

## Vibration effects on remote sensing satellite images

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**Abstract.** Vibration is a source of performance degradation in all optical imaging systems. Performance of high resolution remote sensing payloads is often limited due to satellite platform vibrations. Effects of Linear and high frequency sinusoidal vibrations on the system MTF are known exactly in closed form but the low frequency vibration effects is a random process and must be considered statistically. Usually the vibration MTF budget is defined based on the mission requirements and the overall MTF limitations. For analyzing low frequency effects, designer must know all the systems specifications and parameters. With a good understanding of harmful vibration frequencies and amplitudes in the system preliminary design phase, their effects could be removed totally or partially. This procedure is cost effective and let the designer to eliminate just harmful vibrations and avoids over-designing. In this paper we have analyzed the effects of low-frequency platform vibrations on the payload's modulation transfer function. We have used a statistical analysis to find the probability of imaging with a MTF equal or greater than a pre-defined budget for different missions. The worst and average cases have been discussed and finally we have proposed "look-up figures". Using these look-up figures, designer can choose the electro-optical parameters in such a way that vibration effects be less than its pre-defined budget. Furthermore, using the results, we can propose a damping profile based on which vibration frequencies and amplitudes must be eliminated to stabilize the payload system.

**Keywords:** remote sensing; modulation transfer function (MTF); image quality; vibration analysis; low frequency vibration; statistical analysis; look-up figures

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### 1. Introduction

Vibration of platform is often affects high resolution imaging satellite cameras by blurring in the focus. This kind of blurring should be distinguished from blurring due to system misalignment or an out-of-focus condition (Ahmad 1999). More sever vibration carry the potential for structural failure of the system. In general, operation is not expected at such levels, only survival. There are two important types of vibration that affect the image quality along with the optomechanical systems: Periodic and jitter. Periodic vibration includes low-frequency vibration (1-40 Hz) and high-frequency vibration (2-20 kHz), with different amplitudes caused by the motors used in different satellite subsystems. Table 1 illustrates typical vibration specifications for both high and low frequencies (Xu 2009 and Xu 2003). It is quiet useful to analyze the relationship between the

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Table 1 Typical vibration in satellite imaging missions (Xu 2009, Xu 2003 and Haghshenas 2015)

Vibration mode	Typical Range	Selected value
High Frequency	Frequency = 200-2000 Hz	Frequency = 1250
	Period = 0.005-0.0005 sec	Period = 0.0008 sec
	Amp = 0.2-0.6 $\mu\text{m}$	Amp = 0.4 $\mu\text{m}$ (4 urad)
Low Frequency	Frequency = 1 ~40 Hz	Frequency = 30 Hz
	Period = 1-0.025 sec	Period = 0.0333 sec
	Amp = 10-30 $\mu\text{m}$	Amp = 20 $\mu\text{m}$ (200 urad)

vibration of the whole camera and the image quality in order to determine the tolerance of stabilization in the satellite attitude and make the system design much more cost effective (Rudoler 1991 and Hodar 1992). In this case, the vibrations in internal components of the camera are neglected and only the vibrations of whole camera are analyzed. If the vibration effects on the image quality are known, the image restoration will be done more accurately.

Degradation of image quality caused by vibration or image motion can be described using the modulation transfer function (MTF). Knowledge of vibrational motion MTF can strongly facilitate the image processing and perhaps even reduce stabilization requirements and, hence, the cost<sup>4</sup>.

From the optomechanical point of view, vibration can excite the support structures for the optical elements. This effect can be reduced by mounting the sensor on vibration isolators that filter out the higher frequency vibrations where resonant frequencies for the support structures are located. Typically, the isolators cut off between 10 and 20 Hz with peaking of response at a slightly lower frequency. The major effects of mechanical vibrations in limiting image resolution often derive from the low vibration-frequency components because of their large amplitude (Xu 2003 and Hodar 1992).

The low frequency vibration is complex, because of its random nature. The best engineering tool for comparing different imaging systems is the modulation transfer function (MTF). The total system MTF is limited by the MTF of the weakest link. In the imaging systems involving with vibrations or motions, this weakest link is often the blur caused by the image vibration or motion, rather than that resulting from optical or electronic components. Therefore it is very useful for system designer and engineer to know the exact formulation of such an image blurring due to vibration and its corresponding MTFs (Hodar 1992).

Here, numerical and analytical calculations of MTFs to describe image quality will be considered for sinusoidal vibrations at high and low vibration frequencies. Jitter vibration effects are well-known in closed form and easy to remove, thus it is not considered in this paper.

As the spatial frequency increased, the MTF decreases and accordingly the contrast will degrade. At some relatively high spatial frequency, overall system MTF has decreased to such a low value of contrast that it is below the threshold contrast function of the observer or machine at the output. This means that higher spatial frequency content of the image cannot be resolved by the observer because of the poor contrast. The spatial frequency at which system MTF is just equal to the threshold contrast of the observer or machine defines the maximum useful spatial frequency content of the system. Sometimes, frequencies beyond the system cutoff frequency have non-zero MTF values which referred to spurious or false resolution. This is an interesting phenomenon because it suggests, falsely, that blur radius is smaller than actual blur radius. It is mentioned in many literatures about the effects of relative exposure (the ratio of sensor integration time to the vibrational period) on the relative blur radius and the MTF parameter (Xue 2009, Hodar 1994,

Afkhami *et al.* 2012, Raiter 2003, Yang 2012, Bao 2009 and Robbins 1996).

Studying on both integration times from electro-optical and atmosphere analysis point of view and the relative integration time effects from vibration and MTF aspects, we can define the optimum integration time for a specific mission (Haghshenas 2014). After analyzing the blur effects due to different kinds of vibrations, we have defined the best pixel size of camera considering all vibration situations along with the mission resolution objectives. Optical system designer can use the so-called look-up graphs at the end of this paper to avoid the system MTF degradation.

## 2. General vibration types

Blurred images caused by relative movement between the scene and the sensor may result from mechanical vibration. We can consider the vibration types into three general categories (Ahmad 1999);

- A-Jitter
- B-Linear
- C-Sinusoidal (High and Low frequency)

### A-Jitter (random motion)

With high frequency motion, it is assumed that the image has moved often during the integration time so that the central limit theorem is valid. The central limit theorem says that many random movements can be described by a Gaussian distribution. The Gaussian MTF is

$$MTF_{jitter}(N_i) = e^{-2\pi^2\sigma_R^2N_i^2} \quad (1)$$

Where  $\sigma_R$  is the rms random displacement in millimeter, and  $N_i$  is the spatial frequency (Holst 1998).

### B-Linear motion

If the motion is linear at a constant velocity  $v$  in the image plane, then the noncircular blur radius  $d$  resulting from exposure time  $t_e$  is given by

$$d = v \times t_e \quad (2)$$

Where  $v$  is the uniform relative velocity between the object and sensor (Zhang 2009).

The corresponding optical transfer function (OTF) of the linear motion is given by (Afkhami *et al.* 2012, Holst 1998, Holst 1995);

$$OTF = \text{sinc}(\pi Nd) \quad (3)$$

Where  $N$  is the spatial frequency and  $d$  is the spatial extent of the blur and is equal to  $vt_e$ .

The corresponding MTF is therefore the magnitude of the OTF and it can be written as following

$$MTF = |\text{sinc}(\pi Nvt_e)| \quad (4)$$

### C-Sinusoidal vibration

Sinusoidal vibration is a very critical factor in dynamic imaging systems such as satellite imaging. The sinusoidal image motion is important because of turbines and motors that give rise to mechanical vibrations. In the aerospace and satellite imaging, linear motion is almost always accompanied by vibrations that are often close to being sinusoidal. The sinusoidal motion can be prevented in principle by proper design; in practice, however, it is often the most serious image motion. As we have shown later in this paper, degradation of image quality as a result of sinusoidal motion depends on the ratio of integration time,  $T_{int}$ , to the period of the sinusoidal motion  $T_0$ ; resulting high and low frequencies. In this case, it is necessary to distinguish between these two categories.

Consider a sinusoidal image motion profile as following,

$$x(t) = D \cos\left(\frac{2\pi t}{T_0}\right) \quad (5)$$

#### C1-High frequency sinusoidal vibration

The case of relatively high-frequency sinusoidal motion is defined as concerning a vibration in which, one or more complete vibration cycles ( $T_0$ ) fall within the exposure period. The method of analysis is similar to that used for uniform motion. So the MTF for HF vibration is given by (Rudoler 1991, Hodar 1992)

$$MTF = J_0(2\pi Nd) \quad (6)$$

Where  $J_0$  is the first zero of Bessel function. This form of MTF has been proved by several experimental results (Hodar 1992, Trott 1960).

In HF vibrations, due to the shorter time period than LF, there would be much smaller blur radius for a given dissipated energy.

#### C2-Low frequency sinusoidal vibration

Image motion due to LF vibration effect is characterized by a relatively long vibrational period  $T_0$ , which is longer than the integration time. This means image blur takes place only during a portion of the vibration period rather than during the whole vibration period, as it was in HF case. Image blurring due to low vibration frequencies ( $T_{int} < T_0$ ) is a random process. In this case, the blur radius that occurs for a given  $T_{int}$  depends on when ( $T_x$ ) during the cycle the picture was taken. The time which the vibration happens is a random function. As seen in Fig. 1, the minimum blur occurs when exposure takes place at a vibration extremums, whereas maximum blur occurs when the exposure is centered at  $x(t)=0$  where the motion is extremely close to be linear. In all cases, the shorter the integration time, the smaller the blur radius. The minimum and maximum blur radii for single frequency can be calculated according to the integration time, as following (Wulich 1987),

$$d_{min} = D_{LF} \left(1 - \cos\left[\left(\frac{2\pi}{T_0}\right)\left(\frac{T_{int}}{2}\right)\right]\right) \quad (7)$$

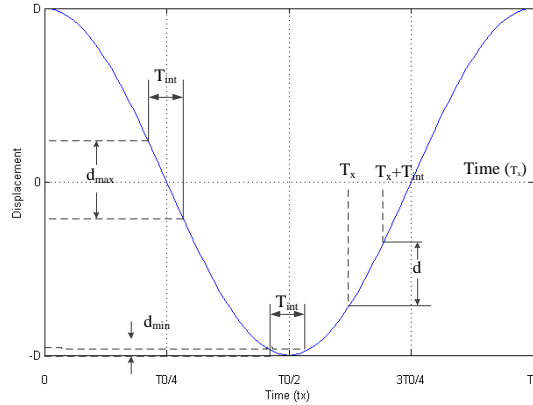


Fig. 1 Blur radius  $d$  versus integration time for low frequency sinusoidal vibration (Haghshenas 2015)

$$d_{\max} = 2D_{LF} \sin\left[\left(\frac{2\pi}{T_0}\right)\left(\frac{T_{\text{int}}}{2}\right)\right] \tag{8}$$

As the ratio of  $T_{\text{int}}/T_0$  decreases the blur radius will decrease. On the other hand, in practice, the integration time cannot be very small because there should be enough light signals at the time of imaging. The minimum integration time limitation is defined by optical and electro-optical specifications of imaging payload, along with the atmosphere conditions and the mission scenario (Haghshenas 2014).

The MTF for low-frequency vibration along optical axis and along-track directions can be expressed as (Rudoler 1991, Hodar 1992, Afkhami *et al.* 2012, Wulich 2017).

$$MTF = |\sin c(\pi Nd)| \tag{9}$$

Where  $N$  is the spatial frequency and  $d$  is blur diameter.

### 3. Limiting resolution

Limiting resolution occurs when the overall system MTF is equal to the output threshold contrast<sup>17</sup>. This quantity is differing according to different applications (typical range is from 0.05 to 0.2). The spatial frequency, at which this function satisfied,  $f_{\text{lim}}$ , represents the limit of resolution. Because of the poor contrast, higher spatial frequency content is not resolved<sup>17</sup>. Only for low vibration frequency image degradation, we have

$$MTF_d = \alpha \tag{10}$$

Where, we can define  $\alpha$  to be 0.6, 0.7 or 0.8 according to the vibration MTF budget. This vibration MTF budget should be defined in such a way to have the overall system MTF to be equal to the output threshold contrast. Considering the MTF to be in sinc function format, we have

$$f_{\text{lim}} = \frac{a}{d} \tag{11}$$

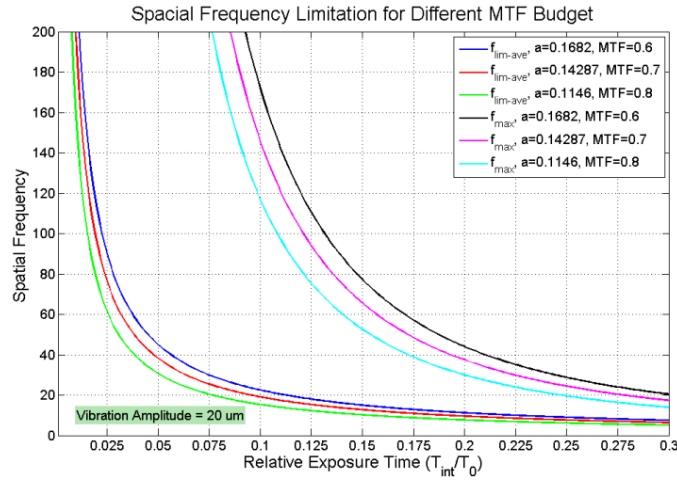


Fig. 2 frequency limitation for different MTF budget. Vibration amplitude is 20 um

Table 2 Typical mission parameters (Haghshenas 2015)

Parameter	Quantity
Resolution (GSD)	40 m
Orbital Height	681 km
Effective Focal Length	110 mm
Pixel Size	6.45 um
F/#	5
Integration time	0.9 ms
Pointing Accuracy	0.5 °
Pointing Stability	0.01 %/sec
Inclination	98.4

Where the maximum possible “a” are 0.1682, 0.14287 and 0.1146 for  $\alpha = 0.6, 0.7$  and  $0.8$ , respectively. The average limiting resolution for low frequency mechanical vibrations can be obtained from (Rudoler 1991, Wulich 1987)

$$f_{lim\_ave} = \frac{a}{d} = \frac{a}{3.75 \times D \times \frac{T_{int}}{T_0}} \tag{12}$$

Thus,  $f_{lim\_ave}$  depends hyperbolically on the relative exposure time ratio. This is shown in Fig. 2 and again emphasizes the advantage of short integration time. Units of  $f_{lim}$  and  $f_{lim\_ave}$  are reciprocal to the units of  $d$  and  $\bar{d}$ , respectively. Maximum useful and usable spatial frequency content is

$$f_{max} = \frac{a}{d_{min}} \tag{13}$$

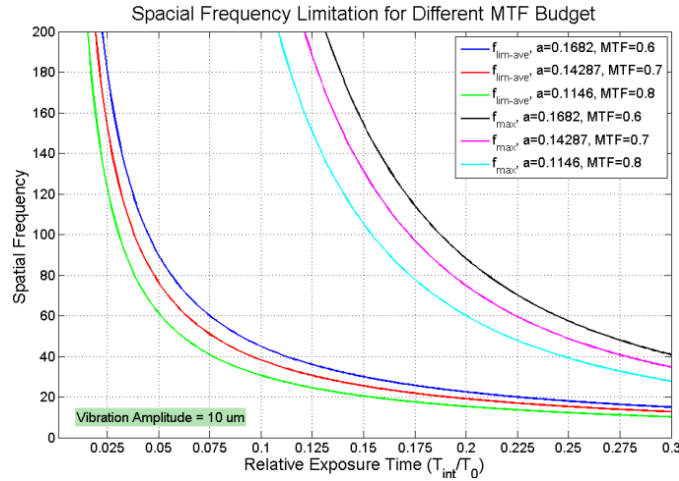


Fig. 3 Spatial frequency limitation for different MTF budget. Vibration amplitude is 10 um

Eq. (13) describes ideal resolution limits for an exposure taking place at an instant of time such that  $d=d_{min}$ . It is clear that both  $d_{min}$  and  $f_{max}$  depend on the relative integration time.

Fig. 2 illustrates that if the vibration MTF budget is set to be 0.8, and if we use table 1 and 2, the maximum resolvable spatial frequency is 40 lp/mm which is equivalent to 12.5 um pixel size. According to these results, if we cannot filter this vibration frequency to reduce its amplitude, the best GSD which this system can achieve is about 77.5 meters! It is about 2 times worse (see Table 2). The area above each line should be considered to avoid by the system designer while below area is the safe place and would not affect the predefined image quality limitation.

If the vibration isolation mechanism do filter 10 um amplitude, then the limitation results for different MTF budget can be find in Fig. 3.

Clearly in this case, if again we use Tables 1 and 2, the limiting resolution would be about 80 lp/mm for 0.8 MTF budget. This spatial frequency equivalent to the Nyquist frequency of a 6.25 um pixel size. Consequently, this system is not vibration limited and therefore its resolution limited by the pixel size.

According to the limitation spatial frequency, the best resolution that can achieve in presence of the vibration, the system limitation, can be calculated. Using these results, designer will avoid over-designing and can select the electro-optical parameters properly.

**Statistical analysis of blur radius and MTF**

Although accelerometers or vibration pickups can be used to determine when vibration acceleration is at an extremum and blur radius thus is minimum, this often may not be feasible in remote sensing satellite imaging because it is the very low vibration frequencies that are of interest. Also, the vibration amplitudes in damped or stabilized systems are of very low amplitude, although not low enough so as not to impair resolution (Hodar 1992).

An additional factor is the weight of the vibration sensors, which can be greater than that of the camera itself. Therefore, the image quality of different shots is not the same. In such situations where the camera system cannot always take its “best shot”, the following analysis may be useful in showing how great the resolution impairment caused by vibrations can be (Rudoler 1991).

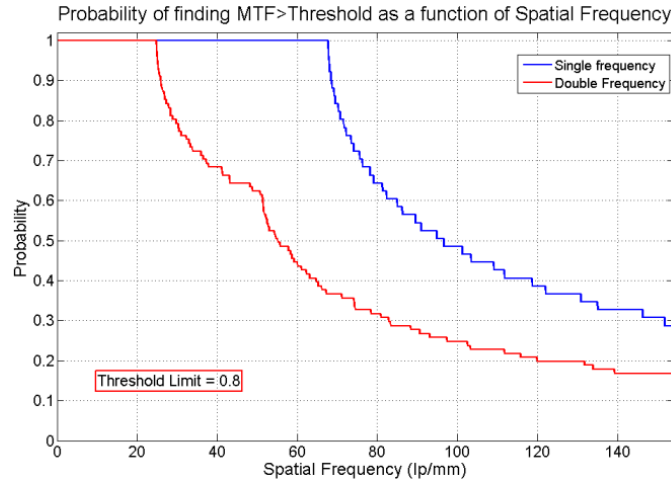


Fig. 4 Probability of finding MTF>0.8 as a function of spatial frequency

Probability of finding  $d^* < d_0^*$  can be written as following,

$$P(d_0^*) = \frac{\text{Number of shots with } d^* < d_0^*}{\text{Total number of shots}} \quad (14)$$

Where,  $d^*$  is relative blur and  $d_0^*$  is the desired maximum relative blur.

Then

$$(1 - Q) = [1 - P(d_0^*)]^N \quad (15)$$

Where, N is the number of shots required to have at least one shot where  $d^* < d_0^*$  and Q is the desired confidence. Finally (Rudoler 1991).

$$N = \frac{\ln(1 - Q)}{\ln(1 - P(d_0^*))} \quad (16)$$

On the other hand, optical engineers using MTF data because it is an easy to use tool and also presents more convenient analysis. Accordingly, we continue this statistical analysis to find the probability of taking an image where its MTF is more than a pre-defined threshold. We also can consider both kind of harmonic vibrations; one and double frequency (Haghshenas 2015).

Fig. 4 illustrates the probability of taking an image where its MTF is bigger than 0.8 versus spatial frequency for the system where specified by Tables 1 and 2. In this case, we have considered a low frequency vibration which amplitude is set to be 10  $\mu\text{m}$ .

#### 4. Look-up figures

It would be very useful for remote sensing payload designers if there was such a “look-up table” to use as a reference to determine the allowable vibrations. Here we have presented some “Look-up Figures” for medium-to-high resolution satellite imagers to be used as a reference for



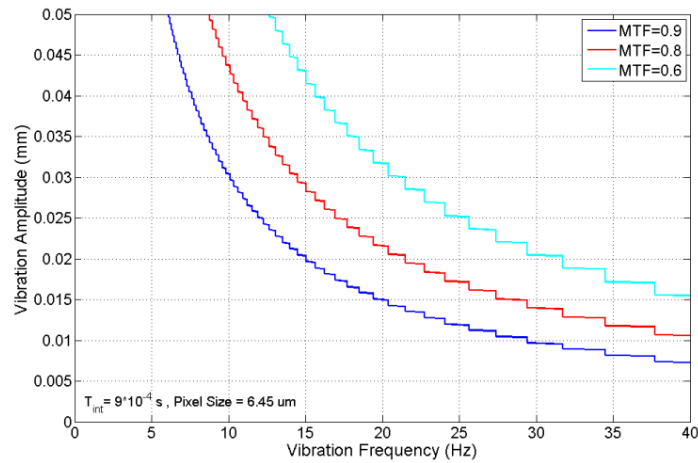


Fig. 5 Lookup graph. Vibration amplitude versus frequency for different MTF limitations. Pixel size and Integration time is supposed to be 6.45  $\mu\text{m}$  and  $9 \times 10^{-4}$  s, respectively

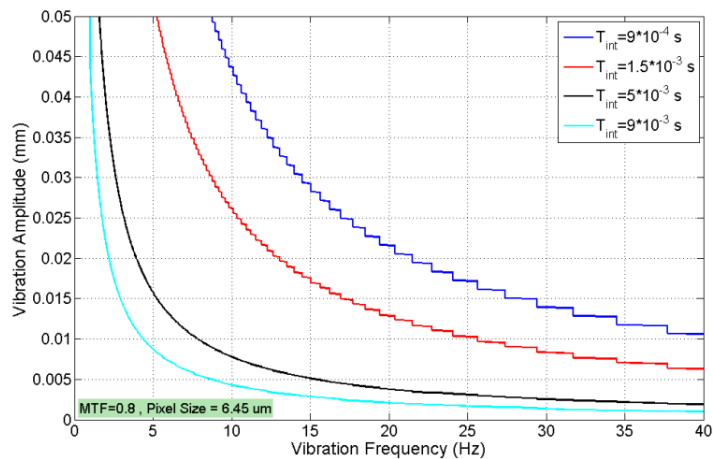


Fig. 6 Lookup graph. Vibration amplitude versus frequency for different integration times. Where  $\text{MTF}_{\text{Vib}}$  budget is 0.8

remote sensing payload designer from vibration degradation point of view.

Fig. 5 represents the look-up figure for a predefined MTF budget. Designer can decide to avoid any different vibration frequency and amplitude where MTF limitation condition is not satisfied. Fig. 6 illustrates vibration effect on the integration time selection. Designer can use this figure to select the proper integration time for his individual payload. Finally Fig. 7 represents how to select the proper pixel size of a typical payload from vibration effects aspect. Proper pixel size selection in turn will redound to proper GSD and F-number selection (Haghshenas 2015).

Again, the area above each line should be considered by designer while below area is the safe place. Therefore if designer has enough data about his systems potential vibration sources, he/she may find if it can disturb the image quality or not.

Fig. 6 illustrates the effects of choosing different integration time for the imaging system of Fig. 5 with a pre-defined MTF value.

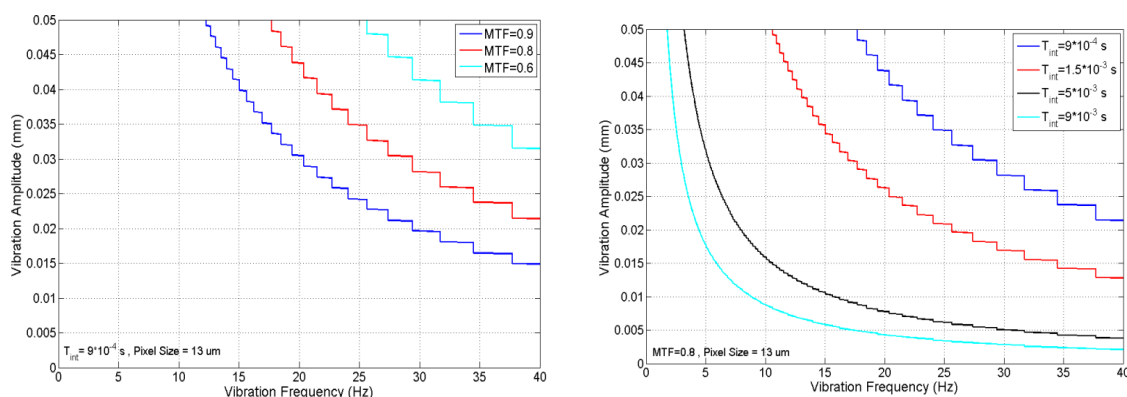


Fig. 7 Vibrational effects on 13 um pixel size

If we choose a 13 um pixel size, then Fig. 7 re-illustrates Figs. 5 and 6 at the new Nyquist frequency.

Clearly as pixel size increase the system is less sensitive to the vibration. So it is more advantageous for high resolution payloads, to utilize larger pixel size along with larger focal length instead to use smaller pixel size and smaller focal length. But, on the other hand, it is not always applicable to use larger focal length because it will increase the mass and dimension budgets of the payload, respectively. So here again it needs to be trade-off between all above mentioned parameters, simultaneously.

## 5. Conclusions

In the system level design of a remote sensing payload, many parameters are correlated together and make a challenging design procedure. In this paper we consider the payload design from vibration limitation point of view. Effects of different kind of vibrations, including linear, high and low frequencies have been considered and analyzed. The majority of this paper deals with the statistical analyzing of low frequency vibration effect on the modulation transfer function of the payload.

We have considered a design case study to show the effects of vibration on the system level design of a remote sensing payload. The results illustrate that maybe the designer have designed a payload with a given GSD resolution but if the vibration effects add to the system, the GSD would be twice or more worst.

The main part of this paper deals with the present of so-called “look-up figures” that guide payload designers to select the main parameters in such a way to avoid the vibration degradation effects.

## References

- Afkhami, H., Mirzaie, M., Haghshenas, J. and Khanmirza, E. (2012), “Calculation of image quality degradation caused by mechanical vibration in satellite push-broom camera”, *Proceedings of the 1st SSD Conference*, Tehran, Iran.

- Ahmad, A. (1999), *Handbook of Optomechanical Engineering*, 1st Edition, CRC Press, U.S.A.
- Bao, G. (2009), "Analysis of vibration on transfer functions of the optical imaging system", *Opt. Prec. Eng.*, **17**(2), 314-320.
- Haghshenas, J. (2014), "Electro-optical design for a space camera based on ModTran data analysis", *SPIE* **9264**, Beijing, China.
- Haghshenas, J. (2015), "Effects of satellite platforms vibration on the image quality of a remote sensing payload: System level design and challenges", *Proc. SPIE*, **9626**, Jena, Germany.
- Haghshenas, J. (2015), "Maximum allowable low-frequency platform vibrations in high resolution satellite missions: Challenges and look-up figures", *Proc. SPIE*, **9626**, Jena, Germany.
- Hodar, O., Dror, I. and Kopeika, N.S. (1994), "Image resolution limits resulting from mechanical vibrations, part IV: Real-time numerical calculation of optical transfer functions and experimental verification", *Opt. Eng.*, **31**(2), 566-578.
- Hodar, O., Fisher, M. and Kopeika, N.S. (1992), "Image resolution limits resulting from mechanical vibrations, part III: Numerical calculation of modulation transfer function", *Opt. Eng.*, **31**(3), 581-589.
- Holst, G.C. (1995), *Electro-Optical Imaging System Performance*, Library of Congress Catalog-in-Publication Data.
- Holst, G.C. (1998), *CCD Arrays, Cameras, and Displays*, 2nd Edition, Winter Park, 292.
- Raiter, S., Stern, A., Hadar, O. and Kopeika, N.S. (2003), "Image restoration from camera vibration and object motion blur in infrared staggered time-delay and integration systems", *Opt. Eng.*, **42**(11), 3253-3264.
- Robbins, H., Novogrozky, Y. and Kaplan, D. (1996), "Image motion restoration from sequence of image", *Opt. Eng.*, **35**(2), 898-904.
- Rudoler, S.O., Hodar, M., Fisher, N.S. and Kopeika, N.S. (1991), "Image resolution limits resulting from mechanical vibrations, part II: Experiment", *Opt. Eng.*, **30**(5), 577-589.
- Trott, T. (1960), "The effects of motion on resolution", *Photogr. Eng.*, **26**, 819-827.
- Wulich, D. and Kopeika, N.S. (1987), "Image resolution limits from mechanical vibration", *Opt. Eng.*, **26**(6), 529-533.
- Xu, P., Hao, Q., Huang, C. and Wang, Y. (2003), "Degradation of modulation transfer function in push-broom camera caused by mechanical vibration", *Opt. Laser Technol.*, **35**, 547-552.
- Xue, B.X. Chen, G. and Ni, G. (2009), "Image quality degradation analysis induced by satellite platform harmonic vibration", *SPIE*, **7513**, Shanghai, China.
- Yang, F.X. Zhang, Y., Huang, W.Hao, B. Guo (2012), "Simulation analysis of space remote sensing image quality degradation induced by satellite platform vibration", *SPIE*, **8557**, Beijing, China.
- Zhong, W.H. Deng, Z.S. and Wu, X. (2009), "Computation model of image motion velocity for space optical remote cameras", *Proceedings of the IEEE International Conference on Mechatronics and Automation*, Changchun, China.