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Analyzing nonlinear vibrations of metal foam nanobeams with symmetric and non-symmetric porosities

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Abstract. This article is concerned with the investigation of geometrically non-linear vibration response of refined thick porous nanobeams. To this end, non-local theory of elasticity has been adopted to provide the nanobeam formulation. Voids or pores can affect the material characteristics of the nanobeam. So, their effects have been considered in this research and also there are various void distributions. The closed form solution of the non-linear problem has been used that is adopted from previous articles. Then, it is focused on the impacts of non-local field, void distribution, void amount and geometrical properties on non-linear vibrational characteristic of a nano-size beam.

Keywords: non-linear vibration; refined beam theory; metal nanobeam; nonlocal elasticity

1. Introduction

There is one type of metal material known as metal foam with low weight due to possessing different variations of porosities in it. The variation of porosities in this material causes a significant difference between metal foams and other perfect metals. In a non-perfect metal, the material characteristics are notably influenced by pore variations. Also, this variation in pores can affect the vibration frequencies of engineering structures made of metal foams. This issue can be understood from the works done by Chen *et al.* 2015 and 2016, Rezaei and Saidi (2016). Different from metal foams, there are also functionally graded (FG) or ceramic-metal materials in which pore variation effect is very important (Mechab *et al.* 2016, Mirjavadi *et al.* 2018, 2019a). In this material, pores may be produced in a phase between ceramic and material. Engineering structures made of this materials are studied to understand their vibration behaviors as reported in the works of Wattanasakulpong *et al.* (2014), Yahia *et al.* (2015), Atmane *et al.* (2015a,b).

Recent studies focus on engineering structures at nano-scales due to their involvement in nano-mechanical systems or devices. However, the main issue in these studies is to select an appropriate elasticity theory accounting for small scale impacts. The impact of size-dependency might be considered with the help of a scale parameter involved in non-local theory of elasticity Eringen (1983). The word "non-local" means that the stresses are not local anymore. This is because we are talking about a stress field of nano-scale structure. Many authors are aware of these facts and they are using this theory to analysis mechanical characteristics of small size engineering

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structures (Natarajan *et al.* 2012, Belkorissat *et al.* 2015, Bounouara *et al.* 2016, Barati *et al.* 2016, Zenkour 2016, Barati 2017a, b, Ebrahimi and Daman 2016, Ebrahimi and Haghi 2018, Ebrahimi and Heidari 2018, Ebrahimi *et al.* 2018, Li *et al.* 2015).

The present research is concerned with the investigation of geometrically non-linear vibrational response of refined thick porous nanobeams. To this end, non-local theory of elasticity has been adopted to provide the nanobeam formulation. Voids or pores can affect the material characteristics of the nanobeam. Therefore, their effects have been considered in this research and also there are various void distributions. The closed form solution of the non-linear problem has been used that is adopted from previous articles. Finally, we try to show the influences of non-local scale, void distribution, void amount and geometrical properties on non-linear vibrational characteristic of a nanobeam made of metal foams.

2. Modeling a porous metal nanobeam

The material characteristics of the metal relies on the type of void/pore distributions. The voids can distribute with uniform or non-uniform schemas. The case of non-uniform template can be divided into symmetric (non-uniform 1) or asymmetric (non-uniform 2). In the following, the expressions for material properties (elastic modulus E and mass density) of a metal foam will be presented (Mirjavadi *et al.* 2019b)

$$\begin{gathered} E = E_2(1-e_0\mathrm{X}), \rho = \rho_2\sqrt{(1-e_0\mathrm{X})} \\ \mathrm{X} = \frac{1}{e_0} - \frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1-e_0} - \frac{2}{\pi} + 1\right)^2 \qquad \text{Uniform} \end{gathered} \tag{1}$$

$$E(z) = E_2(1-e_0\cos\left(\frac{\pi z}{h}\right)), \rho(z) = \rho_2(1-e_m\cos\left(\frac{\pi z}{h}\right)) \quad \text{Non-uniform 1} \tag{2}$$

$$E(z) = E_2(1-e_0\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)), \rho(z) = \rho_2(1-e_m\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)) \quad \text{Non-uniform 2} \tag{3}$$

In above definitions, the index 2 refers to a material property at its highest value. Also, there are two coefficients e_0 and e_m elated to pore amount and mass distribution as

$$e_0 = 1 - \frac{E_2}{E_1} = 1 - \frac{G_2}{G_1}, e_m = 1 - \sqrt{1-e_0} \tag{4}$$

The nanobeam in this study is considered to be thick. So, it is crucial to use a higher order thick beam model. In order to do this, we used a refined one based on following axial and transverse displacements (u_1 and u_3) as

$$u_1(x,z,t) = u(x,t) - (z-z^*)\frac{\partial w_b}{\partial x} - [f(z) - z^{**}]\frac{\partial w_s}{\partial x} \tag{5}$$

$$u_3(x,z,t) = w(x,t) = w_b(x,t) + w_s(x,t) \tag{6}$$

in which w is total deflection and

$$f(z) = -\frac{z}{4} + \frac{5z^3}{3h^2} \tag{7}$$

and

$$z^* = \frac{\int_{-h/2}^{h/2} E(z)\, z dz}{\int_{-h/2}^{h/2} E(z)\, dz}, \quad z^{**} = \frac{\int_{-h/2}^{h/2} E(z)\, f(z) dz}{\int_{-h/2}^{h/2} E(z)\, dz} \tag{8}$$

Based on presented beam theory, many authors have derived its governing equations in the form presented below

$$\frac{\partial N_x}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} - I_3 \frac{\partial^3 w_s}{\partial x \partial t^2} \tag{9}$$

$$\begin{aligned} \frac{\partial^2 M_x^b}{\partial x^2} &= -\frac{\partial}{\partial x}(N_x \frac{\partial (w_b + w_s)}{\partial x}) + k_L (w_b + w_s) - k_P \frac{\partial^2 (w_b + w_s)}{\partial x^2} + k_{NL} (w_b + w_s)^3 \\ &+ I_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} + I_1 (\frac{\partial^3 u}{\partial x \partial t^2}) - I_2 (\frac{\partial^4 w_b}{\partial x^2 \partial t^2}) - I_4 (\frac{\partial^4 w_s}{\partial x^2 \partial t^2}) \end{aligned} \tag{10}$$

$$\begin{aligned} &\frac{\partial^2 M_x^s}{\partial x^2} + \frac{\partial Q_{xz}}{\partial x} + \frac{\partial}{\partial x}(N_x \frac{\partial (w_b + w_s)}{\partial x}) - k_L (w_b + w_s) + k_P \frac{\partial^2 (w_b + w_s)}{\partial x^2} \\ &- k_{NL} (w_b + w_s)^3 = I_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} + I_3 (\frac{\partial^3 u}{\partial x \partial t^2}) - I_4 (\frac{\partial^4 w_b}{\partial x^2 \partial t^2}) - I_5 (\frac{\partial^4 w_s}{\partial x^2 \partial t^2}) \end{aligned} \tag{11}$$

In above equations I_i are mass inertias; k_i (i=L, P, NL) are foundation parameters; N_x is membrane force; M^b and M^s defines the membrane moments which are obtained based on non-local theory as

$$N_x = A[\frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2] \tag{12}$$

$$\begin{aligned} M_x^b &= -D \frac{\partial^2 w_b}{\partial x^2} - E \frac{\partial^2 w_s}{\partial x^2} + (ea)^2 (-N_x \frac{\partial^2 (w_b + w_s)}{\partial x^2} + k_L (w_b + w_s) - k_P \frac{\partial^2 (w_b + w_s)}{\partial x^2} \\ &+ k_{NL} (w_b + w_s)^3 + I_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} - I_2 (\frac{\partial^4 w_b}{\partial x^2 \partial t^2}) - I_4 (\frac{\partial^4 w_s}{\partial x^2 \partial t^2})) \end{aligned} \tag{13}$$

$$\begin{aligned} M_x^s &= -E \frac{\partial^2 w_b}{\partial x^2} - F \frac{\partial^2 w_s}{\partial x^2} + (ea)^2 (-N_x \frac{\partial^2 (w_b + w_s)}{\partial x^2} - \frac{\partial Q_{xz}}{\partial x} + k_L (w_b + w_s) - k_P \frac{\partial^2 (w_b + w_s)}{\partial x^2} \\ &+ k_{NL} (w_b + w_s)^3 + I_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} - I_4 (\frac{\partial^4 w_b}{\partial x^2 \partial t^2}) - I_5 (\frac{\partial^4 w_s}{\partial x^2 \partial t^2})) \end{aligned} \tag{14}$$

$$(1 - (ea)^2 \nabla^2) Q_{xz} = A_s \frac{\partial w_s}{\partial x} \tag{15}$$

in which ea is called non-local coefficient and

$$A=\int_{-h/2}^{h/2}E(z)\,dz,\;\; D=\int_{-h/2}^{h/2}E(z)(z-z^*)^2\,dz,\;\; E=\int_{-h/2}^{h/2}E(z)(z-z^*)(f-z^{**})\,dz$$
$$F=\int_{-h/2}^{h/2}E(z)(f-z^{**})^2\,dz,\quad A_s=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+v)}g^2dz \tag{16}$$

By placing above equations into Eqs.(9-11), one can obtain the governing equation of the nanobeam after doing some mathematical manipulation as can be seen in previous researches

$$\begin{aligned}&-D(\frac{\partial^4 w_b}{\partial x^4})-E(\frac{\partial^4 w_s}{\partial x^4})+A(+\frac{1}{2L}\int_0^L(\frac{\partial w}{\partial x})^2dx)\frac{\partial^2 w}{\partial x^2}-\mu A(+\frac{1}{2L}\int_0^L(\frac{\partial w}{\partial x})^2dx)\frac{\partial^4 w}{\partial x^4}\\&+(1-\mu\frac{\partial^2}{\partial x^2})(-I_0\frac{\partial^2(w_b+w_s)}{\partial t^2}+I_2(\frac{\partial^4 w_b}{\partial x^2\partial t^2})+I_4(\frac{\partial^4 w_s}{\partial x^2\partial t^2})-k_L(w_b+w_s)\\&+k_P\frac{\partial^2(w_b+w_s)}{\partial x^2}-k_{NL}(w_b+w_s)^3)=0\end{aligned} \tag{17}$$

$$\begin{aligned}&-E(\frac{\partial^4 w_b}{\partial x^4})-F(\frac{\partial^4 w_s}{\partial x^4})+A(+\frac{1}{2L}\int_0^L(\frac{\partial w}{\partial x})^2dx)\frac{\partial^2 w}{\partial x^2}-\mu A(+\frac{1}{2L}\int_0^L(\frac{\partial w}{\partial x})^2dx)\frac{\partial^4 w}{\partial x^4}\\&+(1-\mu\frac{\partial^2}{\partial x^2})(-I_0\frac{\partial^2(w_b+w_s)}{\partial t^2}+I_4(\frac{\partial^4 w_b}{\partial x^2\partial t^2})+I_5(\frac{\partial^4 w_s}{\partial x^2\partial t^2})+A_s(\frac{\partial^2 w_s}{\partial x^2})-k_L(w_b+w_s)\\&+k_P\frac{\partial^2(w_b+w_s)}{\partial x^2}-k_{NL}(w_b+w_s)^3)=0\end{aligned} \tag{18}$$

These are two non-linear equations for a non-local non-linear refined beam.

3. Method of solution

Here, we adopted the closed-form solution which was obtained by Barati and Shahverdi (2018). Before that, it is required to simply define the displacements as (Mirjavadi *et al.* 2018)

$$w_b=\sum_{m=1}^{\infty}W_{bm}(t)X_m(x) \tag{19}$$

$$w_s=\sum_{m=1}^{\infty}W_{sm}(t)X_m(x) \tag{20}$$

where W_{bmn} and W_{bmn} are maximum amplitudes and the functions X_m might be defined as

S-S edges
$$\mathrm{X}_m(x)=\sin(\lambda_m x)\qquad \lambda_m=\frac{m\pi}{a} \tag{21}$$

C-C edges
$$\mathrm{X}_m(x)=\sin(\lambda_m x)-\sinh(\lambda_m x)-\xi_m(\cos(\lambda_m x)-\cosh(\lambda_m x))\qquad \xi_m=\frac{\sin(\lambda_m x)-\sinh(\lambda_m x)}{\cos(\lambda_m x)-\cosh(\lambda_m x)} \tag{22}$$

The closed-form expression of non-linear vibration frequency might be presented as

$$\omega_{NL} = \sqrt{\frac{1}{2S_1}\left\{(S_2+S_3)-[(S_2+S_3)^2-4S_1(S_4+S_5)]^{\frac{1}{2}}\right\}} \tag{23}$$

In such a way that

$$\begin{aligned} &S_1 = m_{1,1}m_{2,2}-m_{1,2}^2, \Lambda_2 = k_{1,1}m_{2,2}+k_{2,2}m_{1,1}-2k_{1,2}m_{1,2},\\ &S_3 = \frac{3}{4}W^{*2}k_{1,3}(m_{1,1}+m_{2,2}-2m_{1,2}),\\ &S_4 = k_{1,1}k_{2,2}-k_{1,2}^2,\\ &S_5 = \frac{3}{4}W^{*2}k_{1,3}(k_{1,1}+k_{2,2}-2k_{1,2}), \end{aligned} \tag{24}$$

And the maximum deflections replaced by W* and

$$k_{1,1} = -D\Upsilon_{40} - K_L(\Upsilon_{00}-\mu_0\Upsilon_{20}) + K_P(\Upsilon_{20}-\mu_0\Upsilon_{40}) \tag{25a}$$

$$k_{1,2} = k_{2,1} = -E\Upsilon_{40} - K_L(\Upsilon_{00}-\mu_0\Upsilon_{20}) + K_P(\Upsilon_{20}-\mu_0\Upsilon_{40}) \tag{25b}$$

$$k_{2,2} = -F\Upsilon_{40} - K_L(\Upsilon_{00}-\mu_0\Upsilon_{20}) + K_P(\Upsilon_{20}-\mu_0\Upsilon_{40}) + A_s\Upsilon_{20} \tag{25c}$$

$$G^* = A(\frac{1}{2L}\Upsilon_{11}\Upsilon_{20}) - \mu A(\frac{1}{2L}\Upsilon_{11}\Upsilon_{40}) - K_{NL}(\Upsilon_{0000}-\mu_0(6\Upsilon_{1100}+3\Upsilon_{2000})) \tag{25d}$$

$$m_{1,1} = +I_0\Upsilon_{00} - \mu I_0\Upsilon_{20} - I_2\Upsilon_{20} + \mu I_2\Upsilon_{40} \tag{25e}$$

$$m_{1,2} = m_{2,1} = +I_0\Upsilon_{00} - \mu I_0\Upsilon_{20} - I_4\Upsilon_{20} + \mu I_4\Upsilon_{40} \tag{25f}$$

$$m_{2,2} = +I_0\Upsilon_{00} - \mu I_0\Upsilon_{20} - I_5\Upsilon_{20} + \mu I_5\Upsilon_{40} \tag{25g}$$

in which

$$\{\Upsilon_{00},\Upsilon_{20},\Upsilon_{40},\Upsilon_{11}\} = \int_0^L \{X_mX_m, X_m^{''}X_m, X_m^{''''}X_m, X_m^{'}X_m^{'}\}dx$$

$$\{\Upsilon_{0000},\Upsilon_{1100},\Upsilon_{2000}\} = \int_0^L \{X_mX_mX_mX_m, X_m^{'}X_m^{'}X_mX_m, X_m^{''}X_mX_mX_m\}dx$$

Above calculations might be based on the following normalized quantities

$$\hat{\omega} = \omega L^2\sqrt{\frac{\rho_2 A}{E_2 I}},\ K_L = k_L\frac{L^4}{D},\ K_p = k_p\frac{L^2}{D},\ K_{NL} = k_{NL}\frac{L^4}{A},\ \mu = \frac{e_0 a}{L} \tag{26}$$

Table 1 Verification of normalized frequency based on various non-local parameter (L/h=20)

μ (nm)	Ebrahimi and Salari (2016)	This article
0	9.8594	9.8567
1	9.4062	9.4036
2	9.0102	9.0077
3	8.6603	8.6579

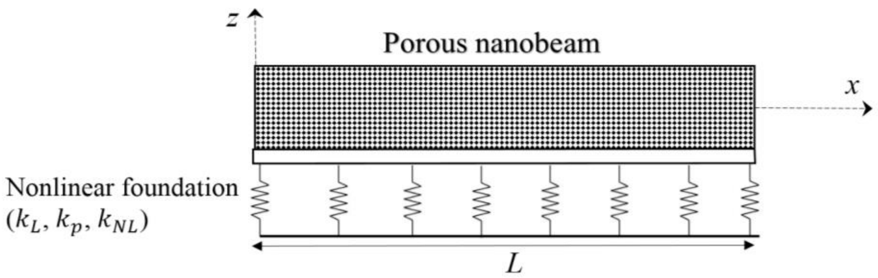


Fig. 1 A metal nanobeam based on elastic foundation

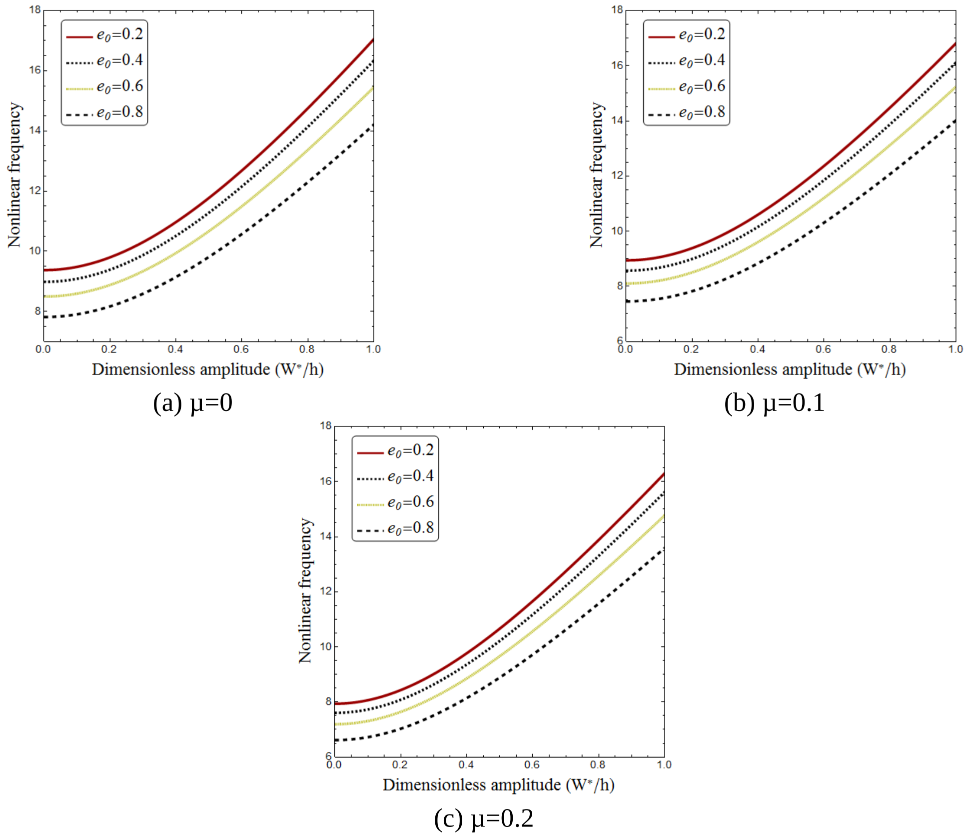
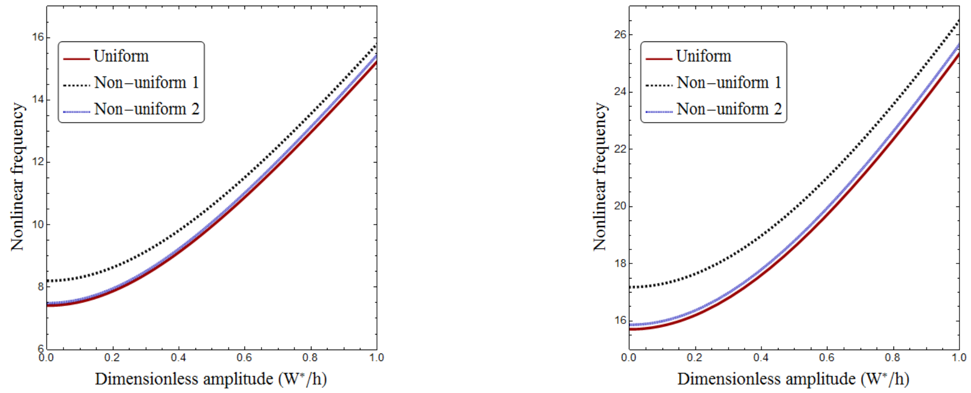
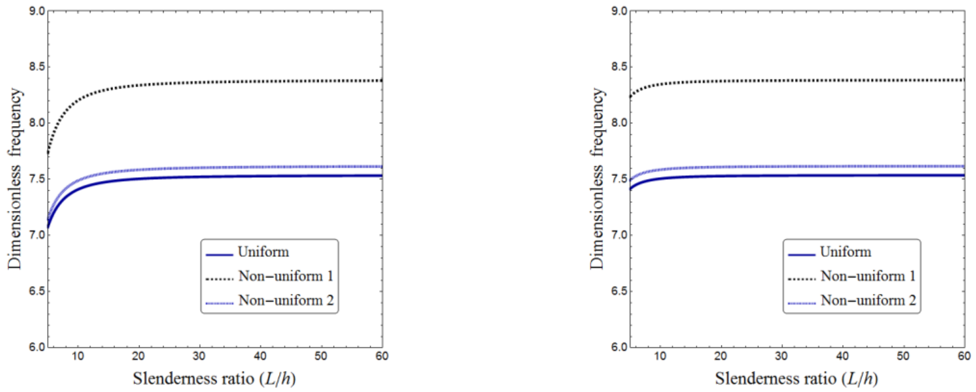


Fig. 2 Normalized non-linear frequency against maximum deflection for various void coefficients and non-local parameters (L/h=10)



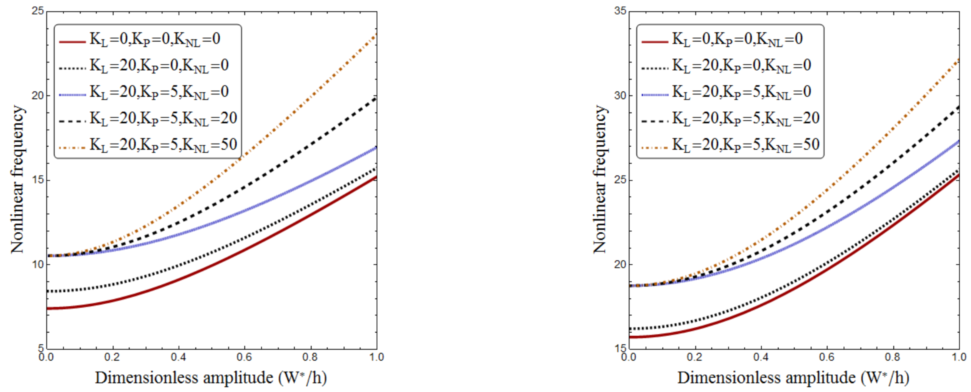
(a) S-S (b) C-C

Fig. 3 Normalized non-linear frequency against maximum deflection for different void distributions (L/h=10, K_L=0, K_p=0, μ=*0.2*)



(a) Higher order refined (b) CBT

Fig. 4 Normalized non-linear frequency against slenderness ratio for different void distributions (K_L=0, K_p=0, μ=0.2, e_0=0.5)



(a) S-S (b) CBT

Fig. 5 Normalized non-linear frequency against maximum deflection for different foundation parameters (L/h=10, μ=0.2, e_0=0.5)

4. Results and discussions

After the derivation of closed-form non-linear vibration frequency of metal foam nanobeams shown in Fig. 1, it is possible to find its dependency on various factors including pore amount/dispersion, elastic foundation, geometrical properties and non-local effects. To do this, set the material properties to $E_2 = 200\ GPa$, $\rho_2 = 7850\ kg/m^3$, $v = 0.33$. Before that, one can see the frequency validation in Table 1. This table shows the accuracy of adopted methodology.

One can see from Fig.2 the variation of non-linear frequency of the nano-size beam against non-local and void coefficients when L=10h. Void or pore dispersion is set as uniform with different values for its coefficient. The vibration frequency of a large-size beam might be achieved by selecting a zero non-local parameter. From the figure, it might be seen that non-local coefficient assigns a stiffness devaluation characteristic together with a smaller vibration frequency. Besides, growth of void coefficient yields a smaller frequency regardless of non-local parameter magnitude.

One can see from Fig.3 the variation of non-linear frequency of the S-S and C-C nano-size beams against void dispersion type at a selected value of non-local coefficient μ=0.2. Gained observations notes that a nano-size beam having void type 1 results in greatest vibration frequency whereas the curves for uniform void type and also void type 2 are near to each other. Such behaviors notes that a nano-size beam with symmetrical void type might achieve the greatest beam stiffness as well as the excellent mechanical properties.

Based on different void types with e_0=0.5, Fig.4 gives the variation of frequency curves according to slenderness ratio (L/h). It is notable that a higher value for slenderness ratio means that the nano-size beam is less rigid. Then, one can conclude that the normalized non-linear frequency will increase with changing of L/h value. However, normalized vibration frequency is more affected by the lower values of slenderness ratio. It might be seen from the figure that shear deformation effect is important at smaller slenderness ratios (larger thickness).

According to different magnitudes for foundation parameters, Fig.5 contains the variation of non-linear frequency with maximum deflection at L=10h. The main conclusion from the figure is the dependency of non-linear foundation coefficient (K_{NL}) to maximum deflection of the nano-size beam. Besides, all foundation parameters might increase the value of non-linear frequency. But, one might see the in-dependency of linear or shear foundations to maximum deflection.

5. Conclusions

Based on the closed form of nonlinear frequency, this paper was devoted to analyze non-linear vibration behavior of a steel nanobeam accounting for foam properties in the presence of porosities. It was seen that non-local coefficient assigned a stiffness devaluation characteristic together with a smaller vibration frequency. Also, a nano-size beam with symmetrical void type might achieve the greatest beam stiffness as well as the excellent mechanical properties. Moreover, normalized vibration frequency became more affected by the lower values of slenderness ratio. Another important conclusion was the dependency of non-linear foundation coefficient to maximum deflection of the nano-size beam.

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