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Mathematical modelling of wind turbine blades through volumetric view

Ali Vardar†

Uludag University, Faculty of Agriculture, Department of Agricultural Machinery, 16059 Bursa, Turkey

Bülent Eker‡

Trakya University, Faculty of Agriculture, Department of Agricultural Machinery, 59030 Tekirdag, Turkey

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Abstract. The demand for energy in the world increases everyday. Blade energy which is wind turbine is a significant resource which must be appreciated in this field. Especially, in places where wind potential is high, the usage of wind energy is a beneficial factor for every country's economy. In this study, first, 6 different miniature rotor were produced by using 6 different NACA profiles. Rotors were produced with three blades. The electrical performance and the speed of start of action values that are provided from each rotor form were established by measuring them in the wind tunnel. The calculation of area and volumetric values of each profile and wind surfaces were made with AutoCad technical drawing program. As a result, it was searched whether there is any relation between electrical performance values and speed of start of motion that rotors produced and volumetric values of rotors. The aim of this study is to find out whether rotor blade volume is one of factors that influences rotor performance. The general tendency observed here is that the increase in the volume of rotor blade leads to an increase in the speed of start of motion and to a decrease in the rotor performance.

Keywords: wind turbine; mathematical modelling; rotor blade.

1. Introduction

The significance of energy in human life is very great. In relation to this, in almost every topic considered, the parameter of energy is evaluated. In the same way, energy is also considered as if they are in the same level with technology. As the existence of technology gains importance with energy, it is possible to create and develop energy through technology. As a result, the increasing needs of human beings leads to a greater amount of usage of energy.

The increase in the demand for the wind energy around the world increased the efforts to improve the structural factors of systems that work with this energy. In these efforts, main goals of engineering were taken as fundamental. Because of this reason, usually two steps stand out. These

† Assistant Professor, Corresponding Author, E-mail: dravardar@uludag.edu.tr
‡ Professor, E-mail: bulek@tu.tzf.edu.tr

are the formation of mathematical model which reflects present turbine systems and the analytical analysis or numerical analysis of the mathematical equation that is found by using approximate numerical values. In the first step of these two, experience, sensation and an efficient mathematical back ground and in the second, together with sensation and information quick and comprehensive calculation tool are required (Civalek and Çatal 2004).

On the other hand, until recently, the concept of three dimensions (3D) was taken in hand when examining the subjects which existed before or formed later in the nature. However, in last years, adding the concept of time lead the concept of time to have been an important parameter in scientific studies. Again, in the same way, when the congruity of subjects is examined, the ability of their having been tested, the explanation of the results of these tests with the help of mathematical formulas is necessary. On the base of this lie the physical rules. After these formulas have been created, the analysis of the situation can be done through creating different models.

Almost in every area, in the base of every mathematical model that is put into action with success, lies the understanding of what the systems are and how they are formed and in relation to this, through logical and scientific rules, transformation of linguistic fictions into patterns that others may understand and benefit from. Mathematical modeling approach is known to be an efficient and economic way to understand the systems in a better way, to analyze and to designing them. As it is known, modeling depends on good explanations to decide on complicated parameters. Because, complicated matters may only be transformed into mathematical expressions only through this method. The high number of influencing factors in wind turbines leads to the transformation of solutions of turbines that are known into complicated structures.

On the other hand, modeling may be defined more as a science than an art (Şen 2002). Modeling studies do not include procedures that may be put into action directly because of their natures. The most important decision for a model builder is to define the relations in choosing the models. Because some relations that are defined may not be much related with the aim of the study. Of course, a model that is defined with rightly chosen relations leads us to more sensitive and better consequences. In the whole structure, showing the internal and external influencing factors of the systems according to the mathematical rules through a couple of procedures and evaluation of the consequence is very simple in mathematical modeling technique. In all the systems, the main goal in evaluation is to show the efficiency of the system. In the studies that are done today, this factor is handled as an indispensable rule. It should not be forgotten that, directly formed models for the systems are mostly weak ones. Because, in the real world, the systems are complicated and indirect structures. Therefore, it can not be claimed that there is only one way to examine wind turbine wind structures. In this stage, mathematical modeling comes forward as an alternative solution.

As it is known, in mathematical models, how much the model reflects the real model depends on measurement level with the real physical event. Most of these models are differential equations on limit values. The most close presentation style of these mathematical equations to physical model is variation problems (Hasanov 2001). The next stage after variation problems is the formation of discrete model.

On the other hand, the experiments that were done before about the same event are helpful for the models framework. Because of this reason, the chance of mathematical model that will be formed and through this model the physical model to be successful will be much more, if healthy solutions are supported with healthy data in wind turbines. As it is known, although benefiting from the wind is a very old method, the simplification of the structures of the turbines that are formed in a modern sense and in relation to this giving the opportunity of expansion of usage of this, modeling technique will save us time. Because of this reason, in this study, improving both functional and structural features of the wind turbine blades was tried to be achieved by depending on the technique of mathematical model.

2. Methodology

The wings that are used in the tests are produced in Trakya University, Agriculture Faculty Agriculture Machines Part Workshop. The wings are made, when considered about the materials, from 'South American forest tree-Balsa' which has very low endurance and have 0,075 g/cm^3 density and which are used in the production of planes. The wings are prepared in 310 mm rotor, 48 mm belly (a part that takes place in the rotor and leads the wind which comes to the rotor center to the wings) diameter and 130 mm wing length.

Certain criteria are bared in mind in choosing the rotor blades. According to these criteria: the number of rotor blade is 3, blade twisting angle is 0 degrees and the angles of blade to rotor surface are 8, 10, 15 and 18 degrees. In fact, rotors are designed with 2, 3 and 4 blades. However, in this study, rotors with 3 blades are used. On the below, the dimensions of the blades are shown in Fig. 1 (Vardar 2002), the profiles of blades are shown on Fig. 2 (Dreese 2000) and the blades that are made according to the profiles are shown in Fig. 3 (Vardar 2002).

Rotor forms are coded for making the process easier. In coding, numerical codes of profiles of NACA are used. In this study, the letters and numbers which are used in wing profiles are defined

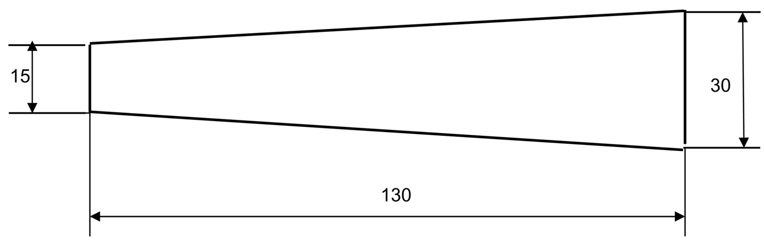

Fig. 1 Blades that are made according to the profiles (Vardar 2002)

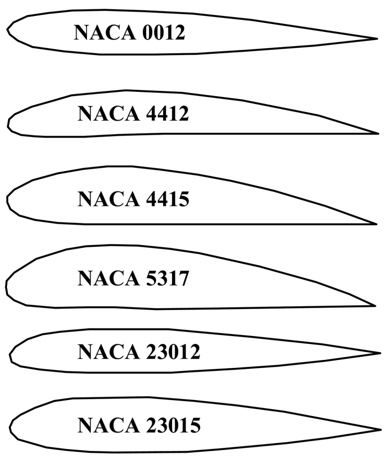

Fig. 2 Blade profiles (Dreese 2000)

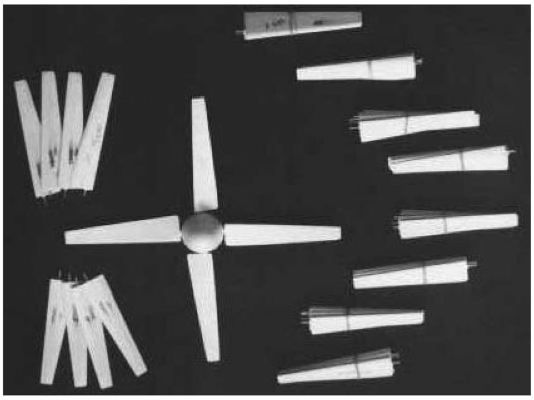

Fig. 3 Different blades for testing (Vardar 2002)

Table 1 Rotor form codes

Rotor form codes	Explanation
0012	NACA 0012 profiled blade
4412	NACA 4412 profiled blade
4415	NACA 4415 profiled blade
5317	NACA 5317 profiled blade
23012	NACA 23012 profiled blade
23015	NACA 23015 profiled blade

according to NACA. In these profiles, the first number that is used is in relation to the highness on the y axis in % ratio (curving highness), which is used according to the probability that the profile takes place in the center of the coordinate system, and the second number is used in % ratio is used according to the position of profile on × axis (curving position) (Fig. 2) (Dreese 2000). Here, the last two numbers are describing the thickness of wing profile in % ratio (Dreese 2000). On the below, data according to the coding are shown (Table 1).

The speed of start of action and electrical performance values were determined by wind tunnel test. In these tests a wind tunnel with total height 1.475 m, total width 0.88 m and total length 2.35 m, tunnel inner length 2 m and diameter 51.35 cm was used (Fig. 4). Height of tunnel center from the floor is 1.11 m. There is a fan above the tunnel with 35 cm diameter, 250 W power and 1400 rpm. There is regulating and venturi pipe inside the channel regulating air flow as a result of movement of the fan. On the other end of the tunnel, there is a mechanism built to test the wings. System works with 220 V and 50 Hz. Depending on temperature and air intensity, it is possible to work the system with 0-5 m/s speed interval as wind speed value. It is possible to measure the speed, temperature, intensity, flow rate and intensity temperature of the air inside the tunnel. Apart from these, it is possible to read voltage and current values of the tested wings - with an electrical motor used in the system - in 23,5 Ω, 57,1 Ω, 102,3 Ω, 151 Ω, 219,1 Ω and 327,3 Ω resistance values. Tests in the wind tunnel were made with at least 10 trials and arithmetic mean of the obtained values were taken (Dreese 2000). The speed of start of action of the rotors and their electrical performance results are seen in Table 2 (Vardar 2002) and Table 3 (Vardar 2002).

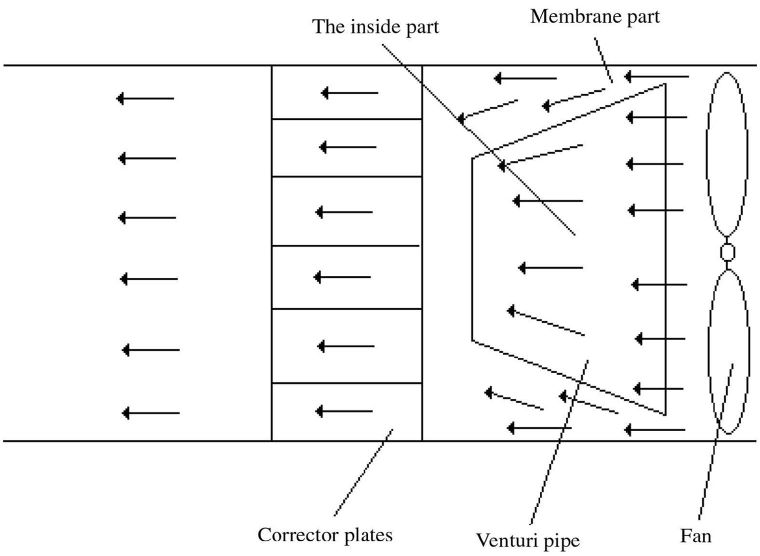

Fig. 4 Wind tunnel wind flowing regulator and venturi pipe (Vardar and Eker 2004)

Table 2 The values of speed of start of motion (Vardar 2002)

Code of rotor blade	The values of speed (m/s)			
	Blade angle: 8	Blade angle: 10	Blade angle: 15	Blade angle: 18
0012	2,79	2,13	1,66	1,57
4412	2,85	2,48	2,3	2,15
4415	1,99	2,3	1,85	1,53
5317	3,7	2,86	2,7	2,51
23012	2,53	2,18	1,99	1,71
23015	2,71	2,2	2	1,85

Table 3 The electrical performance values of rotors in 4 m/s wind speed (Vardar 2002)

Code of rotor blade	Electrical performance (W)			
	Blade angle: 8	Blade angle: 10	Blade angle: 15	Blade angle: 18
0012	85,6	69,1	24,3	16,1
4412	95,16	89,96	80,21	70,7
4415	87,3	68,16	55,66	30,45
5317	54,9	49,95	42,66	38,52
23012	75,73	52,39	43,13	35,24
23015	81,76	65,46	46,22	29,22

Wind speed measurents in the wind tunnel were made by anemometer. In wind tunnel, the values of wind speed that belongs to the rotors at the beginning when rotors started to work are calculated by increasing the wind speed level slowly in the wind tunnel and through reading anemometer values. In wind speed calculations, during 5 minutes, the values on anemometer are recorded and the average is taken as a base.

The anemometer wind speed calculations are in intervals between −20…0 C temperatures are 0…5 m/s, in intervals between 0…+50 C temperatures are 0…10 m/s. Flow calculation interval is 0…99,990 m^3/h in temperature intervals between −20…+50 C. The stability of the device is 0,01 m/s/0,1 C. The system sensibility in calibration when temperature is 25 C is 2 m/s, in wind speed flow calculation it is ± %5, in wind speed calculation it is ± 0,1 m/s and after 2 m/s wind speed in calculation of flow, in ± %5 wind speed calculation it is ± 0,3 m/s (Vardar and Eker 2004).

As a result of the turning of the rotor because of the effect of wind speed, electrical tension and flow is produced, under stable 102,3 Ω resistance in the electric motor which is tied to the rotor with a mile. The electrical tension and flow intensity values here are calculated with two multimeter. The multiplication of values of this electrical tension and flow intensity that is calculated gives the electrical performance values (Erna 1977).

$$N = U \cdot I \tag{1}$$

The experiment technique used in this study is a wind tunnel system. The most important advantage of this system is the opportunity to control wind speed and wind's way of coming on the rotor. Also when the same fan is used, wind speed values are higher compared to open systems. Because in open systems, wind speed will be spread to more volume, thus increasing the loss. Also measurement errors can be minimized, because measurement devices are fixed in the wind tunnel. The biggest disadvantage of the wind tunnel system is the impossiblity to change wind speed interval. To achieve higher wind speed values, the fan must be changed. This means a change in the diameter of the fan. But this is very hard, since the diameter where the fan is connected in the wind tunnel is fixed.

The first thing as a method is volumetric values of blades that form the rotors. When sectional view of rotor blade in Fig. 5 and the areas of surfaces of blade profiles in Fig. 2 (Dreese 2000) are measured, the volumes of blades may be able to be calculated. For this reason, AutoCad technical drawing programs (Nalbant 1998) and the methods below are used.

2.1. The calculations that are made with AutoCad

The measurements of each profile and areas of blade surfaces are made with the help of AutoCad technical drawing program (Nalbant 1998). At the same time, primarily chosen 6 profiles (Fig. 2)

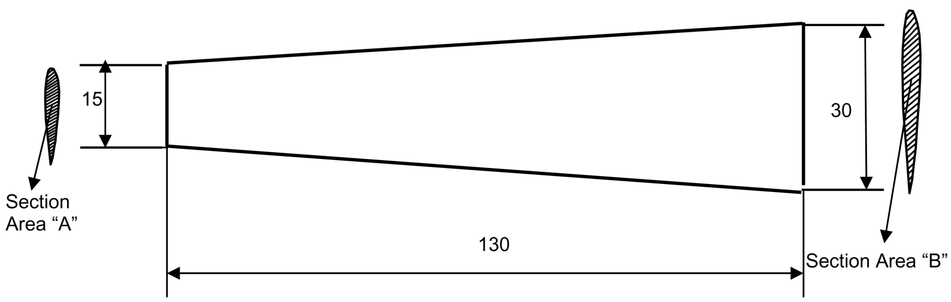

Fig. 5 Sectional view of the blade

(Dreese 2000) and blade surface are drawn with AutoCad program. “PROFILE BLADE VOLUME” and “ROTOR BLADE VOLUME” values for each blade form are observed with the data that is gathered from these and the method described below.

2.2. Volume measurement method

The profiles that are gathered from the AutoCad technical drawing program and surface area data are transformed into volumetric data by using the equation below.

$$RBV = \left[(A \cdot 130) + \left(\frac{A}{4} \cdot 130\right)\right] \cdot BN \tag{2}$$

The value of number of blades in this equation is counted as 3, since rotors have 3 blades. The results that are observed and the data that are shown on Table 2 (Vardar 2002) and Table 3 (Vardar 2002) are reflected on graphs through Excel and shown in visual manner.

3. Results

The results of this study which is made with the aim of mathematical modeling of wind turbine rotor blade volumes are given below. The order of the results are: blade surface area and profile area, measured rotor blade volume values, the relation of rotor’s blade volume and speed of start of motion, the relation of rotor’s blade volume and electrical performance, the relation between the angles which blades are tied to the rotor and electrical performances. Blade surface area and profile area of blades that are observed with AutoCad technical drawing are shown below (in Table 4).

BLADE SURFACE AREA: 2923, 1387 unit^2

Rotor blade volume values that are calculated by using the data given above are shown in Table 5. It is observed that these values are between 8699,925 unit^2 and 12559, 41375 unit^2.

The results that are observed after the calculations are reflected on the graph. On Fig. 6, the relation between blade volume of rotor and speed of start of motion, on Fig. 7, the relation between blade volume of rotor and electrical performance, on Fig. 8, the relation between the angle that blade is tied to rotor and the speed of start of motion and on Fig. 9 the relation between the angle that blade is tied to the rotor and electrical performance are shown. On the graph, a model is made, since a mathematical polynomial relation is assumed (Hamming 1973, Civalek 2005).

Table 4 Area values of profiles

Name of the profile	Area (birim2)
NACA 0012	18,2329
NACA 4412	17,8460
NACA 4415	22,2426
NACA 5317	25,7629
NACA 23012	17,4641
NACA 23015	21,9275

Table 5 Calculated rotor blade volume values

Code of rotor blade	Calculated of rotor blade volume (unit3)
0012	8888,53875
4412	8699,925
4415	10843,2675
5317	12559,41375
23012	8513,74875
23015	10689,65625

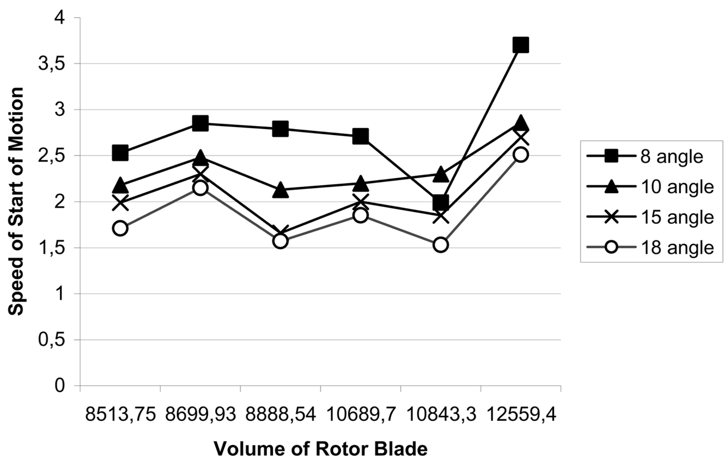

Fig. 6 The relation between blade volume of rotor and speed of start of motion

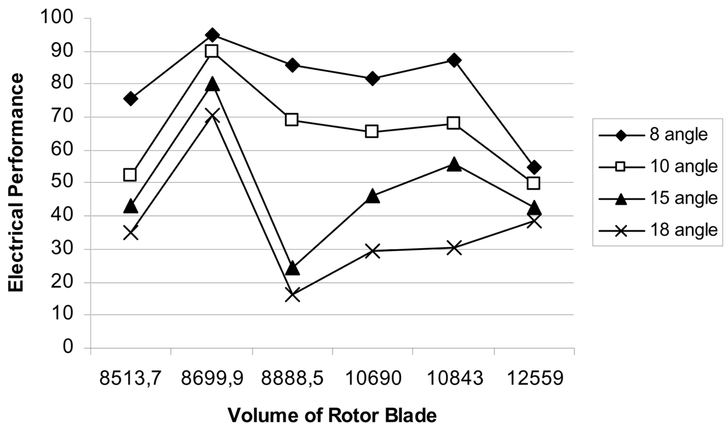

Fig. 7 The relation between blade volume of rotor and electrical performance

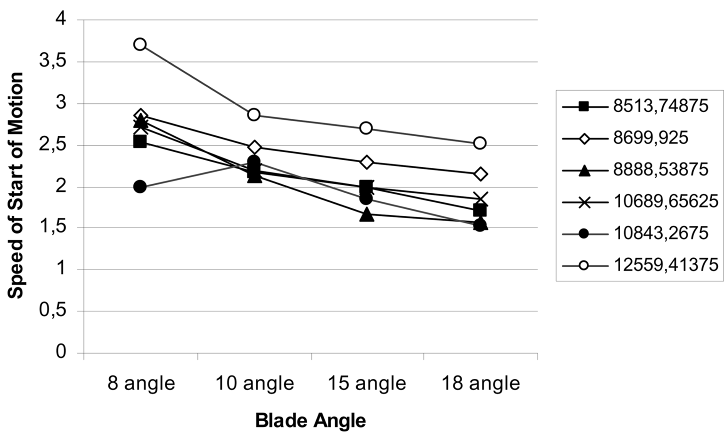

Fig. 8 The relation between the angles of tying of blade to rotor and the speed of start of motion

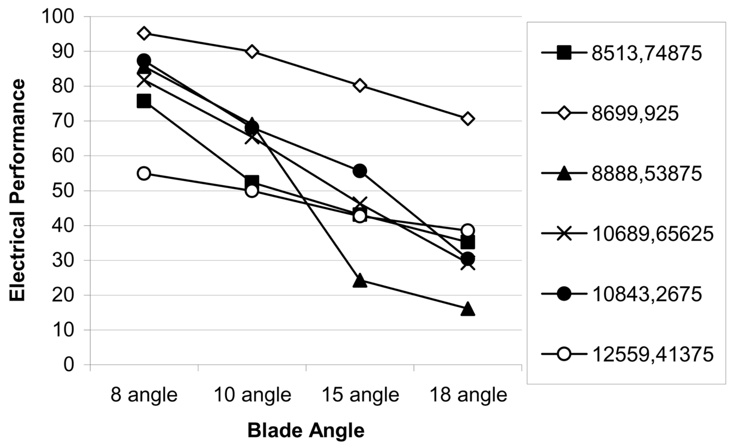

Fig. 9 The relation between the angle of tying of blade to rotor and electrical performances

The equations that are shown below are observed, in the relation between blade volume of rotor and speed of start of motion which are calculated by using polynomial equation (Hamming 1973, Civalek 2005) on the third degree for tying angles of 8, 10, 15 and 18.

For 8 degree of tying angle: $y = 0{,}1129\ x^3 - 1{,}1082\ x^2 + 3{,}1304\ x + 0{,}3167$ (3)

For 10 degree of tying angle: $y = 0{,}0494\ x^3 - 0{,}436\ x^2 + 1{,}0575\ x + 1{,}36$ (4)

For 15 degree of tying angle: $y = 0{,}0494\ x^3 - 0{,}436\ x^2 + 1{,}0575\ x + 1{,}36$ (5)

For 18 degree of tying angle: $y = 0{,}0669\ x^3 - 0{,}6352\ x^2 + 1{,}7208\ x + 0{,}5833$ (6)

The equations that are shown below are observed, in the relation between blade volume of rotor and electrical performance which are calculated by using polynomial Eqs. (9), (10) on the third degree for tying angles of 8, 10, 15 and 18.

For 8 degree of tying angle: $y = -0{,}3127\ x^3 - 0{,}2659\ x^2 + 11{,}172\ x + 67{,}987$ (7)

For 10 degree of tying angle: $y = 1{,}4348\ x^3 - 18{,}363\ x^2 + 66{,}245\ x + 7{,}0267$ (8)

For 15 degree of tying angle: $y = 0{,}7576\ x^3 - 7{,}7583\ x^2 + 21{,}096\ x + 36{,}847$ (9)

For 18 degree of tying angle: $y = 2{,}2747\ x^3 - 22{,}342\ x^2 + 58{,}706\ x + 2{,}9$ (10)

The equations that are shown below are observed, in the relation between the angle of tying of blade to rotor and the speed of start of motion which are calculated by using polynomial Eqs. (9), (10) on the second degree.

For RBV = 8513, 74875 unit^3: $y = 0{,}0175\ x^2 - 0{,}3525\ x + 2{,}8525$ (11)

For RBV = 8699,925 unit^3: $y = 0{,}055\ x^2 - 0{,}503\ x + 3{,}29$ (12)

For RBV = 8888,53875 unit^3: $y = 0{,}1425\ x^2 - 1{,}1255\ x + 3{,}7825$ (13)

For RBV = 10689,65625 unit^3: $y = 0{,}09\ x^2 - 0{,}728\ x + 3{,}335$ (14)

For RBV = 10843,2675 unit^3: $y = -0{,}1575\ x^2 + 0{,}6045\ x + 1{,}5875$ (15)

For RBV = 12559,41375 unit^3: $y = 0{,}1625\ x^2 - 1{,}1855\ x + 4{,}6875$ (16)

The equations that are shown below are observed, in the relation between the angle of tying of blade to rotor and the electrical performance which are calculated by using polynomial Eqs. (9), (10) on the second degree.

For RBV = 8513, 74875 unit^3: $y = 3{,}8625\ x^2 - 32{,}386\ x + 103{,}62$ (17)

For RBV = 8699, 925 unit^3: $y = -1{,}0775\ x^2 - 2{,}9255\ x + 99{,}403$ (18)

For RBV = 8888,53875 unit^3: $y = 2{,}075\ x^2 - 35{,}705\ x + 122{,}48$ (19)

For RBV = 10689, 65625 unit^3: $y = -0{,}175\ x^2 - 16{,}811\ x + 99{,}005$ (20)

For RBV = 10843, 2675 unit^3: $y = -1{,}5175\ x^2 - 10{,}718\ x + 98{,}568$ (21)

For RBV = 12559, 41375 unit^3: $y = 0{,}2025\ x^2 - 6{,}6555\ x + 61{,}628$ (22)

4. Conclusions

The rotor performance in turbine systems that are developed to benefit in an optimum level from the wind energy is related to many factors. These factors can be listed as; material of the wings, weight of the rotor, rotating ability, rotations per minute, wind speed and air intensity. Increase in the number of variables make the evaluation of rotor performance harder. We can fix this problem by mathmatical models derived from physcal rules.

The most important factors that effect rotor performance are the aerodynamic structure of rotor and the wings that made up the rotor. The angles that the wings are tied to the rotor, the angles of the torsion of the wings, the wideness and the shapes of the wings are elements that must be taken into consideration. Also, the lifting and driftage powers of wings are elements that effect the rotor performance in aerodynamic view (Piggott 2000).

The aim of this study is to show whether the rotor wing volume is a factor that affects rotor performance. This topic is leaned upon physical bases with the help of mathematical modeling that is developed in this area. As a result, the general tendency under the light of these findings shows that the increase in the volume in the rotor wing leads to an increase in the value of the starting speed of the rotor to move and a decrease in the rotor performance.

As the angles of tying the blades to the rotor increases, although a positive effect was observed on the speed of start of motion, on the contrary, a negative effect is observed on the electrical performances. According to this, it may be claimed that a conclusion can be reached that the highness of the speed of the start of motion of a rotor leads to a more efficient work on high speed. This situation is clearly seen in the mathematical models that are formed.

If the rotors that have the most appropriate aerodynamic structural blades have low blade volume, this will affect their performance and their speed of start of motion in a positive manner.

Nomenclature

NACA : National Advisory Committee for Aeronautics
N : Electrical Performance, (mW)
U : Electrical Voltage, (V)
I : Electrical Current, (mA)
RBV : Rotor blade volume, ($unit^3$)
A : Sectional area (Area belongs to Section A), ($unit^2$)
BN : Number of blades that will be on the rotor, (number)

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