}^k = G_{13}^k \tag{5}$$

where E_1 and E_2 are the Young moduli in the 1, 2 directions which are according to fibre direction and its in-plane normal, respectively, and G_{12}, G_{13} and G_{23} are the shear moduli in the 1-2, 1-3 and 2-3 planes, respectively, and μ_{ij} are Poisson's ratios.

2.5 Governing equations of equilibrium

The governing equations of equilibrium are derived by using the principle of minimum potential energy. In analytical form it can be written as

$$\delta(U-V)=0 \tag{6}$$

where U is the strain energy, V is the potential energy and δ is the variational symbol. The strain energy of the plate is given by

$$U = \frac{1}{2}\int_0^a \int_0^b \int_{-h/2}^{h/2} \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right) dz\, dy\, dx \tag{7}$$

and the potential energy due to transverse load q(x, y) on the plate is given by

$$V = \int_0^a \int_0^b q(x,y)\, w_0\, dy\, dx \tag{8}$$

Substituting Eqs.(7)-(8) into the Eq.(6), and integrating by parts, the following equations of equilibrium are obtained.

$$
\begin{aligned}
&\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \,, \\
&\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0, \\
&\delta w_0 : \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}\frac{\partial^2 w_0}{\partial x \partial y} + N_y \frac{\partial^2 w_0}{\partial y^2} \\
&\qquad + \left(\frac{\partial w_0}{\partial x}\frac{\partial N_x}{\partial x}\right) + 2\left(\frac{\partial w_0}{\partial x}\frac{\partial N_{xy}}{\partial y}\right) + \left(\frac{\partial w_0}{\partial y}\frac{\partial N_y}{\partial y}\right) + q = 0 \\
&\delta \phi : \frac{\partial P_x}{\partial x} + \frac{\partial P_{xy}}{\partial y} - P_{xz} = 0 \,, \\
&\delta \psi : \frac{\partial P_y}{\partial y} + \frac{\partial P_{xy}}{\partial x} - P_{yz} = 0
\end{aligned} \tag{9}
$$

where the axial force, moment and shear force resultants are expressed as

$$
\begin{aligned}
&N : \left(N_x, N_y, N_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) dz \\
&M : \left(M_x, \mathrm{M}_y, M_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) z\, dz \\
&\left(P_x, P_y, P_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) f(z)\, dz \\
&\left(P_{yz}, P_{xz}\right) = \int_{-h/2}^{h/2} \left(\tau_{yz}, \tau_{xz}\right) f'(z)\, dz
\end{aligned} \tag{10}
$$

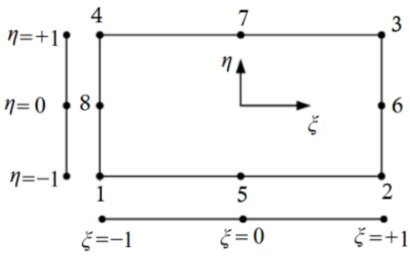

Fig. 2 Eight noded quadratic serendipity element

Table 1 Gauss Legendre integration weights and sampling points

Order of Integration	Weight	Sampling Point
3 Point	$(5/9),(8/9)$ *and* $(5/9)$	$\sqrt{3/5},\ 0$ *and* $-\sqrt{3/5}$
2 Point	1 and 1	$\sqrt{1/3}$ *and* $-\sqrt{1/3}$

2.6 Finite element method

The present theory requires C^0 continuity for its finite element approximation. The displacement field variables (x) for the present theory is shown in Eq. (11).

$$\{x\} = [u_0 \; v_0 \; w_0 \; \phi \; \psi]^T \tag{11}$$

A eight noded element shown in Fig. 2 is used for the finite element modelling. In the FEM the plate domain is discretized into a set of finite elements.

For given element, $N_i(\xi, \eta)$ is shapes (interpolation) function and (ξ, η) are natural coordinates for isoparametric elements. The shape functions in local (natural) coordinates in Eq. (12).

$$[N_i] = \frac{1}{4}\begin{bmatrix} (1-\xi)(1-\eta)(-\xi-\eta-1) \\ (1+\xi)(1-\eta)(+\xi-\eta-1) \\ (1+\xi)(1+\eta)(+\xi+\eta-1) \\ (1-\xi)(1+\eta)(-\xi+\eta-1) \\ 2(1+\xi)(1-\xi)(1-\eta) \\ 2(1+\xi)(1+\eta)(1-\eta) \\ 2(1+\xi)(1-\xi)(1+\eta) \\ 2(1-\xi)(1+\eta)(1-\eta) \end{bmatrix} \tag{12}$$

The displacement variables can be written in terms of shape function in which i=1 to 8,

$$\begin{aligned} u &= \sum_{i=1}^{8} N_i(\xi,\eta)u_i \\ v &= \sum_{i=1}^{8} N_i(\xi,\eta)v_i \\ w &= \sum_{i=1}^{8} N_i(\xi,\eta)w_i \\ \phi &= \sum_{i=1}^{8} N_i(\xi,\eta)\phi_i \\ \psi &= \sum_{i=1}^{8} N_i(\xi,\eta)\psi_i \end{aligned} \tag{13}$$

The displacement field variables $\{x\}$, and displacement field in plate domain at nodes $\{\bar{x}_i\}$ are linked by shape functions N_i mentioned in Eq. (12).

$$\{x\} = \sum_{i=1}^{n=8}[N_i]\{\bar{x}_i\} \tag{14}$$

where $\{\bar{x}_i\}$ and $[N_i]$ are the nodal displacements and shape functions for given element. Using Eq. (6), Eq. (12) and Eq. (14), the system of algebraic equations of the plate using FEM for static analysis has the following form

$$[K]\{\bar{x}_i\} = \{q_s\} \tag{15}$$

$$\begin{bmatrix} [K_{xx}] & [K_{x\psi}] \\ [K_{\psi x}] & [K_{\psi\psi}] \end{bmatrix} \begin{Bmatrix} \bar{x} \\ \bar{\psi} \end{Bmatrix} = \begin{Bmatrix} \{q_s\} \\ \{0\} \end{Bmatrix} \tag{16}$$

where [K_{all}] is the global stiffness matrix, $\{\bar{x}_i\}$ is the displacement vector and [q_s] is the global force vector. The bending terms are calculated using Gauss Legendre Integration with 3 point method and the shear terms are calculated using 2 point method with reduced integration scheme for each element. The values of sampling points and weights for each order are given as below.

On imposing SS-BCs Eq.(15). is solved by Newton-Raphson method for active DoF. For linear problems algebraic equations $[K]\{\bar{x}_i\}=\{q_s\}$ is solved and non-linear systems equations are linearized around equilibrium point and solution is sought by iterative procedure. The Newton Raphson iterations at each force level is continued until residual becomes less than tolerance level.

3. The numerical results

To confirm the applicability and efficacy of the present theory, geometrically linear and nonlinear analysis of laminated composite plate is carried out. The plate is subjected to sinusoidal load as shown in Fig. 3.

The Newton Raphson method is used to solve the succeeding nonlinear finite element equations. The symmetric and asymmetric laminates of the graphite-epoxy material is considered for the present linear and nonlinear analysis.

$$E_1 / E_2 = 25, G_{12} = 0.5E_2, G_{31} = 0.5E_2, G_{23} = 0.2E_2, \mu_{12} = 0.25, \mu_{21} = 0.01. \tag{17}$$

The present results are presented in the form of following non-dimensional stresses and deflection for comparing those with previously published results.

$$\begin{aligned} &\bar{w} = \frac{w}{q_0} \times \frac{100E_2}{a}\left(\frac{h}{a}\right)^3, \bar{\sigma}_x = \frac{\sigma_x}{q_0} \times \left(\frac{h}{a}\right)^2, \bar{\sigma}_y = \frac{\sigma_y}{q_0} \times \left(\frac{h}{a}\right)^2 \\ &\bar{\tau}_{xy} = \frac{\tau_{xy}}{q_0} \times \left(\frac{h}{a}\right)^2, \bar{\tau}_{xz} = \frac{\tau_{xz}}{q_0}\left(\frac{h}{a}\right), \bar{\tau}_{yz} = \frac{\tau_{yz}}{q_0}\left(\frac{h}{a}\right) \end{aligned} \tag{18}$$

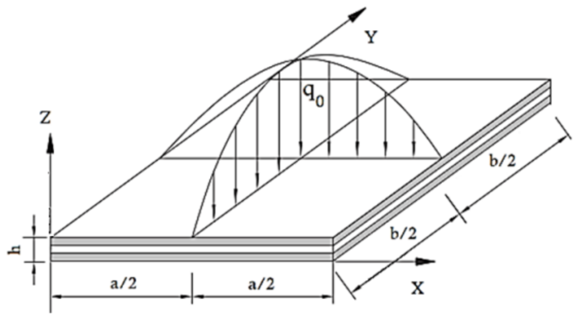

Fig. 3 Laminated composite plate subjected to sinusoidal load

Table 2 Convergence of non-dimensionalized central transverse deflection $\bar{w}$ with mesh size (0◦/90◦/0◦) laminated composite plate

S	Mesh size	Linear $\bar{w}(0)$	Mesh size	Nonlinear $\bar{w}(0)$
4	4×4	1.943	4×4	1.776
	6×6	1.935	6×6	1.784
	8×8	1.924	8×8	1.799
	10×10	1.924	10×10	1.800
			12×12	1.800
	Zenkour (2007)	2.004	Whitney and Pagano (1970)	1.775
10	4×4	0.716	4×4	0.663
	6×6	0.714	6×6	0.674
	8×8	0.713	8×8	0.683
	10×10	0.713	10×10	0.686
			12×12	0.686
	Zenkour (2007)	0.752	Whitney and Pagano (1970)	0.669

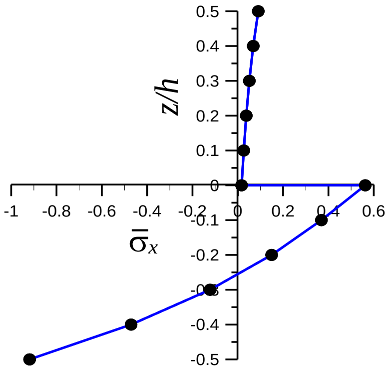

Fig. 4 Through-the-thickness variation of in-plane normal stress ($\bar{\sigma}_x$) for two layered (0°/90°) antisymmetric laminated composite square plate (S=4)

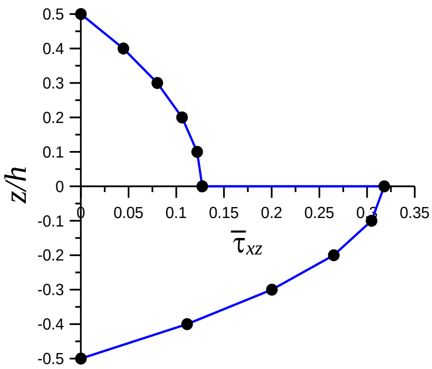

Fig. 5 Through-the-thickness variation of transverse shear stress ($\bar{\tau}_{xz}$) for two layered (0°/90°) antisymmetric laminated composite square plate (S=4)

Table 3 Non-dimensional displacements and stresses for two layered (0°/90°) antisymmetric laminated composite square plate

S	Theory	$\bar{w}$ (0)	$\bar{\sigma}_x$ $(-h/2)$	$\bar{\sigma}_y$ $(-h/2)$	$\bar{\tau}_{xy}$ $(-h/2)$	$\bar{\tau}_{xz}$ (0)	$\bar{\tau}_{yz}$ (0)
4	Present	2.0214	0.8188	0.0981	0.0599	0.127	0.128
	Sayyad and Ghugal (2013b)	1.9424	0.9062	0.0964	0.0562	0.127	0.127
	Mindlin (1951)	1.9682	0.7157	0.0843	0.0525	0.091	0.091
	Kirchhoff (1850)	1.0636	0.7157	0.0843	0.0525	0.000	0.000
	Zenkour (2007)	2.0670	0.8410	0.1090	0.0591	0.120	0.135
10	Present	1.2173	0.7455	0.0888	0.0534	0.112	0.124
	Sayyad and Ghugal (2013b)	1.2089	0.7471	0.0876	0.0530	0.130	0.130
	Reddy (1984)	1.2161	0.7468	0.0851	0.0533	0.127	0.127
	Mindlin (1951)	1.2083	0.7157	0.0843	0.0525	0.091	0.091
	Kirchhoff (1850)	1.0636	0.7157	0.0843	0.0525	0.000	0.000
	Zenkour (2007)	1.2250	0.7302	0.0886	0.0535	0.121	0.125

Table 4 Non-dimensional displacements and stresses for four layered (0°/90°/0°/90°) antisymmetric laminated composite square plate

S	Theory	$\bar{w}$ (0)	$\bar{\sigma}_x$ $(-h/2)$	$\bar{\sigma}_y$ $(-h/2)$	$\bar{\tau}_{xy}$ $(-h/2)$	$\bar{\tau}_{xz}$ (0)	$\bar{\tau}_{yz}$ (0)
4	Present	1.7348	0.7378	0.7258	0.0383	0.1943	0.2199
	Sayyad and Ghugal (2013b)	1.5827	0.4057	0.7088	0.0351	0.1398	0.1398
	Zenkour (2007)	1.9581	0.6146	0.7444	0.0457	0.2325	0.2410
10	Present	0.7097	0.4851	0.5281	0.0287	0.2315	0.2348
	Sayyad and Ghugal (2013b)	0.6847	0.4531	0.5226	0.0266	0.1433	0.1433
	Zenkour (2007)	0.7624	0.4942	0.5308	0.0292	0.2713	0.2714

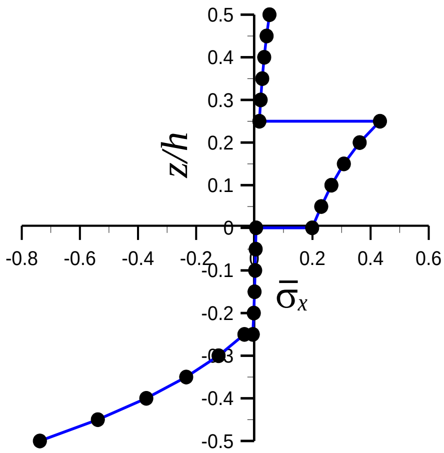

Fig. 6 Through-the-thickness variation of in-plane normal stress $(\bar{\sigma}_x)$ for four layered (0°/90°/0°/90°) antisymmetric laminated composite square plate (S=4)

Table 5 Non-dimensional displacements and stresses for three layered (0°/90°/0°) symmetric laminated composite square plate

S	Theory	$\bar{w}$ (0)	$\bar{\sigma}_x$ $(-h/2)$	$\bar{\sigma}_y$ $(-h/2)$	$\bar{\tau}_{xy}$ $(-h/2)$	$\bar{\tau}_{xz}$ (0)	$\bar{\tau}_{yz}$ (0)
4	Present	1.9248	0.7584	0.08248	0.0499	0.2304	0.2010
	Sayyad and Ghugal (2013b)	1.9015	0.7535	0.0880	0.0496	0.2092	0.1914
	Reddy (1984)	1.9218	0.7345	0.0782	0.0497	0.2024	0.1832
	Mindlin (1951)	1.5681	0.4370	0.0614	0.0369	0.1201	0.1301
	Kirchhoff (1850)	0.4312	0.5387	0.0267	0.0213	–	–
	Zenkour (2007)	2.0046	0.7550	0.0949	0.0505	0.2550	0.2170
10	Present	0.7136	0.5692	0.0417	0.0277	0.3827	0.0982
	Sayyad and Ghugal (2013b)	0.7155	0.5720	0.0411	0.0278	0.2577	0.1070
	Reddy (1984)	0.7125	0.5684	0.0387	0.0277	0.2447	0.1033
	Mindlin (1951)	0.6306	0.5134	0.0353	0.0252	0.1363	0.0762
	Kirchhoff (1850)	0.4312	0.5387	0.0267	0.0213	–	–
	Zenkour (2007)	0.7528	0.5898	0.0418	0.0289	0.3570	0.1200

Table 6 Non-dimensional displacements and stresses for four layered (0°/90°/90°/0°) symmetric laminated composite square plate

S	Theory	$\bar{w}$ (0)	$\bar{\sigma}_x$ $(-h/2)$	$\bar{\sigma}_y$ $(-h/2)$	$\bar{\tau}_{xy}$ $(-h/2)$	$\bar{\tau}_{xz}$ (0)	$\bar{\tau}_{yz}$ (0)
4	Present	2.1229	0.6864	0.6810	0.0478	0.2407	0.2749
	Sayyad and Ghugal (2013b)	1.8784	0.6830	0.6210	0.0442	0.2147	0.2474
	Zenkour (2007)	–	0.7202	0.6625	0.0466	0.2193	0.2915
10	Present	0.7742	0.5460	0.3914	0.02835	0.3231	0.1724
	Sayyad and Ghugal(2013b)	0.7173	0.5484	0.3898	0.0268	0.2783	0.1588
	Zenkour (2007)	–	0.5586	0.4009	0.0275	0.3013	0.1959

Table 7 Non-dimensional transverse displacement ($\bar{w}$) for three layered (0°/90°/0°) symmetric laminated composite square plate with geometric nonlinearity

Theory	*a/h*					
	2	4	10	20	50	100
Present	5.0389	1.8007	0.6860	0.4832	0.4448	0.4448
Hari and Singh (2011)	5.1828	1.9091	0.7101	0.5000	0.4378	0.4283
Savithri and Vardhan (1992)	---	----	0.7031	0.4952	----	0.4350
Kant and Swaminathan (2002)	4.9147	1.8948	0.7151	0.5053	0.4432	0.4343
Kant and Swaminathan (2002)	5.2158	1.9261	0.7176	0.5058	0.4433	0.4343
Reddy(1984)	5.1286	1.9218	0.7125	0.5041	0.4430	0.4342
Senthilnathan (1970)	4.3088	1.4852	0.6041	0.4746	0.4382	0.4330
Whitney and Pagano (1970)	5.2293	1.7758	0.6693	0.4921	0.4411	0.4337

Table 8 Non-dimensional transverse displacements ($\bar{w} = w/h$) for three layered (0°/90°/0°) symmetric laminated composite square plate with geometric nonlinearity for various values of Nondimensional load

	a/h		
$\bar{q} = q_0 S^4 / E_2$	10	20	100
40	0.250	0.220	0.166
80	0.500	0.410	0.361
120	0.722	0.550	0.500
160	0.900	0.722	0.660
200	1.110	0.866	0.750
240	1.230	0.970	0.900
280	1.300	1.110	1.100
320	1.350	1.194	1.112
360	1.410	1.277	1.190
400	1.520	1.361	1.250

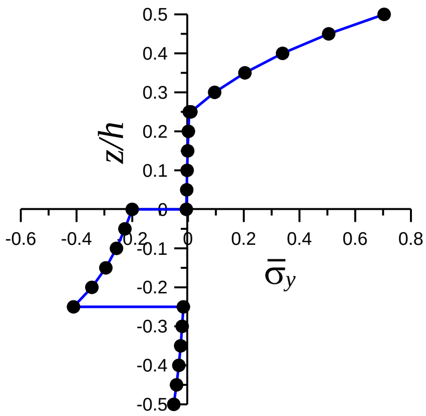

Fig. 7 Through-the-thickness variation of in-plane normal stress $(\bar{\sigma}_y)$ for four layered (0°/90°/0°/90°) antisymmetric laminated composite square plate (S=4)

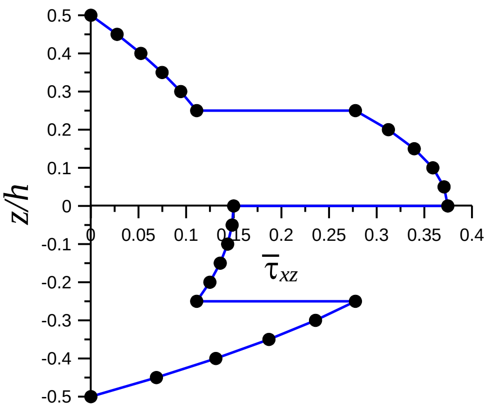

Fig. 8 Through-the-thickness variation of transverse shear stress $(\bar{\tau}_{xz})$ for four layered (0°/90°/0°/90°) antisymmetric laminated composite square plate (S=4)

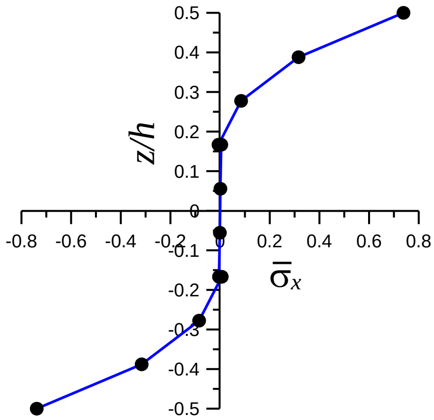

Fig. 9 Through-the-thickness variation of in-plane normal stress $(\bar{\sigma}_x)$ for three layered (0°/90°/0°) symmetric laminated composite square plate (S=4)

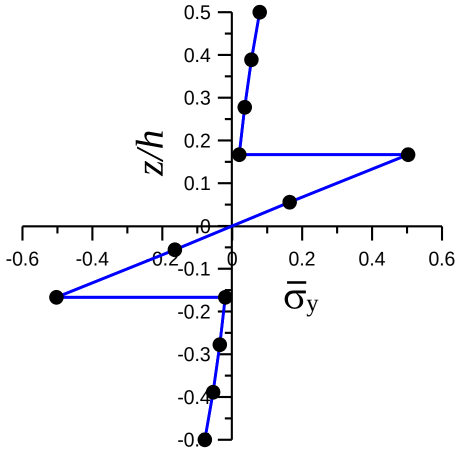

Fig. 10 Through-the-thickness variation of in-plane normal stress $(\bar{\sigma}_y)$ for three layered (0°/90°/0°) symmetric laminated composite square plate (S=4)

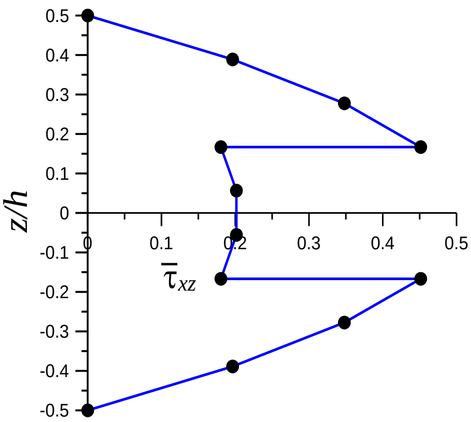

Fig. 11 Through-the-thickness variation of transverse shear stress $(\bar{\tau}_{xz})$ for three layered (0°/90°/0°) symmetric laminated composite square plate (S=4)

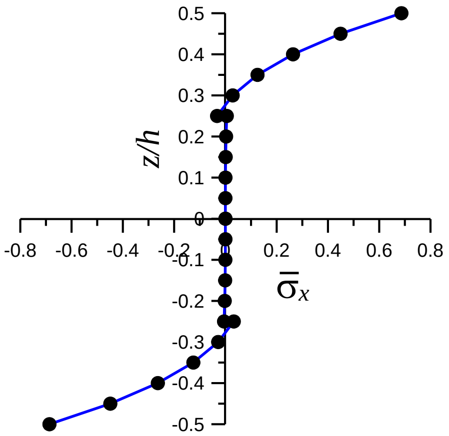

Fig. 12 Through-the-thickness variation of in-plane normal stress $(\bar{\sigma}_x)$ for four layered (0°/90°/90°/0°) symmetric laminated composite square plate (S=4)

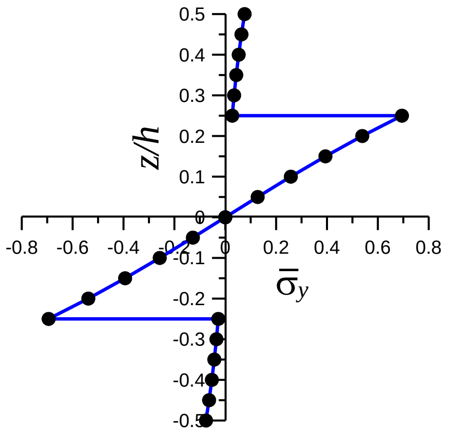

Fig. 13 Through-the-thickness variation of in-plane normal stress $(\bar{\sigma}_y)$ for four layered (0°/90°/90°/0°) symmetric laminated composite square plate (S=4)

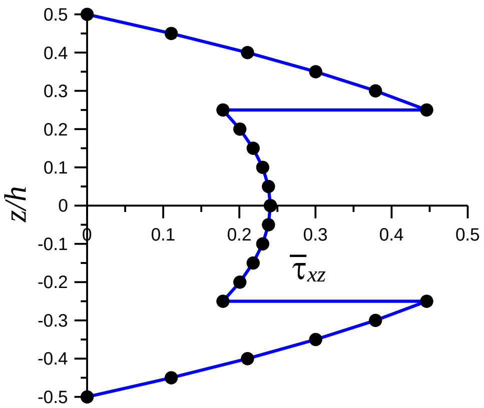

Fig. 14 Through-the-thickness variation of transverse shear stress $(\bar{\tau}_{xz})$ for four layered (0°/90°/90°/0°) symmetric laminated composite square plate (S=4)

4. Discussion on numerical results

A nonlinear finite element code in MATLAB has been developed for the analysis procedure described above. The accuracy of the present FE formulation for linear and nonlinear bending analysis is validated by comparing the results with those available in the literature.

The convergence of nondimensional central transverse deflection based on mesh size is presented in Table 2. The solution process continues until the two subsequent iterations are within the tolerance of 0.0001. Then the convergence is considered to be reached and the corresponding value is the required central displacement of the laminated composite plate. It is noted that the central deflections appear closely converging for mesh size 10×10. The present results are compared with those presented by Zenkour (2007) for linear analysis and with Whitney and Pagano (1970) for nonlinear analysis. Also the given kinematic function based FE model calculate the behavior of deflection with good accuracy with less computational effort than 3D elasticity solution of Zenkour (2007). Hence for all the problems mesh size 10×10 is used to calculate displacements and stresses.

Based on this convergence study presented in Table 2 it is concluded that 8×8 mesh and 10×10 mesh would be sufficient for linear and nonlinear analysis respectively. The Table 2 clearly shows that the performance of the present FE formulation is very good in terms of solution accuracy.

Table 3 shows comparison of non-dimensional transverse displacement and stresses of for two layered (0°/90°) antisymmetric laminated composite square plate subjected to sinusoidal load. Both the layers are of equal thickness i.e., h/2 where h is the overall thickness of the plate. The material properties are stated in Eq. (17). The present results are compared with those presented by Sayyad and Ghugal (2014a), Reddy (1984), Mindlin (1951), Kirchhoff (1850) and 3D elasticity solution of Zenkour (2007). The examination of Table 1 reveals that the present results are in excellent agreement with 3D elasticity solution. It is to be noted that the present results are even better than the well-known theory of Reddy (1984). FSDT of Mindlin (1951) and CLPT of Kirchhoff (1850) under predict the results due to neglect of transverse shear deformation. Figs. 4 and 5 show through the thickness variations of in-plane normal stress and transverse shear stress for (0°/90°) laminated composite square plate. The examination of these figures reveals that the stresses are maximum in zero degree layer whereas minimum in ninety degree layer. This is in fact due to high elastic modulus along the fibre.

Table 4 shows non-dimensional displacements and stresses for four layered (0°/90°/0°/90°) antisymmetric laminated composite square plate. All the layers are of equal thickness i.e. h/4. Same material properties are used in this problem. In this problem also, the present results are in excellent agreement with 3D elasticity solution presented by Zenkour (2007). It is also observed that present trigonometric function shows improvement over trigonometric function suggested by Sayyad and Ghugal (2014a). Figs. 6-8 plot through the thickness distribution of stresses in four layered (0°/90°/0°/90°) laminated composite plate.

Table 5 shows comparison of non-dimensional displacements and stresses for three layered (0°/90°/0°) symmetric laminated composite square plate subjected to sinusoidal load. All layers of plate are of equal thickness (h/3). It is pointed out from Table 5 that the present theory is predicting excellent results compared to those presented by Zenkour(2007) for symmetric lamination scheme also. Similar accuracy can be observed from Table 6 when the present theory is applied for the bending analysis of four layered (0°/90°/90°/0°) symmetric laminated composite square plate subjected to sinusoidal load. In this plate, each layer thickness is h/4. Figs. 9-11 show the plot of distribution of stresses in symmetric laminated plates.

Table 7 shows comparison of geometrically nonlinear displacements obtained for three layer (0°/90°/0°) symmetric laminated composite square plates subjected to sinusoidal load. Numerical results are obtained for various values of aspect ratio (a/h = 2, 4, 10, 20, 50 and 100). The present results are compared with those presented by Hari *et al.* (2011), Savithri and Vardhan (1993), Kant and Swaminathan (2002), Reddy (1984), Senthilnathan (1987) and Whitney and Pagano (1970). The examination of Table 7 reveals that the present results are in good agreement with those presented by Kant and Swaminathan (2002) and Reddy (1984). It is also pointed out that the nondimensional transverse displacement decreases with increase in aspect ratio i.e. thin plate undergoes large dimensional deflection whereas thick plate undergoes small deflection.

Table 8 contains the transverse central deflection for (0°/90°/0) laminated composite square plate in which central deflection is a function of the load parameter and the results are at par with results by Savithri and Vardhan (1993) for various non-dimensional load step (40:40:400) and for different a/h values (10,20,100).

5. Conclusions

In the present study, a new inverse trigonometric shear deformation theory is used for the geometrically linear and nonlinear analysis of laminated composite plates. The theory satisfies traction free boundary conditions and does not need shear correction factor. A simply supported laminated plate is analyzed using finite element method. Finite element codes are developed using MATLAB. Numerical results are obtained for different symmetric and antisymmetric lamination schemes. From the numerical results and discussion it is concluded that the present theory predicts excellent numerical results compared to other higher order shear deformation theories available in the literature. Therefore, the present theory is strongly recommended for the geometrically linear and nonlinear analysis of laminated composite plates.

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