**Advances in Aircraft and Spacecraft Science**, *Vol. 6, No. 1 (2019) 001-018*
DOI: https://doi.org/10.12989/aas.2019.6.1.001


# Small-scale effect on the forced vibration of a nano beam embedded an elastic medium using nonlocal elasticity theory

Samir Belmahi*[1, 2], Mohammed Zidour[1, 3] and Mustapha Meradjah[2]

*[1]Department of Civil Engineering ,University of Ibn Khaldoun, PB 78 Zaaroura, 14000 Tiaret, Algeria.*
*[2]Laboratory of Materials and Hydrology, University of Djillali Liabes,Sidi Bel Abbés, Algeria*
*[3]Laboratory of Geomatics and Sustainable Development, University of Ibn Khaldoun Tiaret, Algeria*

*(Received May 7, 2018, Revised August 4, 2018, Accepted August 6, 2018)*

**Abstract.** This present article represents the study of the forced vibration of nanobeam of a single-walled carbon nanotube (SWCNTs) surrounded by a polymer matrix. The modeling was done according to the Euler-Bernoulli beam model and with the application of the non-local continuum or elasticity theory. Particulars cases of the local elasticity theory have also been studied for comparison. This model takes into account the different effects of the interaction of the Winkler's type elastic medium with the nanobeam of carbon nanotubes. Then, a study of the influence of the amplitude distribution and the frequency was made by variation of some parameters such as (scale effect ($e_0{}^a$), the dimensional ratio or aspect ratio (L/d), also, bound to the mode number (N) and the effect of the stiffness of elastic medium ($K_w$). The results obtained indicate the dependence of the variation of the amplitude and the frequency with the different parameters of the model, besides they prove the local effect of the stresses.

**Keywords:** carbon nanotube; nanocomposite; nanobeam; vibration; Euler-Bernoulli; Winkler

## 1. Introduction

The use of carbon nanotubes in the manufacturing of nanocomposites is one of the most important topics now in materials sciences. Since their discoveries by Sumio Iijima in 1991 (Iijima 1991) are a fundamental part of nanotechnology studies (Ahmadi Asoor *et al*. 2016). Carbon nanotubes have attracted many research activities, which are closely related to their extraordinary electrical, mechanical, thermal, physical and chemical properties (Ahmadi Asoor *et al*. 2016, Kiani 2014a). Carbon nanotube technologies have many applications for to the aerospace industry. It is an advanced composite material of the future in aerospace industry (Mrazova, 2013, Harris 2009). Their use is based on the knowledge at different scales, of their behavior and characteristics (Hurang *et al*. 2010) considering the different factors which are involved (Shehata *et al*. 2011).

The nanocomposites are materials with a nanometric structure (Acton 2013, Okpala 2014) on a scale that lies between 1 and 100 nm (de Azeredo *et al*. 2009). They have the capacity to improve the macroscopic properties of the products (Okpala 2013) as well as the mechanical properties (Sachse *et al*. 2013), without compromising the ductility of the material (Bakis *et al*. 2002). In fact, the small size of these particles does not create a large concentration of constraints (Okpala

*Corresponding author, Ph.D., E-mail: belmasamir@yahoo.fr


http://www.techno-press.org/?journal=aas&subpage=7 ISSN: 2287-528X (Print), 2287-5271 (Online)

2014).

Moreover, they are considered as heterogeneous microscopically and homogenous macroscopically (Gay 1987). The cohesion of the nanocomposite is ensured by the matrix or the charge which can be in the general case a polymer or an organic material (Hurang *et al*. 2010), while the carbon nanotube represents the reinforcement (Okpala 2013, Öchsner *et al*. 2013) which is responsible for these mechanical performances and the overall stability (Hurang *et al*. 2010). The carbon nanotube increases the stiffness which implies the increase of the Young's modulus (Kaushik *et al*. 2015) and the breaking constraint of the nanocomposite (Choi *et al*. 2005). It represents an allotropic form of carbon distinct from diamond and graphite (Kaushik *et al*. 2015) and has only one layer of graphene wound on itself (Harris 1999). CNTs are ultimate reinforcing agent, called nanofibers, in different matrix materials for the development of a new class of nanocomposites that are extremely strong and ultra-light (Kumar *et al*. 2016). So the polymers consist of hydrocarbon macromolecular chains (Okpala 2014, Kumar *et al*. 2003) of different lengths and very high stability (Hina *et al*. 2014).

Carbon nanotubes are currently of interest in several fields of mechanical engineering and for construction. They are currently the basis of most composites materials, so their behavior in a matrix will be linked to this last and the results will be completely different. Several questions have been elaborated on this material, in particularly the free vibration, among which we quote some works on the linear and the non-linear vibration analysis of longitudinal vibration: By use and application of different theories: Ahmadi Asoor *et al*. (2016), Investigation on vibration of single-walled carbon nanotubes by variational iteration method. Togun *et al*. (2016), Nonlinear free vibration of a nanobeam based on non-local Euler-Bernoulli Beam theory, Ansari *et al*. (2013), Nonlinear finite element vibration analysis of DWCNT based on Timoshenko beam theory. Flexural free vibration and buckling analysis of SWCNT using different gradient elasticity theories (Karlicić *et al*. 2015, Rakrak *et al*. 2016). by embedded or related the NTC at different medium: Sound wave and free vibration of CNT embedded in different elastic medium using nonlocal elasticity theory and finite element formulation (Gafour *et al*. 2013, Togun *et al*. 2016, Nguyen *et al*. 2017 Karami *et al*. 2018a, Dihaj *et al*. 2018, Chemi *et al*. 2018, Hamidi *et al*. 2018), Analysis of nonlinear free vibration for DWCNT by Hajnayeb *et al*. (2015), Nonlinear free vibration of SWCNT embedded in viscoelastic medium by Ali-Akbari *et al*. (2015) and nonlinear vibration analysis of the Fluid-Filled SWCNT with the Shell model based on the nonlocal elasticity theory by Soltani *et al*. (2015).

Recently, the continuum mechanics approach has been widely and successfully used to study the responses of nanostructures, such as the static (Yazid *et al*. 2018, Ahouel *et al*. 2016, Zemri *et al*. 2015, Karami *et al*. 2017, Karami *et al*. 2018b), the buckling (Khetir *et al*. 2017, Bellifa *et al*. 2017a, Khetir *et al*. 2017, Houari *et al*. 2018), free vibration (Bounouara *et al*. 2016, Mouffoki *et al*. 2017, Besseghier *et al*. 2017, Al-Basyouni *et al*. 2015), Dynamic analysis, wave propagation and free vibration (Karami *et al*. 2018c, Behrouz *et al*. 2016, Ait Yahia *et al*. 2015, Behrouz *et al*. 2018, Besseghier *et al*. 2017, Bouafia *et al*. 2017, Belkorissat *et al*. 2015, Youcef *et al*. 2018), shear deformation theory by Mokhtar *et al*. (2018).

Also, these studies have been started at different scales on composite structures including FGM plates: By using different theories of deformation: The new theories of vibration, bending and wave propagation, applied on shear deformation of FGM and laminated composite plates (Mahi *et al*. 2015, Bellifa *et al*. 2016, Ait Yahia *et al*. 2015, Boukhari *et al*. 2016) and for refined plate with four variable (Bellifa *et al*. 2017b, Fourn *et al*. 2018) and shells by Zine *et al*. (2018). by new shear deformation theories using only two and three variables: 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate by Belabed *et al*. (2018); shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory by Mokhtar *et al*. (2018); Hachemi *et al*. (2017) three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations and Mouffoki *et al*. (2017) nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory.

Studies on thermal stability of plates and functionally graded sandwich plates (Bousahla *et al*. 2016, Bouderba *et al*. 2016, Menasria *et al*. 2017). Thermal buckling (El-Haina *et al*. 2017, Chikh *et al*. 2017); thermoelastic analysis (Tounsi *et al*. 2013, Attia *et al*. 2018), thermomechanical bending response by Bouderba *et al*. (2013). Hygro-thermo-mécanique (Zidi *et al*. 2014, Beldjelili *et al*. 2016). Thickness stretching effect for the flexure analysis of laminated composite plates (Draiche *et al*. 2016, Bouhadra *et al*. 2018); stretching effect for thermomechanical bending by Hamidi *et al*. (2015). Quasi-3D and 2D shear deformation theories (Younsi *et al*. 2018 and Hebali *et al*. 2014). Bending and free vibration analysis of FGM and composite plates by Abualnour *et al*. (2018); sinusoidal shear deformation theory for functionally graded plates by Benchohra *et al*. (2018). Nonlocal strain gradient 3D elasticity theory and nonlocal strain gradient higher order shell theory by Karami *et al*. (2018a, b). new theories for vibration analysis of five variable of refined plate by Bennoun *et al*. (2016) and higher order shear for composite plates by Bousahla *et al*. (2014) and Belabed *et al*. (2014).

Recent studies and applications in mechanics have shown the need to study boundary conditions. They are usually used to evaluate constants of integration when you are performing an indefinite integral. Abdelaziz *et al*. (2017) utilized various boundary conditions for bending, buckling and free vibration of FGM sandwich plates. A simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions is studied by Ait Amar Meziane *et al*. (2014). Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory has been studied by Kaci *et al*. (2018).

It is clear from this summary above, that works on nanoscale materials are being researched compared to materials at other scales of structure and nanostructure. In particular we have seen that most studies are done under a free vibration hence the importance of studying in this document the forced vibration. This carbon nanotube (CNT) will be integrated in a polymer matrix that represents the elastic medium according to the Winkler model. The nanobeam is represented by the Euler-Bernoulli beam and by application of the non-local elasticity theory. The carbon nanotube considered in this work is of the single-walled (SWCNTs).

## 2. Modeling of the elastic medium

The combination of the nanocomposite indicated above, allowed us to use mechanical modeling where it is supposed that the nanotube is a material embedded on an elastic medium equivalent to the polymer matrix. For this study, several approaches exist, such as Winkler's one in 1867, but also Filonenko and Borodich in 1940, Hetényi in 1950, Winkler-Pasternak in 1954, Vlasov in 1960 and Kerr in 1964 (Iancu 2009a, Mourelatos 1987, Rajpurohit *et al*. 2014, Selvadurai *et al*. 1979). Historically we can say that the Winkler model used without this study has proved its efficiency: Starting with Boussinesqin 1885 who studied the problem of a semi-infinite, homogeneous, isotropic, linear and elastic medium submitted to a concentrated vertical load (P) (Selvadurai *et al*. 1979). Vesic in 1961 compared the results of an infinite beam on an elastic

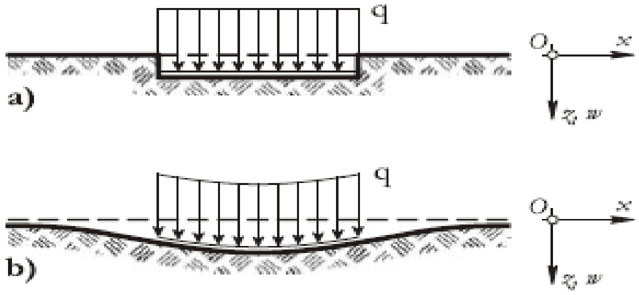


Fig. 1 Deflections of elastic foundations under an uniform load (Mourelatos 1987, Iancu *et al.* 2008): a. Winkler foundation; b. real soil foundations

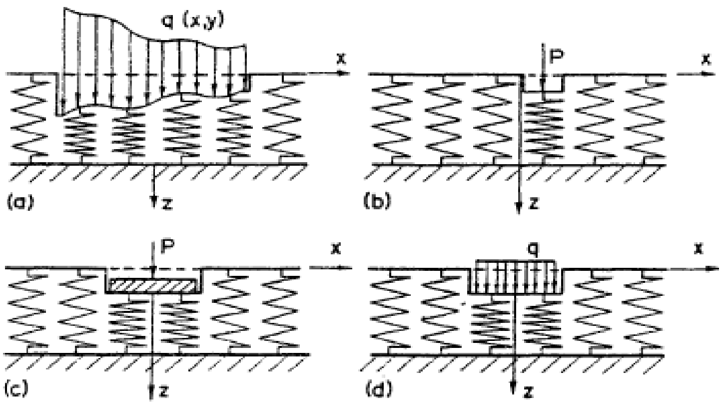


Fig. 2 Winkler model (a) any load; (b) concentrated load, (c) rigid foundation, (d) flexible foundation. (According to Selvadurai, 1979) (Selvadurai *et al.* 1979)

foundation (Chandra *et al.* 1987). Gibson in 1967, confirmed the displacement discontinuity which appears between the loaded portion and the unloaded portion of the foundation surface (Gibson-soil) (Figure 1 and 2 (a and b)), where he determined that displacements are almost constant below the loaded area and are negligible outside this zone and that for different cases of loadings (see Figs. 1(a)-(b)) (Iancu 2009a, Chandra *et al.* 1987).

Winkler supposes in his modeling a beam placed on an elastic support or foundation, modeled by a series of vertical springs, identical, infinitely close, without coupling effect, linearly elastic and with a stiffness noted $k_w(x)$ (Iancu 2009a, Hetenyi 1961, Karasin *et al.* 2014). This model does not consider the transverse shear deformations to simplify the resolution and the obtaining of the analytical solutions (Hetenyi 1961). The discontinuity of the Gibson is validated for this model (see Fig.1) and on the other hand, the settlements of the loaded area in the case of a rigid foundation remain the same in the case of a flexible foundation (see Figs. 2 (c)-(d)) (Selvadurai 1979). This approach presents a linear relationship between the normal algebraic displacement of the structure and the contact pressure between the beam and the elastic foundation (Gorbunov-Posadov *et al.* 1973). Subsequently, Pasternak in 1954 introduced the shearing interaction between the springs by connecting the ends of these last ones to a layer made of incompressible vertical elements that deform only by transverse shear (Mourelatos 1987, Selvadurai *et al.* 1979, Dinev 2012). This is the Winkler-Pasternak model.

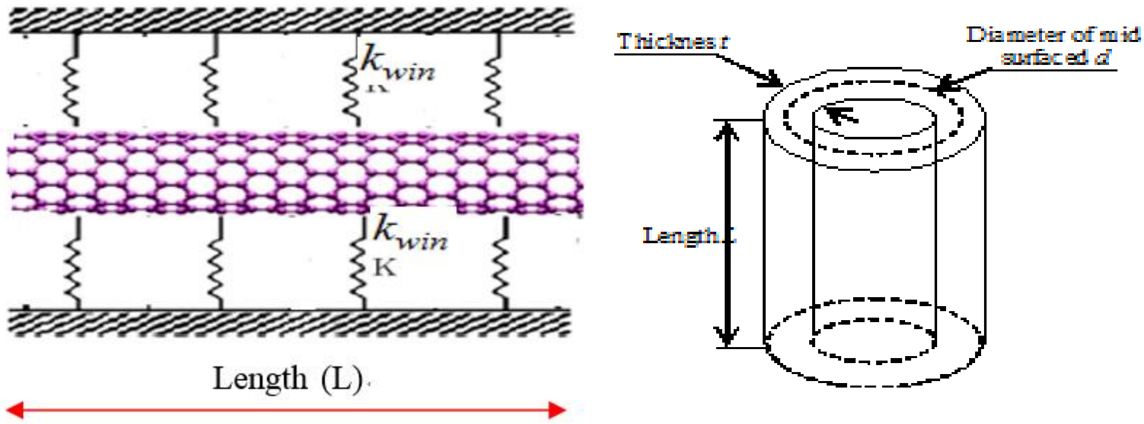


Fig. 3 Geometries and arrangement of a carbon nanotubes incorporated in an elastic medium

The stiffness $k_w(x)$ indicated above is named by the proportionality constant (Iancu 2009b) and known as the support soil reaction module (Selvadurai *et al*. 1979, Iancu *et al*. 2008). In the case of a linear carbon nanotube (Karasin *et al*. 2014) with a constant stiffness (Kacar *et al*. 2011), we can write:

$$f(x) = -k_w w(x) \tag{1}$$

Where $w(x)$ is the transverse displacement of the nanobeam in the $z$ direction and $k_w$ is the Winkler foundation modulus.

## 3. Problem formulation

Considering homogeneous nanobeam of single-walled carbon nanotube with a length (L), thickness (t) a constant section ($A$) and a density ($\rho$). The nanobeam is incorporated an elastic medium with stiffness $k_{win}$ (fig. 3).

Hooke's law for a uni-axial state of stress is given by the following equation (Heireche *et al*. 2008):

$$\sigma(x) - (e_0 a)^2 \frac{\partial^2 \sigma(x)}{\partial^2 x} = E\varepsilon(x) \tag{2}$$

Where $E$ is the modulus of elasticity for the constitutive material of the element; $e_0 a$, is the scale effect ($a$: is an internal characteristic length which represents the length of a C-C bond, and $e_0$ is an adjustable parameter). $\varepsilon(x)$ is the strain given as:

$$\varepsilon(x) = -y \frac{\partial^2 w(x)}{\partial^2 x} \tag{3}$$

The equation of motion of a monolayer carbon nanotube given by Doyle (1997) from which the loading and the elastic medium are added:

$$\frac{\partial V(x)}{\partial x}+q(x)+f(x)-\rho A\frac{\partial^2 w(x)}{\partial^2 t}=0 \tag{4}$$

Where $w(x)$ is the displacement in the z-direction, $(\rho A)$ is the mass per length of nanobeam, $V(x)$ is the shear force, $q(x)$ is the distributed load on the nanobeam and $f(x)$ is the interaction pressure per unite axial length between the nanotube and the surrounding elastic medium represented by equation (1).

From the classical theories of material resistance, the resultant of the shear force $V(x)$ on the cross-section of the nanotube is equal to the derivative of the bending moment with respect to the variable $x$:

$$V(x)=\frac{\partial M(x)}{\partial x} \tag{5}$$

And the resulting bending moment $M(x)$ in a nanobeam section is given as follows:

$$M(x)=\int_A y\sigma dA \tag{6}$$

From Eqs. (2)-(3)-(4)-(5) and (6) the bending moment is equal to:

$$M(x)=-EI\frac{\partial^2 w(x)}{\partial^2 x}+(e_0 a)^2[\rho A\frac{\partial^2 w(x)}{\partial^2 t}-q(x)-f(x)]=0 \tag{7}$$

Where $I$ is the moment of inertia for the cross section of the nanobeam.

From Eqs. (5) and (7) the shear force is equal to:

$$V(x)=-EI\frac{\partial^3 w(x)}{\partial^3 x}+(e_0 a)^2[\rho A\frac{\partial^3 w(x)}{\partial x\partial^2 t}-\frac{\partial q(x)}{\partial x}-\frac{\partial f(x)}{\partial x}] \tag{8}$$

By substituting Eq. (8) into Eq. (4), we obtain the unidimensional general differential equation of the forced vibration of a monolayer carbon nanotube based on the Euler-Bernoulli theory and following the Winkler approach:

$$EI\frac{\partial^4 w(x)}{\partial^4 x}+(1-(e_0 a)^2\frac{\partial^2}{\partial^2 x})(\rho A\frac{\partial^2 w(x)}{\partial^2 t}-q(x)-f(x))=0 \tag{9}$$

Substituting the response of the support $f(x)$ equivalent to Eq. (1) we obtain the governing differential equation (Eq. 10) of forced vibration for the Euler-Bernoulli beam in an elastic medium:

$$EI\frac{\partial^4 w(x)}{\partial^4 x}+(1-(e_0 a)^2\frac{\partial^2}{\partial^2 x})(\rho A\frac{\partial^2 w(x)}{\partial^2 t}-q(x)+k_w w(x))=0 \tag{10}$$

In the case of the local model ($e_0 a$ = 0), not forced (free vibration: ($q(x)$ = 0) ), Eq. (10) will represent the governing differential equation of free vibration for the Euler-Bernoulli beam (Kacar *et al*. 2011) and is expressed as follows:

$$EI\,\frac{\partial^4 w(x)}{\partial^4 x} + \rho A \frac{\partial^2 w(x)}{\partial^2 t} + k_w w(x) = 0 \qquad 0 \le x \le l \tag{11}$$

Returning to Eq. (10), the resolution of the latter with partial derivatives of order four is done by the technique of separation of the variables ($x$ and $t$). Knowing we treat a vibration problem, it will be assumed that displacement $w(x, t)$ and loading $q(x, t)$ are sinusoidal functions of pulsating ($\omega$) (Heireche *et al*. 2008).

So we pose:

$$\begin{cases} w(x,t) = \sum W_{NL} \sin \lambda x \sin \omega t \\ q(x,t) = q_m \sin \lambda x \sin \omega t \end{cases} \qquad \lambda = \frac{N\pi}{l}(m = 1,2,....) \tag{12}$$

The resolution of this differential Eq. (10) gives the amplitude in the model of non-local elasticity theory, $W_{NL}(x)$ such as:

$$W_{NL}(x) = \frac{q_m L^4 (1 + \lambda^2 (e_0 a)^2)}{EI\lambda^4 L^4 - \rho A \omega^2 L^4 + k_w L^4 - (e_o a)^2 \rho A \omega^2 \lambda^2 L^4 + (e_o a)^2 \lambda^2 L^4 k_w} \tag{13}$$

$$W_{NL}(x) = \frac{q_m L^4 \psi}{EI(\beta^4 + \psi\gamma - \psi\Omega^2)} \tag{13a}$$

With

$$\begin{cases} \psi = 1 + \lambda^2 (e_0 a)^2 \\ \beta = \lambda L \\ \Omega^2 = \rho A \dfrac{\omega^2 L^4}{EI} \\ \gamma = \dfrac{k_w L^4}{EI} \end{cases} \tag{14}$$

In the case of local model ($e_0 a = 0$), the amplitude $W_L(x)$ will be expressed as follows:

$$W_L(x) = \frac{q_m L^4}{EI(\beta^4 + \gamma - \Omega^2)} \tag{15}$$

For a free vibration where ($q(x) = 0$), without elastic medium ($k_{win} = 0$) and in the case of the model of local elasticity theory ($e_0 a = 0$), from Eq. (13) we have:

$$EI\lambda^4 - \rho A \omega_n^2 = 0 \tag{16}$$

Where:

$$\omega_n^2 = \frac{EI\lambda^4}{\rho A} \tag{17}$$

Table 1 Results of frequency and amplitude ratios as a function of dimensional ratios (L / d) with an elastic medium and a constant mode number (N = 6)

| r | L/d=10 | L/d=20 | L/d=30 |
|---|---|---|---|
| 2 | 0.930189 | 0.977463 | 0.988388 |
| 2.5 | 0.958901 | 0.987121 | 0.993678 |
| 3 | 0.972650 | 0.991548 | 0.995939 |
| 3.5 | 0.980399 | 0.993990 | 0.997145 |
| 4 | 0.985228 | 0.995492 | 0.997874 |
| 4.5 | 0.988452 | 0.996487 | 0.998351 |
| 5 | 0.990717 | 0.997182 | 0.998682 |
| 5.5 | 0.992370 | 0.997688 | 0.998921 |
| 6 | 0.993616 | 0.998068 | 0.999099 |
| 6.5 | 0.994578 | 0.998360 | 0.999237 |
| 7 | 0.995337 | 0.998591 | 0.999345 |
| 8 | 0.996443 | 0.998926 | 0.999501 |
| 9 | 0.997197 | 0.999154 | 0.999608 |
| 10 | 0.997734 | 0.999317 | 0.999683 |

In the Eq. (14) we have:

$$\Omega^2 = \rho A \frac{\omega^2 L^4}{EI} \tag{18}$$

By substituting the Eq. (17) into Eq. (18) we have:

$$\Omega^2 = \beta^4 r^2 \tag{18a}$$

With:

$$r^2 = \frac{\omega^2}{\omega_n^2} \tag{19}$$

So, ($r$) is the frequency ratio between the forced vibration and the free vibration. To study the effect of locality of the constraints or loading, the ratio between the two amplitudes non-local $W_{NL}(x)$ and local $W_L(x)$ is calculated and worths:

$$\frac{W_{NL}}{W_L} = \frac{\beta^4 \psi (1 - r^2) + \psi\gamma}{\beta^4 (1 - \psi r^2) + \psi\gamma} \tag{20}$$

## 4. Results and discussion

The characteristics of the carbon nanotube used in the calculation are as follows: the modulus of young E = 1TPa, the Poisson's ratio υ = 0.19, the mass density ρ = 2.3 g /cm$^3$ and the effective thickness (t) of NTC is taken equal to 0.3 nm.

To better interpret the results obtained in the calculation during this work, the calculated values are grouped in tables and represented by the figures. So, the Tables 1-3 represent the values of the ratios of the amplitudes between the non-local and local elasticity theory ($W_{NL}$ / $W_L$) computed as a function of the ratios ($r$) between the forced and free frequencies. In Table 1, the effect of the dimensional ratio (L/d) is shown in Fig. 5.

In Table 2, we presented the effect of the mode number (N) shown in Fig. 6.

Table 2 Results of the ratios of frequencies and amplitudes as a function of the mode number with an elastic medium and constant dimensional ratio (L / d = 5)

| r | N=1 | N=3 | N=6 |
|---|---|---|---|
| 2 | 0.988388 | 0.930189 | 0.848146 |
| 2.5 | 0.993678 | 0.958901 | 0.907189 |
| 3 | 0.995939 | 0.972650 | 0.937087 |
| 3.5 | 0.997145 | 0.980399 | 0.954434 |
| 4 | 0.997874 | 0.985228 | 0.965432 |
| 4.5 | 0.998351 | 0.988452 | 0.972856 |
| 5 | 0.998682 | 0.990717 | 0.978111 |
| 5.5 | 0.998921 | 0.992370 | 0.981969 |
| 6 | 0.999099 | 0.993616 | 0.984886 |
| 6.5 | 0.999237 | 0.994578 | 0.987147 |
| 7 | 0.999345 | 0.995337 | 0.988934 |
| 8 | 0.999501 | 0.996443 | 0.991547 |
| 9 | 0.999608 | 0.997197 | 0.993331 |
| 10 | 0.999683 | 0.997734 | 0.994604 |

Table 3 Results of the ratios of frequencies and amplitudes as a function of the parameter of the stiffness of elastic medium and with a mode number and dimensional ratio constants (N= 6; L/d= 10)

| r | without elastic medium | with elastic medium |
|---|---|---|
| 2 | 0.994099 | 0.990800 |
| 2.5 | 0.996620 | 0.995745 |
| 3 | 0.997779 | 0.997432 |
| 3.5 | 0.998419 | 0.998251 |
| 4 | 0.998814 | 0.998722 |
| 4.5 | 0.999075 | 0.999020 |
| 5 | 0.999258 | 0.999223 |
| 5.5 | 0.999391 | 0.999368 |
| 6 | 0.999491 | 0.999475 |
| 6.5 | 0.999568 | 0.999556 |
| 7 | 0.999629 | 0.999620 |
| 8 | 0.999717 | 0.999712 |
| 9 | 0.999777 | 0.999774 |
| 10 | 0.999820 | 0.999818 |

and in the Table 3, the effect of the elastic medium ($k_{win}$), is shown in Fig. 7. A case is shown in Fig. 4, for the constant values (L/d=10, N=6 and without medium elastic).

The Tables 4-5 group successively, summarizes the calculated values of the effect of the dimensional ratio (L/d) and the effect of the mode number (N) on the ratio of the local and non-local amplitudes ($W_{NL}/W_L$) for both cases of elastic and inelastic medium. These results are shown successively in Fig. 8 and Fig. 9.

Table 4 Results of the ratios of amplitudes, according to parameters of dimensional ratios in different cases of stiffness of elastic medium)

| Dimensionnel Ratio (L/d) | without elastic medium | with elastic medium |
|---|---|---|
| 10 | 0.985232 | 0.985228 |
| 20 | 0.995512 | 0 .995492 |
| 30 | 0.997922 | 0 .997874 |
| 40 | 0.998814 | 0 .998722 |
| 50 | 0.999235 | 0 .999072 |
| 60 | 0.999467 | 0 .999161 |

Table 5 Results of the ratios of amplitudes, according to parameters of mode number in different cases of stiffness of elastic medium)

| Numer of mode (N) | without elastic medium | with elastic medium |
|---|---|---|
| 1 | 0.999001 | 0.996838 |
| 2 | 0.996112 | 0 .995939 |
| 3 | 0.991619 | 0 .991548 |
| 4 | 0.985927 | 0 .985890 |
| 5 | 0.979473 | 0 .979451 |
| 6 | 0.972664 | 0 .972649 |

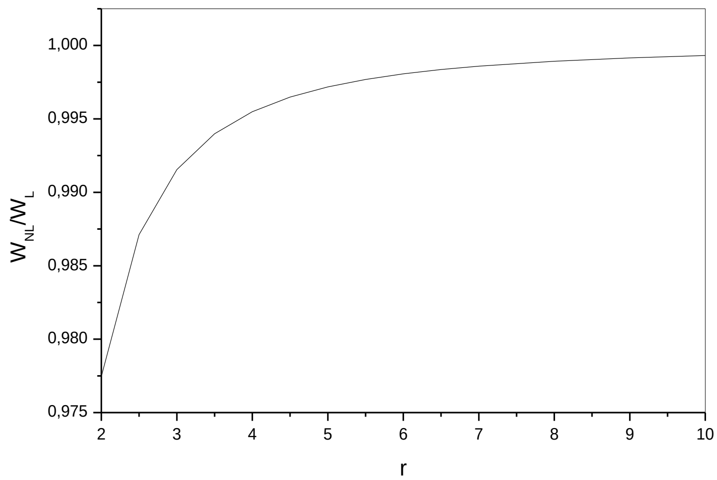


Fig. 4 The relationship between the amplitudes ratio **(**$W_{NL}/W_L$**)** and the frequency ratio (r)

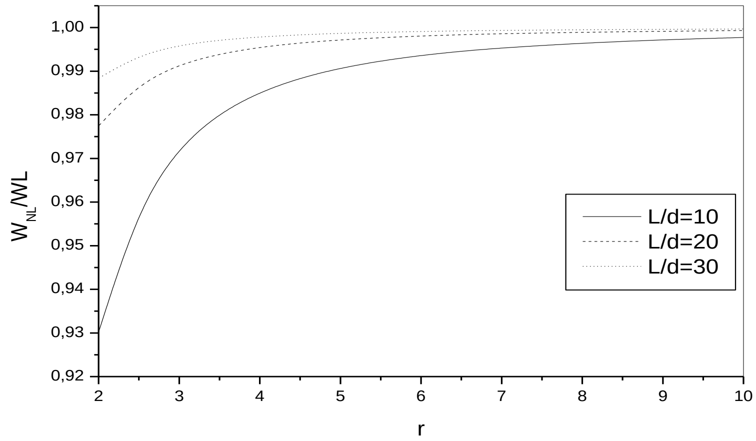


Fig. 5 The relationship between the amplitudes ratio $(W_{NL}/W_L)$ and the frequency ratio (r) for different dimensional ratios (L/d) and mode number constant (N=6)

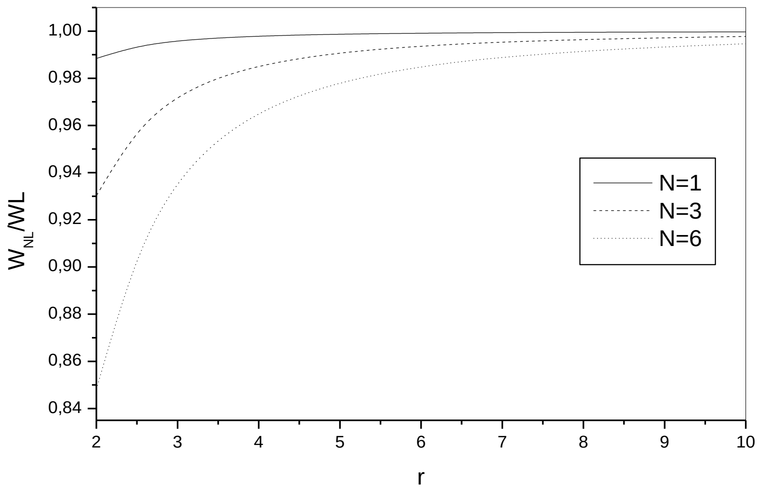


Fig. 6 The relationship between the amplitudes ratio $(W_{NL}/W_L)$ and the frequency ratio (r) for different mode number (N) and dimensional ratio constant (L/d= 10)

### *4.1 Effects of the parameters related to the CNT*

The dependence of the ratio of the amplitude $(W_{NL}/W_L)$ with the frequency ratio (r) of the carbon nanotubes located in an elastic medium is illustrate in the Figs. 4-6.

The ratio $(W_{NL}/W_L)$ will be an index to quantitatively evaluate the scale effect on solutions of carbon nanotube vibrations. Firstly, it is found in the Fig.4 that this variation is important for any frequency value $r \leq 5$, from this value; the dependence becomes very low and begins to stabilize with amplitude close to unity. It means that for the highest forced vibration frequencies ($r > 5$, in our case) we can neglect scale effect and we only based on the local elasticity theory. Thereafter, we see in Fig. 5, that the amplitude $(W_{NL}/W_L)$ has its maximum for the weakest dimensional ratio

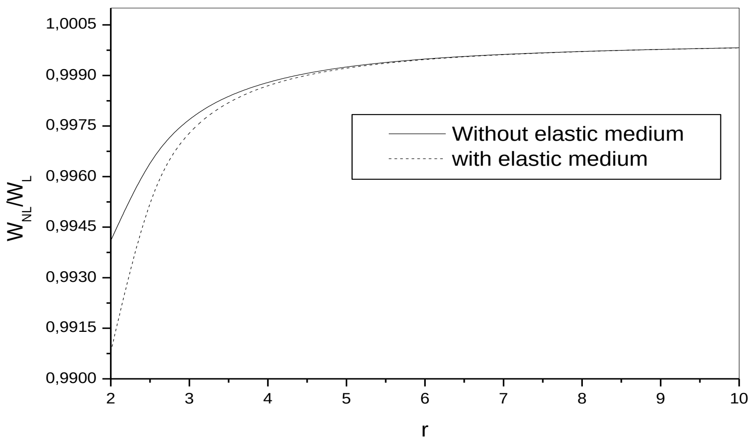


Fig. 7 The relationship between the amplitudes ratio ($W_{NL}/W_L$) and the frequency ratio (r) with and without elastic mediums for the values (N=6 and L/d= 10)

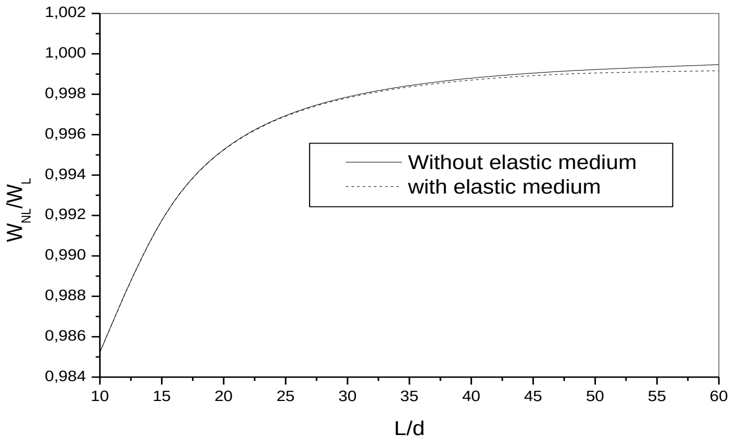


Fig. 8 The relationship between the amplitudes ratio **($W_{NL}/W_L$)** and the dimensional ratio (L/d) with and without elastic mediums, (N=6)

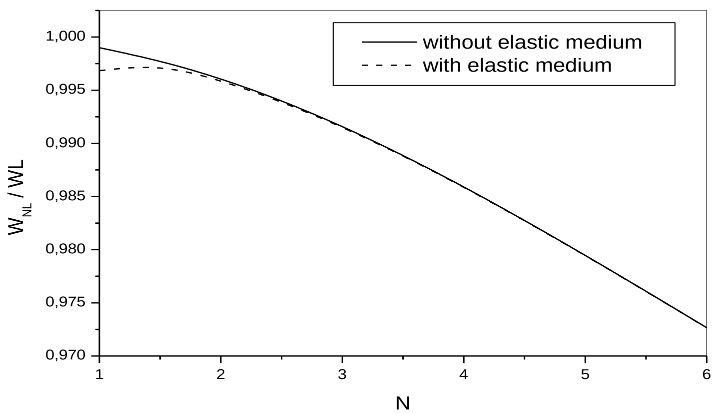


Fig. 9 The relationship between the amplitudes ratio **($W_{NL}/W_L$)** and number of mode (N) with and without elastic mediums, (L/d=10)

(L/d), equivalent at the shortest nanotube knowing that the diameter (d) is taken constant, as well as in the Fig.6, the amplitude has its maximum for the biggest mode, hence the importance of the use of non-local elasticity theory and vice versa.

### *4.2 Effect of the elastic medium*

The Figs. 7-9, illustrate the effect of the elastic medium on the amplitude of the carbon nanotube with the different scale or dimensional ratio (L/d) and the modes number (N).

It can be said that the amplitude of the nanotubes which represents the relationship between the effect of the local and non local continuum theory, depends on the nature of the medium where it is located (see Fig. 7), this dependency is low in the case of the non elastic medium compared to another elastic medium and this for frequency ratios $r \leq 5$; this means that the use of non-local elasticity theory is necessary for the high stiffness. For frequency ratios $r > 5$ the effect of the nature of the elastic medium becomes negligible. Also, it is still evident that the use of non-local elasticity theory is necessary for the case of the weakest dimensional ratios ($L/d \leq 50$, in this study) and that for any values of the stiffness ($k_{win}$) of the medium (see Fig. 8). From the value $L/d > 50$, the local effect is important with a small variation between an elastic medium and a non-elastic medium. The use of non-local elasticity theory is also necessary for the higher modes (N), from the second mode in our study (see Fig. 9) and this for any values of the stiffness ($k_{win}$) of medium.

## 5. Conclusion

In this paper, the study of a single-wall carbon nanotube element (SWCNTs) represented by a Euler-Bernoulli beam model, subjected to a forced vibration and surrounded by a winkler-type elastic medium was presented. Numerous parameters were taken into account including, the dimensional ratio (L/d), the mode number (N) and the stiffness of the elastic medium ($k_{win}$). The results have shown the dependence of the vibration amplitudes on the different parameters cited above. Moreover, it is found that the application of the nonlocal elasticity theory model or theory is important for high mode numbers and short nanotubes. Also, it can be concluded in the case of high rigidities than the non-local theory is necessary.

## References

Abdelaziz, H.H., Meziane, M.A.A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct.*, **25**(6), 693-704

Abualnour, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

Acton, Q.A., (2013), *Issues in Hydrogen, Fuel Cell, Electrochemical, and Experimental Technologies*, Scholarly Edition, Atlanta, Georgia, U.S.A.

Ahmadi Asoor, A.A., Valipour, P. and Ghasemi, S.E. (2016), "Investigation on vibration of single-walled carbon nanotubes by variational iteration method", *Appl. Nanosci.*, **6**(2), 243-249.

Ahouel, M., Houari, M.S.A., Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Yahia, S., Atmane, H.A., Houari, M.S.A. and Tounsi, A., (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, **53**(6), 1143-1165.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Ali-Akbari, H.R. and Firouz Abadi, R.D. (2015), "Nonlinear free vibration of single-walled carbon nanotubes embedded in viscoelastic medium based on asymptotic perturbation method", *J. Sci. Eng.*, **6** (2), 42-58.

Ansari, R. and Hemmatnezhad, M. (2013), "Nonlinear finite element vibration analysis of double-walled carbon nanotubes based on Timoshenko beam theory", *J. Vib. Control*, **19**(1), 75-85.

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2018), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations", *Struct. Eng. Mech.*, **65**(4), 453-464.

Bakis, C.E., Bank, L.C., Brown, V.L., Cosenza, E., Davalos, J.F., Lesko, J.J., Machida, A., Rizkalla, S.H. and Triantafillou, T.C. (2002), "Fiber-reinforced polymer composites for construction-state-of-the-art review", *J. Compos. Construct.*, **6**(2), 73-87.

Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct.*, **14**(2), 103-115.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Bég, O.A. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B Eng.*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017b), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct.*, **25**(3), 257-270.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017a), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech.*, **62**(6), 695-702.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Benchohra, M., Driz, Z., Bakora, A. and Tounsi, A. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech.*, 65(1), 19-31.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst.*, **19**(6), 601-614.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst.*, **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.
Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422.
Bouhadra, A., Tounsi, A., Bousahla, A.A., Benyoucef, S. and Mahmoud, S.R. (2018), "Improved HSDT accounting for effect of thickness stretching in advanced composite plates", *Struct. Eng. Mech.*, **66**(1), 61-73.
Boukhari, A., Atmane, H.A., Tounsi, A., Adda, B. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech.*, **57**(5), 837-859.
Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.
Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.
Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *J. Comput. Methods*, **11**(6), 1350082.
Chandra, S., Madhira, R.M. and Iyengar, N.G.R. (1987), "A new model for nonlinear subgrades", *Proceedings of 5th ICMM*, Roorkee, India.
Chemi, A, Zidour, M., Heireche, H., Rakrak, K. and Bousahla, A.A. (2018), "Critical buckling load of chiral double-walled carbon nanotubes embedded in an elastic medium", *Mech. Compos. Mater.*, **53**(6), 827-836.
Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst.*, **19**(3), 289-297.
Choi, S.M. and Awaji, H. (2005), "Nanocomposites: A new material design concept", *Sci. Technol. Adv. Mater.*, **6**(1), 2-10.
de Azeredo, H.M.C. (2009), "Nanocomposites for food packaging applications", *Food Res. Int.*, **42**(9), 1240-1253.
Dihaj, A., Zidour, M., Meradjah, M., Rakrak, K., Heireche, H. and Chemi, A. (2018), "Free vibration analysis of chiral double-walled carbon nanotube embedded in an elastic medium using non-local elasticity theory and Euler Bernoulli beam model", *Struct. Eng. Mech.*, **65**(3), 335-342.
Dinev, D. (2012), "Analytical solution of beam on elastic foundation by singularity functions", *Eng. Mech.*, **19**(6), 381-392.
Doyle, J.F. (1997), *Wave Propagation in Structures: Spectral Analysis Using Fast Discrete Fourier Transforms*, Springer Science & Business Media, New York, NY, U.S.A.
Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng.*, **11**(5), 671-690.
El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech.*, **63**(5), 585-595.
Fourn, H., Atmane, H.A., Bourada, M., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates", *Steel Compos. Struct.*, **27**(1), 109-122.
Gafour, Y., Zidour, M., Tounsi, A., Heireche, H. and Semmah, A. (2013), "Sound wave propagation in zigzag double-walled carbon nanotubes embedded in an elastic medium using nonlocal elasticity theory", *Physica E*, **48**, 118-123.
Gay, D. (1987), *Materiaux Composites*, *5 ème Edition*, Hermès-Lavoisier, France.
Gorbunov-Posadov, M.I., Malikova, T.A. and Solomin, V.I. (1973), *Analysis of Structures on Elastic Foundation*, Stroiizdat, Moscow, Russia.
Hachemi, H., Kaci, A., Houari, M.S.A., Bourada, M., Tounsi, A. and Mahmoud, S.R. (2017), "A new simple

three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations", *Steel Compos. Struct.*, **25**(6), 717-726.

Hajnayeb, A. and Khadem, S.E. (2015), "An analytical study on the nonlinear vibration of a doublewalled carbon nanotube", *Struct. Eng. Mech.*, **54**(5), 987-998.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 235-253.

Hamidi, A., Zidour, M., Bouakkaz, K. and Bensattalah, T. (2018), "Thermal and small-scale effects on vibration of embedded armchair single-walled carbon nanotubes", *J. Nano Res.*, **51**, 24-38.

Harris, P.J.F. (1999), *Carbon Nanotubes and Related Structures*: *New Materials for the Twenty-first Century*, Cambridge University Press, New York, NY, U.S.A.

Harris, P.J.F. (2009), *Carbon Nanotube Science Synthesis*, *Properties and Applications*, Cambridge University Press, New York, NY, U.S.A.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Bedia, E.A.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**(2), 374-383.

Heireche, H., Tounsi, A., Benzair, A. and Mechab, I. (2008), "Sound wave propagation in single-walled carbon nanotubes with initial axial stress", *J. Appl. Phys.*, **104**(1), 014301.

Hetenyi, M. (1961), *Beams on Elastic Foundations*, University of Michigan Press, Ann Arbor, U.S.A.

Hina, S., Zhang, Y. and Wang, H. (2014), "Characterization of polymeric solutions: A brief overview", *Rev. Adv. Mater. Sci.*, **36**(2), 165-176.

Houari, M.S.A., Bessaim, A., Bernard, F., Tounsi, A. and Mahmoud, S.R. (2018), "Buckling analysis of new quasi-3D FG nanobeams based on nonlocal strain gradient elasticity theory and variable length scale parameter", *Steel Compos. Struct.*, **28**(1), 13-24.

Hu, H., Onyebueke, L. and Abatan, A. (2010), "Characterizing and modeling mechanical properties of nanocomposites, review and evaluation", *J. Minerals Mater. Characterization Eng.*, **9**(4), 275-319.

Iijima, S. (1991), "Helical microtubules of graphitic carbon", *Nature*, **354**(6348), 56-58.

Kacar, A., Tugba, H.T. and Metin, O.K. (2011), "Free vibration analysis of beams on variable winkler elastic foundation by using the differential transform method", *Math. Comput. Appl.*, **16**(3), 773-783.

Kaci, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech.*, **65**(5), 621-631.

Karami, B., Janghorban, M. and Tounsi, A. (2017), "Effects of triaxial magnetic field on the anisotropic nanoplates", *Steel Compos. Struct.*, **25**(3), 361-374.

Karami, B., Janghorban, M. and Tounsi, A. (2018a), "Nonlocal strain gradient 3D elasticity theory for anisotropic spherical nanoparticles", *Steel Compos. Struct.*, **27**(2), 201-216.

Karami, B., Janghorban, M. and Tounsi, A. (2018b), "Variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory", *Thin-Walled Struct.*, **129**, 251-264.

Karami, B., Janghorban, M., Shahsavari, D. and Tounsi, A. (2018c), "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel Compos. Struct.*, **28**(1), 99-110.

Karasin, A. and Gultekin, A. (2014), "An approximate solution for plates resting on winkler foundation", *J. Civil Eng. Technol.*, **5**(11), 114-124.

Karličić, D., Kozić, P. and Pavlović, R. (2015), "Flexural vibration and buckling analysis of single-walled carbon nanotubes using different gradient elasticity theories based on reddy and huu-tai formulations", *J. Theor. Appl. Mech.*, **53**(1), 217-233.

Kaushik, B.K. and Majumder, M.K. (2015), *Carbon Nanotube Based VLSI Interconnects: Analysis and Design*, Springer, Germany.

Khetir, H., Bouiadjra, M.B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech.*, **64**(4), 391-402.

Kiani, K. (2014), "Vibration and instability of a single-walled carbon nanotube in a three-dimensional magnetic field", *J. Phys. Chem. Solids*, **75**(1), 15-22.

Kumar, A. and Gupta, R.K. (2003), *Fundamentals of Polymers Engineering*, *Second edition*, *Revised and Expanded*, CRC Press, Florida, U.S.A.

Kumar, D. and Srivastava, A. (2016), "Elastic properties of CNT- and graphene-reinforced nanocomposites using RVE", *Steel Compos. Struct.*, **21**(5), 1085-1103.

Mahi, A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.* **39**(9), 2489-2508.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct.*, **25**(2), 157-175.

Mokhtar, Y., Heireche, H., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst.*, **21**(4), 397-405.

Mouffoki, A., Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383.

Mourelatos, Z.P. and Parsons, M.G. (1987), "A finite element analysis of beams on elastic foundation including shear and axial effects", *Comput. Struct.*, **27**(3), 323-331.

Mrazova, M. (2013), "Advanced composite materials of the future in aerospace industry", *Incas Bulletin*, **5**(3), 139 - 150.

Nguyen, T.N., Kim, N. and Lee, J. (2017), "Static behavior of nonlocal Euler-Bernoulli beam model embedded in an elastic medium using mixed finite element formulation", *Structural Eng. Mech.*, **63**(2), 137-146.

Öchsner, A. and Shokuhfar, A. (2013), *New Frontier of Nanoparticles and Nanocomposite Materials*, *Novel Principles and Techniques*, Springer, Germany.

Okpala, C.C. (2013), "Nanocomposites: An overview", *J. Eng.*, *Res. Develop.*, **8**(11), 17-23.

Okpala, C.C. (2014), "The benefits and applications of nanocomposites", *J. Adv. Engg. Tech.*, **5**(4), 12-18.

Rajpurohit, V.K., Gore, N.G. and Sayagavi, V.G. (2014), "Analysis of structure supported on elastic foundation", IJEAT, **4**(1), 1-6.

Rakrak, K., Zidour, M., Heireche, H., Bousahla, A.A. and Chemi, A. (2016) "Free vibration analysis of chiral double-walled carbon nanotube using non-local elasticity theory", *Adv. Nano Res.*, **4**(1), 31-44.

Sachse, S., Gendre, L., Silva, F., Zhu, H., Leszczyńska, A., Pielichowski, K., Ermini, V. and Njuguna, J. (2013), "On nanoparticles release from polymer nanocomposites for applications in lightweight automotive components", *J. Physics*, *Conference Series*, **429**(1), 012046.

Selvadurai, A.P.S. (1979), *Elastic Analysis of Soil-Foundation Interaction*, Elsevier Scientific Publishing Company, New York, U.S.A.

Shehata, F., Abdelhameed, M., Fathy, A. and Elmahdy, M. (2011), "Preparation and characteristics of Cu-$Al_2O_3$ nanocomposite", *Open J. Metal*, **1**(2), 25-33.

Soltani, P., Bahramian, R. and Saberian, J. (2015), "Nonlinear vibration analysis of the fluid-filled single walled carbon nanotube with the shell model based on the nonlocal elacticity theory", *J. Solid Mech.*, **7** (1), 58-70.

Teodoru, I.B. (2009a), "EBBEF2p – A computer code for analysing beams on elastic foundations", *Intersections/Intersecţ II*, **6**(1), 28-44.

Teodoru, I.B. (2009b), "Beams on elastic foundation the simplified continuum approach", *Buletinul Institutului Politehnic din lasi*. *Sectia Constructii*, *Arhitectura*, **55**(4), 37- 45.

Teodoru, I.B. and Muşat, V. (2008), "Beam elements on linear variable two-parameter elastic foundation", *Buletinul Institutului Politehnic din lasi*., **54**(2), 69-78.

Togun, N. and Bağdatlı, S.M. (2016), "Nonlinear vibration of a nanobeam on a pasternak elastic foundation based on non local Euler-Bernoulli beam theory", *Math. Comput. Appl.*, **21**(3), 1-19.

Tounsi, A., Houari, M.S.A. and Benyoucef, S. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates”, *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Yazid, M., Heireche, H., Tounsi, A., Bousahla, A.A. and Houari, M.S.A. (2018), “A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium”, *Smart Struct. Syst.*, **21**(1), 15-25.

Youcef, D.O., Kaci, A., Benzair, A., Bousahla, A.A. and Tounsi, A. (2018), “Dynamic analysis of nanoscale beams including surface stress effects”, *Smart Struct. Syst.*, **21**(1), 65-74.

Younsi, A., Tounsi, A., Zaoui, F.Z., Bousahla, A.A. and Mahmoud, S.R. (2018), “Novel quasi-3D and 2D shear deformation theories for bending and free vibration analysis of FGM plates”, *Geomech. Eng.*, **14**(6), 519-532.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), “A mechanical response of functionally graded nanoscale beam: An assessment of a refined nonlocal shear deformation theory beam theory”, *Struct. Eng. Mech.*, **54**(4), 693-710.

Zidi, M., Tounsi, A., Houari, M.S.A. and Bég, O.A. (2014), “Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory”, *Aerospace Sci. Tech.*, **34**, 24-34.

Zine, A., Tounsi, A., Draiche, K., Sekkal, M. and Mahmoud, S.R. (2018), “A novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells”, *Steel Compos. Struct.*, **26**(2), 125-137.

*AP*