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Dynamic modeling of nonlocal compositionally graded temperature-dependent beams

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Abstract. In this paper, the thermal effect on buckling and free vibration characteristics of functionally graded (FG) size-dependent Timoshenko nanobeams subjected to an in-plane thermal loading are investigated by presenting a Navier type solution for the first time. Material properties of FG nanobeam are supposed to vary continuously along the thickness according to the power-law form and the material properties are assumed to be temperature-dependent. The small scale effect is taken into consideration based on nonlocal elasticity theory of Eringen. The nonlocal equations of motion are derived based on Timoshenko beam theory through Hamilton's principle and they are solved applying analytical solution. According to the numerical results, it is revealed that the proposed modeling can provide accurate frequency results of the FG nanobeams as compared to some cases in the literature. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of the several parameters such as thermal effect, material distribution profile, small scale effects, aspect ratio and mode number on the critical buckling temperature and normalized natural frequencies of the temperature-dependent FG nanobeams in detail. It is explicitly shown that the thermal buckling and vibration behaviour of a FG nanobeams is significantly influenced by these effects. Numerical results are presented to serve as benchmarks for future analyses of FG nanobeams.

Keywords: thermal buckling; Timoshenko beam theory; vibration; functionally graded material; nonlocal elasticity theory

1. Introduction

Functionally graded materials (FGMs) are composite materials with inhomogeneous micromechanical structure. They are generally composed of two different parts such as ceramic with excellent characteristics in heat and corrosive resistances and metal with toughness. The material properties of FGMs change smoothly between two surfaces and the advantages of this combination lead to novel structures which can withstand in large mechanical loadings under high temperature environments (Ebrahimi and Rastgoo 2008a, b, c). Presenting novel properties, FGMs have also attracted intensive research interests, which were mainly focused on their static, dynamic

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and vibration characteristics of FG structures (Ebrahimi *et al.* 2009a, b, Ebrahimi 2013, Ebrahimi *et al.* 2016a, Ebrahimi and Zia 2015, Ebrahimi and Mokhtari 2015).

Moreover, structural elements such as beams, plates, and membranes in micro or nanolength scale are commonly used as components in micro/nano electromechanical systems (MEMS/NEMS). Therefore understanding the mechanical and physical properties of nanostructures is necessary for its practical applications. Nanoscale engineering materials have attracted great interest in modern science and technology after the invention of carbon nanotubes (CNTs) by Iijima (1991). They have significant mechanical, thermal and electrical performances that are superior to the conventional structural materials. In recent years, nanobeams and CNTs hold a wide variety of potential applications (Zhang *et al.* 2004, Wang 2005, Wang and Varadan 2006) such as sensors, actuators, transistors, probes, and resonators in NEMSs. For instance, in MEMS/NEMS; nanostructures have been used in many areas including communications, machinery, information technology, biotechnology technologies.

Since conducting experiments at the nanoscale is a daunting task, and atomistic modeling is restricted to small-scale systems owing to computer resource limitations, continuum mechanics offers an easy and useful tool for the analysis of CNTs. However the classical continuum models need to be extended to consider the nanoscale effects and this can be achieved through the nonlocal elasticity theory proposed by Eringen (1972) which consider the size-dependent effect. According to this theory, the stress state at a reference point is considered as a function of strain states of all points in the body. This nonlocal theory is proved to be in accordance with atomic model of lattice dynamics and with experimental observations on phonon dispersion (Eringen 1983).

Moreover, in recent years the application of nonlocal elasticity theory, in micro and nanomaterials has received a considerable attention within the nanotechnology community and Lots of studies have been performed to investigate the size-dependent response of structural systems based on Eringen’s nonlocal elasticity theory (Ebrahimi and Salari 2015a, b, 2016, Ebrahimi *et al.* 2015a, 2016c, Ebrahimi and Nasirzadeh 2015, Ebrahimi and Barati 2016 a, b, c, d, e, f, Ebrahimi and Hosseini 2016 a, b, c). Peddieson *et al.* (2003) proposed a version of nonlocal elasticity theory which is employed to develop a nonlocal Benoulli/Euler beam model. Wang and Liew (2007) carried out the static analysis of micro- and nano-structures based on nonlocal continuum mechanics using Euler-Bernoulli beam theory and Timoshenko beam theory. Aydogdu (2009) proposed a generalized nonlocal beam theory to study bending, buckling, and free vibration of nanobeams based on Eringen model using different beam theories. Phadikar and Pradhan (2010) reported finite element formulations for nonlocal elastic Euler-Bernoulli beam and Kirchhoff plate. Pradhan and Murmu (2010) investigated the flapwise bending-vibration characteristics of a rotating nanocantilever by using Differential quadrature method (DQM). They noticed that small-scale effects play a significant role in the vibration response of a rotating nanocantilever. Civalek *et al.* (2010) presented a formulation of the equations of motion and bending of Euler–Bernoulli beam using the nonlocal elasticity theory for cantilever microtubules. The method of differential quadrature has been used for numerical modeling. Civalek and Demir (2011) developed a nonlocal beam model for the bending analysis of microtubules based on the Euler–Bernoulli beam theory. The size effect is taken into consideration using the Eringen’s nonlocal elasticity theory.

Furthuremore, with the development of the material technology, FGMs have also been employed in MEMS/NEMS (Witvrouw 2005, Lee *et al.* 2006). Because of high sensitivity of MEMS/NEMS to external stimulations, understanding mechanical properties and vibration behavior of them are of significant importance to the design and manufacture of FG

MEMS/NEMS. Thus, establishing an accurate model of FG nanobeams is a key issue for successful NEMS design. Fallah and Aghdam (2012) used Euler-Bernoulli beam theory to investigate thermo mechanical buckling and nonlinear vibration analysis of functionally graded beams on nonlinear elastic foundation. The nonlinear static response of FGM beams under in-plane thermal loading is studied by Ma and Lee (2012). Asghari *et al.* (2010, 2011) studied the free vibration of the FGM Euler-Bernoulli microbeams, which has been extended to consider a size-dependent Timoshenko beam based on the modified couple stress theory. The dynamic characteristics of FG beam with power law material graduation in the axial or the transversal directions was examined by Alshorbagy *et al.* (2011). Ke and Wang (2011) exploited the size effect on dynamic stability of functionally graded Timoshenko microbeams. The free vibration analysis of FG microbeams was presented by Ansari *et al.* (2011) based on the strain gradient Timoshenko beam theory. They also concluded that the value of gradient index plays an important role in the vibrational response of the FG microbeams of lower slenderness ratios. Employing modified couple stress theory the nonlinear free vibration of FG microbeams based on von-Karman geometric nonlinearity was presented by Ke *et al.* (2012). It was revealed that both the linear and nonlinear frequencies increase significantly when the thickness of the FGM microbeam was comparable to the material length scale parameter. Recently, Eltaher *et al.* (2012, 2013a) presented a finite element formulation for free vibration analysis of FG nanobeams based on nonlocal Euler beam theory. They also exploited the size-dependent static-buckling behavior of functionally graded nanobeams on the basis of the nonlocal continuum model (Eltaher *et al.* 2013b). Using nonlocal Timoshenko and Euler–Bernoulli beam theory, Simsek and Yurtcu (2013) investigated bending and buckling of FG nanobeam by analytical method. More recently, vibration behaviour of simply supported Timoshenko FG nanobeams were investigated by Rahmani and Pedram (2014). Most recently Ebrahimi and Barati (2016g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, 2017a, b) and Ebrahimi *et al.* (2017) explored thermal and hygro-thermal effects on nonlocal behavior of FG nanobeams and nanoplates. Furthermore, the common use of FGMs in high temperature environment leads to considerable changes in material properties. For example, Young’s modulus usually decreases when temperature increases in FGMs. To predict the behavior of FGMs under high temperature more accurately, it is necessary to consider the temperature dependency on material properties. It is found that most of the previous studies on vibration analysis of FG nanobeams have been conducted based on the ignorance of the thermal environment effects. As a result, these studies cannot be utilized in order to thoroughly study the FG nanobeams under investigation. However, it is woth mentioning that some of the researchers spent their time in order to present more reliable models to take into consider thermal effects while investigating the mechanical answers of structures. Therefore, there is strong scientific need to understand the vibration behavior of FG nanobeams in considering the effect of temperature changes. Motivated by this fact, in this study, thermal buckling and vibration characteristics of temperature dependent FG nanobeams considering the effect of thermal environment is analyzed. An analytical method called Navier solution is employed for vibration and thermal buckling analysis of size-dependent FG nanobeams for the first time. It is assumed that material properties of the beam, vary continuously through the beam thickness according to power-law form and are temperature dependent. Nonlocal Timoshenko beam model and Eringen’s nonlocal elasticity theory are employed. Governing equations and boundary conditions for the free vibration of a nonlocal FG beam have been derived via Hamilton’s principle. These equations are solved using Navier type method and numerical solutions are obtained. The detailed mathematical derivations are presented while the emphasis is placed on investigating the effect of several parameters such as thermal effects, constituent volume fractions, mode number, aspect ratio and small scale on critical buckling temperature and vibration characteristics of FG nanobeams. Comparisons with analytical solotions and the results from the existing literature are provided for two-constituents metal-ceramic nanobeams and the good agreement between the results of this article and those available in literature validated the presented approach. Numerical results are presented to serve as benchmarks for the application and the design of nanoelectronic and nano-drive devices, nano-oscillators, and nanosensors, in which nanobeams act as basic elements. They can also be useful as valuable sources for validating other approaches and approximate methods.

2. Theory and formulation

2.1 Nonlocal power-law FG nanobeam equations based

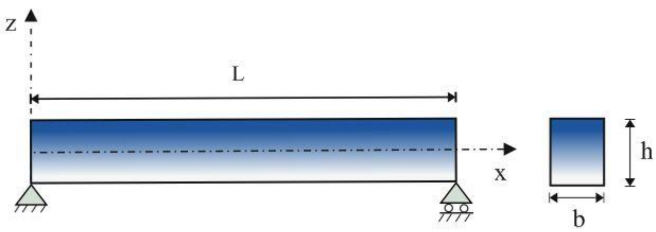


Fig. 1 Geometry and coordinates of Timoshenko FG nanobeam

Consider a FG nanobeam of length *a*, width *b* and uniform thickness *h* in the unstressed reference configuration. The coordinate system for FG nano beam is shown in Fig. 1. The nanobeam is made of elastic and isotropic functionally graded material with properties varying smoothly in the *z* thickness direction only. Indeed, the top surface of the beam is assumed to be made of pure metal and the bottom surface is considered to be consist of pure ceramic. The effective material properties of the FG beam such as Young's modulus E_f, shear modulus G_f and mass density p_f are assumed to vary continuously in the thickness direction (*z*-axis direction) according to a power function of the volume fractions of the constituents.

According to the rule of mixture, the effective material properties, P_f, can be expressed as (Simsek and Yurtcu 2013)

$$P_f = P_c V_c + P_m V_m \tag{1}$$

where P_m, P_c, V_m and V_c are the material properties and the volume fractions of the metal and the ceramic constituents related by

$$V_c + V_m = 1 \tag{2a}$$

The volume fraction of the ceramic constituent of the beam is assumed to be given by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{2b}$$

Here, p is the non-negative variable parameter (power-law exponent) which determines the material distribution through the thickness of the beam and z is the distance from the mid-plane of the FG beam. The FG beam becomes a fully ceramic beam when p is set to be zero. Therefore, from Eqs. (1)-(2), the effective material properties of the FG nanobeam such as Young's modulus (E), mass density (ρ), thermal expansion (α) and Poisson's ratio (v) can be expressed as follows

$$\begin{aligned} E(z) &= \left(E_c - E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p + E_m \\ \rho(z) &= \left(\rho_c - \rho_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p + \rho_m \\ \alpha(z) &= \left(\alpha_c - \alpha_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p + \alpha_m \\ \nu(z) &= \left(\nu_c - \nu_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p + \nu_m \end{aligned} \tag{3}$$

To predict the behavior of FGMs under high temperature more accurately, it is necessary to consider the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in function of temperature $T(K)$ can be expressed as (Touloukian 1967)

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{4}$$

Where $T=T_0+\Delta T$ and $T_0=300\ K$ (ambient or free stress temperature), ΔT is the temperature change, P_0, P_{-1}, P_1, P_2 and P_3 are the temperature dependent coefficients which can be seen in the table of materials properties (Table 1) for Si_3N_4 and SUS304. The bottom surface (z = -h/2) of FG nanobeam is pure metal (SUS304), whereas the top surface (z = h/2) is pure ceramics(Si_3N_4).

Table 1 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient, mass density and Poisson's ratio for Si_3N_4 and SUS304 (Ebrahimi *et al.* 2017)

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α(K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ(Kg/m^3)	2370	0	0	0	0
	ν	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α(K^{-1})	12.330e-6	0	8.086e-4	0	0
	ρ(Kg/m^3)	8166	0	0	0	0
	ν	0.3262	0	-2.002e-4	3.797e-7	0

2.2 Kinematic relations

The equations of motion is derived based on the Timoshenko beam theory according to which the displacement field at any point of the beam can be written as

$$u_x\left(x,z,t\right)=u\left(x,t\right)+z\,\varphi(x,t) \tag{5a}$$

$$u_z(x,z,t)=w(x,t) \tag{5b}$$

where t is time, φ is the total bending rotation of the cross-section, u and w are displacement components of the mid-plane along x and z directions, respectively. Therefore, according to the Timoshenko beam theory, the nonzero strains are obtained as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}+z\,\frac{\partial\varphi}{\partial x} \tag{6}$$

$$\gamma_{xz}=\frac{\partial w}{\partial x}+\varphi \tag{7}$$

where ε_{xx} and γ_{xz} are the normal strain and shear strain, respectively. Based on the Hamilton's principle, which states that the motion of an elastic structure during the time interval [0, t] is such that the time integral of the total dynamics potential is extremum (Tauchert 1974)

$$\int_0^t \delta(U-T+V)\,dt=0 \tag{8}$$

in which U is strain energy, T is kinetic energy and V is work done by external forces. The virtual strain energy can be calculated as

$$\delta U=\int_v \sigma_{ij}\,\delta\,\varepsilon_{ij}\,dV=\int_v(\sigma_{xx}\,\delta\,\varepsilon_{xx}+\sigma_{xz}\,\delta\,\gamma_{xz})\,dV \tag{9}$$

Substituting Eqs. (6) and (7) into Eq. (9) yields

$$\delta U=\int_0^L\left(N\left(\delta\frac{\partial u}{\partial x}\right)+M\left(\delta\frac{\partial\varphi}{\partial x}\right)+Q\left(\delta\frac{\partial w}{\partial x}+\delta\varphi\right)\right)dx \tag{10}$$

in which N is the axial force, M is the bending moment and Q is the shear force. These stress resultants used in Eq. (10) are defined as

$$N=\int_A\sigma_{xx}\,dA,\quad M=\int_A\sigma_{xx}\,z\,dA,\quad Q=\int_A K_s\sigma_{xz}\,dA \tag{11}$$

where K_s is the shear correction factor. Also, the kinetic energy for Timoshenko beam can be written as

$$T=\frac{1}{2}\int_0^L\int_A\rho(z,T)\left(\left(\frac{\partial u_x}{\partial t}\right)^2+\left(\frac{\partial u_z}{\partial t}\right)^2\right)dA\,dx \tag{12}$$

Also, the virtual kinetic energy can be expressed as

$$\delta T = \int_0^L \left[I_0 \left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right) + I_1 \left(\frac{\partial \varphi}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial u}{\partial t}\frac{\partial \delta \varphi}{\partial t}\right) + I_2 \frac{\partial \varphi}{\partial t}\frac{\partial \delta \varphi}{\partial t} \right] dx \tag{13}$$

where (I_0, I_1, I_2) are the mass moment of inertias, defined as follows

$$(I_0, I_1, I_2) = \int_A \rho(z, T)(1, z, z^2)\, dA \tag{14}$$

For a typical FG nanobeam which has been in high temperature environment for a long period of time, it is assumed that the temperature can be distributed uniformly across its thickness so that the case of uniform temperature rise is taken into consideration. In this investigation, initial uniform temperature (T_0=300 K), which is a stress free state, changes to final temperature with ΔT. Hence, the first variation of the work done corresponding to temperature change can be written in the form (Kim 2005, Mahi *et al.* 2010)

$$\delta V = \int_0^L N^T \frac{\partial w}{\partial x} \delta\left(\frac{\partial w}{\partial x}\right) dx \tag{15}$$

where N^T is thermal resultant can be expressed as

$$N^T = \int_{-h/2}^{h/2} E(z, T)\, \alpha(z, T)\, \Delta T\, dz \tag{16}$$

By Substituting Eqs. (10), (13) and (15) into Eq. (8) and setting the coefficients of δu, δw and, $\delta\varphi$ to zero, the following Euler-Lagrange equation can be obtained

$$\frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \varphi}{\partial t^2} \tag{17a}$$

$$\frac{\partial Q}{\partial x} - N^T \frac{\partial^2 w}{\partial x^2} = I_0 \frac{\partial^2 w}{\partial t^2} \tag{17b}$$

$$\frac{\partial M}{\partial x} - Q = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \varphi}{\partial t^2} \tag{17c}$$

Under the following boundary conditions

$$N = 0 \text{ or } u = 0 \text{ at } x = 0 \text{ and } x = L \tag{18a}$$

$$Q = 0 \text{ or } w = 0 \text{ at } x = 0 \text{ and } x = L \tag{18b}$$

$$M = 0 \text{ or } \varphi = 0 \text{ at } x = 0 \text{ and } x = L \tag{18c}$$

2.3 The nonlocal elasticity model for FG nanobeam

Based on Eringen nonlocal elasticity model (Eringen and Edelen 1972), the stress at a reference point x in a body is considered as a function of strains of all points in the near region. This assumption is agreement with experimental observations of atomic theory and lattice dynamics in phonon scattering in which for a homogeneous and isotropic elastic solid the nonlocal stress-tensor components σ_{ij} at any point x in the body can be expressed as

$$\sigma_{ij}(x) = \int_{\Omega} \alpha(|x' - x|, \tau)\, t_{ij}(x')\, d\Omega(x') \tag{19}$$

where $t_{ij}(x')$ are the components of the classical local stress tensor at point x which are related to the components of the linear strain tensor ε_{kl} by the conventional constitutive relations for a Hookean material, i.e.,

$$t_{ij} = C_{ijkl}\,\varepsilon_{kl} \tag{20}$$

The meaning of Eq. (19) is that the nonlocal stress at point x is the weighted average of the local stress of all points in the neighborhood of x, the size of which is related to the nonlocal kernel $\alpha(|x' - x|, \tau)$. Here, $|x' - x|$ is the Euclidean distance and τ is a constant given by

$$\tau = \frac{e_0 a}{l} \tag{21}$$

which represents the ratio between a characteristic internal length, a (such as lattice parameter, C-C bond length and granular distance) and a characteristic external one, l (e.g., crack length, wavelength) trough an adjusting constant, e_0, dependent on each material. The magnitude of e_0 is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics. According to for a class of physically admissible kernel $\alpha(|x' - x|, \tau)$ it is possible to represent the integral constitutive relations given by Eq. (19) in an equivalent differential form as

$$(1 - (e_0 a)^2 \nabla^2)\sigma_{kl} = t_{kl} \tag{22}$$

where ∇^2 is the Laplacian operator. Thus, the scale length e_0a takes into account the size effect on the response of nanostructures. For an elastic material in the one dimensional case, the nonlocal constitutive relations may be simplified as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\,\varepsilon_{xx} \tag{23}$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G\,\gamma_{xz} \tag{24}$$

where σ and ε are the nonlocal stress and strain, respectively. E is the Young's modulus, $G=E/2(1+v)$ is the shear modulus (where v is the poisson's ratio). For Timoshenko nonlocal FG beam, Eqs. (23) and (24) can be rewritten as

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{25}$$

$$\sigma_{xz} - \mu \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G(z)\gamma_{xz} \tag{26}$$

where ($\mu=(e_0a)^2$). Integrating Eqs. (25) and (26) over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal Timoshenko FG beam theory can be obtained as follows

$$N-\mu\frac{\partial^2 N}{\partial x^2}=A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial \varphi}{\partial x} \tag{27}$$

$$M-\mu\frac{\partial^2 M}{\partial x^2}=B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial \varphi}{\partial x} \tag{28}$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2}=C_{xz}\left(\frac{\partial w}{\partial x}+\varphi\right) \tag{29}$$

in which the cross-sectional rigidities are defined as follows

$$(A_{xx},B_{xx},D_{xx})=\int_A E(z,T)(1,z,z^2)dA \tag{30}$$

$$C_{xz}=K_s\int_A G(z)\,dA \tag{31}$$

where the shear correction factor is assumed to be K_s=5/6. The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq. (17a) into Eq. (27) as follows

$$N=A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial \varphi}{\partial x}+\mu\left(I_0\frac{\partial^3 u}{\partial x\,\partial t^2}+I_1\frac{\partial^3 \varphi}{\partial x\,\partial t^2}\right) \tag{32}$$

Also, the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of M from Eq. (17c) into Eq. (28) as follows

$$M=B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial \varphi}{\partial x}+\mu\left(I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\,\partial t^2}+I_2\frac{\partial^3 \varphi}{\partial x\,\partial t^2}+N^T\frac{\partial^2 w}{\partial x^2}\right) \tag{33}$$

By substituting for the second derivative of Q from Eq. (17b) into Eq. (29), the following expression for the nonlocal shear force is derived

$$Q=C_{xz}\left(\frac{\partial w}{\partial x}+\varphi\right)+\mu\left(I_0\frac{\partial^3 w}{\partial x\,\partial t^2}+N^T\frac{\partial^3 w}{\partial x^3}\right) \tag{34}$$

The nonlocal governing equations of Timoshenko FG nanobeam in terms of the displacement can be derived by substituting for N, M and Q from Eqs. (32)-(34), respectively, into Eq. (17) as follows

$$A_{xx}\frac{\partial^2 u}{\partial x^2}+B_{xx}\frac{\partial^2 \varphi}{\partial x^2}+\mu\left(I_0\frac{\partial^4 u}{\partial t^2\partial x^2}+I_1\frac{\partial^4 \varphi}{\partial t^2\partial x^2}\right)-I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^2 \varphi}{\partial t^2}=0 \tag{35a}$$

$$C_{xz}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \varphi}{\partial x}\right)+\mu\left(N^T\frac{\partial^4 w}{\partial x^4}+I_0\frac{\partial^4 w}{\partial t^2\partial x^2}\right)-N^T\frac{\partial^2 w}{\partial x^2}-I_0\frac{\partial^2 w}{\partial t^2}=0 \tag{35b}$$

$$B_{xx}\frac{\partial^2 u}{\partial x^2}+D_{xx}\frac{\partial^2\varphi}{\partial x^2}-C_{xz}(\frac{\partial w}{\partial x}+\varphi)+\mu(I_1\frac{\partial^4 u}{\partial t^2\partial x^2}+I_2\frac{\partial^4\varphi}{\partial t^2\partial x^2})-I_1\frac{\partial^2 u}{\partial t^2}-I_2\frac{\partial^2\varphi}{\partial t^2}=0 \quad (35c)$$

3. Solution procedures

Here, based on the Navier type method, an analytical solution of the governing equations for free vibration and thermal buckling of a simply supported FG nanobeam is presented. The displacement functions are expressed as product of undetermined coefficients and known trigonometric functions to satisfy the governing equations and the conditions at x = 0, L. The following displacement fields are assumed to be of the form

$$u(x,t)=\sum_{n=1}^{\infty}U_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (36)$$

$$w(x,t)=\sum_{n=1}^{\infty}W_n\sin(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (37)$$

$$\varphi(x,t)=\sum_{n=1}^{\infty}\varphi_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \quad (38)$$

where (U_n, W_n, φ_n) are the unknown Fourier coefficients to be determined for each n value; also, ω_n is the natural frequency of the *n*-th mode of the nanobeam. Boundary conditions for simply supported beam are as Eq. (39)

$$u(0)=0\ ,\ \frac{\partial u}{\partial x}(L)=0$$

$$w(0)=w(L)=0\ ,\ \frac{\partial\varphi}{\partial x}(0)=\frac{\partial\varphi}{\partial x}(L)=0 \quad (39)$$

Substituting Eqs. (36)-(38) into Eqs. (35a)-(35c) respectively, leads to Eqs. (40)-(42)

$$(-A_{xx}(\frac{n\pi}{L})^2+I_0(1+\mu(\frac{n\pi}{L})^2)\omega_n{}^2)U_n+(-B_{xx}(\frac{n\pi}{L})^2+I_1(1+\mu(\frac{n\pi}{L})^2)\omega_n{}^2)\phi_n=0 \quad (40)$$

$$(-C_{xz}(\frac{n\pi}{L})^2+I_0(1+\mu(\frac{n\pi}{L})^2)\omega_n{}^2+N^T(\frac{n\pi}{L})^2(1+\mu(\frac{n\pi}{L})^2))W_n-C_{xz}(\frac{n\pi}{L})\phi_n=0 \quad (41)$$

$$\begin{aligned}&(-B_{xx}(\frac{n\pi}{L})^2+I_1(1+\mu(\frac{n\pi}{L})^2)\omega_n{}^2)U_n\\&+(-D_{xx}(\frac{n\pi}{L})^2-C_{xz}+I_2(1+\mu(\frac{n\pi}{L})^2)\omega_n{}^2))\phi_n-C_{xz}(\frac{n\pi}{L})W_n=0\end{aligned} \quad (42)$$

By setting the determinant of the coefficient matrix of the above equations, the analytical solutions can be obtained from the following equations

Table 2 Comparison of the nondimensional fundamental frequency for a S-S FG nanobeam with various gradient indexes when L/h=50

μ	p =0		p =0.2		p =1		p =5	
	Rahmani and Pedram (2014)	Present Analytical	Rahmani and Pedram (2014)	Present Analytical	Rahmani and Pedram (2014)	Present Analytical	Rahmani and Pedram (2014)	Present Analytical
0	9.8631	9.86315733	8.6895	8.68954599	6.9917	6.99174004	5.9389	5.93894397
1	9.4097	9.40973040	8.2901	8.29007206	6.6703	6.67031728	5.6659	5.66592012
2	9.0136	9.01358936	7.9411	7.94106762	6.3895	6.38950303	5.4274	5.42739007
3	8.6636	8.66360601	7.6327	7.63272858	6.1414	6.14140878	5.2166	5.21665314
4	8.3515	8.35146095	7.3577	7.35772548	5.9201	5.92013713	5.0287	5.02869994
5	8.0708	8.07079327	7.1104	7.11045428	5.7212	5.72117899	4.8597	4.85970034

Table 3 Material graduation and aspect ratio effect on the critical buckling temperature $\Delta T_{cr}[K]$ of S-S FG nanobeam with different nonlocality parameters

μ	L/h	Gradient index					
		0	0.2	0.5	1	2	5
0	40	68.6671	57.8509	50.5266	45.4570	41.9905	39.1223
	50	43.9712	37.0448	32.3547	29.1086	26.8894	25.0534
	60	30.5447	25.7331	22.4752	20.2204	18.6790	17.4039
1	40	62.4988	52.6542	45.9878	41.3736	38.2185	35.6080
	50	40.0212	33.7170	29.4482	26.4938	24.4739	22.8029
	60	27.8008	23.4215	20.4562	18.4040	17.0011	15.8405
2	40	57.3473	48.3141	42.1972	37.9634	35.0683	32.6729
	50	36.7224	30.9379	27.0209	24.3100	22.4566	20.9233
	60	25.5093	21.4910	18.7701	16.8870	15.5998	14.5349
3	40	52.9803	44.6350	38.9840	35.0725	32.3979	30.1849
	50	33.9261	28.5820	24.9633	22.4588	20.7466	19.3300
	60	23.5668	19.8545	17.3408	15.6011	14.4119	13.4280
4	40	49.2314	41.4766	36.2254	32.5907	30.1054	28.0490
	50	31.5254	26.5595	23.1969	20.8696	19.2785	17.9622
	60	21.8992	18.4496	16.1137	14.4972	13.3921	12.4779

$$\left\{([K]+\Delta T\,[K_T])-\omega^2[M]\right\}\begin{Bmatrix}U_n\\W_n\\\phi_n\end{Bmatrix}=0 \tag{43}$$

where $[K]$ and $[K_T]$ are stiffness matrix and the coefficient matrix of temperature change, respectively, and $[M]$ is the mass matrix. By setting this polynomial to zero, we can find natural frequencies ω_n and critical buckling temperature ΔT_{cr}.

4. Numerical results and discussions

Through this section, the effect of temperature change, FG distribution, nonlocality effect and thickness ratios on the natural frequencies and critical buckling temperature of the FG nanobeam will be figured out. The functionally graded nanobeam is composed of Steel (SUS304) and Silicon nitride (Si_3N_4) where its properties are given in Table 1. The bottom surface of the beam is pure Steel, whereas the top surface of the beam is pure Silicon nitride. The beam geometry has the following dimensions: L (length) = 10,000 nm, b (width) = 1000 nm and h (thickness) = 100 nm. Relation described in Eq. (44) are performed in order to calculate the non-dimensional natural frequencies

$$\hat{\omega} = \omega L^2 \sqrt{\rho_c A / EI_c} \tag{44}$$

where I=bh^3/12 is the moment of inertia of the cross section of the beam. To evaluate accuracy of the natural frequencies predicted by the present method, the non-dimensional natural frequencies of simply supported FG nanobeam with various nonlocal parameters previously analyzed by Navier method are reexamined. Table 2 compares the results of the present study and the results presented by Rahmani and Pedram (2014) which has been obtained by analytical method for FG nanobeam with different nonlocal parameters (varying from 0 to 5). The reliability of the presented method and procedure for FG nanobeam may be concluded from Table 2; where the results are in an excellent agreement as values of non-dimensional fundamental frequency are consistent with presented analytical solution.

After extensive validation of the present formulation for S-S FG nanobeams, the effects of different parameters such as aspect ratio, nonlocality parameter and gradient index on the thermal buckling of FG nanobeam are investigated.

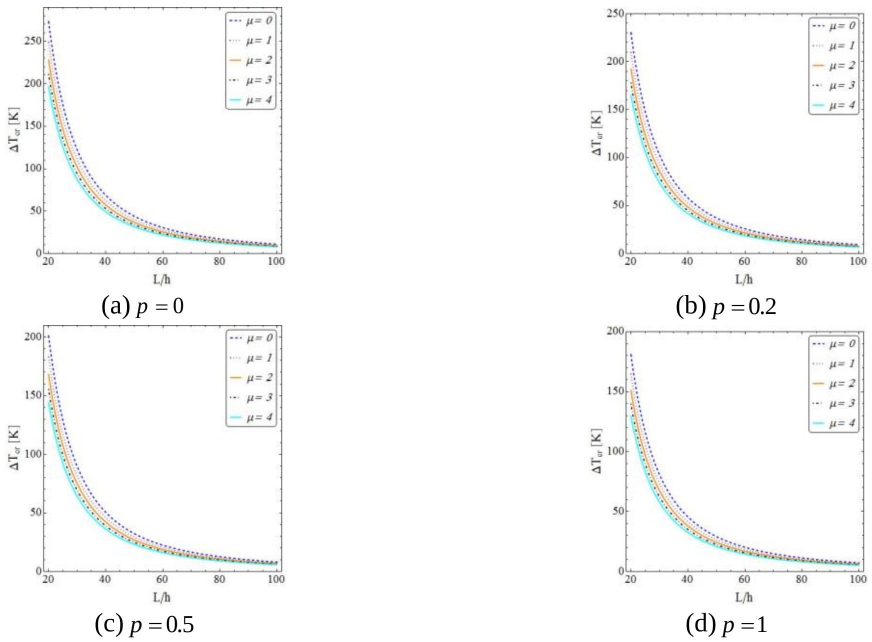
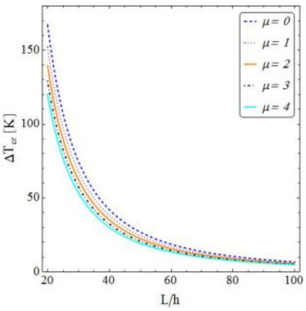
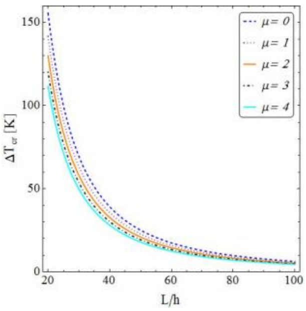


Fig. 2 The variation of the critical buckling temperature of S-S FG nanobeam with aspect ratios and nonlocality parameters for different material graduations

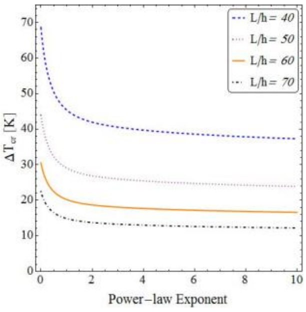


(e) $p = 2$

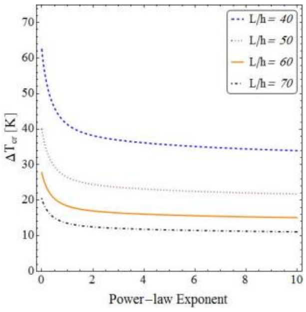


(f) $p = 5$

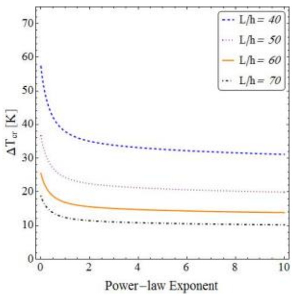
Fig. 2 The variation of the critical buckling temperature of S-S FG nanobeam with aspect ratios and nonlocality parameters for different material graduations



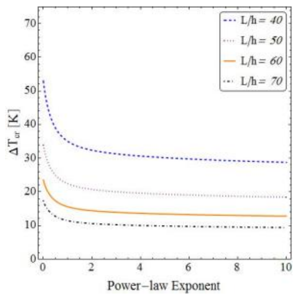
(a) $\mu = 0$



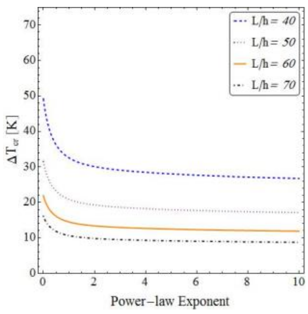
(b) $\mu = 1$



(c) $\mu = 2$



(d) $\mu = 3$



(e) $\mu = 4$

Fig. 3 The variation of the critical buckling temperature of S-S FG nanobeam with material graduations and aspect ratios for different nonlocality parameters

Table 4 Temperature and material graduation effect on first three dimensionless frequency of S-S FG nanobeams with different nonlocality parameters (*L*/*h*=20)

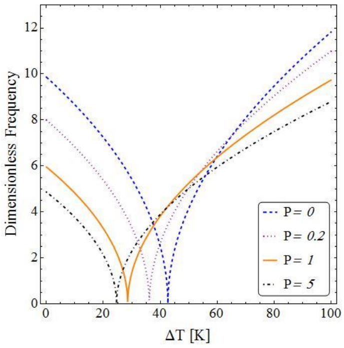
μ	$\hat{\omega}_i$	$\Delta T=10[K]$				$\Delta T=30[K]$				$\Delta T=60[K]$			
		Gradient index				Gradient index				Gradient index			
		0	0.2	1	5	0	0.2	1	5	0	0.2	1	5
0	1	9.6468	7.7926	5.7659	4.6883	9.2623	7.4213	5.4110	4.3483	8.6318	6.8045	4.8070	3.7575
	2	38.6694	31.3157	23.2770	18.9886	38.2938	30.9590	22.9447	18.6744	37.7033	30.3942	22.4123	18.1675
	3	85.5823	69.3223	51.5567	42.0603	85.2047	68.9703	51.2374	41.7617	84.6153	68.4140	50.7229	41.2756
1	1	9.1859	7.4175	5.4848	4.4575	8.7813	7.0264	5.1102	4.0980	8.1135	6.3712	4.4652	3.4644
	2	32.6814	26.4571	19.6535	16.0250	32.2361	26.0321	19.2547	15.6466	31.5324	25.3561	18.6131	15.0333
	3	62.1612	50.3329	37.4103	30.5046	61.6403	49.8398	36.9524	30.0716	60.8230	49.0599	36.2193	29.3733
2	1	8.7825	7.0891	5.2385	4.2552	8.3584	6.6786	4.8446	3.8766	7.6537	5.9855	4.1584	3.1991
	2	28.7981	23.3051	17.3012	14.1002	28.2917	22.8204	16.8443	13.6657	27.4873	22.0451	16.1045	12.9558
	3	51.1639	41.4132	30.7611	25.0703	50.5298	40.8092	30.1952	24.5328	49.5295	39.8499	29.2862	23.6630
3	1	8.4254	6.7984	5.0203	4.0758	7.9824	6.3690	4.6075	3.6785	7.2412	5.6378	3.8793	2.9556
	2	26.0171	21.0469	15.6149	12.7197	25.4554	20.5081	15.1054	12.2341	24.5582	19.6409	14.2738	11.4335
	3	44.4478	35.9639	26.6963	21.7465	43.7164	35.2648	26.0379	21.1193	42.5562	34.1482	24.9737	20.0971
4	1	8.1063	6.5384	4.8251	3.9152	7.6448	6.0907	4.3938	3.4995	6.8673	5.3213	3.6226	2.7293
	2	23.8979	19.3256	14.3287	11.6662	23.2852	18.7368	13.7703	11.1331	22.3008	17.7828	12.8512	10.2451
	3	39.7998	32.1912	23.8803	19.4427	38.9812	31.4069	23.1389	18.7349	37.6756	30.1465	21.9314	17.5709

In Table 3, critical buckling temperature of the simply supported FG nanobeams are presented for various values of the gradient index (p =0,0.2,0.5,1,2,5), nonlocal parameters (μ=0,1,2,3,4) and three different values of aspect ratio (*L*/*h*=40, 50, 60) based on analytical Navier solution method. It can be concluded from the results of the table that an increase in nonlocal scale parameter gives rise to a decrement in the critical buckling temperature. In addition, it is seen that the ΔT_{cr} decrease by increasing gradient index and aspect ratio (*L*/*h*) and it can be stated that nonlocality parameter has a notable effect on the critical buckling temperature, so that by fixing other parameters and increasing nonlocal parameter from 0 to 4 the ΔT_{cr} decreases about 28%.

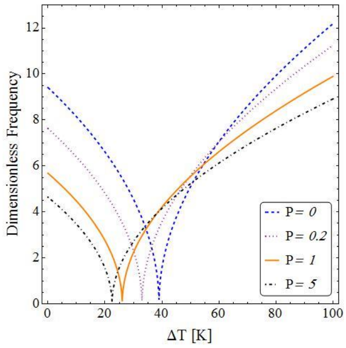
Variations of the critical buckling temperature ΔT_{cr} of the simply supported FG nanobeams with respect to aspect ratio for different values of gradient indexes and nonlocal parameters are depicted in Fig. 2. Observing the figure, it is easily deduced for S-S FG nanobeams that, an increase in aspect ratio parameter gives rise to a decrease in the critical buckling temperature for all gradient indexes. In addition, it is deduced that the buckling temperature decreases by increasing nonlocality parameters.

The critical buckling temperature versus the gradient index of S-S FG nanobeam for different values of nonlocality parameters and aspect ratio is illustrated in Fig. 3. It can be observed from the figure that with an increase of the beam aspect ratio and nonlocal parameter, critical buckling temperature decreases. Also in this diagram it is noticed that, the buckling temperature reduces with high rate where the power exponent in range from 0 to 2 than that where power exponent in range between 2 and 10.

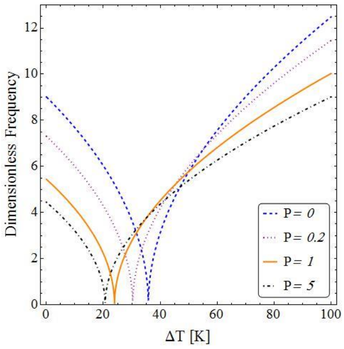
In order to investigate the vibration characteristics of the Timoshenko FG nanobeam, the first three non-dimensional fundamental frequencies of simply-supported FG nanobeam is presented in



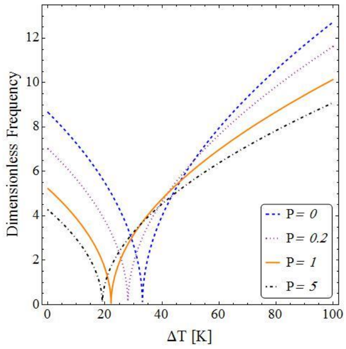
(a) $\mu = 0$



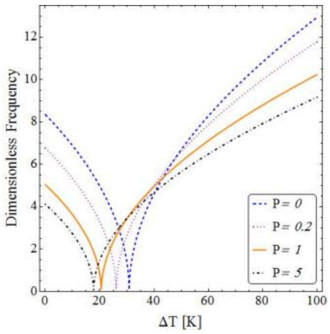
(b) $\mu = 1$



(c) $\mu = 2$



(d) $\mu = 3$



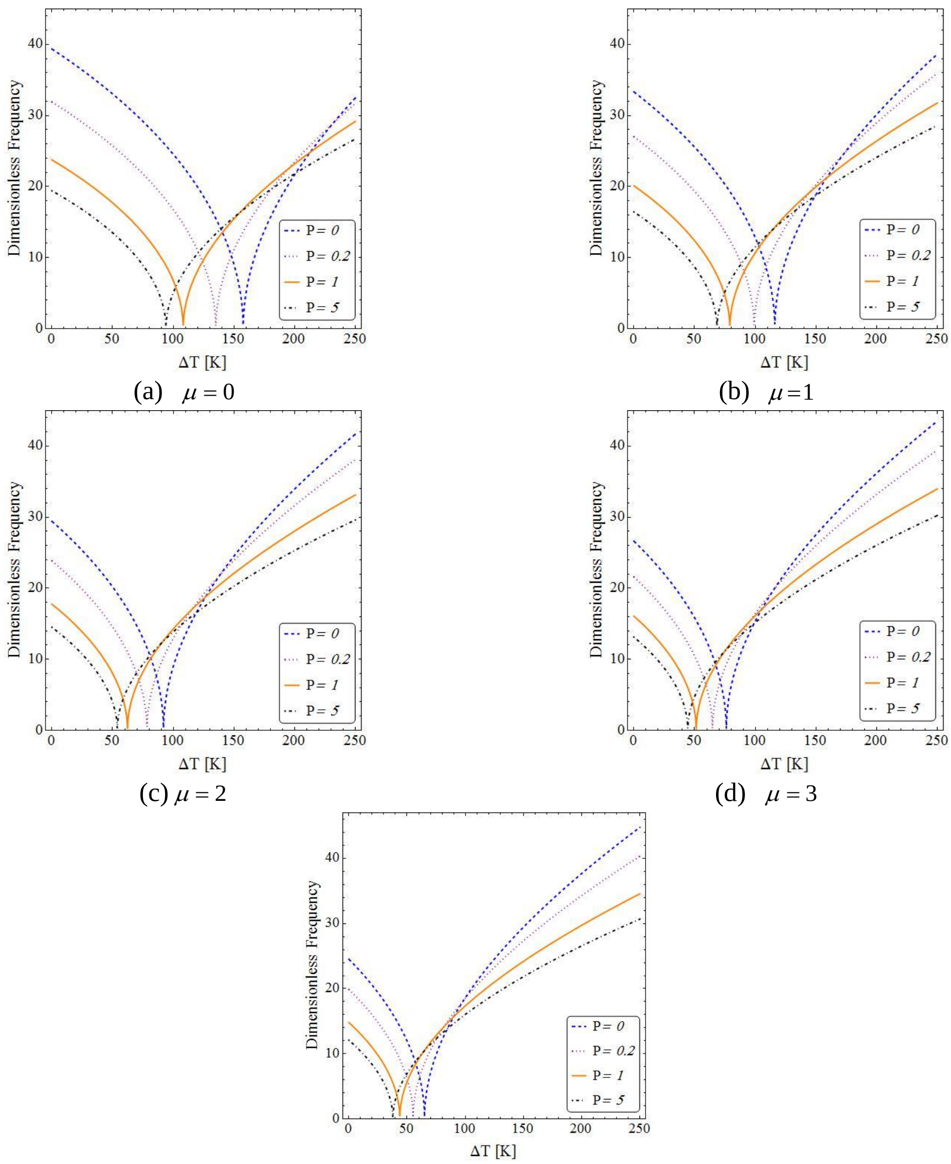
(e) $\mu = 4$

Fig. 4 Variations of the first dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of gradient indexes and nonlocal parameters (L/h=50)

Table 4, which figures out the effect of nonlocal parameter (varying from 0 to 4), gradient index (varying from 0 to 5) and three different values of temperature changes (ΔT=10, 30, 60) for L/h =20 on the natural frequency characteristics of FG nanobeam.

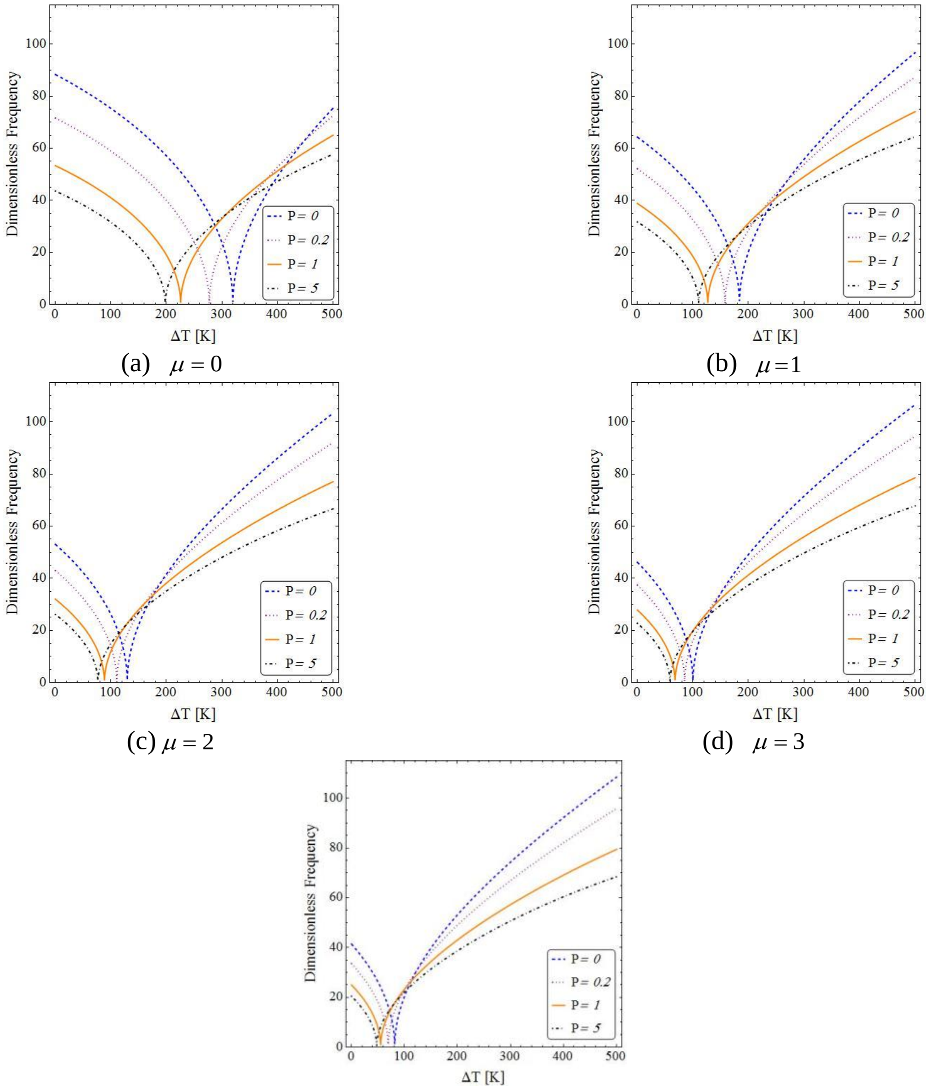
First of all, when the two parameters vanish (μ=0 and p=0) the classical isotropic beam theory is rendered. Furthermore, the effects of temperature change, nonlocal parameter and gradient indexes on the dimensionless frequencies are presented in this table. It can be concluded from the results of the Table 4 that increasing the gradient index parameter yields the reduction in dimensionless frequencies for every nonlocality parameter and temperature change, which highlights the significance of the material distribution parameter. However, the increasing of nonlocal parameter causes the decreasing in fundamental frequency, at a constant material graduation index. In addition, it is seen that the first three dimensionless natural frequencies decrease by increasing temperature change and it can be stated that temperature change has a significant effect on the dimensionless natural frequencies, especially for lower mode numbers.

Variations of the first three dimensionless natural frequencies of the simply supported FG nanobeams with respect to temperature changes for different values of gradient indexes and nonlocal parameters are plotted in Figs. 4, 5 and 6, respectively.



(a) $\mu = 0$ (b) $\mu = 1$ (c) $\mu = 2$ (d) $\mu = 3$ (e) $\mu = 4$

Fig. 5 Variations of the second dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of gradient indexes and nonlocal parameters (L/h=50)



(a) $\mu = 0$ (b) $\mu = 1$

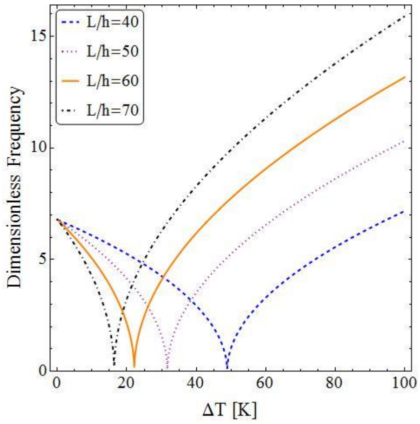
(c) $\mu = 2$ (d) $\mu = 3$

(e) $\mu = 4$

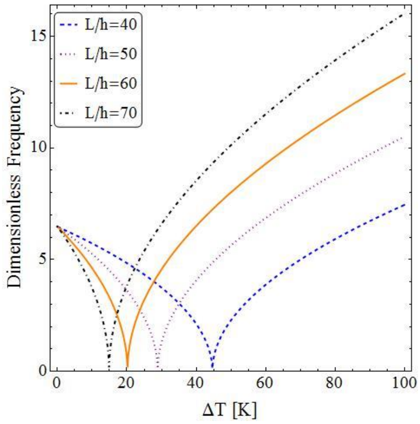
Fig. 6 Variations of the third dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of gradient indexes and nonlocal parameters (L/h=50)

It is seen from the figures that the fundamental frequency of FG nanobeam decreases with the increase of temperature until it approaches to the critical buckling temperature. This is due to the reduction in total stiffness of the beam, since geometrical stiffness decreases when temperature rises. Frequency reaches to zero at the critical temperature point. The increase in temperature yields in higher frequency after the branching point.

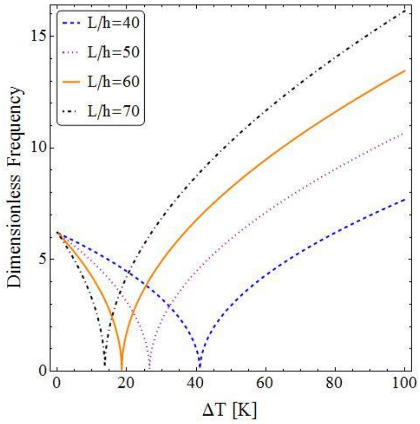
One important observation within the range of temperature before the critical temperature, it is seen that the FG nanobeams with lower value of gradient index (higher percentage of ceramic phase) usually provide larger values of the frequency results. However, this behavior is opposite in the range of temperature beyond the critical temperature. It is also observable that the branching



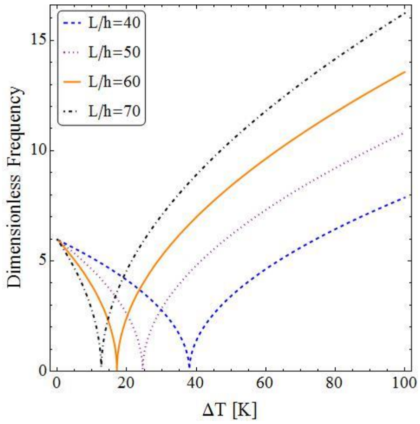
(a) $\mu = 0$



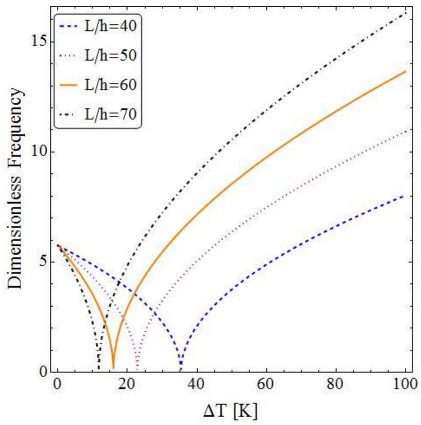
(b) $\mu = 1$



(c) $\mu = 2$



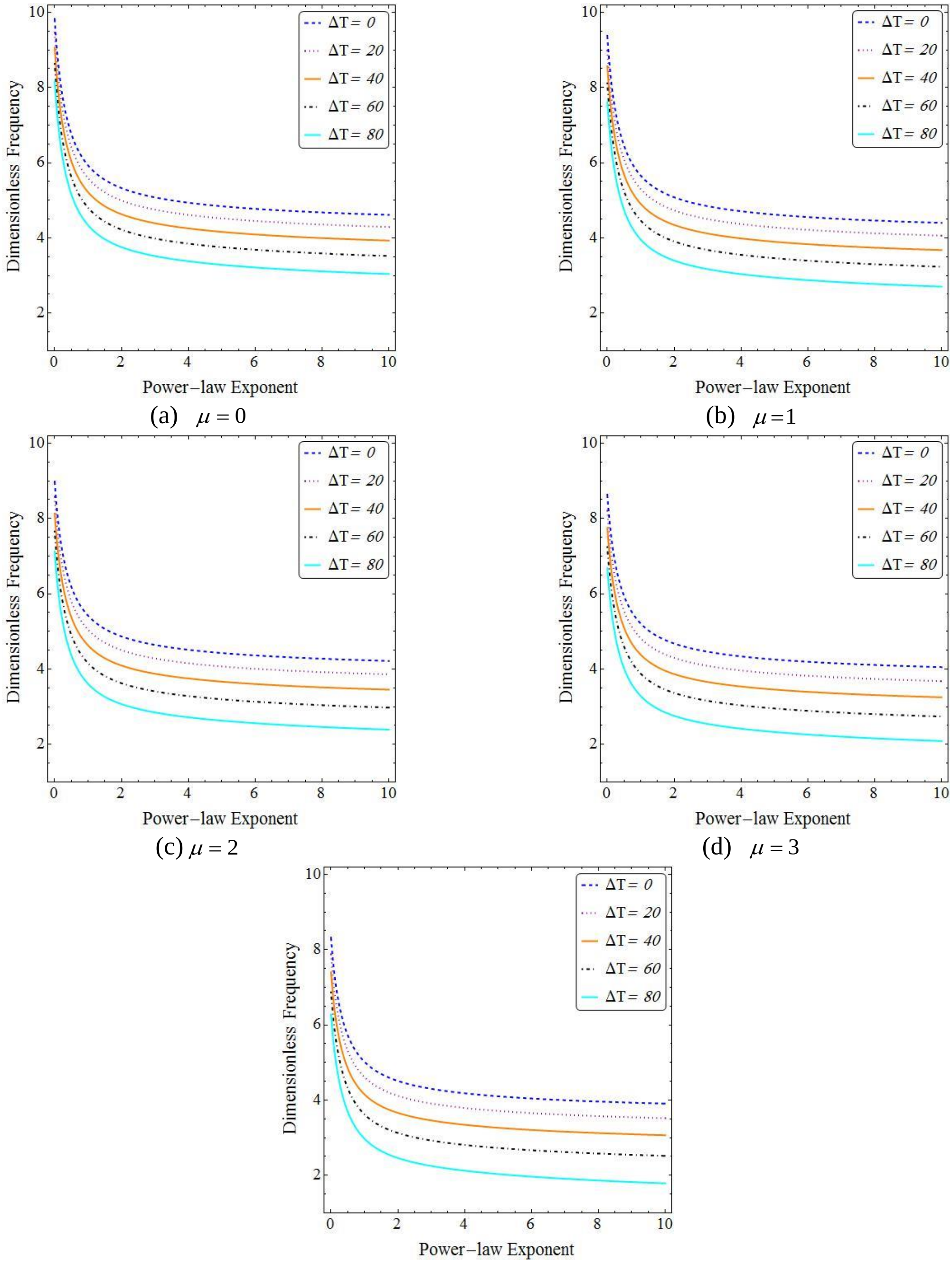
(d) $\mu = 3$



(e) $\mu = 4$

Fig. 7 Variations of the first dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of nonlocal parameters and aspect ratios (p=0.5)

point of the FG nanobeam is postponed by consideration of the lower gradient indexes due to the fact that the lower gradient indexes result in the increase of stiffness of the structure.



(a) $\mu = 0$ (b) $\mu = 1$

(c) $\mu = 2$ (d) $\mu = 3$

(e) $\mu = 4$

Fig. 8 The variation of the first dimensionless frequency of S-S FG nanobeam with material graduation and temperatures for different nonlocality parameters (L/h=20)

In addition, Fig. 7 demonstrates the variation of the first dimensionless natural frequency of FG nanobeam respect to temperature changes with varying of the aspect ratio and nonlocality parameter at p = 0.5. It is shown that, by increasing the nonlocal parameter and aspect ratio, the critical buckling temperature decreases at a constant material distribution.

The fundamental frequency parameter as a function of power law indexes and temperature rise is presented in Fig. 8 for the FG nanobeam with S–S boundary condition. It can be easily seen that all of the previous discussed phenomena can be understood based on this figure. In fact, the decreasing influence of nonlocality, gradient index and temperature change can be observed.

5. Conclusions

Thermal buckling and vibrational behavior of the temperature-dependent FG nanobeams in thermal environment are investigated on the basis of nonlocal elasticity theory in conjunction with Navier analytical method. Eringen's theory of nonlocal elasticity together with Timoshenko beam theory are used to model the nanobeam. Thermo-mechanical properties of the FG nanobeams are assumed to be functions of both temperature and thickness. The governing differential equations and related boundary conditions in thermal environment are derived by implementing Hamilton's principle. Accuracy of the results is examined using available date in the literature. Finally, through some parametric study and numerical examples, the effect of different parameters are investigated. The effects of small scale parameter, material property gradient index, mode number, temperature change and aspect ratio on critical buckling temperature and fundamental frequencies of FG nanobeams are investigated.

It is concluded that various factors such as nonlocal parameter, gradient index, temperature-dependent material properties, thermal environment and aspect ratio play important roles in dynamic behavior of FG nanobeams. It is illustrated that presence of nonlocality leads to reduction in natural frequency and buckling temperature. It is observed that the fundamental frequency decreases with the increase in temperature and tends to the minimum point closing to zero at the critical temperature. This decrease in frequency with thermal load is attributed to the fact that the thermally induced compressive stress weakens the beam stiffness. However, after the critical temperature region, the fundamental frequency increases with the increment of temperature. Also, it is concluded that under temperature rise, with the increase in the gradient index value leads to the decrease in frequency; however, the trend is reversed after the pre-buckling stage is passed. Moreover, it is revealed that critical buckling temperature decreases with the increase in aspect ratio and gradient index.

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