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A lower bound analytical estimation of the fundamental lateral frequency down-shift of items subjected to sine testing

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Abstract. The dynamic coupling between shaker and test-article has been investigated by recent research through the so called Virtual Shaker Testing (VST) approach. Basically a VST model includes the mathematical models of the test-item, of the shaker body, of the seismic mass and the facility vibration control algorithm. The subsequent coupled dynamic simulation even if more complex than the classical hard-mounted sine test-prediction, is a closer representation of the reality and is expected to be more accurate. One of the most remarkable benefits of VST is the accurate quantification of the frequency down-shift (with respect to the hard-mounted value), typically affecting the first lateral resonance of heavy test-items, like medium or large size Spacecraft (S/Cs), once mounted on the shaker. In this work, starting from previous successful VST experiences, the parameters having impact on the frequency shift are identified and discussed one by one. A simplified analytical system is thus defined to propose an efficient and effective way of calculating the lower bound frequency shift through a simple equation. Such equation can be useful to correct the S/C lateral natural frequency measured during the test, in order to remove the contribution attributable to the shaker in use. The so-corrected frequency value becomes relevant when verifying the compliance of the S/C w.r.t. the frequency requirement from the Launcher Authority. Moreover, it allows to perform a consistent post-test correlation of the first lateral natural frequency of S/C FE model.

Keywords: vibration testing; seismic mass; frequency shift; Virtual Shaker Testing

1. Introduction

In the aerospace industry the base-shake sine testing is a standard technique employed to ensure that the structure will withstand the low frequency vibration loads. Ideally, the input accelerations on the tested structure would be the same as the ones declared in the sine test specification. However, this ideal situation is not practical for multiple reasons. Among others, one of the most impacting effects is given by the dynamic coupling between the shaker and the test article, which grows as the modal effective masses of the article increase. Since the numerical sine test-prediction is commonly obtained by imposing the nominal acceleration of the test specification directly to the base of the Spacecraft (S/C), the numerically predicted structural responses of the tested structure are typically not in good agreement with the ones from the test. Some works (Allen et. al. 2011, Mayes 2012, Mayes *et al.* 2014) look into the differences between the fixed-

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base modes and the dynamic behaviour on a shaker and provide a methodology to recover the fixed-base modes from shaker run acquisitions.

The Virtual Shaker Testing (VST) approach is a promising way to include the dynamic coupling between the shaker and the test article in the numerical sine test-prediction. Basically, VST consists of a refined modeling that includes the item to be tested, the shaker with its coil (the excitation means), the seismic mass and the facility vibration control system altogether. The subsequent transient simulation will be representative of the vibration test on the specific facility which is used for it, accounting for all the parameters which may impact or alter the dynamic behaviour of the payload (e.g. S/C-shaker dynamic coupling, choice of compression factor, sine sweep rate effect and beatings). In principle, multiple VST simulations may be conducted by considering the same test-item but referring to different test facilities. The results would help the program manager in choosing the best solution for conducting the test campaign and mitigate the risks. VST can also prevent the possible overtesting of large S/Cs and can lead to better manage the dynamic coupling between specimen and shaker. Moreover, VST allows performing a physically consistent S/C FE model updating from test responses. The latter being a critical task for the S/C-Launch Vehicle final Coupled Load Analysis.

Initial experiences of VST were performed by Appolloni and Cozzani (2007), followed by other works (Ricci *et al.* 2008, Ricci *et al.* 2009). The idea at the basis of these works is to initially identify the parameters that may influence the test. A simplified model of the test-item and of the shaker is thus considered under dynamic coupling. The vibration control system is also included in the modeling, by referring to an integrated computational environment. In these cases, the authors concluded that, by adequate upgrade of the controller and acting on the parameters identified as critical for the dynamic coupling, it is possible to enhance the quality of the test and the performances of the facility.

More recently, some papers (Waimer *et al.* 2015a, Waimer *et al.* 2015b, Waimer *et al.* 2016, Waimer *et al.* 2018) were published about the experimental system identification of an electrodynamic shaker, which is a topic that should be addressed in VST for reducing the error of modeling. Additional works (Remedia *et al.*, 2017) also demonstrated how VST can be used to significantly improve post-test FE model correlation. A Special Issue of AAS edited by Nali *et al.* in 2018 provides a number of papers written between 2016 and 2018 on the topic of vibration testing issues and the VST approach. A disseminating overview of the VST approach and the description of the first VST application to a flight S/C can be found in Nali and Bettacchioli 2017 and Nali *et al.* 2018a, respectively.

To the best of the authors' knowledge, there is no literature about a simplified formulation able to provide or anticipate some relevant results from a possible complete VST analysis. As a consequence, this work aims at making available a simple analytical formulation useful to calculate at once the shift typically affecting the first lateral resonance of heavy test-items (like medium or large size S/Cs), without having to set up a dedicated VST analysis for the purpose.

The benefits of quantifying the test-item frequency shift that is attributable to the shaker assembly in use lie in the following two major points.

1. Having a consistent justification for the frequency down-shift of the S/C main lateral mode (e.g. towards the frequency requirement from the Launcher Authority).
2. Making possible a consistent post-test FE model correlation for the S/C first lateral natural frequency.

Some preliminary results of this work were presented in 2018 (Nali *et al.* 2018b). Limitations and assumptions will be detailed throughout the paper, together with the analytical development required to obtain the proposed set of equations. Hence, the analytical results will be compared and discussed with respect to previous well correlated VST analyses and test data.

2. Analytıcal formulatıon for calculatıng the natural frequency shıft

In this section several analytical developments are illustrated by referring to S/C sine test in lateral configuration, where a Vibration Test Adapter (VTA) is employed. The starting point of the analytical formulation is shared with a previous work (Nali *et al.* 2018b), but a number of assumptions are now removed in order to refine the accuracy of the results. The final aim is to make available an easy-to-use analytical formulation for calculating the natural frequency shift of the first lateral mode of the S/C+VTA assembly attributable to the shaker assembly in use.

The following hypotheses are made hereafter.

a) In-plane motion.
b) Simple harmonic motion.
c) The seismic mass is assumed as a rigid body.

The assumption a) was confirmed to be in line with real test results also for unbalanced seismic masses, as illustrated in Nali *et al.* 2018b. The assumption b) lead to have agreement w.r.t. test results in case of slow sine sweep rate. The correction available in the literature can be considered in case of medium or high sine sweep rates (Lollock 2002, Roy and Girard 2012, ECSS-E-HB-32-26A 2013, Nali and Bettachioli 2016, Roy *et al.* 2018).

The impact of the assumption c) was quantified by running a number of complete VST analyses, in terms of a slight frequency shift (Nali *et al.* 2018b). In this work, the assumption c) is retained, at the cost of extrapolating (through some simple algebraic formulas) just a lower bound value for the natural frequency of the test-item. Such easy-to-obtain information will be useful on its own, when complete VST analysis is not available.

2.1 Calculating the torsion stiffness of the slip table, bearings and oil meatus assembly

The S/C+VTA assembly, when vibrating at its first lateral natural frequency expresses a certain torsion stiffness k_t to its hard-mounted base. By increasing the number of components considered in the assembly, the underneath system composed by the slip table, the slip table bearings and the pressurized oil meatus (located between slip table and seismic mass) introduces a given stiffness relief $\Delta k_t < 0$. Let k_{te} be the equivalent torsion stiffness of the assembly resulting at the level just below the bearings and the oil meatus (that is, excluding the seismic mass flexibility and its motion from the problem, by constraining all the DOFs of the seismic mass):

$$k_{te} = k_t + \Delta k_t. \tag{1}$$

The first natural frequency of the S/C+VTA assembly f_n is given by:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_t}{m_{S/C}h_{1d}^2}}, \tag{2}$$

where $m_{S/C}$ is the mass of the test-item+VTA assembly and h_{1d} is its dynamic CoG, calculated for the first lateral mode[a].

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[a]For the first lateral S/C+VTA mode, h_{1d} often falls between 15 h_1 and 35 h_1, depending on the dynamic properties of the S/C+VTA assembly, and it is normally calculated as the ratio between the lateral base moment and lateral base force occurring at the frequency of interest.

k_t can thus be calculated as follows:

$$k_t = 4\pi^2 m_{S/C} h_{1d}^2 f_n^2. \tag{3}$$

By expressing k_t as a Taylor series expansion about f_n:

$$k_t = 4\pi^2 m_{S/C} h_{1d}^2 \left(f_n^2 + 2 f_n \Delta f_n + \Delta {f_n}^2\right),$$

Δk_t can be calculated by retaining only the terms having Δf_n:

$$k_t = 4\pi^2 m_{S/C} h_{1d}^2 f_n^2. \tag{4}$$

It is useful to calculate the equivalent torsion stiffness of the assembly given by slip table stiffness, the slip table bearings and the pressurized oil meatus, which will be indicated as k_m. Its value mainly depends on the pressurized oil meatus and the VTA footprint onto the slip table. k_m thus represents an intrinsic property of the shaker and the VTA in use.

Since k_{te} is the stiffness corresponding to series of k_t and k_{tm}, the following system can be written:

$$\begin{cases} k_{te} = k_t + \Delta k_t \\ k_{te} = \frac{k_t k_{tm}}{k_t + k_{tm}} \end{cases}, \tag{5}$$

Leading to k_{tm}:

$$k_{tm} = -\frac{k_t^2 + k_t \Delta k_t}{\Delta k_t}. \tag{6}$$

The resulting new natural frequency, decreased by the effect given by k_m is:

$$f_n' = \frac{1}{2\pi}\sqrt{\frac{k_{te}}{m_{S/C} h_{1d}^2}}, \tag{7}$$

where " ' " stands for "updated value" and "not for derivative".

f_n' is the frequency of the S/C+VTA assembly inclusive of the effect of slip table stiffness, slip table bearings and oil meatus.

The following equality is also respected:

$$f_n' = f_n + \Delta f_n. \tag{8}$$

2.2 Calculating the torsion stiffness of the slip table, bearings and oil meatus assembly

Fig. 1 illustrates the simplified system useful to evaluate the effect of the seismic mass inertia on the first lateral frequency measured during the sine test. $m_{S/C}$ and $I_{S/C}$ indicate respectively the mass of the S/C+VTA assembly and its inertia w.r.t. the VTA base. h_3 gives the distance between the longitudinal axis of the S/C+VTA assembly and the CoG of the seismic mass. m_{SM} and I_{SM} indicate respectively the mass and moment of inertia of the seismic mass with respect to its CoG. The translation of m_{SM}, is neglected in the following since it doesn't significantly impact the results.

Fig. 2 describes the relevant rotations of the system dynamics. Counterclockwise rotations are taken as positive: θ_1 and θ_2 are always opposite in sign for equilibrium reasons. The system is considered as free-free since the constraint represented by the seismic mass mounting suspensions

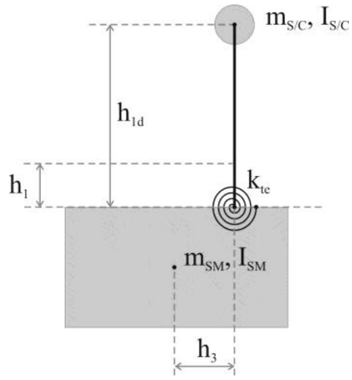

Fig. 1 Geometrical scheme and parameters of the system

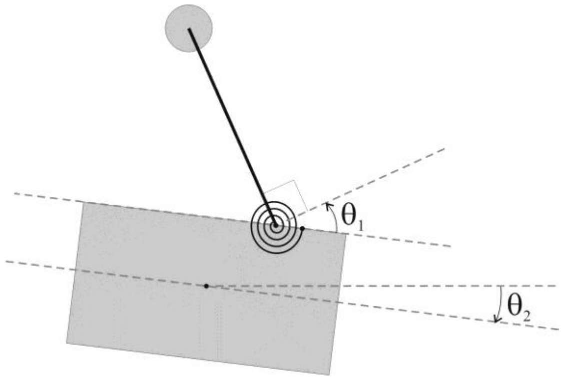

Fig. 2 Rotations characterizing the system, counterclockwise rotations are positives

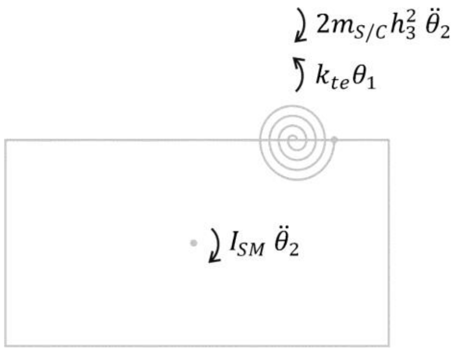

Fig. 3 Equilibrium of moments around the CoG of the seismic mass

do not significantly affect the results, having an impact only at very low frequencies (below 6 Hz in most shakers).

The equilibrium of moments around the CoG of the seismic mass in Fig. 1 can be written as follows, according to the scheme in Fig. 3 (sliding is considered between slip table and seismic mass, implying that no lateral force is developed at the location of torsion spring):

$$\left(I_{SM} + 2m_{S/C}h_3^2\right)\ddot{\theta}_2 - k_{te}\theta_1 = 0\,. \tag{9}$$

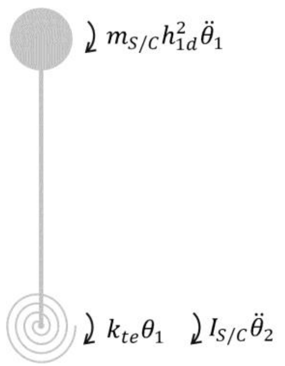

Fig. 4 Equilibrium of moments around the S/C-VTA assembly

Rotations θ_1 and θ_2 are in relation as they respect the following equilibrium of moments around the S/C-VTA assembly, according to Fig. 4:

$$m_{S/C}h_{1d}^2\ddot{\theta}_1 + I_{S/C}\ddot{\theta}_2 + k_{te}\theta_1 = 0. \tag{10}$$

The geometrical location of moments in Fig.s 3 and 4 is worthless and it is defined in order to make the picture more clear to the reader.

It is worth mentioning that the term $2mh_3^2\,\ddot{\theta}_2$ in Eq. (9) gives the moment provided by the reciprocal longitudinal movements of the S/C-VTA assembly and the seismic mass. Such movements are driven by the longitudinal stiffness of the pressurized oil meatus and bearings below the slip table. Such displacements are practically very small since the above longitudinal stiffness is very high (they are not represented in Fig.s 2). The factor 2 indicates that the above contribution is given by two mutual longitudinal forces equal in modulus ($mh_3^2\,\ddot{\theta}_2$) and opposite in sign (opposite-phase motion). As a consequence, forces occurring on the top and the bottom face of the pressurized oil meatus are balanced out. The dynamic behaviour described above is explained by the fact that, once in lateral configuration, no longitudinal excitation is provided to the assembly and therefore the forces applied to the oil meatus have to be balanced along the longitudinal direction.

From Eq. (9) we have that:

$$\ddot{\theta}_2 = \frac{k_{te}}{I_{SM}+2m_{S/C}h_3^2}\theta_1. \tag{11}$$

Eq. (10) can thus be rewritten as:

$$\ddot{\theta}_1 + \left\{\frac{I_{S/C}\,k_{te}}{m_{S/C}h_{1d}^2(I_{SM}+2m_{S/C}h_3^2)} + \frac{k_{te}}{m_{S/C}h_{1d}^2}\right\}\theta_1 = 0. \tag{12}$$

The circular frequency from Eq. (12) is given by the square root of the coefficient multiplying θ_1. It follows that the lateral frequency of the entire system f_n'', which is inclusive of the effect of the seismic mass inertia, is the following:

$$f_n'' = \frac{1}{2\pi}\sqrt{\frac{k_{te}\{I_{S/C}+I_{SM}+2m_{S/C}h_3^2\}}{m_{S/C}h_{1d}^2(I_{SM}+2m_{S/C}h_3^2)}}. \tag{13}$$

The first lateral frequency shift, given by $f_n - f_n''$, is now attributable to the specific shaker assembly in use:

$$f_{shift} = f_n'' - f_n. \tag{14}$$

Note. f_{shift} is here found analytically by simply assuming the seismic mass as a rigid body. It is understood that, in the real case, the elastic behaviour of the seismic mass leads to a certain reduction of f_n'', growing the actual frequency shift. The relevant information provided by f_{shift}, intended just as a difference between values, will be illustrated in the section of numerical results.

2.3 Calculating the torsion stiffness of the slip table, bearings and oil meatus assembly

After the sine test of a generic S/C, f_n can be calculated from the $f_n^{sine\_test}$, which is the one measured on the shaker, possibly corrected by the sine sweep rate effect (Lollock 2002, Roy and Girard 2012, ECSS-E-HB-32-26A 2013, Nali and Bettachioli 2016).

At this point, writing

$$f_n^{sine\_test} < f_n'' \tag{15}$$

recalls that, in the formulation presented here, the flexibility of the seismic mass (leading to decrease the measured resonant frequency) is not considered, according to the above assumption c).

From Eqs. (7),(13),(15), f_n' can be rewritten as a function of $f_n^{sine\_test}$:

$$f_n' > f_n^{sine\_test} \sqrt{\frac{I_{SM}+2m_{S/C}h_3^2}{I_{S/C}+I_{SM}+2m_{S/C}h_3^2}}, \tag{16}$$

where $m_{S/C}$, h_{1d}, h_3 and $I_{S/C}$ are related to the specific S/C+VTA under test. The stiffness k_{te} can now be calculated as:

$$k_{te} > 4\pi^2 m h_{1d}^2 f_n^{sine\_test\ 2}, \tag{17}$$

which, from the second equation of the system in Eqs. (5) gives:

$$k_t > \frac{k_{te}k_{tm}}{k_{tm} - k_{te}}. \tag{18}$$

k_{tm} has to come from a previous correlation performed with a test-item using the same VTA. Finally, the lower bound value of f_n is given by Eq.s (2),(18):

$$f_n > \frac{1}{2\pi}\sqrt{\frac{k_t}{m_{S/C}h_{1d}^2}}. \tag{19}$$

As mentioned in the introduction, the value of f_n extrapolated by test data, can be used to:

a) have an argument to refine the compliance verification of the hard-mounted S/C lateral frequency w.r.t. the frequency requirement from the Launcher Authority;
b) perform a consistent post-test correlation of the S/C FE model of the S/C+VTA assembly for the first lateral natural frequency.

2.4 Limitations of the proposed analytical formulation w.r.t. the VST FEM analysis

The VST FEM analysis can provide additional information on top of the natural frequency shift, which is here handled through a simplified analytical formulation. As a result, the VST FEM analysis should be performed if one or more points among the following list are of interest.

1. Cross-talk quantification (typically due to the seismic motion or to the test-item CoG in-plane offset).
2. Prediction of possible vibration control issues and identification of measures to mitigate them:
   a. by refining the notching strategy and the abort thresholds, depending on the predicted overshoots;
   b. by making a specific choice of the compression factor and the sine sweep rate (which might be tailored for different frequency ranges) in order to improve the test pilot profile.
3. A more accurate sine test-prediction is required w.r.t. the classical hard-mounted one.
4. Multiple VST analysis may be performed by referring to various shaker-assemblies in aiming at identifying the most suitable facility for the test (if multiple options are under consideration).
5. Prediction of the effectiveness of possible shaker assembly refurbishments.

3. Numerical results: use of the proposed analytical formulation

In this section the analytical formulation presented above is assessed towards a test experience held on V994 shaker assembly of Thales Alenia Space in Rome. Numerical results are obtained by adopting a numerical precision greater than the one given by the number of significant digits reported in the following steps. The tested structure is illustrated in Fig. 5. Input parameters are given in Table 1.

Table 1 Input parameters

I_{SM}	$471139\ kgm^2$
$I_{S/C}$	$7877\ kgm^2$
$m_{S/C}$	$2564\ kg$
h_{1d}	$18.65\ m$
h_3	$1.385\ m$

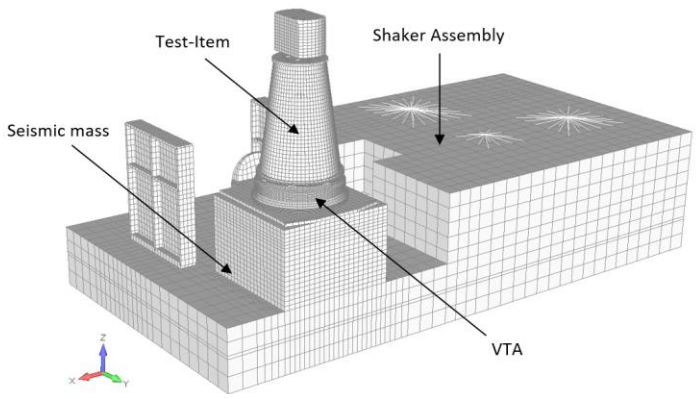

Fig. 5 Structure tested on the shaker, lateral configuration; excitation along Y

3.1 Case 1: obtaining k_{tm} for the first time; f_n and f_n' are available (before the sine test)

Let's consider first the case where f_n and f_n' are available as input from a correlated VST FEM analysis: Table 2.

From Eq. (3): $k_t = 1.3372E10\ Nm/rad$.

$\Delta f_n = f_n' - f_n = -0.777\ Hz$.

From Eq. (4) : $\Delta k_t = -1.0450E9\ Nm/rad$.

By using the first and the second equation in the system in Eq. (5):

$k_{te} = 1.2327E10\ Nm/rad$; $k_{tm} = 1.5774E11\ Nm/rad$.

The above k_{tm} has to be considered as a stiffness property of the shaker-VTA (as a function of the slip table bearings, the pressurized oil meatus and the footprint of the VTA). That is, once characterized, it can be used when testing other S/Cs, provided that the same shaker and VTA are used[b]. Output parameters of present case are summarized in Table 3.

At this point, the steps given in the below case 2 should follow.

3.2 Case 2: k_{tm} is available from heritage (before the sine test)

If k_{tm} is available from heritage (e.g. from an experience regarding another test-item and in line with the above *case 1*, provided that the same VTA shaker assembly is employed), Eq.s (13),(14) lead to:

$f_n'' = 18.864\ Hz$; $f_{shift} = f_n'' - f_n = -0.625\ Hz$.

It is confirmed that the above calculated value for f_n'' is in good agreement with the value of $18.836\ Hz$, which is obtained through the VST FEM analysis with the seismic mass set as infinitely rigid (error less than 0.15 %). This confirms the consistency of the analytical formulas, where the same assumptions are retained.

Output parameters of present case 2 are summarized in Table 4, while the accuracy of the here calculated f_n'' w.r.t. the VST FEM result is discussed in Table 5.

Table 2 Input parameters, case1

f_n	$19.489\ Hz$
f_n'	$18.712\ Hz$

Table 3 Output parameters, case 1

k_t	$1.3372E10\ Nm/rad$
Δf_n	$-0.777\ Hz$
Δk_t	$-1.0450E9\ Nm/rad$
k_{te}	$1.2327E10\ Nm/rad$
k_{tm}	$1.5774E11\ Nm/rad$

[b]In general, it might be useful to know the longitudinal stiffness per unit area between slip table and seismic mass: in this case, after the correlation, this value matched $1.26E10\ N/m^3$ (quantity physically given by the pressurized oil meatus and the bearings below the slip table).

Table 4 Output parameters, case 2

f_n''	$18.864\ Hz$
f_{shift}	$-0.625\ Hz$

Table 5 Accuracy of the calculated f_n'', case 2

f_n'' analytical	f_n'' from VST FEM	Error %
$18.864\ Hz$	$18.836\ Hz$	< 0.15

Table 6 Frequency from the sine test

$f_n^{sine\_test}$	$\mathbf{18.710\ Hz}$

Table 7 Output parameters, after the sine test

f_n'	$> 18.559\ Hz$
k_{te}	$> 1.2126E10\ Nm/rad$
k_t	$> 1.3136E10\ Nm/rad$
f_n	$\mathbf{> 19.316\ Hz}$
$f_n - f_n^{sine\_test}$	$\mathbf{\sim 0.6\ Hz}$

3.3 After the sine test

After running the sine test, the $f_n^{sine\_test}$ in Table 6 was obtained.

From Eq. (16): $f_n' > 18.559\ Hz$.

By using Eq. (17), $k_{te} > 1.2126E10\ Nm/rad$, while from Eq. (18) $k_t > 1.3136E10\ Nm/rad$.

Finally, the lower bound value of f_n is found from Eq. (19): $f_n > 19.316\ Hz$.

It is worth to point out that the above inequality is true since $f_n = 19.489\ Hz$, as given in *Case 1*. This is a relevant point when verifying the compliance of the S/C lateral frequency w.r.t. the frequency requirement from the Launcher Authority, which is expressed in terms of minimal natural frequency for the S/C in hard-mounted configuration. In fact, even if in practice the frequency requirement is simply compared to $f_n^{sine\_test}$, considering the above lower bound of f_n for the comparison makes the assessment more consistent and advantageous for the S/C developer. In fact in this case $f_n - f_n^{sine\_test} \sim 0.6\ Hz$, which is a noteworthy difference. Furthermore, in case of very marginal design, f_n could be further corrected from the little down-shift caused by the VTA (this can be done by FE analysis, provided that the FE model of the VTA is validated). The above results are summarized in Table 7.

For the sake of completeness, it should be noted that the main reason for which the expression derived in Eq. (19) features an inequality, rather than an equality, is given by the flexibility of the seismic mass, which can be predicted through a dedicated VST analysis or estimated by heritage (if the elastic behaviour of the seismic mass in use is well-known). In any case, such contribution is noticeable after the sine test and it is equal to $f_n'' - f_n^{sine\_test}$, that is $0.155\ Hz$ in the case here described. By taking this additional contribution into account, the uncertainty in the value of f_n is effectively eliminated and the inequality $f_n > 19.316\ Hz$ can be converted to the following equality, $f_n = 19.471\ Hz$, which matches closely with the starting value of $19.489\ Hz$ given as data in *Case 1*.

The so-extrapolated f_n could be further used in order to perform a consistent post-test correlation of the S/C FE model, which is an important pre-condition to perform an accurate S/C-Launch Vehicle final Coupled Load Analysis.

4. Conclusions

An analytical methodology to calculate a lower bound frequency down-shift of the main lateral modes of large test-items, like medium or large size S/Cs, is formulated and proposed in this paper. Numerical results have been commented by referring to a test experience held in TAS.

The benefits of quantifying the test-item frequency shift that is attributable to the shaker assembly in use lie in the following two points.

1. It provides, if required, a consistent justification for the frequency down-shift of the S/C main lateral mode (e.g. towards the frequency requirement from the Launcher Authority).
2. It allows to perform a consistent post-test correlation of the S/C FE model of the S/C+VTA assembly for the first lateral natural frequency.

The analytical formulation presented in this paper offers both the above benefits effortlessly, by referring to just a few handy formulas and without the need for performing a dedicated VST FEM analysis for the purpose. The methodology is general and it can be applied to any shaker in lateral configuration. All the parameters required in the equations are illustrated throughout the paper.

Finally, the relevant formulas of the proposed formulation can be taken into consideration when defining the design of new shakers-assemblies, with a view to minimizing the intrinsic dynamic coupling with the test-items.

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