

Advanced numerical tool for composite woven fabric preforming

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Abstract. In this paper, geometrical and mechanical approaches are proposed for the simulation of the draping of woven fabric onto complex parts. The geometrical discrete approach allows to define the ply shapes and fibres orientation in order to optimize the composite structural properties and the continuum meso-structural mechanical approach allows to take into account the mechanical properties of fibres and resin and the various dominating mode of deformation of woven fabrics during the forming process. Some numerical simulations of forming process are proposed and compared with the experimental results in order to demonstrate the efficiency of our approaches.

Keywords: woven fabric; draping, geometrical approach; continuum finite element analysis

1. Introduction

Composite reinforced by woven fabric is known to have high specific stiffness and, in combination with automatic manufacturing processes, make it possible to fabricate complex components (aircraft, boat, automotive and military). The main objective of aerospace industries is to reduce to half the amount of fuel by 2020 and at least 70% less by 2025. The weight saving to increase payload and the reductions of the cost/time of the production cycle are imperative targets. Aircraft and rotorcraft parts are made from high-performance composite materials due to their increased strength, reduced weight, and increased service life. For these reasons, most aircraft part designers choose to create new parts out of carbon-fibre-reinforced plastic despite the challenges involved, namely that the manufacture of CFRP parts is more complex than traditional materials and can therefore make parts more costly to produce Bannister (2001), Campbell (2004).

One of the most attractive properties of the fabric reinforcements is that they are easily handled and automatically processed, which can lower the production cost. As known, the substitution of metal alloys by composite materials, in general, reduces structural mass by 20-30%. The mass increase is due also to the numerous variety of semi-products (roving, fabrics, knitted fabrics, braids pre-impregnated or not) permitting the development of new structures. Fabrication processes, also, have undergone substantial evolution in recent years. Although the traditional lay-

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up process will remain the process of choice for some applications, new developments in Resin Transfer Molding (RTM), Liquid Composite Moulding (LCM) or Sheet Molding Compound (SMC), low temperature curing pre-pregs and low pressure molding compounds have matured significant are reached, and are now being exploited in high technology areas such as aerospace industry, Hou *et al.* (1997), Campbell (2004).

For example, by using such composites, the aerospace industry can realize improved fuel economy through plane-weight reduction by replacing the currently used aluminum parts with thermoplastic woven fabric composites. The choice of manufacturing process depends on the type of matrix and fabric, the temperature required to form the part and the cost effectiveness of the process. In particular, thermo-forming is a promising manufacturing process for producing high-volume low-cost composite parts using commingled fibre glass/polypropylene woven fabrics, Hou *et al.* (1997).

The composite manufacturing process involves large displacements and rotations and large shear of weft and warp fibres, which can have a significant effect on the processing and structural properties of the finished product. The effective modeling of the forming properties of woven fabric polymer composite materials has been the focus of research for more than a decade. The formulation of new and more efficient numerical models for the simulation of the shaping composite processes must allow for reduction in the delay in manufacturing of complex parts and an optimization of costs in an integrated design approach. Several modeling approaches have been developed to account for the evolution of the orthotropic directions during high shearing, and these approaches include the geometrical and the finite element approaches; Hou *et al.* (1997), ElHami *et al.* (2009), Warby *et al.* (2003), Teik-Cheng Lim *et al.* (2002), Cherouat *et al.* (2005), Mark *et al.* (1991), Taha *et al.* (2012).

In this context, simulation methods are needed to anticipate the performance of the final part but also to predict the reinforcement preforming and the resin injection. Early methods, based on a geometric approach of the fishnet type algorithm emerged a few decades ago. The geometrical approach so called fishnet algorithms is used to determine the deformed shape of draped fabrics. The main assumptions are that the fibers are inextensible, intersection points between warp and weft yarns are fixed during preforming and the rotations between warp and weft yarns are free. This method, where the fabric is placed progressively from an initial line, provides a close enough resemblance to handmade draping, Van Der Ween (1991), Long and Rudd (1994), Hancock (2005).

These methods were originally developed for prepreg hand draping. They are very fast and fairly efficient in many prepreg draping cases. Nevertheless, this method has major drawbacks. They account neither for the mechanical behavior of the fabric nor for the static boundary conditions. This last point is very important in the case of forming with punch and die (such as in the pre-forming of the RTM process). The loads on the tools, especially on the blank-holder, influence the quality of the shaping operation, and therefore, need to be considered in simulations and therefore, need to be considered in simulations. More recently finite element methods have been used for the draping simulation of composite reinforcements, Boisse *et al.* (1995), Trochu *et al.* (2006), Rajiv Asthana *et al.* (2006), Prodromou *et al.* (2011).

The alternative to the geometrical approach consists of a mechanical analysis of the fabric deformation under the boundary conditions prescribed by the forming process. This requires a specific model of the woven reinforcement and its mechanical behavior. The mechanical behavior of woven fabrics is complex due to the intricate interactions of the fibres. It is a multi-scale problem. The macroscopic behavior is very much dependent on the interactions of fibres at the

meso-scale (scale of the woven unit cell) and at the micro-scale (level of the fibres constituting yarns). Despite of a great amount of work in the field, there is no widely accepted model that accurately describes all the main aspects of fabric mechanical behavior. The main model families come from the multi-scale nature of the textile. A first family of models is obtained by homogenizing the mechanical behavior of the underlying meso-structure and considering the fabric as an anisotropic continuum Boisse *et al.* (1995), Trochu *et al.* (2006).

If these models can easily be integrated in standard finite element using conventional shell or membrane elements, then the identification of homogenized material parameters is difficult, especially because these parameters change when the fabric is strained and when, consequently, the directions and the geometry (crimp, transverse sections...) of the fibres change. Some of these approaches will be described, especially a non-orthogonal constitutive model and an anisotropic hypo-elastic continuous behavior for fibrous material based on an objective derivative using the rotation of the fibre. Conversely, some authors present finite element models to model the warp and the weft fibres behavior. Each yarn or each fibre is modeled and is assumed to be a straight or a curved beam or truss. Sometimes they are modeled as 3D domains. Springs are often used to model warp and weft yarn interactions. In the objective of fabric forming simulations, some authors extend the finite element modeling to the whole textile structure that is represented by a network of interwoven trusses or beams with different tensional and rotational springs. Accounting for the simplicity of each component, the whole textile structure deformation can be computed, Vanclooster *et al.* (2009), Potluri *et al.* (2001), Peng *et al.* (2003), Sharma and Sutcliffe (2004), Fan *et al.* (2006).

Nevertheless, the computational effort needed is relatively significant. At present this method is restricted to simple geometry of the local yarn and relatively simple mechanical behavior. When a fine model of the fibrous yarns is used, the analysis can only consider a small part of the textile reinforcement such as a few woven or knitted cells. The semi-discrete approach is a compromise between the above continuous and discrete approaches Boisse *et al.* (2001). A finite element method is associated to a mesoscopic analysis of the woven unit cell. Specific finite elements are defined that are made of a discrete number of woven unit cells. The mechanical behaviour of these woven cells is obtained by experimental analyses or from 3D FE computations of the woven cell. The nodal interior loads are deduced from this local behavior and the corresponding strain energy in the element deformation.

In this study, geometrical and finite element approaches are used to simulate the deformation of preforming of woven fabric using discrete geometrical and continuum mechanical approaches. These approaches, while giving good results and being efficient in terms of computing time, are generally somewhat complex and sometimes very challenging to implement into commercially available FEA packages.

2. Discrete geometrical approach

The draping of woven fabric using a continuum approach requires the resolution of mechanical equilibrium problems (see section 3). In general, in the case of complex surfaces, the boundary conditions are not well defined and the contact between the surface and the fabric is difficult to manage. Furthermore, the resolution of such a problem can be too long in CPU time and is detrimental to the optimization stage of draping regarding the initial fibre directions. All of these facts lead us to consider rather a discrete approach which is very fast and more robust allowing

simultaneously to define the stratification sequences and the flat pattern for different plies and to predict difficult impregnated areas which involve manual operation like dart insertion or, on the contrary, the shortage of fabric.

Mapping schemes are most commonly employed in commercial packages to predict draping. A layer of fabric is represented by a square mesh which is fitted onto the drape surface. The mapping scheme is based on the assumption that the fabric mainly deforms due to shear deformation and fibre extension can be neglected. The resin, if present, is also neglected during the simulation. The fabric always remains in a fixed position on the draping surface after being mapped. The shape of the product must be represented in algebraic expressions when modelling draping with a mapping scheme.

Several methods are used for predicting the fibre reorientation of the fabric. The geometrical model, also referred to as the kinematics or fishnet model is a widely used model to predict the resulting fibre reorientation for doubly curved fabric reinforced products. The model based on a pinned-joint description of the weave assumes inextensible fibres pinned together at their crossings, allowing free rotation at these joints. They analytically solved the fibre redistribution of a fabric orientated in the bias direction on the circumference of simple surfaces of revolution, such as cones, spheres and spheroids. The resulting fibre orientations were solved as a function of the constant height coordinate of the circumference.

These last twenty years, many authors presented numerically based drape solutions, based on the same assumptions. The author refers to Robertson *et al.* (1984), Long and Rudd (1994), Trochu *et al.* (2006). Typically, draping starts from an initial point and two initial fibre directions. Further points are then generated at a fixed equal distance from the previous points creating a mesh of quadrilateral cells. There is no unique solution for the geometrical drape method. This problem is generally solved by defining two fibre paths on the drape surface.

Based on technical criteria (mold surface covering, fabric drape covering and fibre angular distortion), this approach can constitute the pre-dimensioning or the pre-optimization stage for the manufacturing of complex composite parts. The geometrical approach is based, in general, on the fishnet method for which a fabric mesh element is subjected only to shear deformations. The difficulty of such a method is the mapping of the fabric mesh element onto any surface Cherouat *et al.* (2001).

In this study, we propose a new geometrical algorithm which takes into account the true geometry of the fabric mesh element plotted onto the surface. Such a fabric mesh element is then defined by a curved quadrilateral whose edges are geodesic lines with the same length plotted onto the surface to drape. Given three vertices of the fabric mesh element on the surface, we propose an optimization algorithm to define the fourth vertex of the fabric mesh element. This algorithm allows us to drape the complex surface using an advancing front approach from the data of an initial start point between the fabric and the surface and the initial fibre directions at this point. In this section, geometrical approach based of the mould mesh discretization is presented. For the second formulation, we propose an algorithm of composite fabric draping without any approximation on the geometry of surface to be draped.

First, we present the mathematical formulation of the geometrical draping and then we propose an algorithm scheme to solve the draping problem. Let denote by Σ the surface of the part to drape and we assume that a geometrical mesh \mathbf{T}_Σ of surface is known. Let Φ be the woven composite fabric modeled by two families (warp and weft) of mutually orthogonal and inextensible fibre described by the local coordinates $\mathbf{x}=(\zeta,\eta)$. These families constitute regular quadrilateral fabric mesh \mathbf{T}_Φ of the fabric Φ (Fig. 1 gives example of draping steps of complex part). The problem of

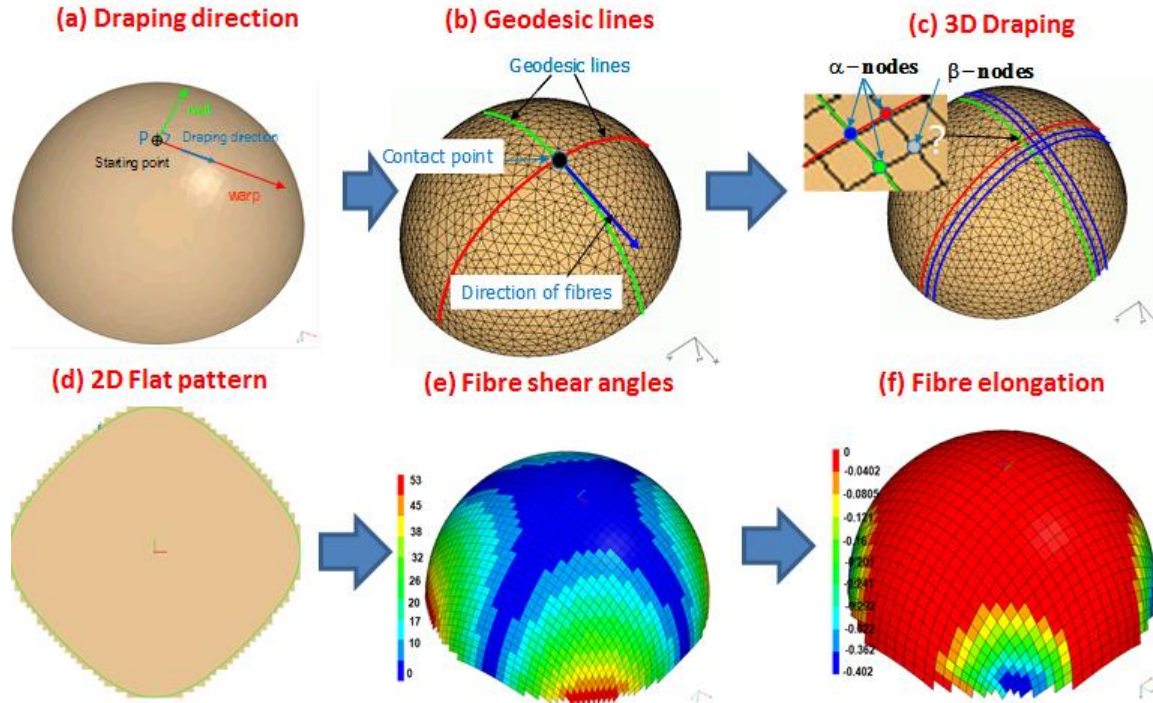


Fig. 1 Geometrical draping steps of woven composite fabric

geometrical draping of Φ onto the surface Σ consists of calculating each node displacement of fabric mesh \mathbf{T}_F with a point of the surface mesh \mathbf{T}_Σ such that the lengths of the edge of the corresponding mesh \mathbf{T}_F^Σ on the surface are preserved (no extensible). This problem presents infinity of solutions depending on:

1. Starting point associated with a node of fabric \mathbf{T}_F^Σ .
2. Initial warp and weft orientation α .

Thus, to ensure a unique solution, we suppose that the points of impact on the part surface as well as the fabric orientation are given. The draping scheme is given by the following step Cherouat *et al.* (2001).

1. associate a starting point (corresponding to the point of impact of the machine: to drape) on the surface on geometrical part mesh $\mathbf{x}_0^\Sigma = (\xi_0, \eta_0)$ (Fig 1(a)),
2. compute numerically step by step the warp nodes of \mathbf{T}_F^Σ , classified as α -nodes, from the starting point, associated with nodes (ξ, η_0) of \mathbf{T}_F (Fig 1b),
3. compute numerically step by step the weft nodes of \mathbf{T}_F^Σ , classified also as α -nodes, from the starting point, associated with nodes (ξ_0, η) of \mathbf{T}_F (Fig 1c),
4. compute numerically cell by cell all the other nodes of \mathbf{T}_F^Σ , classified as β -nodes, from x_0 and the nodes associated with nodes (ξ, η_0) and (ξ_0, η) of \mathbf{T}_F (Fig 1c).

The nodes of \mathbf{T}_F^Σ associated with nodes (ξ, η_0) and (ξ_0, η) of \mathbf{T}_F and the α -nodes are located on the surface along the geodesic lines emanating from the point of impact. Regarding the β -nodes, various algorithms are proposed Cherouat *et al.* (2001). Most of them use an analytical

expression of the surface and formulate the draping problem in terms of non-linear partial differential equations. Other algorithms are also proposed to simplify these equations by using a finite element discretization of the surface by flat triangular face (i.e., a mesh of the surface). Based on this latter approach we propose a new algorithm. The β -nodes are computed by solving an optimization problem corresponding to determine a vertex of an equilateral quadrilateral plotted on the surface from the data of the three other vertices. This optimization problem formulates the direction of the geodesic lines emanating from the searched vertex.

Consequently, two problems arise:

- Problem 1: determine the geodesic exit of a given point of surface according to a given orientation.
- Problem 2: determine the geodesic exits of these points intersecting itself mutually according to given two points of surface and lengths (this geodesic is given according to their orientations).

The developed algorithm is implemented in Geom-Drape tool Cherouat *et al.* (2001). This software provides a fibre quality chart (showing the fibre distortions, the rate of falling and the rate of draped surface) to predict difficult impregnated regions. It can be used to optimise the draping process (with respect to the above quality measure) by improving the lay-up directions or the marker data location. The lay-up of complex curved surfaces can be made in a few seconds. The use of draping tool should not only allow for a more detailed assessment of the draped fabric (Fig 1(c)), but also for an optimization in terms of total fabric shear deformation (Fig 1(e)), flat pattern of the initial fabric (Fig 1(d)), and fibre stretching (Fig 1(f)) and the optimization of the best layup start point (Fig 1(a)).

3. Continuum mechanical approach

Several constitutive models were proposed for modelling fabric draping. They are based on elastic material models, viscous material models or multi-component models. Most constitutive material laws are formulated in plate or shell theory. Generally, the models are implemented in FE formulations. The models are described below. Boisse *et al.* (2005) modelled the bi-axial fabric behaviour in forming processes. The undulation of the yarns in the fabric was accounted for in the bi-axial weave model, assuming fibres with stiffness in the fibre direction only. Recently, Ivanov and Tabiei (2002) developed an elastic material model based on the RVE of a plain weave fabric. A homogenization technique accounts for the weaves microstructure, with the assumption of transversely isotropic yarns. The shear properties of the fabric were neglected up to the locking angle, and then the yarn shear properties were used. Elastic material laws are fairly simple. The implementation of these models in FE packages is therefore, compared to more advanced material models, reasonably simple too. Generally, the resin material behaviour is viscoelastic. The properties of the resin cannot be taken into account correctly with a purely elastic material law.

Draping can be modelled using the combination of continuum based material models and the FE method with or without remeshing. If the draping surface is defined, and material properties are known, the FE method can generate a solution for the draping problem, incorporating the boundary conditions, contact condition and loading required for shaping. The accuracy depends among others, on the assumptions in the material model and the solution scheme. Implicit schemes are known to give more reliable results than dynamic explicit schemes. The drape modelling should be considered as a nonlinear problem due to the large deformations of the fabric during draping.

During the draping process of woven fabric, the two main mode of deformation at the mesoscopic scale are the stretching of the fibres due fibres undulation and the in-plane shearing of the fabric resulting in a change of the angle between the warp and the left yarns. The evolution of two straight lines draws alternatively on warp and weft fibre directions during the forming deformation become curved but remain continuous. The assumption is that each cross connection of straight warp and weft fibre before deformation remains cross connected during the deformation. The basic assumptions for the mechanical forming are that the woven fabric is considered as a continuous 3D material. The warp and weft fibres are represented by a truss which connecting points are hinged and the membrane resin is coupled kinematically to the fabric at these connecting points. Two reference frames have to be considered. The \mathbf{e}_i unit vectors define the local orthogonal reference frame that rotates with the continuum material, and the \mathbf{g}_i basis vectors form a non-orthogonal frame that follows the fibre direction. Here, $(\mathbf{g}_1, \mathbf{g}_2)$ correspond to warp and weft directions, respectively. In this case, for each connecting point $\bar{\mathbf{X}}^f$ of warp and weft yarns is associated a material position space of a resin $\bar{\mathbf{X}}^m$.

$$\begin{cases} \mathbf{F}_{ij}^f = \lambda_L \mathbf{g}_i \otimes \mathbf{g}_{0j} & \text{fibres} \\ \mathbf{F}_{ij}^m = \frac{\partial \mathbf{x}^m}{\partial \mathbf{X}^m} \mathbf{e}_{0i} \otimes \mathbf{e}_{0j} & \text{resin} \end{cases} \quad (1)$$

where \mathbf{F}^f and \mathbf{F}^m are the deformation gradient tensor of fibre and resin respectively, λ_L^f is the longitudinal elongation of each fibre and \mathbf{g}_{0i} and \mathbf{g}_i are respectively the fibre orientations in the initial C_0 and the current C_t configurations and are the resin. The relative rotation of fibre can be associated to the rotation of the rigid body of the median line of the fibre $\mathbf{R} = \mathbf{g}_i \otimes \mathbf{g}_{0i}$. The relationship of the no sliding inter-fibre can write at each connecting point as $\bar{\mathbf{x}} = \bar{\mathbf{x}}(\bar{\mathbf{X}}, t) / \bar{\mathbf{x}}^f = \bar{\mathbf{x}}^m = \bar{\mathbf{x}}$. The gradient of transformation of the pre-impregnated woven fabric \mathbf{F} and the pseudo gradient of transformation of the fibre \mathbf{F}^f are defined by the function. Using the above assumptions, the mechanical deformation of composite fabric depends on the relative movement of fibres and the deformation of not polymerized resin. The shaping problem imposes the use of incremental formulation in finite deformations. In finite deformation analysis a careful distinction has to be made between the coordinate systems that can be chosen to describe the behaviour of the body. The rate constitutive equations for finite strain use objective derivatives. The problem of the integration of strain rate tensors is a central one in large deformations. The rate of deformation tensor of woven fabric is obtained by

$$\begin{cases} \mathbf{D}^f = \begin{pmatrix} \dot{\lambda}_i^f \\ \dot{\lambda}_i^f \end{pmatrix} (\mathbf{g}_i \otimes \mathbf{g}_i) & \text{fibres} \\ \mathbf{D}^m = \frac{1}{2} (\dot{\mathbf{F}}^m \mathbf{F}^{m-1} + \mathbf{F}^{m-T} \dot{\mathbf{F}}^{mT}) (\mathbf{e}_{0i} \otimes \mathbf{e}_{0i}) & \text{resin} \end{cases} \quad (2)$$

The frame associated with Green-Naghdi's derivative is defined, at the material point considered. The rotation \mathbf{R}^f is used to update the initial constitutive axes of warps or weft fibres $\mathbf{g}_{0i} \otimes \mathbf{g}_{0i}$ to the current constitutive axes and the rotation \mathbf{R} is used to update the initial constitutive axes of resin $\mathbf{e}_{0i} \otimes \mathbf{e}_{0i}$. The stretching tensors Eq. (5) are written in the rigid body rotation frames. The longitudinal component $\bar{\mathbf{D}}_L^f = \dot{\lambda}_L^f / \lambda_L^f$ and the transversal components $(\bar{\mathbf{D}}_T^f, \bar{\mathbf{D}}_3^f)$ are obtained

by the unidirectional behaviour of fibre as $\bar{\mathbf{D}}_T^{\text{R}} = \bar{\mathbf{D}}_3^{\text{R}} = -\nu_{\text{LT}} \bar{\mathbf{D}}_L^{\text{R}} \Rightarrow \lambda_T^f = \lambda_3^f = (\lambda_L^f)^{-\nu_{\text{LT}}}$ and ν_{LT} is the Poison's ratio of fibre. Using Green-Naghdi's objective tensor stress, the stress rate $\bar{\boldsymbol{\sigma}}_L^f$, depending on the stretching deformation $\bar{\mathbf{D}}_L^f$, and the stress rate tensor of the membrane resin $\boldsymbol{\sigma}^m$, depending on the tensor deformation rate \mathbf{D}^m and the elastic properties \mathbf{C}^m , can be written at each time as

$$\boldsymbol{\sigma}^f = \left(\sum_{\text{weft}} \mathbf{E}_L^f(\lambda_1^f) \mathbf{D}_{11}^f - \sum_{\text{weft}} \nu_{\text{LT}} \mathbf{E}_L^f(\lambda_2^f) \mathbf{D}_{22}^f \right) + \left(\sum_{\text{warp}} \mathbf{E}_L^f(\lambda_2^f) \mathbf{D}_{22}^f - \sum_{\text{warp}} \nu_{\text{LT}} \mathbf{E}_L^f(\lambda_1^f) \mathbf{D}_{11}^f \right) \quad (3)$$

The stress and strain relationship of the woven fabric composites are defined in the non-orthogonal material coordinate frame. Therefore, coordinate transformations of stress and strain into the orthogonal coordinate system should be considered in the non-orthogonal constitutive model as

$$\boldsymbol{\sigma}^m = \begin{pmatrix} \frac{\mathbf{E}^m}{1-\nu_m^2} \mathbf{D}_{11}^m & \frac{\nu_m \mathbf{E}^m}{1-\nu_m^2} \mathbf{D}_{22}^m & 0 \\ \frac{\nu_m \mathbf{E}^m}{1-\nu_m^2} \mathbf{D}_{11}^m & \frac{\mathbf{E}^m}{1-\nu_m^2} \mathbf{D}_{22}^m & 0 \\ 0 & 0 & \mathbf{G}^m \mathbf{D}_{12}^m \end{pmatrix} \quad (4)$$

The constitutive law of fibres is nonlinear and is written in terms of longitudinal modulus of stretching $\mathbf{E}_L^f(\lambda_L^f)$, the compressive stiffness ($\lambda_L^f \leq 0$) of fiber is supposed negligible $\mathbf{E}_L^f = 0$. Later is function of elongation of warp and weft fibre (λ_1^f, λ_2^f), effective elastic modulus of fibre $\bar{\mathbf{E}}_f$ and undulation factor ε_{sh} ; (\mathbf{E}^m, ν_m) are the membrane elastic properties. To determine the stress state in a membrane material at a given time, the deformation history must be considered. For linear viscoelastic materials, a superposition of hereditary integrals describes the time dependent response. Let $\mathbf{G}^m(t)$ be the shear stress relaxation modulus of the not polymerized resin and $\mathbf{G}^{m^\infty} = \mathbf{G}^m(t = \infty)$ the limit value. The viscoelastic behaviour of not polymerized resin is formulated in the time domain by the hereditary integral and using the relaxation time τ_k and the shear modulus relaxation, which are material parameters \mathbf{G}^{m^k} . Hereditary integrals with Prony series kernels can be applied to model the shear behavior of the not polymerized resin. The behavior of fibre and resin can be written as

$$\begin{cases} \mathbf{E}_L^f(\lambda_L^f) = \bar{\mathbf{E}}_f \left(1 - \exp\left(\frac{-\dot{\lambda}_L^f}{\lambda_L^f \varepsilon_{\text{sh}}}\right) \right) & \text{fibres} \\ \mathbf{G}^m(t) = \mathbf{G}^m(\infty) + \sum_1^k \mathbf{G}^{m^k} \exp\left(\frac{-t}{\tau_k}\right) & \text{resin} \end{cases} \quad (5)$$

Each material point is moving as in a continuum, ensured by the non-sliding of fibers due to fabric weaving and resin behavior. Therefore, a nodal approximation for the displacement can be used. The deformation of composite fabric is described within the frame of membrane

assumptions. The global equilibrium of the fabric is obtained by minimizing the total potential energy. The effect of spatial equilibrium of composite material on the actual configuration is established in terms of nonlinear equations: kinematic non-linearity, material non-linearity and contact with friction non-linearity. It is linearized for each load increment by an iterative Newton method. It should be emphasized that during the motion, nodes and elements are permanently attached to the material points with which they were initially associated. Consequently, the subsequent motion is fully described in terms of the current nodal positions as

$$\mathbf{x} = \sum_{k=1}^m \mathbf{N}^k(\xi, \eta) \mathbf{X}^k + \sum_{k=1}^m \mathbf{N}^k(\xi, \eta) \mathbf{u}^k \quad (6)$$

where \mathbf{u}^k are the nodal displacements of each connecting point, $\mathbf{N}^k(\xi, \eta)$ are the standard shape functions (of membrane of truss element) and (m) denotes the number of nodes.

In a finite element approximation, the only independent variables in the equations of linearized virtual work are the displacements of the material points. Substituting the element coordinate and displacement interpolations into the equilibrium equations, for a given set of elements we obtain the state that forces acting on a fabric equals the mass times the acceleration of the body

$$\sum_e [\mathbf{M}^e] \{\ddot{\mathbf{u}}\} + \sum_e \left(\{\mathfrak{R}_{\text{int}}^e\} - \{\mathfrak{R}_{\text{ext}}^e\} \right) = \{\mathbf{0}\} \quad (7)$$

where $[\mathbf{M}^e]$ is the consistent composite mass matrix and $\left(\{\mathfrak{R}_{\text{int}}^e\} - \{\mathfrak{R}_{\text{ext}}^e\} \right)$ is the so called quasi-static equilibrium residual

$$\left\{ \begin{aligned} [\mathbf{M}^e] &= \rho_r h_0 \int_{S^m} [\mathbf{N}^e]^T [\mathbf{N}^e] d\Gamma + \sum_{\text{fibres}} \rho_f S_0^f \int_L [\tilde{\mathbf{N}}^e]^T [\tilde{\mathbf{N}}^e] ds \\ \{\mathfrak{R}_{\text{int}}^e\} - \{\mathfrak{R}_{\text{ext}}^e\} &= h_0 \int_{S^m} [\mathbf{B}_{\text{resin}}^e]^T \{\boldsymbol{\sigma}^m\} d\Gamma - \sum_{\text{fibres}} S_0^f \int_{L^f} \mathbf{E}_L^f [\mathbf{B}_{\text{fibres}}^e]^T \{\boldsymbol{\sigma}^f\} ds - \int_{\Gamma_f} [\mathbf{N}^e]^T \{\bar{\mathbf{t}}\} d\Gamma \end{aligned} \right. \quad (8)$$

$[\mathbf{N}^e]$ and $[\tilde{\mathbf{N}}^e]$ are the matrix of the nodal interpolation functions both associated elements, $\left([\mathbf{B}_{\text{resin}}^e], [\mathbf{B}_{\text{fibres}}^e] \right)$ are the strain interpolation function of resin and fibre, (h_0, S_0^f) are the initial thickness and surface of resin and fibres, and $\bar{\mathbf{t}}$ is the external surface load. The index e refers to the e^{th} element.

According to the different modes of deformation occurring in the prepreg fabric during the shaping process, bi-component finite elements are developed to characterize the mechanical behavior of thin composite structures. The bi-component element is based on an association of 3D linear membrane finite elements combined with complementary truss linear finite elements. The global stiffness of composite fabric is obtained by the summation of elementary stiffness matrix of warp fiber, elementary stiffness matrix of weft fiber and elementary stiffness matrix of resin. The nonlinear constitutive equation of fibre behavior is implemented in the Abaqus/Explicit using VUMAT user's subroutine (see Fig. 2). The governing equilibrium Eq. (7) is solved as a dynamic problem using explicit integration. This is achieved by using the central difference method to approximate the velocity and the acceleration in the next time step, using only information from the previous step, where all state variables are known. This approach has proven to be, in particular, suitable to highly nonlinear geometric and material problems, particularly where a large

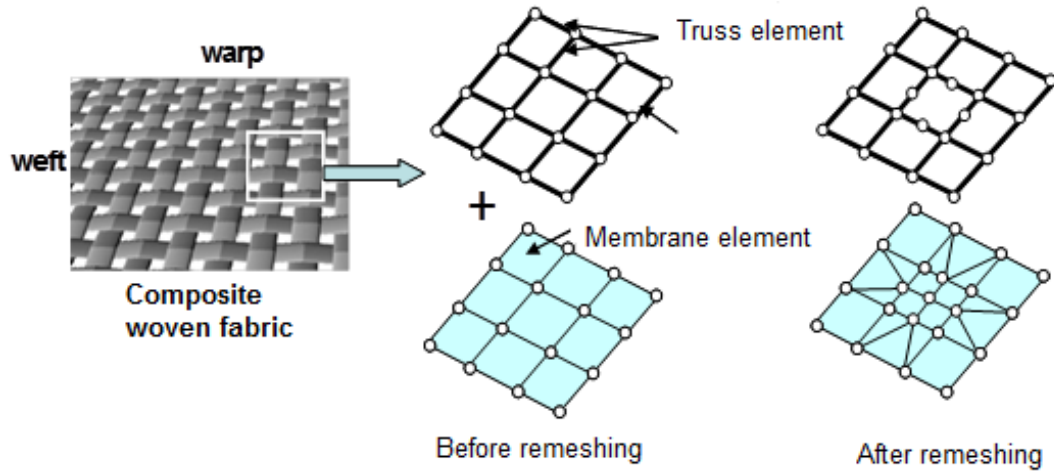


Fig. 2 Bi-component finite element for composite woven fabric

amount of contact between different structural parts occurs. In the present work, the Dynamic Explicit (DE) resolution procedure is used within the general purpose FE code Abaqus/Explicit.

3. Applications

Three geometrical and mechanical draping simulation examples are given. These simulations are performed using the geometrical analysis computer code GeomDrap and Abaqus FEA software *et al.* (2001). For each example, we assume that a mesh of the part to drape is given. In the first example, we study the effect of the excessive fiber distortion and the cutting chisel on the draping process. The second example shows the influence of the fibre orientations on the flat pattern of the woven fabric and the last example we study the deep-drawing of dry glass woven fabric using complex rigid tools.

3.1 3D draping of complex cylindrical part

The first example concerns the geometrical draping of a complex piece composed of two half hemispheres with a radius of 38.8mm that are connected by a half cylinder with a length of 170mm Vanclooster *et al.* (2009). The centroid of the part is chosen as the starting point for which two different fibre orientations are specified. Fig. 3 shows the resulting 3D surface lay-up for the $(0^\circ/90^\circ)$ fibre orientation (Fig. 3(a)) and the corresponding 2D woven flat patterns (Fig. 3(c)). Likewise, Fig. 3(b) shows the draping results for the $(\pm 45^\circ)$ fibre orientation and Fig. 5(d) the corresponding 2D woven flat pattern. One can notice that, in the considered cases, the surface of the piece is not globally draped because the shear angles between fibers ($>80^\circ$) are very excessive and induce defects in the weaving pattern.

In order, to drape completely the proposed surface without excessive fibre distortion ($<50^\circ$), it is necessary to make cuts chisel along the line C2 for $(0^\circ/90^\circ)$ fibre orientation (Fig. 3(c)) and along the line C1 for $(\pm 45^\circ)$ fibre orientation (Fig. 3(d)). With the cutting chisel operation, the shear limit reached 38° in the case of $(0^\circ/90^\circ)$ fibre orientation and 68° in the case of $(\pm 45^\circ)$.

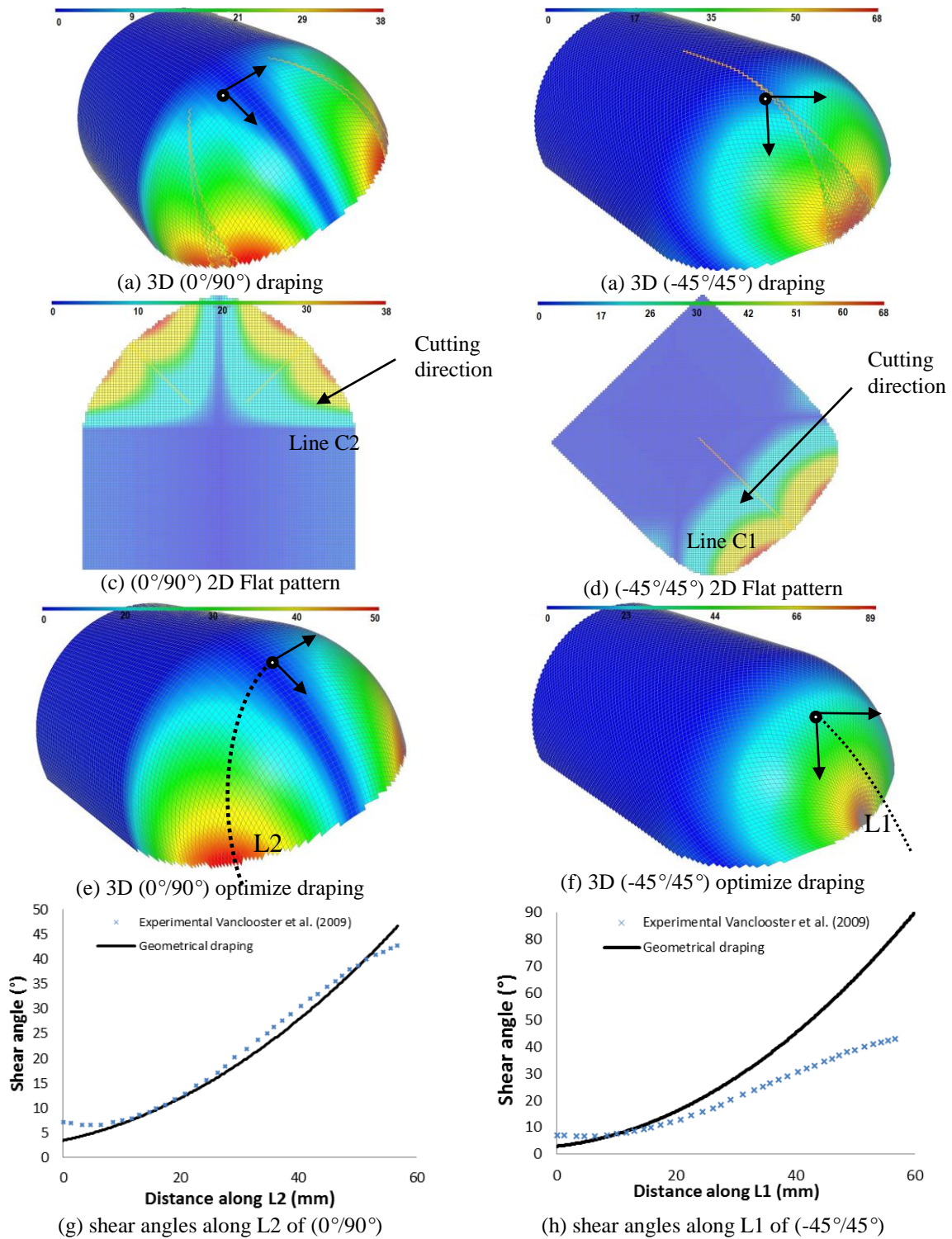


Fig. 3 Geometrical draping of cylindrical part in comparison with experimental results (Vanclooster *et al.* 2009)

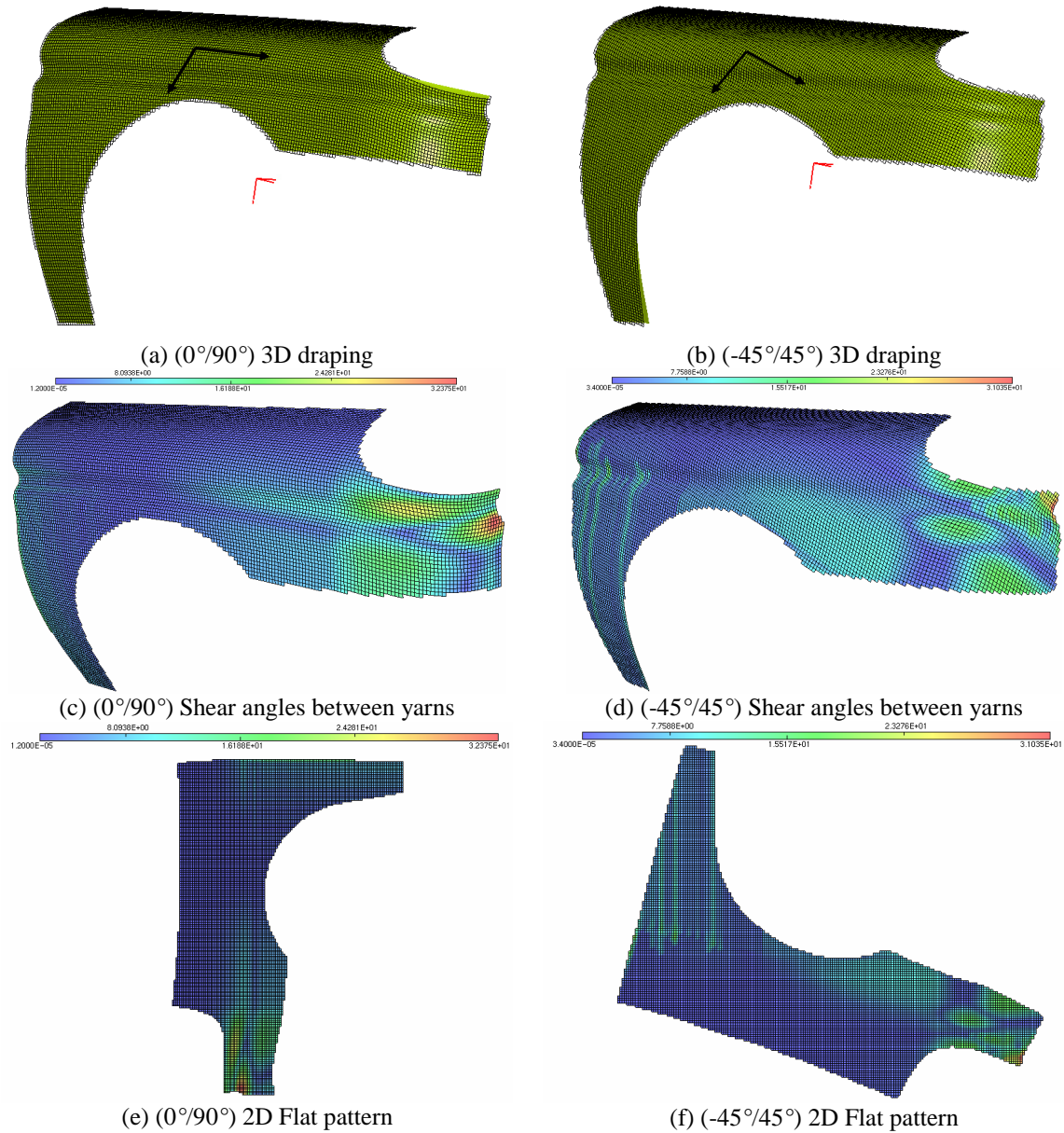


Fig. 4 Geometrical draping of composite car part

Figs. 3(e) and 3(f) give the result of the optimized draping without cutting the fabric minimizing distortion between the fibers but allowing warp and weft fiber elongations of 20%. Without the cutting chisel operation, the parts are completely draped and the shear limit reached 52° in the case of $(0^\circ/90^\circ)$ fibre orientation and 89° in the case of $(\pm 45^\circ)$. To compare the experimental shear angles given by Vanclooster *et al.* (2009) with the geometrical results, two cross-sections where most shearing occurs are examined. Figs. 3(e) and 3(f) give an overview of the two cross-sections along the symmetrical L1 and the median line L2. From Fig. 3(g), it can be

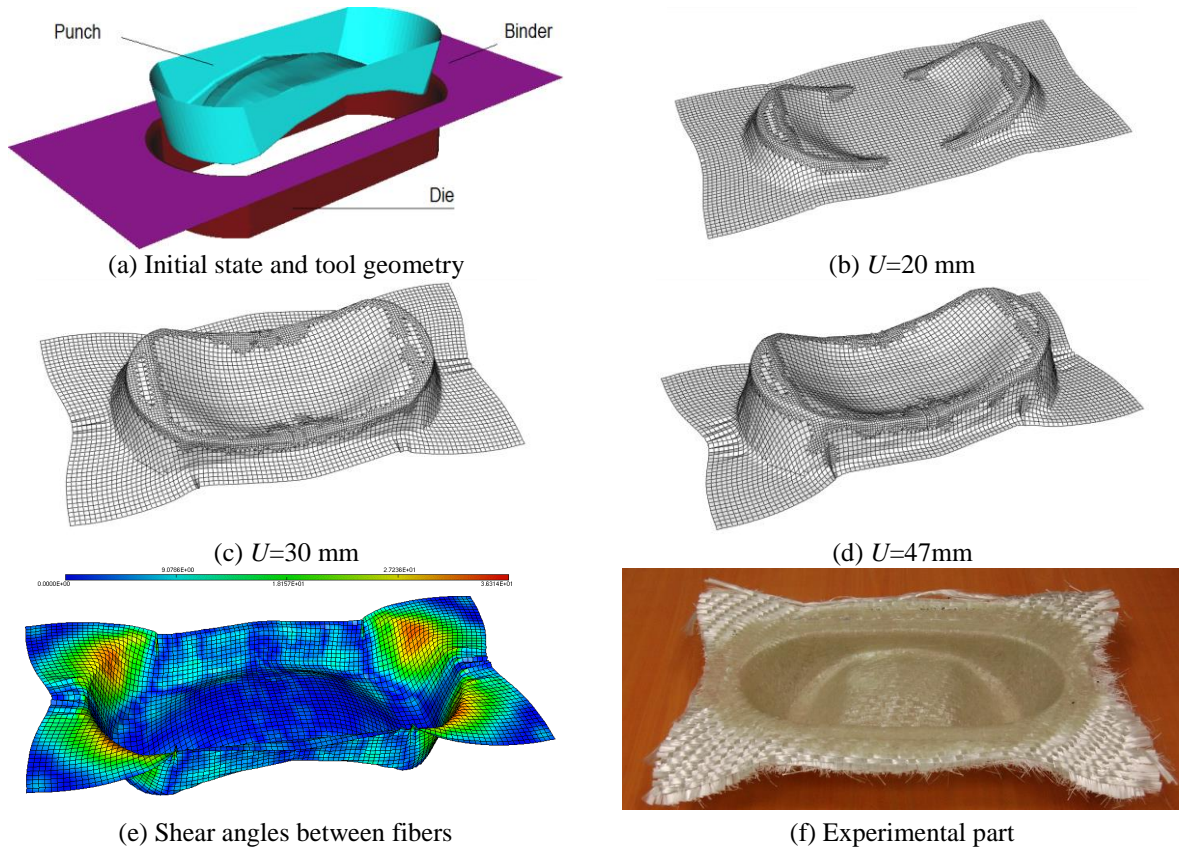


Fig. 5 Deformed meshes for each punch displacement and shear angles between fibres

concluded that for the ($0^\circ/90^\circ$) draping orientation the agreement between the experimental shear angles and the geometrical results is good. On the other side, with the drape orientation ($\pm 45^\circ$), the results from the geometrical model do not agree at all with the experimental values (see Fig. 3(h)). This is due to the fact that the locking angle of the fabric is achieved and the possibility of complete draping with this orientation is impossible. In this case, the geometrical approach is not appropriate for predicting the fiber directions because it takes no account the mechanical properties of fibers that have a significant impact on the draping process when fiber distortions are very important.

3.2 3D geometrical draping of composite car part

The second example concerns the 3D draping of complex shape. The centroid of the part is chosen as the starting point from which the ($0^\circ/90^\circ$) and ($-45^\circ/45^\circ$) fibre orientations are specified. Fig. 4 shows the resulting 3D draping for the two orientations Fig. 4(a) for ($0^\circ/90^\circ$) fabric orientation and Fig. 4(b) for ($45^\circ/45^\circ$) fabric orientation and the corresponding 2D flat pattern in Figs. 4(e) and 5(f) respectively. We can note that all part surface is completely draped with acceptable fiber distortions. Fig. 4(c) presents shaded contours interpolated from the map of the fiber distortions for ($0^\circ/90^\circ$) fibre orientation and Fig. 4(d) presents fiber distortions for ($\pm 45^\circ$)

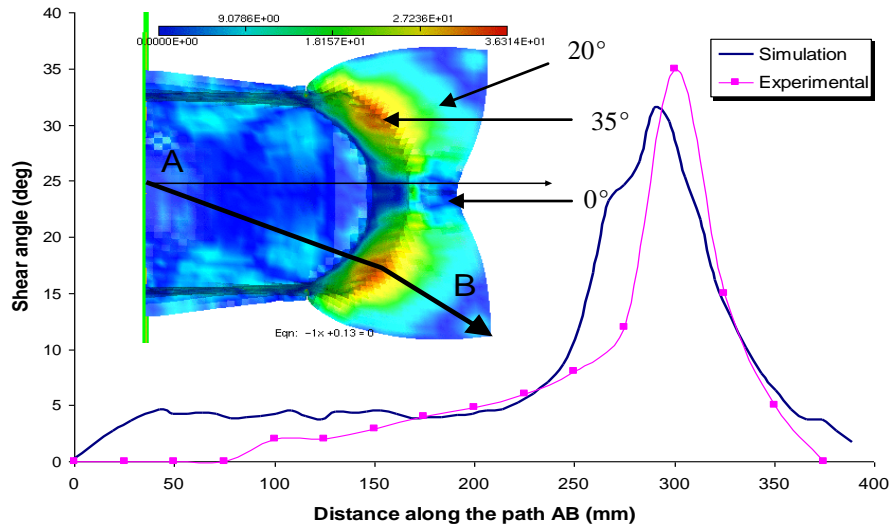


Fig. 6 Experimental and predicted shear angles along the diagonal line AB

fiber orientations. The fiber distortions for both ($0^\circ/90^\circ$) and ($\pm 45^\circ$) draping are relatively small (30°) but the maximum shear angle localization and the 2D fabric flat pattern are different.

3.3 Deep-drawing of glass woven fabric

The last example concerns the numerical forming of glass fabric using complex tools using the mechanical approach and the computational remeshing procedure. The initial shape of the taffetas glass woven fabric is (700×350 mm). The high tensile stiffness along the warp respectively weft yarn direction is introduced via truss elements that connect the nodes of the membrane element. The Young's modulus of warp and weft yarns is 70 GPa. The geometry of the deformation simulation is shown in Fig. 5. The maximum binding force is 30N and the total punch stroke is 47 mm. The friction coefficient between the glass fabric and the steel tools is estimated $\mu=0.3$. The fibre direction is assumed ($0^\circ/90^\circ$). The final deformed shape of ($0^\circ/90^\circ$) fabric for $U=20, 30$ and 47 mm of punch displacement is shown in Figs. 5(b), 5(c) and 5(d) respectively and is compared to the experimental one (Fig. 5(f)). The shear angle distribution was plotted in Fig. 5(e). We can note that, the adapting meshes of the warp and the weft to the curved shape of the punch. As shown in the Fig. 6, maximum shear angle (35°) was observed at the highly curved area and such positions were almost same for all simulation cases. Good agreement between the predicted and experimental preforms and shear angles between warp and weft fibres.

4. Conclusions

An efficient numerical approach (geometrical and mechanical) has been presented to simulate accurately the draping of composite fabric. The objective of this work is to show the influence of the woven fabric orientation on the prediction of the final shape. The geometrical discrete approach is based on a modified MOSAIC algorithm, which is suitable to generate a regular quad

mesh representing the lay-up of the curved surfaces. The mechanical approach is based on a continuum meso-structural model. It allows us to take into account the mechanical properties prepreg fabric during the forming process. Numerical examples concerning the deep-drawing or draping of woven composite reinforcement demonstrated the efficiency of the proposed approaches.

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