Advances in Aircraft and Spacecraft Science, *Vol. 6, No. 3 (2019) 207-223*
DOI: https://doi.org/10.12989/aas.2019.6.3.207

Improvement of thermal buckling response of FG-CNT reinforced composite beams with temperature-dependent material properties resting on elastic foundations

Ismail Bensaid*[1] and Bachir Kerboua[2a]

[1]IS2M Laboratory, Faculty of Technology, Department of Mechanical Engineering, University Abou Beckr Belkaid (UABT), Tlemcen, Algeria
[2]Mechanical engineering Department, University Abou Beckr Belkaid (UABT), Tlemcen, Algeria

(Received June 19, 2018, Revised August 28, 2018, Accepted December 6, 2018)

Abstract. Current investigation deals with the thermal stability characteristics of carbon nanotube reinforced composite beams (CNTRC) on elastic foundation and subjected to external uniform temperature rise loading. The single-walled carbon nanotubes (SWCNTs) are supposed to have a distribution as being uniform or functionally graded form. The material properties of the matrix as well as reinforcements are presumed to be temperature dependent and evaluated through the extended rule of mixture which incorporates efficiency parameters to capture the size dependency of the nanocomposite properties. The governing differential equations are achieved based on the minimum total potential energy principle and Euler-Bernoulli beam model. The obtained results are checked with the available data in the literature. Numerical results are supplied to examine the effects of numerous parameters including length to thickness ratio, elastic foundations, temperature change, and nanotube volume fraction on the thermal stability behaviors of FG-CNT beams.

Keywords: FG-CNT beams; thermal buckling; elastic foundation; thermal effects

1. Introduction

Recent development in the science of carbon fiber leads to invention of a new class of these last namely carbon nanotubes (CNTs) which have extraordinary thermal, mechanical, and electrical properties (Thostenson *et al*. 2001, Esawi and Farag 2007, Besseghier *et al*. 2015, Liew *et al*. 2015,Adda Bedia *et al*. 2015). The carbon nanotube-reinforced composites (CNTRCs) have a colossal potential of improving the strength and stiffness compared to the classical carbon fiber-reinforced polymer composites. Some contributions have been provided to investigate the benefits of using aligned carbon nanotube CNT reinforced polymer composites (Ajayan *et al*. 1994, Odegard *et al*. 2003, Thostenson and Chou 2003, Griebel and Hamaekers 2004, Fidelus *et al*. 2005, Hu *et al*. 2005, Zhu *et al*. 2007, Bakhti *et al.* 2013, Barzoki *et al.* 2015, Bidgoli *et al.* 2015). (Ashrafi and Hubert 2006) determined the elastic properties of CNTRCs using a finite element investigation. Xu et *al.* (2006) researched the thermal characteristic of polymer-matrix composites

*Corresponding author, Ph.D., E-mail: bensaidismail@yahoo.fr
[a]Professor, E-mail: kerbouac@yahoo.fr

http://www.techno-press.org/?journal=aas&subpage=7 ISSN: 2287-528X (Print), 2287-5271 (Online)

reinforced by SWCNT. The elastic properties of CNTRCs were estimated by Han and Elliott 2007, by using the molecular dynamics simulation (MDS). It has been demonstrated in the previous studies that studies proved that adding a small amount of carbon nanotube can significantly improve the mechanical, electrical, and thermal properties of polymeric composites. The idea of functionally graded carbon nanotube-reinforced composites (FG-CNTRC) was first pioneered by Shen 2009. Ke *et al.* (Ke *et al.* 2010, Ke *et al.* 2013) explored the impact of FGCNT volume fraction on dynamic stability the nonlinear vibration and of composite beams. Wang and Shen (Wang and Shen 2011) studied the free vibration of CNTRC plates in thermal environments. They showed that the CNTRC plates with symmetrical pattern of CNTs leads to lower natural frequencies compared to unsymmetrical or uniform distribution of CNTs. Zhu *et al*. (2012) developed a model based on the finite element method to study the free vibration of FG-CNT reinforced composite plates. The bending, buckling and vibration behaviors of carbon nanotube-reinforced composite (CNTRC) beams were studied and discussed in details by Wattanasakulpong and Ungbhakorn (2013) based on several higher-order shear deformation theories. Rafiee et *al*. (2013) developed a closed-form solution to investigate the thermal postbuckling of temperature dependent beams via the classical Euler–Bernoulli beam model. Shen and Xiang (2013) researched the thermal postbuckling behavior of FG-CNTRC beams considering both edges simply supported. Lei *et al*. (2013) investigated the free vibration of FG-CNT reinforced composite rectangular plates in a thermal environment by employing the element-free kp-Ritz method. Alibeigloo (2014) contributed to the deflection behavior of a CNT reinforced composite rectangular host plate attached to thin piezoelectric layers exposed to thermal load and or electric range. The vibration response of aligned carbon nanotube reinforced composite beams were studied by Aydogdu (2014) using the Ritz method. Arefi (2015) provided an analytical solution to study static bending of a curved beam made of functionally graded materials and taking into account different cross sections. Yang *et al.* 2015 explored the dynamic buckling responses of FG nanocomposite beams reinforced by CNT as a core and incorporated with two surface bonded piezoelectric layers. Mirzaei and Kiani (2016) studied the thermal buckling characteristic of temperature dependent FG-CNT-reinforced composite plates. Wu *et al.* 2015 used the Timoshenko beam model to investigate the free vibration and buckling performance of sandwich beams reinforced with FG-CNTRCs face sheets. Tagrara et *al*. 2015 contributed to the bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams based on a novel refined trigonometric beam theory. Phung-Van et *al*. 2015 developed an isogeometric model and higher-order shear deformation theory (HSDT) to investigate the static and dynamic behavior of functionally graded carbon nano-reinforced composite plates. Kiani Y (2016) determined the shear buckling behaviour of FG-CNTRC composite plates with various types of boundary conditions via the Chebychev–Ritz technique. In another work Kiani 2016 analyzed thermal postbuckling response of a sandwich beam made of a stiff host core and carbon nanotube (CNT)-reinforced face sheets by using the first-order shear deformation theory and von Kármán type of geometrical nonlinearity. Recently, Hadji et *al*. (2017) extended the work of Tagrara et *al*. (2017) by developing a simple quasi-3D sinusoidal shear deformation theory by means of stretching effect for carbon nanotube-reinforced composite beams resting on elastic foundation. Arefi and Zenkour (2017i) analyzed the bending response of a three-layer sandwich nanoplate integrated with two piezoelectric face-sheets subjected to thermo-electro-mechanical loadings. The trigonometric shear and normal deformation theory is used in term of kinematic equation. More recently, Vo-Duy et *al*. (2019) studied for the first time the free vibration response of laminated FG-CNT reinforced composite beams using finite element method, but they neglected neither the temperature dependency of the material properties nor the thermal environment effects on the beam.

In addition, considerable progression in the utilization of structural elements such as beams and plates with micro and nano scales after the invention of carbon nanotubes (CNTs) by Iijima (1991), due to providing outstanding mechanical, chemical, and electronic characteristics compared to the conventional structural materials. Lots of studies have been also performed to investigate mechanical responses of the nano/micro structures subjected to multi field load; one can cite the works of Arefi and his co-workers (Arefi and Zenkour 2017a, b, c, d, e, f, g, h, Arefi and Soltan Arani 2018) they used an improved analytical models in their investigations. They also discussed the effects of various physical fields and small scale parameter on the bending, wave propagation and dynamic behavior of single and multilayered micro/nano beams.

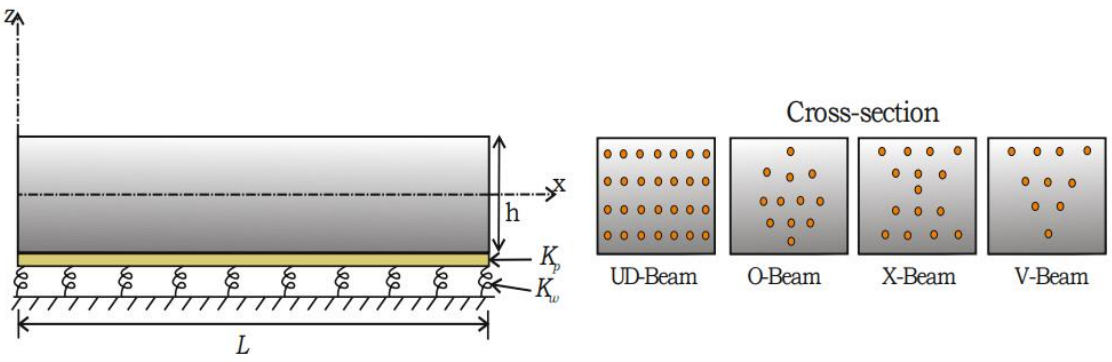
One can see from above discussions that most of the previous studies on the bending, buckling and vibration analysis of FG-CNT reinforced beams and plates have been conducted based on the ignorance of the effects of dependency of the thermal material properties of CNTRC and surrounding elastic foundation. It is noted that in some cases the use of FGCNT structures in high temperature environment leads to considerable changes in material properties. For example, Young's modulus usually decreases when temperature increases in FG-CNT. To predict the behavior of FGMs under high temperature more correctly, it is necessary to consider the temperature dependency on material properties (Arefi et *al*. 2015, Arefi and Nahas 2014, Arefi and Rahimi 2010, Arefi and Zenkour 2017) for more reliable design while investigating the thermal buckling response of FG-CNT reinforced beams.

In this research paper, thermal stability response of FG-CNTC beams subjected to uniform temperature rise loading is explored within the framework of Euler-Bernoulli beam theory. The material properties of carbon nanotubes are supposed to vary in the thickness direction in a functionally graded pattern and are temperature dependent. The Governing equations of motions and related boundary conditions have been formulated using the minimum total potential energy principle, and solved analytically for simply supported supports. After carrying out some comparison studies to check the efficiency and correctness of the present model, parametric studies are provided to investigate the effects of various parameters such as volume fraction and distribution contour of CNTs, elastic foundations, and geometrical parameters of the beam on the thermal buckling characteristics of FG-CNTRC beams.

2. FG-CNTRCs beams on elastic foundations

Let consider a FG-CNTRC beam surrounded on elastic foundations. The length, width, and total thickness of the beam are L, b, and h, respectively Fig. 1(a). The beam is made by a polymeric matrix reinforced with single walled carbon nanotube (SWCNT). The SWCNT are dispersed through the beam thickness, and may be uniform (referred to as UD) or functionally graded (referred to as FG) (Wattanasakulpong and Ungbhakorn 2013, Tagrara et *al*. 2015). In this study, three types of FG distribution of CNTs and the UD case are considered. FG-V, FG-O and FG-X CNTRC are the functionally graded distribution of carbon nanotubes through the thickness direction of the composite beam Fig. 1(b).

The thermo-mechanical properties of CNTRC beams can be computed employing the extended rule of mixture. However to account for the scale dependent properties of nanocomposite model, efficiency parameters are incorporated. The effective Young's modulus and shear modulus of CNTRC beams may be written as (Kaci *et al*. 2012, Bakhti *et al*. 2013, Wattanasakulpong and



(a) (b)

Fig. 1 Geometry of a CNTRC beam on elastic foundation, (a) and cross sections of different patterns of reinforcement, (b)

Ungbhakorn 2013, Tagrara, Tounsi et *al*. 2015, Mirzaei and Kiani 2016).

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \tag{1a}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \tag{1b}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G_p} \tag{1c}$$

in which, η_1, η_2 and η_3 are the so called efficiency parameters and as mentioned previously are incorporated to take into consideration the size dependent material properties of the beam. These constants are determined by matching the elastic moduli of CNTRCs estimated by the MD simulation with the numerical results determined by the rule of mixture (Han and Elliott 2007). Moreover, $E^{cn}{}_{11}$, $E^{cn}{}_{22}$ and $G^{cn}{}_{22}$ are the Young's modulus and shear modulus of SWCNTs, respectively. In addition, E_m and G_m specify the corresponding properties of the isotropic matrix.

The volume fraction of CNTs and matrix, which are denoted by V_{cn} and V_m, respectively in Eq. (1) are determined from the following relation

$$V_{cn} + V_m = 1 \tag{2}$$

As discussed previously, three types of functionally graded CNTRC beams have been selected. These types along with the UD type are the considered patterns of CNT dispersion through the thickness of the beam. In Table 1 distribution function of CNTs across the beam thickness is given, the same value of volume fraction for all.

It is noted that, in FG-X type distribution of CNT is highest near the top and bottom surfaces whereas the mid-plane is free of CNT. For FG-O, however, bottom and top surfaces are free of CNTs and the mid-surface of the plate is enriched with CNTs. In FG-V, the top surface is enriched with CNT and the bottom one is free of CNT, and in this case no thermal bifurcation type buckling

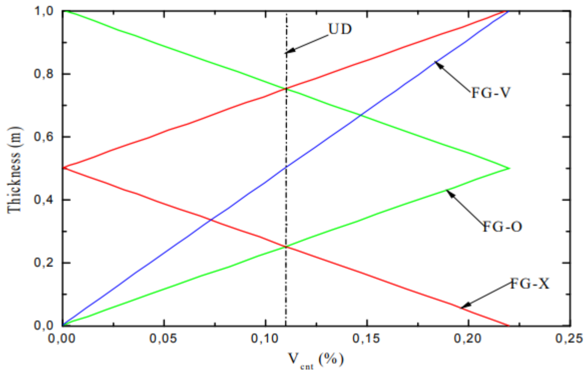


Fig. 2 Variation of volume fraction with respect to the thickness for V^*_{cnt}=0.11 and h = 0.002 m

appeared (Shen 2009, Kiani 2016). In UD type, each surface of the beam across the thickness has the same volume fraction of CNTs as is presented in Fig. 2. By employing the same rule, Poisson's ratio (ν) of the CNTRC beams is expressed as

$$\nu = V^*_{cn}\nu^{cnt} + V_m\nu^m \;; \tag{3}$$

where ν_{cnt} and ν_m are the Poisson's ratios of the CNT and polymer matrix respectively. Explicit mathematical term of CNTs volume fraction in every case of distribution is given in Table 1.

Following the Shapery model, longitudinal and transverse thermal expansion coefficients are expressed as (Shen 2011)

$$\alpha_{11} = \frac{V_{cn}E^{cn}_{11}\alpha^{cn}_{11} + V_m E^m \alpha^m}{V_{cn}E^{cn}_{11} + V_m E^m} \;; \tag{4}$$

$$\alpha_{22} = \left(1+\nu^{cn}_{12}\right)V_{cn}\alpha^{cn}_{22} + \left(1+\nu^m\right)V_m\alpha^m - \nu_{12}\alpha_{11} \tag{5}$$

in which, $\alpha^{cn}{}_{11}$; $\alpha^{cn}{}_{22}$ and α^m in the above equation are the thermal expansion coefficients of the constituents.

3. Basic formulation

3.1 Kinematics and constitutive equations

The equations of motion are formulated based on the classical beam model Euler-Bernoulli beam theory in which the displacement field at every point of the beam can be expressed as (Barati and Zenkour 2018, Arefi and Zenkour 2017e)

$$u\left(x,z,t\right) = u_0(x,t) - z\frac{\partial w_0}{\partial x} \tag{6a}$$

$$w\left(x,z,t\right) = w_0\left(x,t\right) \tag{6b}$$

Table 1 Volume fraction of CNTs as a function of thickness coordinate for various cases of CNTs distribution (Shen 2009, Mirzaei and Kiani 2016)

CNTs distribution	V_{cn}
UD CNTRC	V_{cn}^{*}
FG-V CNTRC	$V_{cn}^{*}\left(1+2\frac{z}{h}\right)$
FG-O CNTRC	$2V_{cn}^{*}\left(1-2\frac{\lvert z\rvert}{h}\right)$
FG-X CNTRC	$4V_{cn}^{*}\frac{\lvert z\rvert}{h}$

where t is time, u_0 and w_0 are displacement components of the mid-plane along x and z directions, respectively. So, according to the Euler-Bernoulli beam model, the nonzero strains are written as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}=\frac{\partial u_0}{\partial x}-z\frac{\partial^2 w_0}{\partial x^2}; \tag{7}$$

By supposing that the material of CNTRC beam obeys Hooke's law, the stresses field can be written as a linear function of strain field and temperature change as

$$\sigma_x=Q_{11}\left(z\right)\varepsilon_x-\alpha_{11}\Delta T \tag{8}$$

3.2 Equations of motion

Through minimum total potential energy's principle, the governing equations can be derived as follows (Reddy 2002, Simsek 2013, Larbi Chat et *al*. 2015)

$$\delta\prod=\delta\left(U_{\text{int}}-W_{ext}\right)=0 \tag{9}$$

in which Π is the total potential energy. δU_{int} is the virtual variation of the strain energy; and δW_{ext} is the variation of work induced by external forces. The first variation of the strain energy is given as (Arefi and Zenkour 2017a, c Bensaid et *al*. 2018)

$$\delta U_{\text{int}}=\int_{v}\sigma_{ij}\delta\varepsilon_{ij}dV=\int_{v}\left(\sigma_{xx}\delta\varepsilon_{xx}\right)dV \tag{10}$$

$$=\int_0^L\left(N\frac{d\delta u_0}{dx}+M\frac{d^2\delta w}{dx^2}\right)dx, \tag{11}$$

by which N is the axial force and M is the bending moment. These stress resultants used in Eq.

(11) are defined as

$$N = \int_{-h/2}^{h/2} \sigma_{xx} dz, \quad M = \int_{-h/2}^{h/2} \sigma_{xx} z\, dz \tag{12}$$

The first-order variation of the work to temperature change can be obtained by

$$\delta W = \int_0^L \left(N^T \frac{\partial w}{\partial w} \frac{\partial \delta w}{\partial x} - k_w \delta w + k_p \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} \right) dx, \tag{13}$$

Also, N^T, M^T are defined as the thermally induced force and moment produced which are acquired upon calculation of stress resultants as (Ebrahimi and Fardshad 2018, Bensaid and Bekhadda 2018)

$$\left(N^T, M^T\right) = \int_{-h/2}^{h/2} E\left(z,T\right) \alpha\left(z,T\right)\left(T - T_0\right)(1,z) dz, \tag{14}$$

where T_0 in Eq .(14) is the reference temperature.

It must be emphasized that as discussed previously, under uniform temperature rise and for FG-X, FG-O and UD types of CNTs repartition, no thermal bending moments are created. Also, due to the symmetric distribution of CNTs through the thickness in these three cases, the stretching-bending coupling stiffness components, i.e., B_{ij}'s are all eliminated. FG-V type of CNT arrangement and even in uniform heating, thermal moments are generated. Clearly, the induced thermal moments impose the beam to deflect unless the thermally induced moments are carried by the supports.

Substituting the expressions for δU_{int}, and δW_{ext} from Eqs. (11) and (13) into Eq. (9) and integrating by parts, and collecting the coefficients of δu_0 and δw_0 the following governing equations of thermally induced FG-CNTs beam can be obtained (Simsek 2013, Bensaid and Bekhadda 2018)

$$\delta u_0 = \frac{dN}{dx} = 0 \tag{15a}$$

$$\delta w_0 : \frac{\partial^2 M}{\partial x^2} - N^T \frac{\partial^2 w}{\partial x^2} - k_w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \tag{15b}$$

By substituting Eq. (7) into Eq. (8) and the following results into Eq. (12), the constitutive equations for the stress resultants are obtained as

$$N = A_{11} \frac{du_0}{dx} - B_{11} \frac{d^2 w_0}{dx^2}; \tag{16a}$$

$$M = B_{11} \frac{du_0}{dx} - D_{11} \frac{d^2 w_0}{dx^2}; \tag{16b}$$

in which the cross-sectional rigidities are specified as follows

$$\left(A_{11}, B_{11}, D_{11}\right) = \int_{-h/2}^{h/2} Q_{11}\left(1, z, z^2\right) dz \tag{17}$$

Eq. (16) can be expressed in terms of displacements (u_0, w_0) by using Eqs. (17) and (16) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} = 0 \tag{18a}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} - N^T\frac{\partial^2 w}{\partial x^2} - k_w + k_p\frac{\partial^2 w}{\partial x^2} = 0 \tag{18b}$$

4. Solution procedure

In this investigation, an analytical solution based on the Navier type method is presented to solve the governing equations for thermal buckling behavior of a simply supported FG-CNTs beam. The displacement terms are given as product of undetermined coefficients and known trigonometric functions to satisfy the governing equations and the conditions at $x = 0, L$.

$$\begin{Bmatrix} u_0 \\ w_0 \end{Bmatrix} = \sum_{m=1}^{\infty}\begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ W_m = \sin(\lambda x) e^{i\omega t} \end{Bmatrix}; \tag{19}$$

where U_m and W_m, are arbitrary parameters to be determined, ω is the eigenfrequency associated with m^{th} eigenmode, and $\lambda = m\pi / L$.

Inserting the expansions of u_0 and w_0 from Eq. (19) into the equations of motion Eq. (18), the analytical solutions can be obtained from the next of systems equations

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix}\begin{Bmatrix} U_n \\ W_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{20}$$

where

$$\begin{aligned} &k_{11} = A_{11}\lambda^2;\; k_{22} = -B_{11}\lambda^3 \\ &k_{22} = -D_{11}\lambda^4 - N^T\lambda^2 + k_w + k_p\lambda^2 \end{aligned} \tag{21}$$

in which k_{ij}*'s* are stiffness and temperature change matrices coefficients, respectively. The above system of equations is solved by calculating its determinant and making it equal to zero, with the help of the Eq. (14), we can obtain the thermal buckling loads ΔT_{cr}.

5. Results and discussion

In this section, numerical results are presented in the aims to analyse the thermal buckling behaviour of FG-CNTRC beams with temperature dependent material properties. At first, a

Table 2 Thermo-mechanical properties of (10,10) armchair SWCNT at particular temperatures Shen and Xiang (2013)

$T(K)$	E^{cn}_{11} (TPa)	E^{cn}_{22} (TPa)	G^{cn}_{22} (TPa)	ν^{cn}_{12}	$\alpha^{cn}_{12}(10^{-6}/K)$	$\alpha^{cn}_{22}(10^{-6}/K)$
300	5.6466	7.0800	1.9445	0.175	3.4584	5.1682
400	5.5679	6.9814	1.9703	0.175	4.1496	5.0905
500	5.5308	6.9348	1.9643	0.175	4.5361	5.0189
700	5.4744	6.8641	1.9644	0.175	4.6677	4.8943

validation studies are conducted. Afterwards, parametric studies are made to inspect the influences of involved parameters. Poly (methyl methacrylate), referred to as PMMA, is selected for the matrix with material properties E^m=(3.52-0.0034T) GPa, ν^m=0.34 and α^m= 45(1+0.0005ΔT)10^{-6}K. In calculation of elasticity modulus of matrix $T=T_0+\Delta T$ where T_0= 300K is the reference temperature and T is measured in Kelvin. The (10,10) armchair SWCNT is chosen as the reinforcement. Elasticity modulus, shear modulus, Poisson's ratio and thermal expansion coefficient of SWCNT are extremely dependent to temperature. Shen and Xiang (2013) reported these properties at four certain temperature levels, i.e., T =300; 400; 500 and 700 K. The magnitudes of E_{11}; E_{22}; G_{12}; α_{11}; α_{22} and ν_{12} for CNTs at these four defined temperatures are given in Table 2.

The thermomechanical properties of the CNTs to obtain the properties of CNT as a function of temperature dependency may be written as (Nguyen et *al*. 2017, Mirzaei and Kiani 2016)

$$
\begin{aligned}
&E_{11}^{cn}(T)(TPa)=6.3998-4.338417\times10^{-3}T+7.43\times10^{-6}T^2-4.458333\times10^{-9}T^3\\
&E_{22}^{cn}(T)(TPa)=8.02155-5.420375\times10^{-3}T+9.275\times10^{-6}T^2-5.5625\times10^{-9}T^3\\
&G_{12}^{cn}(T)(TPa)=1.40755+3.476208\times10^{3}T-6.965\times10^{-6}T^2+4.479167\times10^{-9}T^3\\
&\alpha_{11}^{cn}(T)(10^{-6}/K)=-1.12515+0.02291688T-2.887\times10^{-5}T^2+1.13625\times10^{-8}T^3\\
&\alpha_{22}^{cn}(T)(10^{-6}/K)=5.43715-0.984625\times10^{4}T+2.9\times10^{-7}T^2+1.25\times10^{-11}T^3\\
&\nu_{12}^{CN}=0.175
\end{aligned}
\tag{22}
$$

The parameters η_i (i = 1, 3) of CNT employed in Eq. (1) are evaluated by corresponding Young's modulus E_{11} and E_{22} and the shear modulus G_{12} of FG-CNTRC material obtained by the extended law of mixture to molecular simulation results. For three different volume fractions of CNTs in Table 1, these parameters are as: η_1 = 0.137, η_2 = 1.022, η_3 = 0.715 for the case of V^*_{cnt}= 0.12 (12%); η_1 = 0.142, η_2 = 1.626, η_3 = 1.138 for the case of V^*_{cnt} = 0.17 (17%), and η_1 = 0.141, η_2 = 1.585, η_3 = 1.109 for the case of V^*_{cnt} = 0.28 (28%) (Shen HS 2011, Yas and Samadi 2012).

5.1 Comparison studies

To evaluate precision of the obtained results predicted by the present method, the critical bucking temperature values of simply supported FG beam without CNTs reinforcement for various power law index and length to thickness ratios are validated with those existing in the literature. Fig. 3 and Table 3 both show a comparison between the results of the present study and the results

Table 3 Comparison of critical buckling temperature of S–S FG-CNTs beam with various gradient indexes

L/*h*	*p*=0		*p*=0.2		*p*=0.5		*p*=1	
	Present	Ref(a)	Present	Ref(a)	Present	Ref(a)	Present	Ref(a)
5	1111.4419	1111.4414	803.9227	794.8324	629.7134	607.0711	516.3452	516.3449
20	277.8604	277.8603	200.9806	198.7081	157.4283	151.7577	129.0863	129.0862

(a) Taken from Kiani and Eslami (2012)

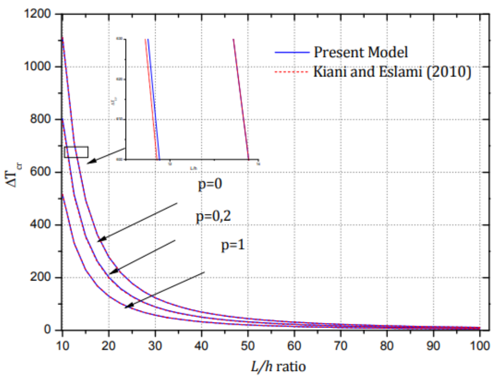


Fig. 3 Validation of critical buckling temperature of FG-CNTs beam with various gradient indexes

Table 4 the critical buckling temperature of S–S FG-CNTs beam with various volume fraction of CNTs, different graded pattern

L/*h*	Type	V^*_{cnt} =0.12	V^*_{cnt} =0.17	V^*_{cnt} =0.28
10	UD	2289.1833	2318.1118	2346.1247
	FG-X	3396.5747	3452.4298	3504.5204
	FG-O	1172.2563	1178.6998	1185.5652
20	UD	572.2958	579.5279	586.5311
	FG-X	849.1436	863.1074	876.1301
	FG-O	293.0640	294.6745	296.3913

given by Kiani and Eslami (2012) which has been obtained by analytical procedure for FG beam with different gradient index parameters (varying from 0 to 1) and aspect ratio (*L*/*h*). One can see that, the results match well with the aforementioned works which indicates the precision and effectiveness of the proposed solution and formulation in the current investigation.

5.2 Parametric studies

After validation of the obtained results from present study for S-S FG-CNTs beams with the existing data in the open literature, the effects of different parameters will be explored in detail.

In Table 4, variation of the critical buckling temperature of S-S supported edges of FG-CNTRs beams are presented for various values of volume fractions of CNTs, trough thickness CNT dispersion profile and two different values of length to thickness ratio (*L*/*h*=10, 20) based on analytical solution method. It is seen that the critical buckling temperature of the FG beam increases with an increase in the CNT volume fraction V^*_{cnt} but decreases permanently as the pattern of the CNT across the thickness changes in order from FG-X to UD and finally FG-O. This

Table 5 Variation of critical buckling temperature for FG-CNTs beam with and without elastic foundation under different graded pattern of CNT

L/h		K_w=0, K_p=0			K_w=0.1 , K_p=0.02		
	V^*_{cnt}	UD	O	X	UD	O	X
10	0.28	2346.1247	1185.5652	3504.5204	2353.28	1189.1847	3515.2198
20	0.28	586.5311	296.3913	876.1301	588.3218	297.2961	878.8049

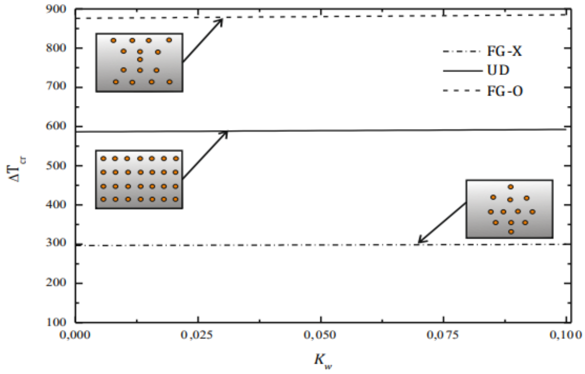


Fig. 4 Impact of Winkler modulus parameter on the critical buckling loads of CNTRC beams (L/h = 10; K_p= 0; V^*_{cnt} = 0.28)

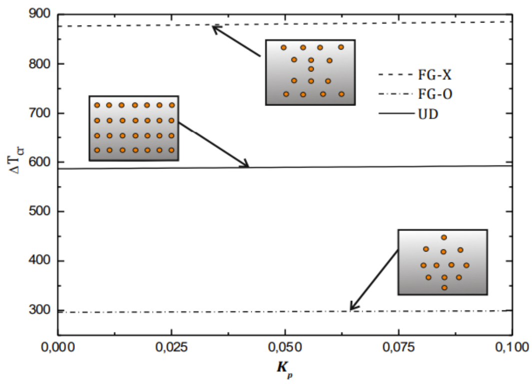


Fig. 5 Impact of Pasternak modulus parameter on the critical buckling loads of CNTRC beams (L/h = 10; K_w= 0; V^*_{cnt} = 0.28)

is due to, the bending rigidity of the beam are much more in FG-X case since in this case the upper and lower surfaces that are far from the mid-plane are more enriched with CNT. Besides, it is shown that the buckling temperature decreases by increasing aspect ratios (L/h).

Table 5 presents the effects of Winkler (K_w) and Pasternak (K_p) parameters on the variation of critical buckling temperature of S-S FG-CNTs beam for various CNT dispersion patterns , and length to thickness ratios at V^*_{cnt}=0.28. One can see that the existence of elastic medium enhances considerably the rigidity of the FG-CNTs beams and increases the buckling temperature. Also, as started before FG-X results in higher values of buckling temperature than others ones. Furthermore, impact of Pasternak layer on FG beam is more important than Winkler layer.

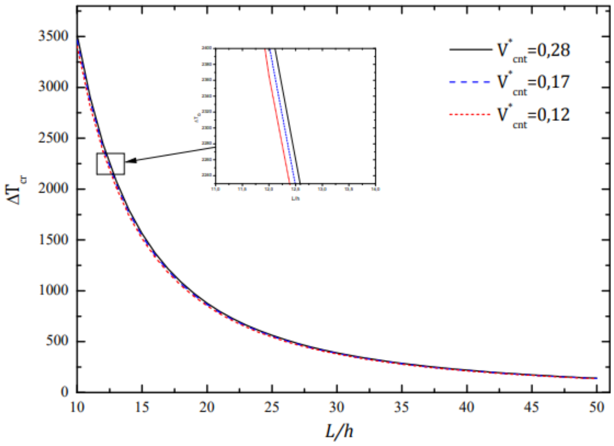


Fig. 6 Variation of critical buckling temperature versus length to thickness ratios of S–S FG-CNTs beam with various volume fractions of CNTs with (TID) properties, and FG-X pattern

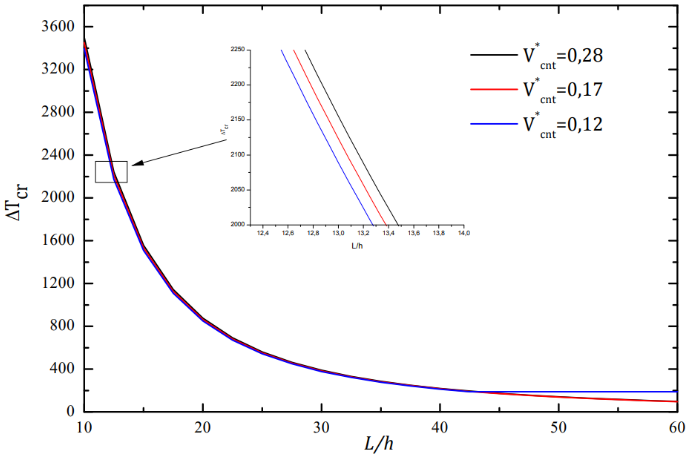


Fig. 7 Variation of critical buckling temperature versus length to thickness ratios of S–S FG-CNTs beam with various volume fractions of CNTs with (TD) properties, and FG-X pattern

Figs. 4 and 5, respectively, present the effect of both Winkler modulus parameter and the Pasternak shear modulus on the max value of critical buckling temperature of different patterns of the CNT dispersion. It can be seen that the buckling loads increase linearly as the increase the elastic medium parameters. This is due to the stiffness enhancement of FG-CNTs beam when it is laid on elastic medium.

Figs. 6 and 7 illustrate the variation of critical buckling temperature of FG beam resting on elastic medium versus slenderness ratio (L/h) with temperature independent (TID) and temperature dependent (TD) materials properties respectively, and for various values of the volume fraction of CNTs at FG-X CNTs type K_w=0.1and K_p =0.02. The increase in the amount of the volume fraction of CNTs V^*_{cnt} leads to rise in the thermal buckling load for each two cases (TID, TD). Also in Figs. 6 and 7 it is noticed that, critical buckling temperature decreases with the increase of the length to thickness ratios (L/h) for all volume fraction of CNTs. In addition, it is deduced that the buckling temperature decreases by taking into account temperature dependent (TD) materials properties.

6. Conclusion

Thermal buckling response of functionally graded carbon nanotube reinforced composite beams resting on elastic foundation has been inspected in this study based on the Euler-Bernoulli beam theory and analytical procedure. It was supposed that the mechanical properties of the CNTs and the polymeric matrix depend on temperature and change in the thickness direction by uniform or functionally graded pattern. An enriched rule of mixtures approach is established to capture the size dependent features of CNTs. The governing differential equations and related boundary conditions are formulated by using the minimum total potential energy principle. Accuracy of obtained results is checked with available data in the literature. Finally, a parametric study and numerical examples were conducted, to investigate the effect of different parameters, such as volume fraction of CNTs, dispersion pattern of CNTs, Winkler–Pasternak elastic foundation, thermal environment, and length to thickness ratio on critical buckling temperature. It is found that, in all previous parametric studies, FG-X distribution of CNTs is the most proficient type for thermal buckling analysis because under such distribution, critical buckling temperature is higher compared to the other types of distribution of CNTs.

References

Adda Bedia, W., Benzair, A., Semmah, A., Tounsi, A. and Mahmoud, S.R. (2015), "On the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity", *Braz. J. Phys.*, **45**(2), 225-233. https://doi.org/10.1007/s13538-015-0306-2.

Ajayan, P.M., Stephan, O., Colliex, C. and Trauth, D. (1994), "Aligned carbon nanotube arrays formed by cutting a polymer resin–nanotube composite", *Science*, **256**(5176), 1212-1214. https://doi.org/10.1126/science.265.5176.1212.

Alibeigloo, A. (2014), "Three-dimensional thermoelasticity solution of functionally graded carbon nanotube reinforced composite plate embedded in piezoelectric sensor and actuator layers", *Compos. Struct.*, **118**, 482-495. https://doi.org/10.1016/j.compstruct.2014.08.004.

Arefi, M. (2015), "Elastic solution of a curved beam made of functionally graded materials with different cross sections", *Steel Compos. Struct.*, **18**(3), 659-672. https://doi.org/10.1016/10.12989/scs.2015.18.3.659.

Arefi, M. (2015), "Nonlinear thermal analysis of a hollow functionally graded cylinder with temperature variable material properties", *J. Appl. Mech. Tech. Phys.*, **56**(2), 267-273. https://doi.org/10.1134/S0021894415020121.

Arefi, M. and Arani, A.H.M. (2017), "Higher order shear deformation bending results of a magnetoelectrothermoelastic functionally graded nanobeam in thermal, mechanical, electrical, and magnetic environments", *Mech. Base. Desig. Struct. Machi.*, **46**(6), 669-692. https://doi.org/10.1080/15397734.2018.1434002.

Arefi, M. and Nahas, I. (2014), "Nonlinear electro thermo elastic analysis of a thick spherical functionally graded piezoelectric shell", *Compos. Struct.*, **118**, 510-518. https://doi.org/10.1016/j.compstruct.2014.08.002.

Arefi, M. and Rahimi, G.H. (2010), "Thermo elastic analysis of a functionally graded cylinder under internal pressure using first order shear deformation theory", *Academ. J.*, **5**(12), 1442-1454.

Arefi, M. and Zenkour, A.M. (2017a), "Vibration and bending analysis of a sandwich microbeam with two integrated piezo-magnetic face-sheets", *Compos. Struct.*, **159**(1), 479-490. https://doi.org/10.1016/j.compstruct.2016.09.088.

Arefi, M. and Zenkour, A.M. (2017b), "Transient sinusoidal shear deformation formulation of a size dependent three-layer piezo-magnetic curved nanobeam", *Acta Mechanica*, **228**(10), 3657-3674. https://doi.org/10.1007/s00707-017-1892-6.

Arefi, M. and Zenkour, A.M. (2017c), "Size-dependent vibration and bending analyses of the piezomagnetic three-layer nanobeams", *Appl. Phys. A*, **122**(10), 880. https://doi.org/10.1007/s00339-017-0801-0.

Arefi, M. and Zenkour, A.M. (2017d), "Wave propagation analysis of a functionally graded magneto-electro elastic nanobeam rest on Visco-Pasternak foundation", *Mech. Res. Commun.*, **79**, 51-62. https://doi.org/10.1016/j.mechrescom.2017.01.004.

Arefi, M. and Zenkour, A.M. (2017e), "Transient analysis of a three-layer microbeam subjected to electric potential", *Int. J. Smart Nano Mater.*, **8**(1), 20-40. https://doi.org/10.1080/19475411.2017.1292967.

Arefi, M. and Zenkour, A.M. (2017f), "Influence of magneto-electric environments on size-dependent bending results of three-layer piezomagnetic curved nanobeam based on sinusoidal shear deformation theory", *J. Sandw. Struct. Mater.* https://doi.org/10.1177/1099636217723186.

Arefi, M. and Zenkour, A.M. (2017g), "Influence of micro-length-scale parameters and inhomogeneities on the bending, free vibration and wave propagation analyses of a FG Timoshenko's sandwich piezoelectric microbeam", *J. Sandw. Struct. Mater.* https://doi.org/10.1177/1099636217714181.

Arefi, M. and Zenkour, A.M. (2017h), "Analysis of wave propagation in a functionally graded nanobeam resting on visco-Pasternak's foundation", *Theor. Appl. Mech. Lett.*, **7**(3), 145-151. https://doi.org/10.1016/j.taml.2017.05.003.

Arefi, M. and Zenkour, A.M. (2017i), "Thermo-electro-mechanical bending behavior of sandwich nanoplate integrated with piezoelectric face-sheets based on trigonometric plate theory", *Compos. Struct.*, **162**, 108-122. https://doi.org/10.1016/j.compstruct.2016.11.071.

Ashrafi, B. and Hubert, P. (2006), "Modeling the elastic properties of carbon nanotube array/polymer composites", *Compos. Sci. Technol.*, **66**(3), 387-396. https://doi.org/10.1016/j.compscitech.2005.07.020.

Aydogdu, M. (2014), "On the vibration of aligned carbon nanotube reinforced composite beams", *Adv. Nano Res.*, **2**(4), 199-210. https://doi.org/10.12989/anr.2014.2.4.199.

Bakhti, K., Kaci, A., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2013), "Large deformation analysis for functionally graded carbon nanotube-reinforced composite plates using an efficient and simple refined theory", *Steel Compos. Struct.*, **14**(4), 335-347. https://doi.org/10.12989/scs.2013.14.4.335.

Barati, M.R. and Zenkour, A.M. (2018), "Analysis of postbuckling of graded porous GPLreinforced beams with geometrical imperfection", *Mech. Adv. Mater. Struct.*, 1-9. https://doi.org/10.1080/15376494.2017.1400622.

Barzoki, A.A.M., Loghman, A. and Arani, A.G. (2015), "Temperature-dependent nonlocal nonlinear buckling analysis of functionally graded SWCNT-reinforced microplates embedded in an orthotropic elastomeric medium", *Struct. Eng. Mech.*, **53**(3), 497-517. https://doi.org/10.12989/sem.2015.53.3.497.

Bensaid, I. and Bekhadda, A. (2018), "Thermal stability analysis of temperature dependent inhomogeneous size-dependent nano-scale beams", *Adv. Mater. Res.*, **7**(1), 1-16. https://doi.org/10.12989/amr.2018.7.1.001.

Bensaid, I., Bekhadda, A. and Kerboua, B. (2018), "Dynamic analysis of higher order shear-deformable nanobeams resting on elastic foundation based on nonlocal strain gradient theory", *Adv. Nano Res.*, **6**(3), 279-298. https://doi.org/10.12989/anr.2018.6.3.279.

Besseghier, A., Heireche, H., Bousahla, A.A., Tounsi, A. and Benzair, A. (2015), "Nonlinear vibration properties of a zigzag single-walled carbon nanotube embedded in a polymer matrix", *Adv. Nano Res.*, **3**(1), 29-37. https://doi.org/10.12989/anr.2015.3.1.029.

Bidgoli, M.R., Karimi, M.S. and Arani, A.G. (2015), "Viscous fluid induced vibration and instability of FGCNT-reinforced cylindrical shells integrated with piezoelectric layers", *Steel Compos. Struct.*, **19**(3), 713- 733. https://doi.org/10.12989/scs.2015.19.3.713.

Ebrahimi, F. and Fardshad, R.E. (2018), "Dynamic modeling of nonlocal compositionally graded temperature-dependent beams", *Adv. Aircraft Spacecraft Sci.*, **5**(1), 141-164. https://doi.org/10.12989/aas.2018.5.1.141.

Esawi, A.M.K. and Farag, M.M. (2007), "Carbon nanotube reinforced composites: Potential and current

challenges", *Mater. Des.*, **28**(9), 2394-2401. https://doi.org/10.1016/j.matdes.2006.09.022.
Fidelus, J.D., Wiesel, E., Gojny, F.H., Schulte, K. and Wagner, H.D. (2005), "Thermo-mechanical properties of randomly oriented carbon/epoxy nanocomposites", *Compos. Part A*, **36**(11), 1555-1561. https://doi.org/10.1016/j.compositesa.2005.02.006.
Griebel, M. and Hamaekers, J. (2004), "Molecular dynamics simulations of the elastic moduli of polymer-carbon nanotube composites", *Comput. Method. Appl. Mech. Eng.*, **193**(17-20), 1773-1788. https://doi.org/10.1016/j.cma.2003.12.025.
Hadji, L., Zouatnia, N., Mezian, A.A.M. and Kassoul, A. (2015), "A simple quasi-3D sinusoidal shear deformation theory with stretching effect for carbon nanotube-reinforced composite beams resting on elastic foundation", *Earthq. Struct.*, **13**(5), 509-518. https://doi.org/10.12989/eas.2015.13.5.509.
Han, Y. and Elliott, J. (2007), "Molecular dynamics simulations of the elastic properties of polymer/carbon nanotube composites", *Comput. Mater. Sci.*, **39**(2), 315-323. https://doi.org/10.1016/j.commatsci.2006.06.011.
Hu, N., Fukunaga, H., Lu, C., Kameyama, M. and Yan, B. (2005), "Prediction of elastic properties of carbon nanotube reinforced composites", *Proc. Royal. Soc. A Math Phys. Eng. Sci.*, **461**(2058), 1685-1710. https://doi.org/10.1098/rspa.2004.1422.
Kaci, A., Tounsi, A., Bakhti, K. and Adda Bedia, E.A. (2012), "Nonlinear cylindrical bending of functionally graded carbon nanotube-reinforced composite plates", *Steel Compos. Struct.*, **12**(6), 491-504. https://doi.org/10.12989/scs.2012.12.6.491.
Ke, L.L., Yang, J. and Kitipornchai, S. (2010), "Nonlinear free vibration of functionally graded carbon nanotube-reinforced composite beams", *Compos. Struct.*, **92**(3), 676-683. https://doi.org/10.1016/j.compstruct.2009.09.024.
Ke, L.L., Yang, J. and Kitipornchai, S. (2013), "Dynamic stability of functionally graded carbon nanotubereinforced composite beams", *Mech. Adv. Mater. Struct.*, **20**(1), 28-37. https://doi.org/10.1080/15376494.2011.581412.
Kiani, Y. (2016), "Shear buckling of FG-CNT reinforced composite plates using Chebyshev-Ritz method", *Compos Part B Eng.*, **105**, 176-87. https://doi.org/10.1016/j.compositesb.2016.09.001.
Kiani, Y. (2016), "Thermal postbuckling of temperature dependent sandwich beams with carbon nanotube reinforced face sheets", *J. Therm. Stresses*, **39**(9), 1098-110. https://doi.org/10.1080/01495739.2016.1192856.
Kiani, Y. and Eslami, M.R, (2010), "Thermal buckling analysis of functionally graded material beams", *Int. J. Mech. Mater. Des.*, **6**(3), 229-238. https://doi.org/10.1007/s10999-010-9132-4.
Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425-442. https://doi.org/10.12989/scs.2015.18.2.425.
Lei, Z.X., Liew, K.M. and Yu, J.L. (2013), "Free vibration analysis of functionally graded carbon nanotube-reinforced composite plates using the element-free kp-Ritz method in thermal environment", *Compos. Struct.*, **106**, 128-138. https://doi.org/10.1016/j.compstruct.2013.06.003.
Liew, K.M., Lei, Z.X. and Zhang, L.W. (2015), "Mechanical analysis of functionally graded carbon nanotube reinforced composites: A review", *Compos Struct.*, **120**, 90-97. https://doi.org/10.1016/j.compstruct.2014.09.041.
Mirzaei, M. and Kiani, Y. (2016), "Thermal buckling of temperature dependent FG-CNT reinforced composite plates", *Meccanica*, **51**(9), 2185-2201. https://doi.org/10.1007/s11012-015-0348-0.
Nguyen, V.T., Nguyen, D.K., Ngo, D.T., Phuong, T. and Nguyen, D.D. (2017), "Nonlinear dynamic response and vibration of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) shear deformable plates with temperature-dependent material properties and surrounded on elastic foundations", *J. Therm. Stresses*, **40**(10), 1254-1274. https://doi.org/10.1080/01495739.2017.1338928.
Odegard, G.M., Gates, T.S., Wise, K.E., Park, C. and Siochi, E.J. (2003), "Constitutive modelling of nanotube-reinforced polymer composites", *Compos. Sci. Technol.*, **63**(11), 1671-1687. https://doi.org/10.1016/S0266-3538(03)00063-0.

Phung-Van, P., Abdel-Wahab, A., Liew, K.M., Bordas, S.P.A. and Nguyen-Xuan, H. (2015), "Isogeometric analysis of functionally graded carbon nanotube-reinforced composite plates using higher-order shear deformation theory", *Compos. Struct.*, **123**, 137-149. https://doi.org/10.1016/j.compstruct.2014.12.021.

Rafiee, M., Yang, J. and Kitipornchai, S. (2013), "Thermal bifurcation buckling of piezoelectric carbon nanotube reinforced composite beams", *Comput. Math. Appl.*, **66**(7), 1147-1160. https://doi.org/10.1016/j.camwa.2013.04.031.

Reddy, J.N. (2002), *Energy Principles and Variational Methods in Applied Mechanics*, John Wiley & Sons Inc., New York, U.S.A.

Shen, H.S. (2009), "Nonlinear bending of functionally graded carbon nanotube-reinforced composite plates in thermal environments", *Compos. Struct.*, **91**(1), 9-19. https://doi.org/10.1016/j.compstruct.2009.04.026.

Shen, H.S. (2011), "Postbuckling of nanotube-reinforced composite cylindrical shells in thermal environments, part I: Axially-loaded shells", *Compos. Struct.*, **93**(8), 2096-2108. https://doi.org/10.1016/j.compstruct.2011.02.011.

Shen, H.S. and Xiang, Y. (2013), "Nonlinear analysis of nanotube reinforced composite beams resting on elastic foundations in thermal environments", *Eng. Struct.*, **56**, 698-708. https://doi.org/10.1016/j.engstruct.2013.06.002.

Şimşek, M. and Yurtçu, H.H. (2013), "Analytical solutions for bending and buckling of functionally graded nanobeams based on the nonlocal Timoshenko beam theory", *Compos. Struct.*, **97**, 378-386. https://doi.org/10.1016/j.compstruct.2012.10.038.

Tagrara, S.H., Benachour, A., Bouiadjra, M.B. and Tounsi, A. (2015), "On bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams", *Steel Compos. Struct.*, **19**(5), 1259-1277. https://doi.org/10.12989/scs.2015.19.5.1259.

Thostenson, E.T. and Chou, T.W. (2003), "On the elastic properties of carbon nanotube-based composites: Modelling and characterization", *J. Phys. A Appl. Phys.*, **36**(5), 573-582.

Thostenson, E.T., Ren, Z.F. and Chou, T.W. (2001), "Advances in the science and technology of carbon nanotubes and their composites: A review", *Compos. Sci. Technol.*, **61**(13), 1899-1912. https://doi.org/10.1016/S0266-3538(01)00094-X.

Vo-Duy, T., HO-Huu, V. and Nguyen-Thoi, T. (2019), "Free vibration analysis of laminated FG-CNT reinforced composite beams using finite element method", *Front. Struct. Civ. Eng.*, **13**(2), 324-336. https://doi.org/10.1007/s11709-018-0466-6.

Wang, Z.X. and Shen, H.S. (2011), "Nonlinear vibration of nanotube-reinforced composite plates in thermal environments", *Comput. Mater. Sci.*, **50**(8), 2319-2330. https://doi.org/10.1016/j.commatsci.2011.03.005.

Wattanasakulpong, N. and Ungbhakorn, V. (2013), "Analytical solutions for bending, buckling and vibration responses of carbon nanotube-reinforced composite beams resting on elastic foundation", *Comput. Mater. Sci.*, **71**, 201-208. https://doi.org/10.1016/j.commatsci.2013.01.028.

Wu, H., Kitipornchai, S. and Yang, J. (2015), "Free vibration and buckling analysis of sandwich beams with functionally graded carbon nanotube-reinforced composite face sheets", *J. Struct. Stabil. Dyn.*, **15**(7), 1540011. https://doi.org/10.1142/S0219455415400118.

Xu, Y., Ray, G. and Abdel-Magid, B. (2006), "Thermal behavior of single-walled carbon nanotube polymermatrix composites", *Compos. Part A Appl. Sci. Manufact.*, **37**(1), 114-121. https://doi.org/10.1016/j.compositesa.2005.04.009.

Yang, J., Ke, L.L. and Feng, C. (2015), "Dynamic buckling of thermo-electro-mechanically loaded FG CNTRC beams", *J. Struct. Stabil. Dyn.*, **15**(8), 1540017. https://doi.org/10.1142/S0219455415400179.

Yas, M.H. and Samadi, N. (2012), "Free vibrations and buckling analysis of carbon nanotube-reinforced composite Timoshenko beams on elastic foundation", *Int. J. Pres. Vess. Pip.*, **98**, 119-128. https://doi.org/10.1016/j.ijpvp.2012.07.012.

Zhu, P., Lei, Z.X. and Liew, K.M. (2012), "Static and free vibration analyses of carbon nanotube reinforced composite plates using finite element method with first order shear deformation plate theory", *Compos. Struct.*, **94**(4), 1450-1460. https://doi.org/10.1016/j.compstruct.2011.11.010.

Zhu, R., Pan, E. and Roy, A.K. (2007), "Molecular dynamics study of the stress-strain behavior of carbon-nanotube reinforced Epon 862 composites", *Mater. Sci. Eng. A*, **447**(1-2), 51-57.

https://doi.org/10.1016/j.msea.2006.10.054.

EC