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Simplified formulas for evaluation of across-wind dynamic responses of rectangular tall buildings

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Abstract. Tall buildings under wind action usually oscillate simultaneously in the along-wind and across-wind directions as well as in torsional modes. While several procedures have been developed for predicting wind-induced loads and responses in along-wind direction, accurate analytical methods for estimating across-wind and torsional response have not been possible yet. Simplified empirical formulas for estimation of the across-wind dynamic responses of rectangular tall buildings are presented in this paper. Unlike established empirical formulas in codifications, the formulas proposed in this paper are developed based on simultaneous pressure measurements from a series of tall building models with various side and aspect ratios in a boundary layer wind tunnel. Comparisons of the across-wind responses determined by the proposed formulas and the results obtained from the wind tunnel tests as well as those estimated by two well-known wind loading codes are made to examine the applicability and accuracy of the proposed simplified formulas. It is shown through the comparisons that the proposed simplified formulas can be served as an alternative and useful tool for the design and analysis of wind effects on rectangular tall buildings.

Keywords: tall building; wind effect; wind-induced response; wind tunnel test.

1. Introduction

Modern tall buildings are usually constructed of high-strength or light weight materials, or both, and tend to be more flexible and lightly damped than those in the past. As a result, the sensitivity of these tall buildings to dynamic excitations, such as strong winds, has increased. Therefore, the emphasis in the design of modern tall buildings has shifted to satisfy the requirements to control building movements and to limit wind-induced accelerations which may adversely affect occupant comfort (Tallin and Ellingwood 1984). The oscillations of tall buildings caused by wind action have been found to occur in the along-wind and across-wind directions as well as in torsional modes. For tall buildings with aspect ratios over 5, their across-wind responses usually exceed along-wind responses and can even reach several times of along-wind responses in many cases (Li, *et al*. 2000,

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2003, 2004); hence it is very important to evaluate across-wind responses, including across-wind dynamic displacement, acceleration and vibration-induced dynamic loads for the design and analysis of tall buildings.

It is well known that the mechanisms of across-wind dynamic loads on tall buildings are much more complex than those of along-wind dynamic loads. It has been recognized (Melbourne 1975, Solari 1985) that across-wind dynamic loads on tall buildings are induced by three major mechanisms: along-wind turbulence, across-wind turbulence and wake excitation, and wake excitation is the main contributor. Wake excitation mechanism, i.e., the mechanism of vortex formation and shedding in the separation wake of a tall building is strongly correlated with side ratio, aspect ratio, shape of the cross section of the building and the turbulence intensity of incident flow, thus it is very difficult to establish an analytic model for estimation of across-wind dynamic loads on tall buildings. Therefore, strictly speaking, accurate analytical calculation methods to evaluate across-wind dynamic responses of tall buildings are not available in the literatures, while several procedures, e.g., gust factor approach (Davenport 1967), have been developed for predicting loads and response in along-wind direction. At present, wind tunnel tests, including aeroelastic model test, high frequency force balance model test and simultaneous pressure measurements on model surfaces, are still the most effective approaches for evaluating across-wind loads and responses of tall buildings. Nevertheless, due to the constraints of time and cost in conducting wind tunnel tests, it is desirable to propose effective approximate calculation methods or simplified empirical formulas for estimating across-wind dynamic responses of tall buildings with regular shapes in typical surrounding exposures for design purposes. Since the 1970s, great efforts have been made to propose approximate analytical approaches and simplified empirical formulas for evaluation of across-wind dynamic responses of tall buildings (Melbourne 1975, Solari 1985). Several experimental techniques in boundary layer wind tunnels have been developed to determine fluctuating wind forces on buildings and structures (Kwok 1977, Reinhold 1977, Kareem 1985, Islam 1988, Li 2000, Holmes 2001, Liang, *et al*. 2002, 2003). The following two main methodologies were usually adopted to establish simplified empirical formulas in wind codes: (1) Based on extensive wind tunnel test data of across-wind responses measured from aeroelastic models, simplified formulas were established as a function of several key parameters such as structural geometry, dimension, mass, natural frequency, damping ratio and surrounding exposure etc., e.g., National Building Code of Canada (1995), in which the formulas for estimating structural across-wind responses are applicable to tall buildings with all sorts of cross sections; (2) Empirical formulas were developed according to the database of across-wind loads measured from high frequency force balance models or simultaneously measured pressures from rigid model surfaces, e.g., the formulas recommended by the Architecture Institute of Japan (1996), which are applicable to rectangular tall buildings with all kinds of side and aspect ratios.

The simplified empirical formulas established on the basis of wind tunnel measurements of across-wind loads from high frequency force balance models may have three obvious shortcomings: (1) there is no information on variations of across-wind dynamic loads along building height; (2) there is lack of information on correlations between across-wind dynamic loads at different building heights; (3) the fundamental mode shape of a tall building is assumed to be linear. These three shortcomings may cause errors in the estimation of across-wind effect on a tall building, and these errors may become rather large as the building height increases. On the other hand, construction of an aeroelastic model is more costly and time consuming than building a rigid model. Comparing with these two experimental methods (high frequency force balance and aeroelastic model techniques), simultaneous measurements of pressures from rigid model surfaces can not only determine the integral fluctuating wind forces on a building model but also provide information on the spatial and temporal distributions of wind loads over the surfaces of the building. It is thus desirable to propose simplified formulas for estimating structural across-wind responses based on simultaneous pressure measurement approach. However, to the authors' best knowledge; all the existing empirical formulas in wind codes for evaluation of across-wind dynamic response of tall buildings were developed on the basis of the force balance or aeroelastic model techniques. Therefore, there is a need to propose corresponding simplified empirical formulas on the basis of the simultaneous pressure measurement approach.

In the light of wind tunnel test data from simultaneously measured pressures on surfaces of a series of rigid models, a mathematical model for across-wind dynamic loads on rectangular tall buildings in frequency domain was proposed by the authors (Liang, *et al*. 2002). On the basis of this mathematical model and applying random vibration theory, simplified empirical formulas for evaluating across-wind dynamic responses of rectangular tall buildings with various side and aspect ratios are presented in this paper. In order to examine the accuracy of the proposed formulas, comparative studies on the across-wind responses determined by the proposed formulas and the results obtained from wind tunnel tests as well as those estimated from well-known wind loading codes such as National Building Code of Canada (1995) and the Architecture Institute of Japan (1996), are presented. In addition, a parametric study is carried out to investigate the variations of across-wind dynamic responses of rectangular tall buildings with different side ratios, aspect ratios, fundamental natural frequency, and damping ratio under different surrounding exposures and wind speeds. Finally, the effects of the side ratios, exposure conditions, mean wind speed and fundamental natural frequency of a rectangular tall building on its across-wind dynamic responses are presented and discussed.

2. Simplified empirical formulas

The formulas proposed in this paper are developed on the basis of simultaneously measured pressures on the surfaces of several rectangular rigid models with different aspect and side ratios employing wind tunnel tests and random vibration method in frequency domain. Similar to the approach for evaluating along-wind dynamic responses, across-wind response can also be described in terms of the background and resonant components. Therefore, the across-wind dynamic responses of a rectangular tall building can be expressed as algebraic expressions, which consist of several parameters such as mean wind speed or pressure atop the building, geometrical dimensions, damping ratio, fundamental natural frequency and mode shape of the building, root mean square (RMS) lift coefficient and expression of fluctuating across-wind force spectrum for the building.

In this study, the RMS lift coefficient and expression of fluctuating across-wind force spectrum for rectangular tall buildings are determined from simultaneous measurements of surface pressures on rigid models. Fig. 1 shows a rigid model in a boundary layer wind tunnel with measuring section of 1.4×1.4 m, which is the property of China Aerodynamic Research Centre. The mean wind speed and turbulence intensity profiles above the measuring section for the wind tunnel test are shown in Fig. 2. Building models for the wind tunnel test are 3-D rectangular cylinders with four different side ratios, denoted as $\text{depth}/\text{width} = 1.0,\ 2.0,\ 3.0$ and 4.0, and two different heights denoted as $H = 0.4$, 0.8 m are selected for each cross section of the models. The approaching flow is perpendicular to the side face of every model in the wind tunnel tests. The models were made of balsa and their natural frequencies are high enough to be regarded as rigid models. Fluctuating wind

Fig. 1 A model in wind tunnel

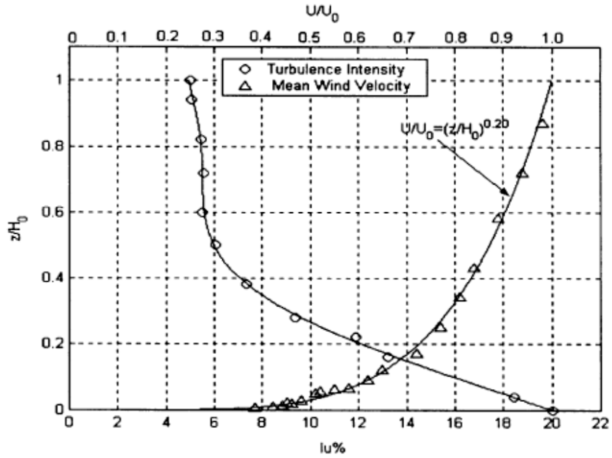

Fig. 2 Mean wind speed and turbulence intensity profiles

pressures on the surfaces of the models were measured by pressure transducers placed at five levels. The instantaneous across-wind force at each level on a building model can be obtained by integrating fluctuating pressure data recorded simultaneously on the model surfaces. Once the time histories of across-wind force at each level are measured, RMS lift coefficient and power spectrum of across-wind force can be obtained by data processing software, and coherence function between two levels can also be determined. Fig. 3 shows typical examples of the fluctuating across-wind force spectra from FFT analysis of the measured data. Empirical formulas can be established by applying the least-square method to fit the experimental data. It is observed from Fig. 3 that the empirical formulas fitted the results of the wind tunnel test very well. As Fig. 3 indicates, when $1/4 < D/B < 3$, the distribution of the across-wind force spectrum is single peak shaped, and when $3 \le D/B \le 4$, there are two peaks in the graph of across-wind force spectrum. Normally, when the fundamental natural frequency of a tall building in across-wind direction decreases, the mechanical admittance will approach to the peak of the across-wind force spectrum from high frequency range, then the across-wind responses will increase. Nevertheless, when $3 \le D/B \le 4$, the mechanical admittance may move down in the spectrum between the two spectral peaks from the second spectral peak; in these cases, across-wind response will decrease as the fundamental natural frequency of a tall building in across-wind direction decreases. More empirical formulas for the power spectra of across-wind force on tall buildings with various side and aspect ratios can be found in the authors' previously published paper (Liang, *et al*. 2002). In general, the reduced wind speed $\{\bar{V}(H)/n_1 B\}$ for a tall building with its height less than 200 m is much less than 10; in such a case, the aerodynamic damping in across-wind direction is positive and can be conservatively neglected (Vickery and Steckley 1993), where $\bar{V}H$ is the mean wind speed at the top of the building; n_1 is the fundamental natural frequency of the building in across-wind direction; B is the windward width of the building. Thus,

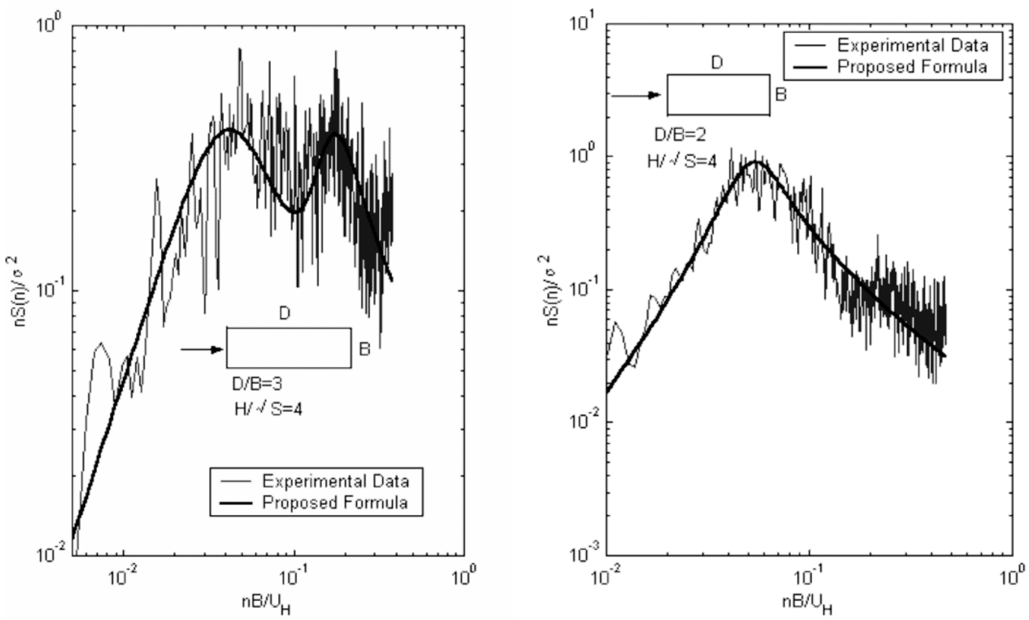

Fig. 3 Comparison of the across-wind force spectra between the proposed formulas and experiment results

in the next step of this study we will establish simplified formulas for estimating across-wind dynamic responses of rectangular tall buildings on the basis of the simultaneously measured surface pressures on the rigid building models in the wind tunnel tests.

In the light of employing random vibration method in frequency domain, the expression of RMS across-wind displacement of a rectangular tall building at height z can be derived as follows.

When wind incident direction is perpendicular to a side face of a rectangular tall building, the governing differential equation of motion in across-wind direction of the building, simplified as a continuous flexural bar is:

$$m(z)\frac{\partial^2 y}{\partial t^2} + c(z)\frac{\partial y}{\partial t} + \frac{\partial^2\left(EI\left(\frac{\partial^2 y}{\partial t^2}\right)\right)}{\partial z^2} = P_y(z, t) \tag{1}$$

where $m(z)$, $c(z)$ and $P_y(z, t)$ are mass, damping coefficient and across-wind dynamic load intensity at height z; EI is the flexural rigidity of the bar.

By using the mode superposition method, the differential equation of the generalized coordinates governing the across-wind motion of the building can be deduced from Eq. (1) as follows

$$\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j = \frac{\int_0^H P_y(z, t)\phi_{yj}(z)dz}{\int_0^H m(z)\phi_{yj}^2(z)dz} \qquad j = 1, \infty \tag{2}$$

where ζ_j, ω_j and ϕ_{yj} are the damping ratio, circular natural frequency and mode shape of the j-th mode, respectively; H is the building height.

According to the random vibration theory, the spectral density of the structural displacement responses in the across-wind direction can be expressed as

$$S_y(z,\omega) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\phi_{yj}(z)\phi_{yk}(z)H_{yj}(-i\omega)H_{yk}(i\omega)S_{F_{yj}F_{yk}}(\omega) \tag{3}$$

where $H_{yj}(i\omega)$ and $H_{yk}(i\omega)$ are frequency response functions of the j-th and k-th mode, respectively.

If the empirical formula of the power spectral density for across-wind dynamic force proposed by the authors (Liang, *et al*. 2002) is adopted herein, we have:

$$S_{F_{yj}F_{yk}}(\omega) = \frac{\int_0^H\int_0^H \phi_{yk}(z')\phi_{yj}(z)B^2\bar{C}_L^2\frac{1}{4}\rho^2V^2(z)V^2(z')r(z,z')\frac{f_y(\omega)}{\omega}dzdz'}{\int_0^H m(z)\phi_{yj}^2dz\int_0^H m(z)\phi_{yk}^2dz} \tag{4}$$

where ρ and $V(z)$ are air density and mean wind velocity at height z; $\bar{C}_L$ is the RMS lift coefficient, which can be expressed as (Liang, *et al*. 2002)

$$\bar{C}_L = 0.045(D/B)^3 - 0.335(D/B)^2 + 0.868(D/B) - 0.174$$

$r(z,z')$ is vertical coherence function which can be expressed as follows (Liang, *et al*. 2002):

$$r(z,z') = \exp\left[-\left(\frac{|z-z'|}{B\varepsilon}\right)^2\right] \tag{5}$$

in which B and D are the width and depth of the rectangular building, the value of ε can be referred to Liang, *et al*. (2002); $f_y(\omega)$ is a coefficient of the across-wind force spectrum, it can be expressed as a function of the side ratio and aspect ratio of a rectangular tall building as follows (Liang, *et al*. 2002):

(1) For $\frac{1}{4} \le D/B < 3$

$$f_y(\omega) = A\cdot\frac{H(C_1)\cdot\bar{n}_1^2}{(1-\bar{n}_1^2)^2 + C_1\cdot\bar{n}_1^2} + (1-A)\cdot\frac{0.906\cdot\bar{n}_1^3}{[(1-\bar{n}_1^2)^2 + 2\bar{n}_1^2]} \tag{6}$$

$$H(C_1) = 0.179\cdot C_1 + 0.65\cdot\sqrt{C_1} \tag{7}$$

$$C_1 = [0.47\cdot(D/B)^{2.8} - 0.52\cdot(D/B)^{1.4} + 0.24]/(H/\sqrt{S}) \tag{8}$$

The values of A and $\bar{n}_1$ can be determined according to the following equations

(i) For $\frac{1}{4} \le D/B < \frac{1}{2}$,

$$A = (H/\sqrt{S})\cdot[-0.6\cdot(D/B)^2 + 0.29\cdot(D/B) - 0.06] + [9.84\cdot(D/B)^2 - 5.86\cdot(D/B) + 1.25] \tag{9}$$

$$\bar{n}_1 = \frac{\frac{\omega}{2\pi}B}{0.094V^*} \tag{10}$$

where $S = B \times D$ is area of cross section; 0.094 is the Strouhal number for $\frac{1}{4} \le D/B < \frac{1}{2}$, and V^* is the mean wind speed at the 2/3 height of the building.

(ii) For $\frac{1}{2} \le D/B < 3$,

$$A = (H/\sqrt{S}) \cdot [-0.118 \cdot (D/B)^2 + 0.358 \cdot (D/B) - 0.214] + [0.066 \cdot (D/B)^2 - 0.26 \cdot (D/B) + 0.894] \tag{11}$$

$$\bar{n}_1 = \frac{\frac{\omega}{2\pi}B}{[0.002(D/B)^2 - 0.023(D/B) + 0.105]V^*} \tag{12}$$

where the Strouhal number $S_t = 0.002(D/B)^2 - 0.023(D/B) + 0.105$, for $\frac{1}{2} \le D/B < 4$.

(2) For $3 \le D/B \le 4$

$$f_y(\omega) = A \cdot \frac{1.275 \cdot \bar{n}_1^2}{(1-\bar{n}_1^2)^2 + 2\bar{n}_1^2} + (1-A) \cdot \frac{0.906\bar{n}_1^3}{\left[k^{2.5}\left(1-\left(\frac{\bar{n}_1}{k}\right)^2\right)^2 + \frac{2\bar{n}_1^2}{k^{0.5}}\right]} \tag{13}$$

where $k = -0.175(H/\sqrt{S}) + 4.7$ (14)

$$\begin{aligned} A &= a \cdot (I'_u)^b \\ a &= 0.17 \cdot (H/\sqrt{S}) + 3.32 \\ b &= 0.18 \cdot (D/B) + 0.26 \end{aligned} \tag{15}$$

where I'_u is the turbulence intensity at 2/3 height of the building. The expression of $\bar{n}_1$ in Eq. (13) is the same as that in Eq. (12).

Therefore, the RMS value of the dynamic displacement response can be determined by

$$\sigma_y(z) = \left[\int_0^\infty S_y(z,\omega)d\omega\right]^{\frac{1}{2}} \tag{16}$$

If only the contribution of the fundamental mode to the across-wind response is considered, Eqs. (3) and (4) can be simplified as follows:

$$S_y(z,\omega) = \phi_{y1}^2(z)\left|H_{y1}(i\omega)\right|^2 S_{F_{y1}F_{y1}}(\omega) \tag{17}$$

$$S_{F_{yj}F_{yk}}(\omega) = \left(\frac{1}{M_1}\right)^2 \int_0^H \int_0^H \phi_{y1}(z')\phi_{y1}(z) B^2 \bar{C}_L^2 \frac{1}{4}\rho^2 V^2(z) V^2(z') r(z,z') \frac{f_y(\omega)}{\omega} dz dz' \tag{18}$$

where $M_1 = \int_0^H m(z)\phi_{y1}^2 dz$ is the equivalent modal mass for the first mode.

If the fundamental mode shape is taken as a straight line and the mass density $m(z)$ is regarded as a constant m, then we have $M_1 = mH/3$.

If the coherence of across-wind fluctuating forces between different levels is neglected and the fundamental mode shape is taken as a straight line, then Eq. (18) can be further simplified as:

$$S_{F_{yj}F_{yk}}(\omega) \approx \frac{f_y(\omega)}{\omega}\left(\frac{1}{m}\right)^2 \left(\frac{1}{2}\rho V_H^2 B \bar{C}_L\right)^2 \left(\frac{3}{2+2\alpha}\right)^2 = \frac{f_y(\omega)}{\omega}\left(\frac{1}{m}\right)^2 \left(\frac{1}{2}\rho V_H^2 B \bar{C}_L\right)^2 (\beta_L)^2$$
$$= (\beta_L)^2 \frac{f_y(\omega)}{\omega}\left(\frac{1}{m}\right)^2 (w_H B \bar{C}_L)^2 \tag{19}$$

where α is coefficient related to terrain roughness, V_H, w_H are the mean wind speed and wind pressure atop the building, respectively; $\beta_L = 3/(2+2\alpha)$ is a parameter related to terrain roughness.

Then Eq. (16) can be simplified as follows:

$$\sigma_y(z) = \beta_L \phi_{y1}(z) \frac{1}{m} w_H B \bar{C}_L \left[\int_0^\infty |H_{y1}(i\omega)|^2 \frac{f_y(\omega)}{\omega}\right]^{\frac{1}{2}}$$
$$\approx \beta_L \phi_{y1}(z) \frac{w_H}{m\omega_1^2} B \bar{C}_L \left[\int_0^\infty \frac{f_y(\omega)}{\omega} d\omega + \frac{\pi f_y(\omega_1)}{4\zeta_1}\right]^{\frac{1}{2}}$$
$$= \beta_L \phi_{y1}(z) \frac{w_H}{m\omega_1^2} B \bar{C}_L \sqrt{1 + \frac{\pi f_y(\omega_1)}{4\zeta_1}} \tag{20}$$

where $f_y(\omega)/\omega$ is the normalized expression of across-wind dynamic force spectrum under the condition that $\int_0^\infty \frac{f_y(\omega)}{\omega} d\omega = 1$, and $\omega_1 = 2\pi n_1$, in which n_1 is the fundamental natural frequency of the building in across-wind direction (*Hz*), ζ_1 is the damping ratio of the first mode of the building in across-wind direction.

The expression for the fundamental mode shape ϕ_{y1} in across-wind direction is adopted as $\sin\left[\frac{\pi}{2}\left(\frac{z}{H}\right)^{1.8}\right]$, which was obtained by applying the least-square method to fit the measured and calculated results of the fundamental translational mode shape of more than 20 tall buildings (Li, *et al*. 1994, 1996).

Finally the simplified formula for estimating the RMS across-wind displacement of a rectangular tall building at height z can be expressed as:

$$\sigma_y(z) = \beta_L \sin\left[\frac{\pi}{2}\left(\frac{z}{H}\right)^{1.8}\right]\frac{w_H}{m\omega_1^2}B\bar{C}_L\sqrt{1+\frac{\pi f_y(\omega_1)}{4\zeta_1}} \quad \text{(m)}$$

$$= \beta_L \sin\left[\frac{\pi}{2}\left(\frac{z}{H}\right)^{1.8}\right]\frac{w_H}{m\omega_1^2}B\bar{C}_L\sqrt{1+R_L} \tag{21}$$

where $w_H = (1/2)\rho\bar{V}^2(H)$ (kN/m^2) and $R_L = \pi f_y(\omega_1)/(4\xi_1)$

In the above equation, R_L in the radical sign represents the contribution of the resonant component, and the value of 1 in the radical sign of Eq. (21) denotes the contribution of the background component. For acceleration response of a tall building, the background component can be reasonably neglected as several well-known wind loading codes did. Therefore, the RMS across-wind acceleration response of a rectangular tall building at height z can be expressed as:

$$\sigma_{\ddot{y}}(z) = \beta_L \sin\left[\frac{\pi}{2}\left(\frac{z}{H}\right)^{1.8}\right]\frac{w_H\bar{C}_L B}{m}\sqrt{R_L} \quad (\text{m/s}^2) \tag{22}$$

Similarity, the wind-induced dynamic load on a rectangular tall building at height z in across-wind direction can be written as

$$P(z) = \mu\,\beta_L \sin\left[\frac{\pi}{2}\left(\frac{z}{H}\right)^{1.8}\right]w_H\,\bar{C}_L B\sqrt{R_L} \quad (\text{kN/m}) \tag{23}$$

where μ is the peak factor, letting $\mu = 3.5$.

3. Comparisons and discussions

3.1. Comparison of the results from the simplified formulas and those from direct integration calculation

In order to examine the applicability and accuracy of the proposed empirical formulas, it is necessary to present comparisons between the results obtained from the proposed simplified formulas and those determined from the direct integration calculation based on the same mathematical model of across-wind dynamic loads which was used to derive the simplified formulas. Table 1 lists several examples of the comparisons for typical tall buildings with steel structures. Generally, the error range between the two sets of results is approximately within 5%, thus suggesting that the results obtained from the simplified formulas are fairly accurate. Most of the results calculated by the simplified formulas are somewhat greater than those obtained by the direction integration calculation. In other words, the proposed simplified formulas provide slightly conservative results. The results in Table 1 also show that when a narrow side of a rectangular tall building is windward, the across-wind dynamic response is much greater than when its broad side is windward, and the difference increases as the ratio of the broad side to the narrow side increases.

Table 1 Across-wind dynamic responses obtained by the simplified formulas (SF) and the integration calculation (IC) (Steel structures with mass density of 180 kg/m^3 and damping ratio of 0.02, located in open terrain)

Wind pressure at 10 m height (kN/m^2)	Building height (m)	Side ratio D/B	Building width (m)	The first natural frequency (Hz)	Tip RMS acceleration (gal)	Tip RMS displacement (mm)
0.5	120	1/3	60	0.714	4.56(SF)	2.73(SF)
0.5	120	1/3	60	0.714	4.43(IC)	2.66(IC)
0.5	160	4	20	0.4545	38.18(SF)	48.05(SF)
0.5	160	4	20	0.4545	35.96(IC)	45.30(IC)
0.5	80	1	20	0.714	18.10(SF)	11.25(SF)
0.5	80	1	20	0.714	17.27(IC)	10.92(IC)
0.6	120	2	30	0.556	13.68(SF)	13.42(SF)
0.6	120	2	30	0.556	13.47(IC)	13.29(IC)
0.6	160	1/4	80	0.714	1.57(SF)	1.00(SF)
0.6	160	1/4	80	0.714	1.52(IC)	0.95(IC)
0.6	120	3	20	0.556	39.09(SF)	33.1 (SF)
0.6	120	3	20	0.556	37.19(IC)	31.5 (IC)

3.2. Comparison of the results from the simplified formulas with those from two wind loading codes

Three tall buildings with different side ratios are considered in this part to compare the across-wind dynamic responses obtained by the simplified formulas with those determined by two wind loading codes [National Building Code of Canada (NBCC), 1995; the Architectural Institute of Japan (AIJ), 1996]. The structural parameters of the three buildings are listed in Table 2. The tall buildings are of steel structures, whose densities are $\rho_b = 180$ kg/m^3. The comparisons of the across-wind RMS accelerations determined from the simplified formulas with those obtained from the wind loading codes are illustrated in Fig. 4.

As shown in Fig. 4, there are small differences among the across-wind RMS accelerations at the top of the square building (building 1) obtained by NBCC, AIJ and the simplified formulas (SF) proposed in this paper, though the across-wind RMS acceleration curve obtained by AIJ increases more sharply at high wind speeds than the other curves do. When the side ratio of a tall building is equal to 3 (building 3), the across-wind RMS accelerations at the top of building 3 obtained by NBCC and AIJ are almost identical, while those obtained by SF are somewhat larger. When the side ratio is equal to 1/3 (building 2), the differences among the results obtained by NBCC, AIJ and

Table 2 Structural parameters of three tall buildings (steel structures with mass density of 180 kg/m^3 and damping ratio of 0.02, located in urban terrain)

	Building height (m)	Side ratio D/B	Building width (m)	The first natural frequency in across-wind direction (Hz)
Building 1	150	1	30	0.32
Building 2	150	1/3	60	0.455
Building 3	150	3	20	0.25

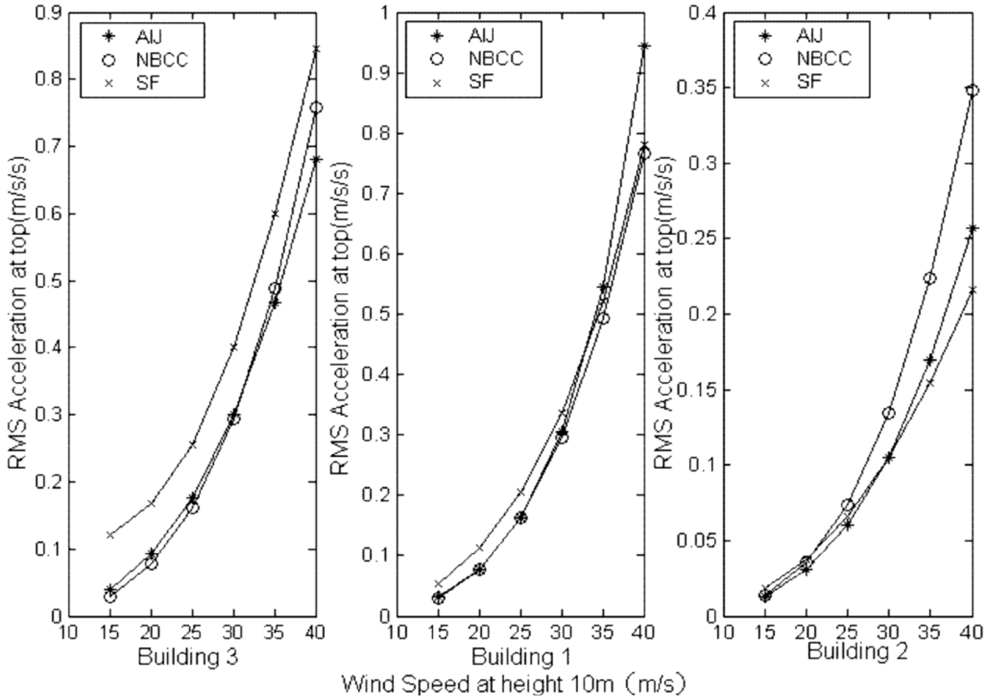

Fig. 4 Across-wind RMS accelerations of tall buildings determined from two wind loading codes and the simplified formulas (SF)

SF are minor when wind speed is low, and as wind speed increases, the differences among the across-wind RMS accelerations at the top of buildings 2 obtained by NBCC, AIJ and SF are rather obvious. Especially the results obtained by NBCC are much greater than those obtained by AIJ and SF. It is shown through the comparisons that when a narrow side or a broad side of a rectangular tall building is windward, the dynamic responses of a rectangular tall building in across-wind direction obtained by SF are larger, while those obtained by NBCC are smaller. Though the formulas recommended in NBCC, AIJ and this study were developed based on different wind tunnel test techniques, Fig. 4 indicates that the differences among the results obtained from the three approaches are acceptable in engineering practices for most cases. Therefore, the proposed simplified formulas for evaluating across-wind dynamic responses can be an alternative and useful tool for the analysis of tall buildings at preliminary design stages.

3.3. Comparison of the results from the simplified formulas with wind tunnel test data

Wind tunnel test has been recognized as an effective approach to estimate wind-induced vibrations of tall buildings. In order to further verify the accuracy of the proposed simplified formulas, it is desirable to compare the results determined by the proposed simplified formulas with the available wind tunnel measurements. Table 3 shows the comparisons of across-wind acceleration responses of several rectangular tall buildings with different side and aspect ratios. Meanwhile, the across-wind acceleration responses of the tall buildings evaluated from wind loading codes such as AIJ and NBCC are also presented in this table for comparison purposes.

Table 3 Across-wind acceleration responses obtained from the simplified formulas and wind tunnel tests for four tall buildings

Building	Wind speed atop building (m/s)	RMS by wind tunnel test (gal)	RMS by the simplified formulas (gal)	RMS obtained from AIJ (gal)	RMS obtained from NBCC (gal)
Building 1 (Islam 1988)	Building density = 190 kg/m^3, damping ratio = 1%, n_1 = 0.20 Hz, building height = 180 m, building width (B) = 30 m, D/B = 1.0, terrain exponent α = 0.16				
	24.00	4.262	6.53	5.69	4.903
	25.99	5.698	8.07	7.22	6.383
	28.02	7.097	9.880	9.080	8.172
	30.00	9.214	11.890	11.270	10.238
Building 2 (Islam 1988)	Building density = 190 kg/m^3, damping ratio = 1%, n_1 = 0.15 Hz, building height = 240 m, building width (B) = 30 m, D/B = 1.0, terrain exponent α = 0.16				
	27.00	9.537	9.620	11.710	10.511
	28.98	12.177	11.890	15.110	13.276
	31.01	15.635	14.680	19.530	16.592
	32.99	18.360	17.950	24.990	20.360
Building 3 (Kijewski and Kareem 1999)	Building density = 192.22 kg/m^3, damping ratio = 1%, n_1 = 0.2 Hz, building height = 182.22 m, building width (B) = 30.48 m, D/B = 1.0, terrain exponent α = 0.333				
	24.25	6.07	6.19	4.73	5.36
	25.99	8.07	7.44	5.92	6.73
	27.71	9.82	8.82	7.35	8.33
	31.19	13.50	12.11	11.01	12.32
Building 4 (Cheung, *et al*. 1993)	Building density = 210.0 kg/m^3, damping ratio = 1%, n_1 = 0.1245 Hz, building height = 323.0 m, building width (B) = 46.8 m, D/B = 1.0, terrain exponent α = 0.280				
	10.97	0.8597	0.4561	0.442	0.226
	12.65	0.9055	0.6523	0.6160	0.3565
	25.26	4.873	3.987	3.957	3.497
	30.08	7.588	6.410	6.830	6.220
	45.27	25.00	21.25	31.05	23.98
	54.91	41.30	42.80	76.74	45.32

It can be seen from Table 3 that the across-wind acceleration responses obtained from the simplified formulas are closer to the results of wind tunnel tests than those determined from the two wind loading codes (AIJ and NBCC) in most cases. Therefore, the comparison results further suggest that the simplified formulas proposed in this paper can be served as an alternative and useful tool in engineering practice.

Table 4 Calculated results of wind-induced dynamic loads on RC tall buildings in across-wind direction (damping ratio of 0.05)

Basic wind pressure (kN/m^2)	Width × depth (m^2)	Terrain category	Result	Building height (m)											
				80			110			150			200		
				The fundamental natural period in across-wind direction (s)											
				1	1.2	1.5	1.5	1.7	2	2	2.2	2.5	2.5	3	3.5
0.5	20×60	B	R_L	0.3372	0.4542	0.5699	0.7221	0.8207	0.9680	1.2460	1.3705	1.5612	2.0087	2.4135	2.8094
			$P(z)$	12.510	13.728	15.377	19.166	20.433	22.190	27.803	29.159	31.121	38.705	42.426	45.774
		C	R_L	0.3267	0.3932	0.4930	0.6377	0.7246	0.8543	1.1217	1.2337	1.4053	1.8413	2.2126	2.5763
			$P(z)$	9.204	10.098	11.307	14.794	15.770	17.123	22.490	23.586	25.172	32.702	35.848	38.683
		D	R_L	0.2677	0.3221	0.4036	0.5363	0.6092	0.7180	0.9678	1.0643	1.2120	1.6270	1.9553	2.2773
			$P(z)$	5.9992	6.5803	7.3658	10.279	10.955	11.893	16.632	17.442	18.613	25.628	28.095	30.320
	30×30	B	R_L	0.3644	0.4437	0.5660	0.5306	0.6111	0.7356	0.6538	0.7315	0.8563	0.7061	0.9063	1.1371
			$P(z)$	31.134	34.354	38.802	41.598	44.644	48.981	50.993	53.939	58.359	58.105	65.827	73.736
		C	R_L	0.3133	0.3807	0.4843	0.4633	0.5326	0.6390	0.5789	0.6463	0.7536	0.6312	0.8032	0.9962
			$P(z)$	22.824	25.160	28.375	31.928	34.232	37.498	40.909	43.223	46.676	48.481	54.687	60.906
		D	R_L	0.2546	0.3087	0.3913	0.3844	0.4409	0.5272	0.4896	0.5452	0.6331	0.5410	0.6818	0.8355
			$P(z)$	14.814	16.312	18.366	22.036	23.599	25.805	29.955	31.609	34.061	37.418	42.006	46.501
	60×20	B	R_L	1.9615	2.6758	4.1347	4.3102	5.5793	7.2747	7.1289	7.4543	6.5799	6.5983	4.6348	3.5994
			$P(z)$	72.090	84.200	104.67	118.33	134.62	153.72	168.06	171.85	161.45	177.27	148.57	130.93
		C	R_L	1.1761	1.5645	2.3261	2.5934	3.3397	4.6332	4.8405	5.5592	5.8888	5.7745	4.7172	3.7566
			$P(z)$	44.132	50.900	62.064	75.39	85.55	100.77	118.06	126.52	130.22	146.35	132.27	118.04
		D	R_L	0.3499	0.4625	0.6696	0.9195	1.1605	1.6003	1.9826	2.3878	2.9744	3.2236	3.5178	3.2764
			$P(z)$	17.331	19.926	23.976	34.012	38.210	44.870	60.157	66.0202	73.684	91.160	95.229	91.903

Table 5 Calculated results of wind-induced dynamic loads on steel tall buildings in across-wind direction (damping ratio of 0.02)

Basic wind pressure (kN/m^2)	Width × depth (m^2)	Terrain category	Result	Building height (m)											
				80			100			150			200		
				The fundamental natural period in across-wind direction (s)											
				1.5	2	2.5	2.5	2.8	3.2	3.2	3.5	4	4	4.5	5
0.5	20×60	B	R_L	1.4248	1.9156	2.4136	3.0394	3.4153	3.9101	5.0025	5.4808	6.2794	8.0271	9.0221	9.9842
			$P(z)$	24.313	28.192	31.645	39.321	41.682	44.598	55.709	58.3111	62.415	77.373	82.029	86.291
		C	R_L	1.2326	1.6554	2.0834	2.6810	3.0116	3.4464	4.5021	4.7161	5.0484	7.3649	8.2843	9.1779
			$P(z)$	17.879	20.719	23.2440	30.333	32.149	34.392	45.056	47.161	50.484	65.403	69.366	73.011
		D	R_L	1.0089	1.3533	1.7011	2.2516	2.5282	2.8918	3.8823	4.2533	4.8738	6.5129	7.3313	8.1303
			$P(z)$	11.646	13.488	15.122	21.062	22.318	23.869	33.313	34.868	37.325	51.276	54.402	57.290
	30×30	B	R_L	1.4150	1.9584	2.5450	2.3952	2.7555	3.2614	2.9436	3.3403	4.0952	3.5621	4.4928	5.7534
			$P(z)$	61.351	72.176	82.278	88.38	94.80	103.13	108.20	115.26	127.62	130.51	146.57	165.86
		C	R_L	1.2107	1.6664	2.1519	2.0682	2.3692	2.7861	2.5619	2.8885	3.4915	3.0691	3.7773	4.6672
			$P(z)$	44.865	52.636	59.814	67.459	72.202	78.297	86.06	91.38	100.47	106.90	118.60	131.83
		D	R_L	0.9783	1.3385	1.7170	1.6951	1.9333	2.2587	2.1268	2.3827	2.8425	2.5320	3.0461	3.6467
			$P(z)$	29.039	33.966	38.469	46.271	49.415	53.412	62.430	66.079	72.2174	80.953	88.791	97.150
	60×20	B	R_L	10.337	17.968	17.282	16.721	12.861	9.3142	9.5739	8.2793	7.5765	8.3170	8.6173	9.3572
			$P(z)$	165.49	218.19	213.98	233.06	204.40	173.94	194.75	181.11	173.25	199.02	202.58	211.10
		C	R_L	5.8151	10.498	14.442	14.804	13.448	10.438	10.331	8.8929	7.9386	8.6322	8.9260	9.7399
			$P(z)$	98.13	131.85	154.65	180.12	171.68	151.25	172.48	160.02	151.19	178.93	181.95	190.06
		D	R_L	1.6740	2.887	4.5657	6.2090	7.2444	7.5167	8.1530	7.6693	7.2913	8.0507	8.6388	9.6777
			$P(z)$	37.910	49.784	62.607	88.382	95.467	97.245	121.99	118.32	115.37	144.06	149.23	157.95

Note: Unit for $P(z)$ is: kN/m, and the peak factor in Eq.(23) is 3.5.

3.4. Wind-induced dynamic loads in across-wind direction

By adopting the simplified formulas proposed in this paper, wind-induced dynamic loads in across-wind direction acting on rectangular tall buildings with different aspect ratios, side ratios, natural frequencies, damping ratios and locations with respect to different surrounding exposures as well as under different wind speeds can be evaluated. Some selected results are presented in Tables 3 and 4. In the calculations, the damping ratio of the fundamental mode for steel tall buildings is assumed to be 0.02, and that for reinforced concrete (RC) tall buildings is taken to be 0.05; the mass density of steel tall buildings is assumed to be 180 kg/m^3, and that of RC tall buildings is 250 kg/m^3. In the Load Code for the Design of Building Structures in China (GBJ9-1987), the surrounding exposure is divided into four categories: A, B, C and D, according to terrain roughness; the terrain roughness related exponent for A, B, C and D terrain categories, is 0.12, 0.16, 0.22 and 0.3, respectively. In Tables 4 and 5, the basic wind pressure represents the mean wind pressure at 10 m height at B terrain category, as specified in the Load Code of China (GB J9-1987). It is shown in Tables 4 and 5 that the wind-induced dynamic loads in across-wind direction decrease as surrounding terrain becomes rougher.

4. Conclusions

The main conclusions from this study are summarized as follows:

(1) The across-wind dynamic responses of isolated rectangular tall buildings can be evaluated with acceptable accuracy by the simplified formulas proposed in this paper. The proposed formulas should be useful to help structural engineers to improve their designs at preliminary design stage for tall buildings and to decide if wind tunnel test is necessary.

(2) Through comparisons with the results obtained by the direct integration calculation based on the wind tunnel tests, it is shown that the across-wind dynamic responses of isolated rectangular tall buildings estimated by the proposed formulas are fairly good approximations.

(3) The numerical results of across-wind acceleration responses obtained from the simplified formulas are closer to the wind tunnel measurements than those obtained from two well-known wind loading codes (AIJ and NBCC) for many cases. Therefore, the simplified formulas proposed in this paper can be served as an alternative and useful tool to evaluate the wind-induced across-wind response of rectangular buildings.

(4) Though wind tunnel study on estimating across-wind responses of tall buildings can be conducted by three experimental approaches, i.e., aeroelastic model test, high frequency force balance model test and simultaneous measurements of model surface pressures, the pressure measurement approach can provide more detailed information on the spatial and temporal distributions of wind loads over the surfaces of a tall building for developing the simplified formulas than the other two test techniques. Therefore, it is desirable to propose simplified formulas for engineering applications based on simultaneous pressure measurement approach.

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