Advances in Aircraft and Spacecraft Science, *Vol. 9, No. 1 (2022) 59-68*
https://doi.org/10.12989/aas.2022.9.1.059

Mathematical modeling of the local temperature effect on the deformation of the heat-shielding elements of the aircraft

Boris A. Antufiev[1a], Ying Sun*[2], Olga V. Egorova[1b] and Nikolay M. Bugaev[3c]

[1]Department of Resistance of Materials Dynamics and Strength of Machines, Moscow Aviation Institute (National Research University), Moscow, Russian Federation
[2]Department of Mechanical Engineering, Hangzhou Xiaoshan Technician College, Hangzhou Zhejiang, People's Republic of China
[3]Moscow Aviation Institute (National Research University), Moscow, Russian Federation

(Received August 21, 2020, Revised January 15, 2022, Accepted January 18, 2022)

Abstract. The physical and mathematical foundations of the heat-shielding composite materials functioning under the conditions of aerodynamic heating of aircraft, as well as under the conditions of the point effect of high-energy radiation are considered. The problem of deformation of a thin shallow shell under the action of a local temperature field is approximately solved. Such problems arise, for example, in the case of local destruction of heat-protective coatings of aircraft shells. Then the aerodynamic heating acts directly on the load-bearing shell of the structure. Its destruction inevitably leads to the death of the entire aircraft. A methodology has been developed for the numerical solution of the entire complex problem on the basis of economical absolutely stable numerical methods. Multiple results of numerical simulation of the thermal state of the locally heated shallow shell under conditions of its thermal destruction at high temperatures have been obtained.

Keywords: composite materials; heat and mass transfer problems; influence functions; reusable space system; temperature field

1. Introduction

During the operation of a reusable space system, about twenty external factors act on its heat-shielding elements (Bulychev and Rabinskiy 2019a, Astapov *et al.* 2019a, Antufev *et al.* 2018, Antufev *et al.* 2019a, Kuznetsova and Makarenko 2019, Kuznetsova and Rabinskiy 2019a, Kuznetsova and Rabinskiy 2019b, Kuznetsova and Rabinskiy 2019c, Rabinsky and Kuznetsova 2019, Formalev *et al.* 2019a, Formalev and Kolesnik 2007, Formalev and Kolesnik 2019). These include: intense aerodynamic heating with a different chemical composition of the atmosphere, high and low temperatures caused by gas dynamics of supersonic flows, cosmic radiation, solar radiation, etc. As a rule, heat-shielding structural elements are multifunctional composite layered materials with complex properties, saturated with various binding substances. In addition, the outboard edges

*Corresponding author, Engineer, E-mail: sun5588-1@tanu.pro
[a]Professor, E-mail: b.antufiev7@tanu.pro
[b]Associate Professor, E-mail: ov-egorova@nuos.pro
[c]Head of the Laboratory, E-mail: n.bugaev@toronto-uni.com

http://www.techno-press.org/?journal=aas&subpage=7 ISSN: 2287-528X (Print), 2287-5271 (Online)

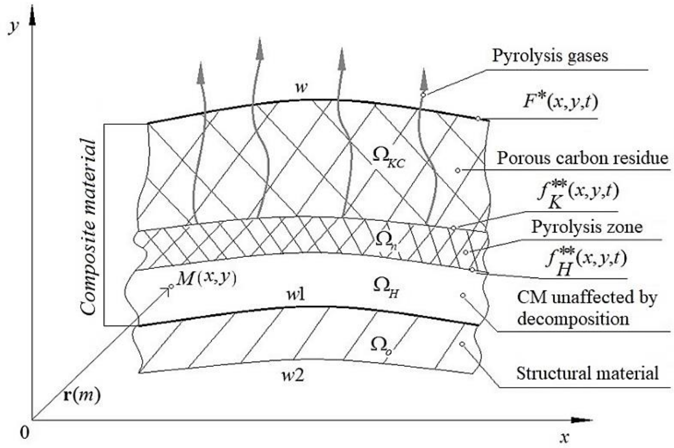


Fig. 1 Modeling of heat and mass transfer in anisotropic composite materials under high-temperature loading

and some inside surfaces of these elements have various heat-shielding coatings, which significantly increase its service life (Antufev *et al.* 2019b, Antufev *et al.* 2019c, Astapov *et al.* 2019b, Bulychev and Rabinskiy 2019b, Makarenko and Kuznetsova 2019, Rabinskiy 2019). In the scientific literature, the heat-protective coating is interpreted as an inertial layer that mainly changes the dynamic characteristics of the structure and the effect of this destruction on its natural oscillation frequencies (Cinefra *et al.* 2015, Carrera and Valvano 2017, Carrera and Valvano 2019, Molodetska 2021). In the proposed paper, a completely different aspect of this problem is considered i.e., the effect of a local temperature field resulting from aerodynamic heating of the carrier shell in the damage zone of the protective layer.

During the operation of space-based technology, the heat-shielding coating is directly affected by all of the previously listed factors (Antufev *et al.* 2018, Antufev *et al.* 2019a, Astapov *et al.* 2019a, Bulychev and Rabinskiy 2019a, Kuznetsova and Makarenko 2019), however, the most dangerous from the standpoint of aging and destruction are high-intensity energy flows and high temperatures. Therefore, in this paper, first of all, the effect of high-intensity heat fluxes of energy on the surface of heat-shielding composite materials is considered.

Composite materials are widely used in aviation and rocket and space technology as structural and heat-shielding materials due to their unique properties resulting from their manufacturing technology (Milenin *et al.* 2021). The matrix of fine-fibre fillers is saturated with binding substances that are easily decomposed at moderate temperatures (Formalev *et al.* 2018a, Formalev *et al.* 2019b, Formalev *et al.* 2019c, Formalev *et al.* 2019d, Grabar *et al.* 2020). When such materials are used as heat shielding materials at hypersonic flight speeds, heat fluxes from high-temperature boundary layers are absorbed (Formalev *et al.* 2018b, Formalev *et al.* 2018c, Formalev *et al.* 2019e, Formalev *et al.* 2019f, Formalev *et al.* 2019g, Formalev *et al.* 2020, Kolesnik and Bulychev 2020, Kolesnik *et al.* 2019, Senchenkov *et al.* 2021). The heat and mass transfer process are presented in Fig. 1.

From the Fig. 1 it follows that at temperatures up to ~600K the heat-shielding material operates due to its thermophysical characteristics, namely, the volumetric heat capacity and heat transfer inside the material due to its thermal conductivity. Above this temperature, decomposition of the binding agent begins under the action of endothermic chemical reactions with the formation of gaseous decomposition products and a porous residue consisting of filler fibers and carbon residue

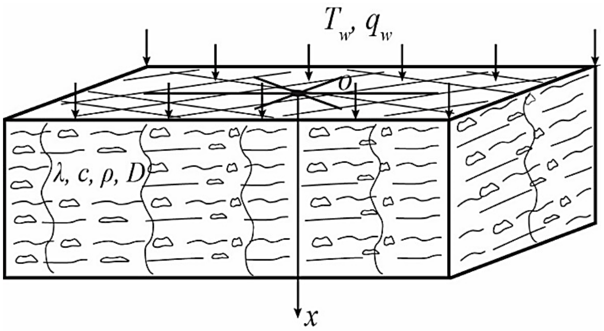


Fig. 2 Diagram of loading on aircraft structural element

from the decomposition of the binding substance. Due to decomposition of the binding substance the decomposition area, limited by the coordinates of the beginning and end of decomposition, becomes very thin and separates the material unaffected by decomposition and the porous residue in which there is no trace of decomposition.

Further, through the porous residue, under the action of a pressure gradient between the decomposition area, where the pressure due to the low filtration rate of the pyrolysis gases is considered the stagnation pressure, and the outer boundary, where the pressure is equal to the ambient pressure, the pyrolysis gases are filtered, absorbing thermal energy due to convection. At the same time, to implement the filtration process, the stagnation pressure must exceed the ambient pressure by the value of the hydraulic resistance of the porous residue. In this case, pyrolysis gases are blown into the high-temperature boundary layer, pushing it away from the outer boundary and decreasing its temperature, which entails a decrease in convective heat fluxes to the outer boundary. When the porous residue reaches the ablation temperature, the mass removal of carbon residue of the composite material begins at the outer boundary due to physical and chemical transformations.

2. Physical and mathematical model of heat and mass transfer in heat-shielding composite materials under high-temperature loading

Let us consider the effect of a high-intensity heat flux of energy on a structural element of a modern FVs, consisting of a composite material (Fig. 2).

When developing a mathematical model of heat and mass transfer, preliminary assumptions must be made that limit its applicability and the laws of decomposition of adhesive and nonlinear filtration of pyrolysis gases through a porous residue must be formulated. To study the process of heat and mass transfer, we will formulate assumptions that allow, in the first approximation, to analytically evaluate the main parameters of the process.

1. For the problem under consideration, the computational space is assumed to be two-dimensional in spatial variables with an outer boundary w described by an implicit function $F^{*}(x,y,t)=0$, which can be nonstationary under the action of mass carryover when the temperature of the outer surface exceeds the temperature of carryover masses.

2. A structural element made of CM is exposed to a high-intensity flow of thermal energy (Fig. 2), the computational space of which is assumed to be one-dimensional (x-axis).

3. Ω_n – the pyrolysis zone is limited by unsteady moving surfaces of the beginning $f_n^{**}(x,y,t)$

and the end $f_k^{**}(x,y,t)$ the binding substance decomposition with the formation of a gas component and a porous residue occurs.

4. Between the porous frame and pyrolysis gases, thermodynamic equilibrium is established at each point of the porous region bounded by the surfaces $f_k^{**}(x,y,t)$ and $F^*(x,y,t)$, where convective-conductive heat transfer occurs with account for non-isothermal multidimensional filtration of pyrolysis gases.

5. It is assumed that during filtration of pyrolysis gases, the pressure in the adhesive decomposition zone is considered as the stagnation pressure, since the filtration rate there is practically zero.

6. Injection of pyrolysis gases into the gas-dynamic boundary layer is taken into account by a decrease in gas-dynamic heat fluxes depending on the injection parameter, which is the product of the density of the injected gases and the filtration rate at the outer boundary $F^*(x,y,t)$;

Taking into account the accepted assumptions, the mathematical model of unsteady heat and mass transfer in the two-dimensional region of CM at high temperatures includes the following relations:

• Balance of convective-conductive and radiant heat fluxes and heat fluxes absorbed due to mass entrainment at the boundary w in contact with the high-temperature gas-dynamic boundary layer

$$\left(\frac{\alpha}{c_p}\right)_w (I_b - I_w) - \Lambda gradT|_w - \varepsilon_w \cdot \sigma \cdot T_w^4 - \left(P \cdot c_p \cdot \rho \cdot V\right)_g \cdot T_w = \dot{m} \cdot Q^* \left(I_e\right) \cdot \eta\left(T_w - T^*\right),$$
$$(x,y) \in w,\ t > 0, \tag{1}$$

where $\left(\frac{\alpha}{c_p}\right)_w$ – coefficient of heat transfer from the boundary layer, referred to the heat capacity of the gas-dynamic flow at the wall temperature, taking into account the injection

$$\left(\frac{\alpha}{c_p}\right)_w = \left(\frac{\alpha}{c_p}\right)_{w0} - \beta \cdot \left(\rho_g V_g\right)_w \tag{2}$$

where β – injection parameter

$$\beta = 0{,}56\left(\frac{M_a}{\bar{M}}\right)^{0{,}29} \cdot \left(\frac{I_w}{I_b}\right) \tag{3}$$

where $\bar{M}$ – weighted-average molar mass of pyrolysis gases mixture; M_a – molar mass of air; I_b – effective enthalpy of the boundary layer

$$I_b = c_{pn} \cdot T_n + \frac{V_n^2}{2}, \tag{4}$$

c_{pn}, T_n, V_n – respectively, the heat capacity, temperature, speed of the oncoming gas-dynamic flow; I_w – enthalpy of the gas at the wall temperature, $I_w = \int_0^{T_w} c_p(T)dT$; $\varepsilon_w, \sigma, Q^*, T^*$ – respectively, the degree of surface emissivity, Stefan-Boltzmann constant, effective enthalpy of CM, temperature of CM mass carryover; Λ – thermal conductivity tensor of the CM, $\dot{m}$ – mass velocity of mass carryover, $\dot{m} = \rho \cdot \dot{n}^*$, where $\dot{n}^*$ – linear velocity of mass loss in the direction normal to the surface $F^*(x,y,t)$; $\eta(z)$ – the Heaviside step function.

• Heat transfer equation in structural material

$$c_0(T)\rho_0 \frac{\partial T}{\partial t} = div(\Lambda_0 gradT),\ (x,y) \in \Omega_0,\ t > 0, \tag{5}$$

where Λ_0 – structural material thermal conductivity tensor.

• Mathematical model of coupled thermoelasticity for determination of the stress-strain state of a heat-shielding composite element (Fig. 2) (Kuznetsova and Rabinskiy 2019c, Formalev and Kolesnik 2019, Makarenko and Kuznetsova 2019, Astapov *et al.* 2019b, Rabinskiy 2019)

$$\frac{\partial^2 u}{\partial \tau^2} = \frac{\partial^2 u}{\partial x^2} - b_u \frac{\partial T}{\partial x}\frac{\partial T}{\partial \tau} = k\frac{\partial^2 T}{\partial x^2} - b_T \frac{\partial^2 u}{\partial x \partial \tau} \quad (6)$$

where $u, T -$ displacement, temperature field, concentration change, respectively; k – heat capacity coefficients, $x, \tau -$ time and coordinate; $b_t -$ coefficients connecting the combined thermoelasticity problem.

• Initial conditions for temperature and moving boundaries

$$T(x, y, 0) = T(x, y), (x, y) \in \Omega \quad (7)$$

$$f_n^{**}(x, y, 0) = \varphi(x, y), (x, y) \in f_n^{**}(x, y, 0) \quad (8)$$

$$f_k^{**}(x, y, 0) = \psi(x, y), (x, y) \in f_k^{**}(x, y, 0) \quad (9)$$

where Ω – initial region between the boundaries $w2$ and w, not subject to phase transformations.

• Continuity of heat fluxes and temperatures at the boundary $f_{w1}(x, y)$ between CM and structural material

$$\Lambda gradT|_{f_{w1+0}} - \Lambda_0 gradT|_{f_{w1-0}} = 0, T|_{f_{w1+0}} = T|_{f_{w1-0}}, (x, y) \in w1, t > 0 \quad (10)$$

• Balance of convective-conductive and radiant heat fluxes on the inner free boundary $w2$

$$\alpha_{w2}(T_{b2} - T_{w2}) + \Lambda_0 gradT|_{w2} - \varepsilon_{w2}\sigma T_{w2}^4 = 0, (x, y) \in w2, t > 0 \quad (11)$$

• Thermal conductivity equation with allowance for filtration in the porous residue

$$c_{bff}(T) \cdot \rho_{bff}\frac{\partial T}{\partial t} = div(\Lambda_{eff} gradT) - P(c_p\rho \cdot V)_g gradT, (x, y) \in \Omega_{kc}, t > t_k^{**} \quad (12)$$

where t_k^{**} – time of occurrence of the boundary $f_k^{**}(x, y, t)$.

• Energy equation in the adhesive decomposition zone with account for physicochemical transformations with thermal effect Q^{**} and filtration

$$c_{bff}(T) \cdot \rho_{bff}(T)\frac{\partial T}{\partial t} = div(\Lambda_{bff} gradT) - \dot{\rho}(x, y, t) \cdot Q^{**} - P^{**}(x, y)(c_p\rho V)_g gradT, (x, y) \in \Omega_n, t > t_n^{**} \quad (13)$$

where $P^{**}(x, y)$ changes linearly in the direction normal to the boundary $f_n^{**}(x, y, t)$ from the value $P^{**} = 0$ on this boundary to the value $P^{**} = P$ on the boundary $f_k^{**}(x, y, t)$ – equation of state of pyrolysis gases.

$$\rho_g = \frac{p_g \cdot \bar{M}_g}{R_g \cdot T}, (x, y) \in \Omega_{kc}\ t > t_k^{**} \quad (14)$$

For the coupled thermoelasticity problem (6), the standard conditions for the first initial-boundary value problem are satisfied. Further, to the given system of equations, it is necessary to add the law of changes in the CM density in the adhesive decomposition zone and the Stefan conditions on the nonstationary moving boundary of the beginning and end of the adhesive decomposition, as well as the continuity equation for filtration of pyrolysis gases and the law of nonlinear filtration of pyrolysis gases (Formalev and Kolesnik 2007, Formalev and Kolesnik 2019).

The mathematical model (Eqs. (1)-(14)) is complex, consisting of several mathematical models:

- heat conduction problem in anisotropic medium with heat conduction tensor Λ_0;
- heat and mass transfer in porous carbon residue Ω_{cb}, limited by two nonstationary moving boundaries $f_k^{**}(x, y, t)$ and $F^*(x, y, t)$ taking into account mass carryover, filtration of pyrolysis gases, and their injection into the boundary layer, emission and tensor nature of nonlinear heat and mass transfer;
- non-isothermal multidimensional filtration of pyrolysis gases in anisotropic porous residue by determining the fields u and v of the filtration components rate and pressure of pyrolysis gases.
- problem of coupled thermoelasticity for a composite rod in one-dimensional formulation.

The mathematical model is closed by the equation for determining the CM density in the pyrolysis zone and the stagnation pressure of pyrolysis gases, as well as the nonlinear law of filtration of pyrolysis gases in a porous medium.

3. Solution of multidimensional problems of heat and mass transfer in anisotropic composite materials under high-temperature loading

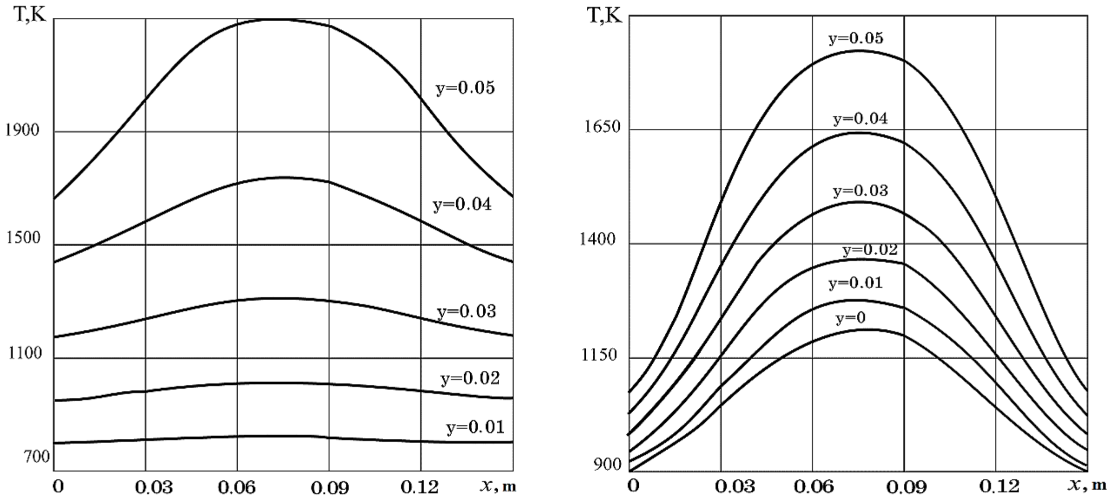
The presented mathematical model is a problem of nonlinear anisotropic coupled thermoelasticity in the presence of convective-conductive and radiant types of heat transfer at free boundaries. In the problems of heat and mass transfer, due to the significant nonlinearity of the complex problem, only implicit numerical schemes are used, as a result of which we can expect the stability of the numerical solution and its convergence to the exact one.

It should be noted that in problems of heat and mass transfer with phase transformations, the temperature distribution significantly affects the rate of phase transformations, and phase transformations, in turn, significantly change the temperature fields. As a result, in such problems the step of numerical integration over time should be related to the rate of phase transformations or to the velocities of motion of phase transformations boundaries even when using completely implicit numerical schemes. This is confirmed by numerical experiments (Antufev *et al.* 2019b, Antufev *et al.* 2019c, Astapov *et al.* 2019b, Bulychev and Rabinskiy 2019b, Makarenko and Kuznetsova 2019, Rabinskiy 2019).

If the step of numerical integration in time is not consistent with the velocities of motion of the phase transformations boundaries, then the solution falls apart due to the fact that, at an arbitrary step in time, the boundaries of phase transformations can move deep into the composite so the temperature field, which depends on the coordinates of these boundaries, will not be connected with the temperature field at the previous time layers.

Further, the results of numerical modelling of heat and mass transfer in anisotropic composite materials are presented on the basis of the developed software package that implements the developed complex mathematical model. The following input data were taken for calculations: $l_1 = 0{,}15$ m; $l_2 = 0{,}05$ m; $T_0(x, y) = 300$ K $= const$; $\alpha_{wi} = 1$ kBm/m$^2 \cdot$ K; $T_{wi} = 300$ K, $i = 2{,}3{,}4$; $c^{(1)} = 1$ kJ/kg $\cdot$ K; $\rho^{(1)} = 2500\,\frac{\text{kg}}{\text{m}^3}$; $c^{(2)} = 0,$kJ/kg $\cdot$ K; $\rho^{(2)} = 2000\,\frac{\text{kg}}{\text{m}^3}$; $T^* = 800$ K; $Q^* = 1000$ kg; $\lambda_\xi^{(1)} = 1$ kV/m $\cdot$ K; $\lambda_\eta^{(1)} = 0{,}5$ kV/m $\cdot$ K.

Fig. 3 reveals the results of calculations of a Stefan-type problem in a two-dimensional anisotropic rod (Vasil'ev and Vasilyeva 2020) with a nonstationary moving boundary of phase transformations in the form of curves $T(x, y)$ and nonstationary moving boundaries $y = y(x, t)$ $\lambda_\xi^{(2)} = \lambda_\eta^{(2)} = 1$ kV/m $\cdot$ K $t_{con} = 10$ c, where index (2) refers to the reacted phase, and (1) – to the



(a) (b)

Fig. 3 Temperature field at time moment $t_{con} = 10c$ (a) phase transformations for the orthotropic case $\varphi = 0$ (a) and $\varphi = \frac{\pi}{2}$ (b)

initial phase.

In the orthotropic case $\lambda_{11} = \lambda_{\xi}$, $\lambda_{22} = \lambda_{\eta}$, if the angle φ, orienting the main axes, is equal to zero and $\lambda_{11} = \lambda_{\eta}$, $\lambda_{22} = \lambda_{\xi}$, if $\varphi = \frac{\pi}{2}$. It can be seen from these figures that an increase (decrease) in the λ_{11} component increases the heating and the speed of the boundary movement $y = f(x^*(t), t)$. The heat transfer parameters and the characteristics of the thermal conductivity tensor were taken as follows: $\alpha = 5\ \mathrm{kV/(m^2 \cdot K)}$, $T_{b1} = 10^4$ K, $\lambda_{11} = 0.8\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$, $\lambda_{12} = 0.25\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$, $\lambda_{22} = 0.2\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$.

The input data were taken as follows: in a two-dimensional anisotropic region made of composite material $\lambda_{\xi} = 0.0009\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$; $\lambda_{\eta} = 0.0006\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$; $\lambda_{\xi_{ks}} = 0.0012\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$; $\lambda_{\eta_{ks}} = 0.0025\frac{\mathrm{kV}}{(\mathrm{m \cdot K})}$; $\rho_n = 1600\frac{\mathrm{kg}}{\mathrm{m^3}}$; $\rho_k = 1250\frac{\mathrm{kg}}{\mathrm{m^3}}$; $T^{**} = 750$ K ; $Q^{**} = 1030\frac{\mathrm{kJ}}{\mathrm{kg}}$; $\varepsilon_1 = 0.8$; $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0$; $l_1 = 0.15$ m; $l_2 = 0.05$ m; $p_{w1} = 10^5$ Pa; $P = 0.4$; $k = 10^{-14}$ m^2; $\overline{M} = 0.4\frac{\mathrm{kg}}{\mathrm{mole}}$; $t_{con} = 20$ c; $\tau = 0.5$ c; $T_{b1} = 4000$ K; $T_{b2} = T_{b3} = T_{b4} = 300$ K; $T_n = 300$ K.

4. Conclusions

The calculation results have revealed that:

- the stronger the thermal load, the stronger the shift of the above-mentioned maxima in the direction of deviation of the temperature field in the direction of the greater thermal conductivity;
- the temperature of deeper points from the border is higher than the temperature of points closer to the border;
- a significant asymmetry of the boundary of phase transformations and the temperature field is

visible even with a small degree of anisotropy;

- the temperature field has maxima shifted from the axis of symmetry $x = \frac{l_1}{2}$ of the heat load in the direction of the $O\xi$ with a large thermal conductivity coefficient, and the deeper from the boundary $y = l_2$, on which the principal axis with a large thermal conductivity coefficient is also set.

Acknowledgments

The work was carried out with the financial support of RFBR grant 20-01-00517 A.

References

Antufev, B.A., Egorova, O.V. and Rabinskiy, L.N. (2019b), "Quasi-static stability of a ribbed shell interacting with moving load", *INCAS Bull.*, **11**, 33-39.

Antufev, B.A., Egorova, O.V. and Rabinskiy, L.N. (2019c), "Dynamics of a cylindrical shell with a collapsing elastic base under the action of a pressure wave", *INCAS Bull.*, **11**, 17-24. https://doi.org/10.13111/2066-8201.2019.11.S.3.

Antufev, B.A., Egorova, O.V., Medvedskii, A.L. and Rabinskiy, L.N. (2019a), "Dynamics of shell with destructive heat-protective coating under running load", *INCAS Bull.*, **11**, 7-16. https://doi.org/10.13111/2066-8201.2019.11.S.2.

Antufev, B.A., Egorova, O.V., Orekhov, A.A. and Kuznetsova, E.L. (2018), "Dynamics of thin-walled structure with high elongation ratio and discrete elastic supports on the rigid surface under moving loads", *Periodico Tche Quimica*, **15**(1), 464-470.

Astapov, A.N., Kuznetsova, E.L. and Rabinskiy, L.N. (2019b), "Operating capacity of anti-oxidizing coating in hypersonic flows of air plasma", *Surf. Rev. Let.*, **26**(2), 1850145. https://doi.org/10.1142/S0218625X18501457.

Astapov, A.N., Lifanov, I.P. and Rabinskiy, L.N. (2019a), "Perspective heat-resistant coating for protection of CF/SIC composites in air plasma hypersonic flow", *High Temperat.*, **57**(5), 744-752. https://doi.org/10.1134/S0018151X19050018.

Bulychev, N.A. and Rabinskiy, L.N. (2019a), "Surface modification of titanium dioxide nanoparticles with acrylic acid/isobutylene copolymer under ultrasonic treatment", *Periodico Tche Quimica*, **16**(32), 338-344.

Bulychev, N.A. and Rabinskiy, L.N. (2019b), "Ceramic nanostructures obtained by acoustoplasma technique", *Nanosci. Technol.*, **10**(3), 279-286. https://doi.org/10.1615/NanoSciTechnolIntJ.2019031161.

Carrera, E. and Valvano, S. (2017), "A variable kinematic shell formulation applied to thermal stress of laminated structures", *J. Therm. Stress.*, **40**(7), 803-827. https://doi.org/10.1080/01495739.2016.1253439.

Carrera, E. and Valvano, S. (2019), "A variable ESL/LW kinematic plate formulation for free-vibration thermoelastic analysis of laminated structures", *J. Therm. Stress.*, **42**(4), 452-474. https://doi.org/10.1080/01495739.2018.1474513.

Cinefra, M., Valvano, S. and Carrera, E. (2015), "Heat conduction and thermal stress analysis of laminated composites by a variable kinematic MITC9 shell element", *Curv. Layer. Struct.*, **2**, 301-320. https://doi.org/10.1515/cls-2015-0017.

Formalev, V.F. and Kolesnik, S.A. (2007), "Conjugate heat transfer between wall gasdynamic flows and anisotropic bodies", *High Temperat.*, **45**(1), 76-84. https://doi.org/10.1134/S0018151X07010105.

Formalev, V.F. and Kolesnik, S.A. (2019), "On thermal solitons during wave heat transfer in restricted areas", *High Temperat.*, **57**(4), 498-502. https://doi.org/10.1134/S0018151X19040047.

Formalev, V.F., Bulychev, N.A., Kuznetsova, E.L. and Kolesnik, S.A. (2019d), "The thermal state of a packet of cooled microrocket gas-dynamic lasers", *Tech. Phys. Let.*, **46**(3), 245-248.

https://doi.org/10.1134/S1063785020030074.
Formalev, V.F., Kartashov, É.M. and Kolesnik, S.A. (2019c), "Simulation of nonequilibrium heat transfer in an anisotropic semispace under the action of a point heat source", *J. Eng. Phys. Thermophys.*, **92**(6), 1537-1547. https://doi.org/10.1007/s10891-019-02074-7.
Formalev, V.F., Kartashov, É.M. and Kolesnik, S.A. (2020), "On the dynamics of motion and reflection of temperature solitons in wave heat transfer in limited regions", *J. Eng. Phys. Thermophys.*, **93**(1), 10-15. https://doi.org/10.1007/s10891-020-02085-9.
Formalev, V.F., Kolesnik, S.A. and Kuznetsova, E.L. (2018b), "Analytical study on heat transfer in anisotropic space with thermal conductivity tensor components depending on temperature", *Periodico Tche Quimica*, **15**(1), 426-432.
Formalev, V.F., Kolesnik, S.A. and Kuznetsova, E.L. (2018c), "Analytical solution-based study of the nonstationary thermal state of anisotropic composite materials", *Compos.: Mech., Comput., Appl.*, **9**(3), 223-237.
Formalev, V.F., Kolesnik, S.A. and Kuznetsova, E.L. (2019a), "Effect of components of the thermal conductivity tensor of heat-protection material on the value of heat fluxes from the gasdynamic boundary layer", *High Temperat.*, **57**(1), 58-62. https://doi.org/10.1134/S0018151X19010085.
Formalev, V.F., Kolesnik, S.A. and Kuznetsova, E.L. (2019e), "Approximate analytical solution of the problem of conjugate heat transfer between the boundary layer and the anisotropic strip", *Periodico Tche Quimica*, **16**(32), 572-582.
Formalev, V.F., Kolesnik, S.A. and Kuznetsova, E.L. (2019f), "Identification of new law for decomposition of bonding heat-shielding composite materials", *Asia Life Sci.*, **1**, 139-148.
Formalev, V.F., Kolesnik, S.A. and Kuznetsova, E.L. (2019g), "Mathematical modeling of a new method of thermal protection based on the injection of special coolants", *Periodico Tche Quimica*, **16**(32), 598-607.
Formalev, V.F., Kolesnik, S.A. and Selin, I.A. (2018a), "Local non-equilibrium heat transfer in an anisotropic half-space affected by a non-steady state point heat source", *Herald Bauman Moscow State Tech. Univ., Ser. Nat. Sci.*, **80**(5), 99-111.
Formalev, V.F., Kolesnik, S.A., Kuznetsova, E.L. and Rabinskiy, L.N. (2019b), "Origination and propagation of temperature solitons with wave heat transfer in the bounded area during additive technological processes", *Periodico Tche Quimica*, **16**(33), 505-515.
Grabar, I., Vodyanitsky, G., Slyusarenko, I. and Tymkiv, V. (2020), "Improvement of the feed preparation and dispensing unit by the method of morphological analysis-synthesis", *Sci. Horiz.*, **4**, 55-64.
Kolesnik, S.A. and Bulychev, N.A. (2020), "Numerical analytic method for solving the inverse coefficient problem of heat conduction in anisotropic half-space", *J. Phys.: Conf. Ser.*, **1474**(1), 012024.
Kolesnik, S.A., Bulychev, N.A., Rabinskiy, L.N. and Kazaryan, M.A. (2019), "Mathematical modeling and experimental studies of thermal protection of composite materials under high-intensity effects of laser radiation", *XIV International Conference on Pulsed Lasers and Laser Applications*, **9**, 113221R. https://doi.org/10.1117/12.2553955.
Kuznetsova, E.L. and Makarenko, A.V. (2019), "Mathematical model of energy efficiency of mechatronic modules and power sources for prospective mobile objects", *Periodico Tche Quimica*, **16**(32), 529-541.
Kuznetsova, E.L. and Rabinskiy, L.N. (2019a), "Numerical modeling and software for determining the static and linkage parameters of growing bodies in the process of non-stationary additive heat and mass transfer", *Periodico Tche Quimica*, **16**(33), 472-479.
Kuznetsova, E.L. and Rabinskiy, L.N. (2019b), "Numerical modeling and software for determining the static and linkage parameters of growing bodies in the process of non-stationary additive heat and mass transfer", *Periodico Tche Quimica*, **16**(33), 472-479.
Kuznetsova, E.L. and Rabinskiy, L.N. (2019c), "Heat transfer in nonlinear anisotropic growing bodies based on analytical solution", *Asia Life Sci.*, **2**, 837-846.
Makarenko, A.V. and Kuznetsova, E.L. (2019), "Energy-efficient actuator for the control system of promising vehicles", *Russ. Eng. Res.*, **39**(9), 776-779. https://doi.org/10.3103/S1068798X19090144.
Milenin, O.S., Velykoivanenko, O.A., Rozynka, G.P. and Pivtorak, N.I. (2021), "Multiscale procedure of numerical evaluation of damage and technical state of structures from fiber composite materials", *Paton*

Weld. J., **11**, 48-52.
Molodetska, T.I. (2021), "Investigation of the stress-strain state of the bending process of coated materials", *Sci. Herald Uzhhorod Univ. Ser. Phys.,* **50**, 15-24.
Rabinskiy, L.N. (2019), "Non-stationary problem of the plane oblique pressure wave diffraction on thin shell in the shape of parabolic cylinder", *Periodico Tche Quimica*, **16**(32), 328-337.
Rabinsky, L.N. and Kuznetsova, E.L. (2019), "Simulation of residual thermal stresses in high-porous fibrous silicon nitride ceramics", *Pow. Metall. and Met. Ceram.*, **57**(11/12), 663-669. https://doi.org/10.1007/s11106-019-00029-4.
Senchenkov, I.K., Ryabtsev, I.O., Chervinko, O.P. and Babinets, A.A. (2021), "Calculation of residual stress-strain state of deposited steel sheet plates", *Paton Weld. J.,* **5**, 46-50.
Vasil'ev, V. and Vasilyeva, M. (2020), "An accurate approximation of the two-phase Stefan problem with coefficient smoothing", *Math.*, **8**, 1924. https://doi.org/10.3390/math8111924.

GY