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Numerical simulation of flow past 2D hill and valley

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Abstract. Numerical simulation of flow past two-dimensional hill and valley is presented. Application of three turbulence models – the standard and modified (Kato-Launder) k-ε models and standard k-ω model – is discussed. The computational methodology is briefly described. The mean velocity and turbulence intensity profiles, obtained from numerical simulations of flow past the hill, are compared with the experimental data acquired in a boundary-layer wind tunnel at Colorado State University. The mean velocity, turbulence kinetic energy and Reynolds shear stress profiles from numerical simulations of flow past the valley are compared with published experimental data. Overall, the results of simulations employing the standard k-ε model were found to be in a better agreement with the experimental data than those obtained using the modified k-ε model and the k-ω model.

Keywords: numerical simulation; k-ε model; k-ω model; ASCE 7-98; wind speed-up ratio.

1. Introduction

Modeling of turbulent flow over complex terrain and isolated hills has been extensively studied over the past years. Experimental and/or numerical simulations were presented by a number of researchers (Counihan 1975, Mason and King 1985, Gong and Ibbetson 1989, Carpenter and Locke 1999, Ishihara, *et al*. 1999, Uchida and Ohya 1999, Kim, *et al*. 2000, Chung and Bienkiewicz 2001, and Pradoto, *et al*. 2001). Most of theoretical studies have been restricted to two-dimensional cases.

Due to an increasing power of computers, numerical simulation has become a useful tool for studying flow past complex terrain. Although significant progress has been accomplished in this area, three-dimensional computation of an unsteady atmospheric flow past complex topography remains a difficult and time-consuming task (Uchida and Ohya 1999).

A balance between the accuracy of the results and time required/costs involved is sought in practical applications of computational simulations. Three methods of modeling turbulence - the standard and modified k-ε models and the k-ω model - have been proven to offer a reasonable compromise for such cases. The standard k-ε model has been the most commonly used. Studies presented by Carpenter and Locke (1999), Kim, *et al*. (2000) and Lee, *et al*. (2002) are recent examples of application of this model in simulations of flow past complex terrain.

The isotropic eddy-viscosity concept employed by the k-ε model leads to modeling inaccuracies in non-isotropic flow regions, such as those associated with flow impingement and separation. To

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remedy this constraint, Kato and Launder (1993) proposed modifications of this model. The resulting modified k-ε model imposes reduction in the production of turbulence kinetic energy k in the flow impingement and separation regions. The performance of this model in simulation of flow past two-dimensional topography is tested in the paper. The results of calculations employing the standard and modified k-ε models are compared with experimental data. They are also compared with simulations resulting from the use of the k-ω model proposed by Wilcox (1993).

First, the three models and their implementation in an in-house developed computer code are presented. Next, a companion experimental study is briefly described. Subsequently, representative results of computations of turbulent flow past a two-dimensional hill are presented and compared with the experimental data. Profiles of the mean velocity and turbulence intensity are discussed. The numerical and experimental wind speed-up ratios on the hilltop are compared with provisions of the ASCE7-98 Standard, ASCE (1998). Finally, the results of numerical simulation of flow past a two-dimensional valley – the mean velocity, kinetic energy of turbulence and the Reynolds shear stress – are presented and compared with the experimental data published by Khurshudyan, *et al*. (1990).

2. Numerical model

2.1. Standard and modified k-ε model

The standard k-ε model (STKE) is a semi-empirical turbulence model based on transport equations for the turbulence kinetic energy k and its dissipation rate ε. The model transport equation for k was derived from the exact equation for turbulence kinetic energy, while the transport equation for ε was obtained using physical reasoning. The above variables (k and ε) are used to calculate the eddy viscosity $\nu_t = C_\mu k^2/\varepsilon$, where C_μ is an empirical constant. The transport equations of the STKE are as follows:

$$U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \varepsilon \tag{1}$$

$$U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon} P_k - C_{2\varepsilon}\frac{\varepsilon^2}{k} \tag{2}$$

In these equations, $P_k = -\overline{u_i u_j}(\partial U_i / \partial x_j)$ is the rate of production of kinetic energy of turbulence, resulting from the interaction of turbulent stresses and the mean velocity gradient. Due to the isotropic eddy-viscosity concept imbedded in the STKE, an over-prediction of turbulence kinetic energy is produced in flow impingement regions. Kato and Launder (1993) proposed modification of the formula prescribing the production term P_k. In the resulting turbulence model, denoted hereafter KLKE, the original production term is modified and it reads $P_k = \nu_t \varepsilon S \Omega$, where the dimensionless strain and vorticity parameters are respectively defined as follows:

$$S = k/\varepsilon\left(\sqrt{1/2(\partial U_i/\partial x_j + \partial U_j/\partial x_i)}\right);\ \ \Omega = k/\varepsilon\left(\sqrt{1/2(\partial U_i/\partial x_j - \partial U_j/\partial x_i)}\right) \tag{3}$$

This modification leads to a marked reduction in turbulence kinetic energy in flow stagnation regions. It also causes mathematical inconsistency between terms expressing the energy transport from the mean flow to turbulence, in the transport equations for the kinetic energy of the mean flow and turbulence.

Eqs. (1) and (2) involve a number of parameters characterizing turbulent flow. In these equations, σ_k and σ_ε are respectively the turbulent Prandtl numbers for k and ε, ν is molecular kinetic viscosity, ν_t is eddy viscosity, while $C_{1\varepsilon}$ and $C_{2\varepsilon}$ are empirical constants. The commonly used values of the above parameters (C_μ=0.09, σ_k=1.0, σ_ε=1.3, $C_{1\varepsilon}$=1.44, and $C_{2\varepsilon}$=1.92) were assumed in simulations presented herein.

2.2. k-ω model

The core of the k-ω model (KW) are the transport equations for k and ω. The equation for k is a modified transport equation employed by the k-ε model, Eq. (1). The transport equation for ω is similar to that for ε, Eq. (2). The transport equations of the k-ω model are as follows:

$$U_j\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\nu+\sigma^*\nu_t)\frac{\partial k}{\partial x_j}\right] + \left(-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}\right) - \beta^* k\omega \tag{4}$$

$$U_j\frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\nu+\sigma\nu_t)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}\left(-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}\right) - \beta\omega^2 \tag{5}$$

where $\nu_t = k/\omega$ is the eddy viscosity. It follows from the definition of ν_t that ω represents the specific dissipation rate, i.e., the turbulence dissipation per unit turbulent kinetic energy. The empirical constants proposed by Wilcox (1993) - σ^*=1/2, β^*=0.09, σ=0.5, α=5/9, and β=3/40 - were employed in simulations presented herein.

2.3. Grid system, geometry and computational domain

Selection of an appropriate grid system to discretize a computational domain is one of the most important factors affecting the success of the numerical simulation of flow past terrain of complex topography. In this study, body-fitted, non-orthogonal grids were employed. Such grid systems can be adapted to any geometry and they lead to faster convergence of calculations than those associated with orthogonal curvilinear grids (Ferziger and Peric 1996). A staggered grid

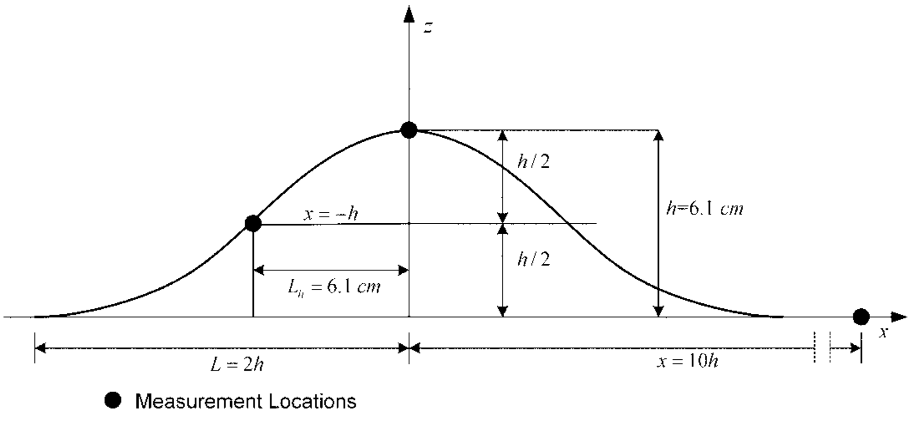


Fig. 1 Two-dimensional 1:2 hill used in experimental and numerical simulations

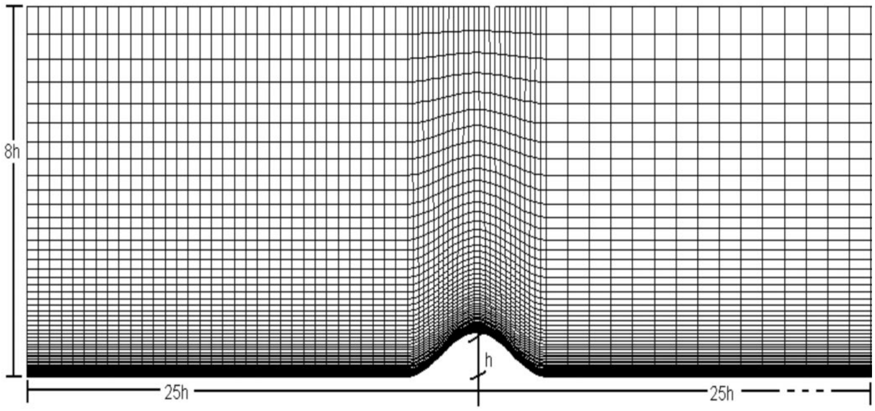


Fig. 2 Computational domain used in numerical simulation of flow past hill

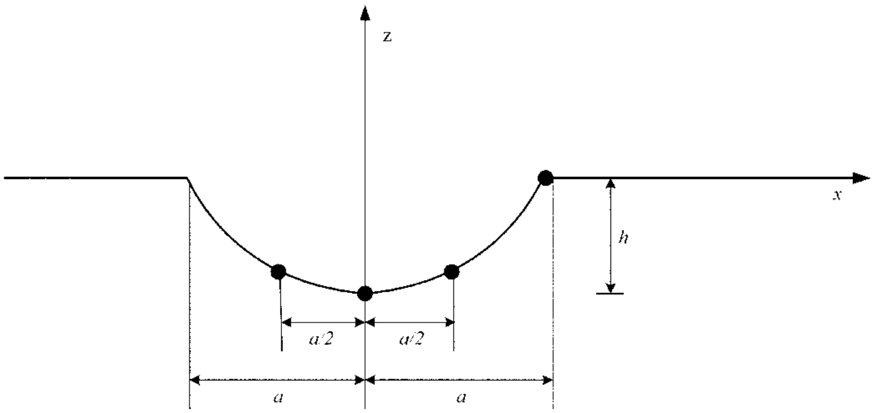


Fig. 3 Two-dimensional valley used in numerical simulation

arrangement (Harlow and Welsh 1965) was employed to discretize the flow velocity and pressure. Near the surface of the modeled terrain, the grid resolution was higher than that away from the flow-terrain interface.

Fig. 1 shows schematically the geometry of an isolated two-dimensional hill considered in numerical and experimental simulations. Locations where measurements of the flow were carried out are indicated in the figure as solid circles. The hill shape was sinusoidal, with the hill aspect ratio of 1:2. The aspect ratio is defined as the ratio of the hill height h to the hill width $2L_h$, where L_h is defined at the elevation of the hill half-height $h/2$. The grid of computational domain used in calculation of flow past the hill is shown in Fig. 2. It had a horizontal span of 50h and a height of 8h. The first grid point (closest to the hill surface) was located at a distance of 0.00164h from the hill surface. The grid was expanded in a vertical direction, with a uniform expansion ratio of 1.1:1. The numerical calculations were carried out for a mesh size of 178×60.

Fig. 3 shows the geometry and flow measurement positions employed for an isolated two-dimensional valley. The aspect ratio of the valley was 1:3. The aspect ratio is defined as the ratio of the valley depth h to the half-width a.

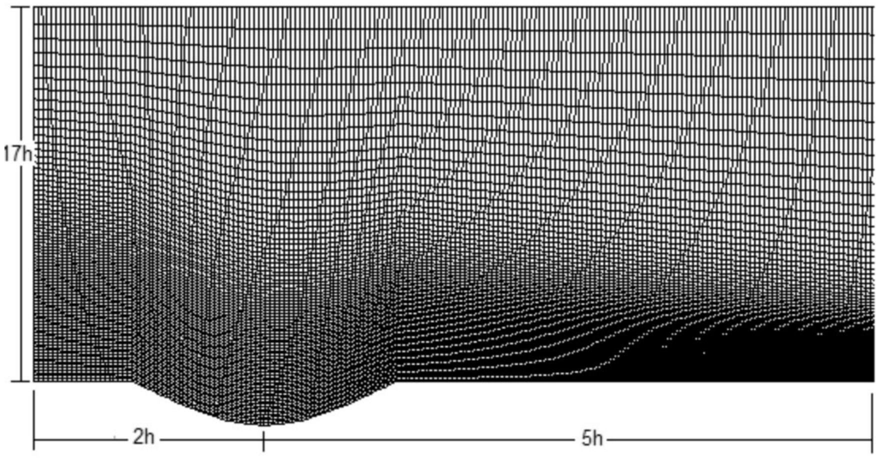


Fig. 4 Computational domain used in simulation of flow past valley

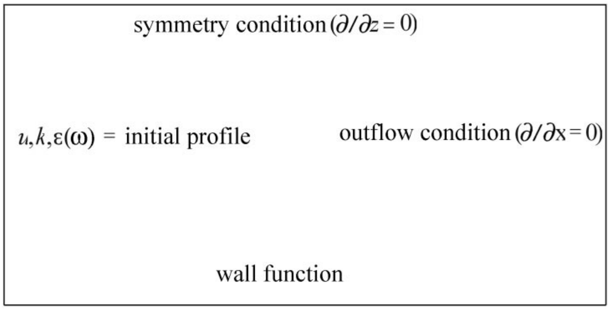


Fig. 5 Boundary conditions used in numerical simulation

Fig. 4 shows the grid of computational domain employed in this case. The domain spanned from $2h$ upwind of the center of the valley through $5h$ downwind. Its height was $17h$. The first grid point was located at a distance of $0.00855h$ from the surface of the valley. The mesh size was 184×50.

2.4. Numerical approach and boundary conditions

The governing equations were discretized using the finite volume method. The velocity and pressure coupling was implemented using the SIMPLE algorithm (Patankar 1980). Fig. 5 shows the boundary conditions employed in computations. The symmetry condition, i.e., the flow velocity normal to the plane and the normal gradient of the remaining velocity components set to zero, was enforced on the top of the domain. At the outflow boundary, zero-gradient conditions were imposed for all variables. The wall function approach was applied at the bottom boundary.

The mean velocity and turbulence intensity profiles obtained in an experimental study at Colorado State University were used as inflow conditions in simulation of flow past the hill. In the two-dimensional valley calculations, the computational inflow profiles were the same as those employed by Maurizi (2000). The mean velocity profile was logarithmic, with the friction velocity u_*=0.19 m/s

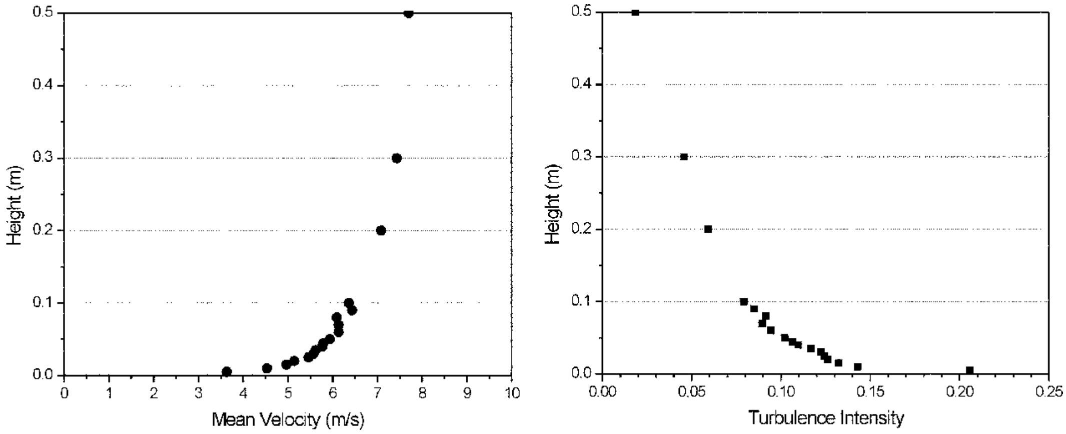


Fig. 6 Mean velocity and turbulence intensity profiles

and the roughness length $z_0 = 1.6\times10^{-4}$ m. The free stream velocity was $U_\infty = 4$ m/s. The remaining boundary conditions were the same as those implemented for flow past the hill.

3. Experimental study

An open circuit boundary-layer wind tunnel (the Environmental Wind Tunnel) at the Wind Engineering and Fluid Laboratory, Colorado State University, was used to model flow past the two-dimensional isolated hill. The approach flow profiles used in the experimental study and employed as inflow boundary conditions in numerical simulations are shown in Fig. 6. A power law with the exponent of 0.136 was found to be a good empirical fit for the mean wind profile, with the reference velocity of 6.36 m/s at the elevation of 0.1 m above the wind tunnel floor.

4. Results and discussion

4.1. Flow past hill

The experimental and numerical profiles of the mean velocity and turbulence intensity are shown in Figs. 7 and 8, respectively. The data is presented for the following locations - the foot of the hill ($x=-h$), the hilltop ($x=0$) and downstream of the hill ($x=10h$) - and the three discussed turbulence models: the standard and modified k-ε (STKE and KLKE, respectively) models and the k-ω (KW) model.

Fig. 7 shows that the mean velocity profiles obtained using the STKE and KLKE models are in an overall good agreement with the experimental data. The agreement for the results of simulations employing the KW model is good upstream of the hill. However, it deteriorates at low elevations on the hilltop and downstream of the hill.

The numerical and experimental profiles of the turbulence intensity are depicted in Fig. 8. They are in a worse agreement with the experimental data than the profiles of the mean velocity in Fig. 7. It can be seen that, at the low-elevation at the foot of the hill ($x=-h$, Fig. 8a), the best agreement is exhibited by the STKE and KW models. On the hilltop ($x=0$, Fig. 8b), the KLKE model appears to be in the best agreement with the experimental data. Downstream of the hill ($x=10h$, Fig. 8c), the

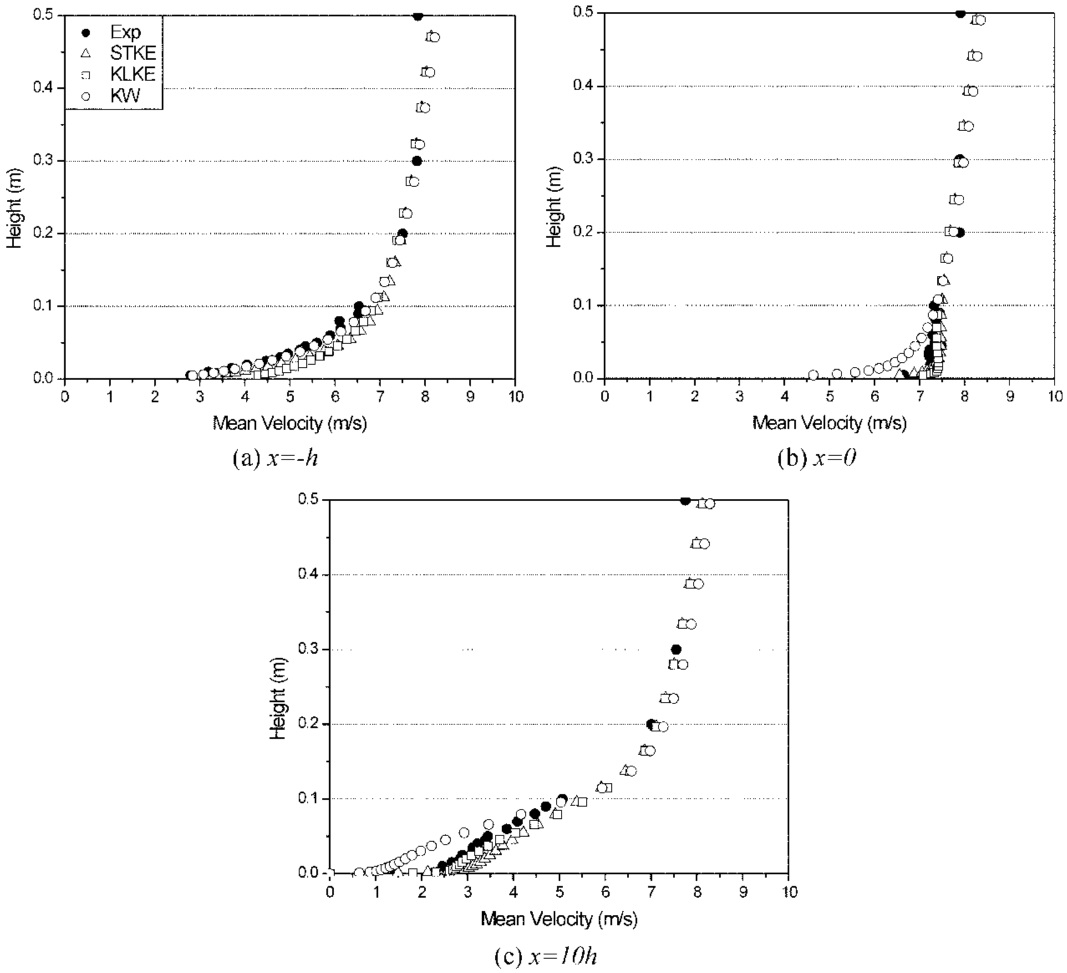


Fig. 7 Numerical and experimental mean velocity, 2D hill

value of the low-elevation experimental turbulence intensity falls between the results of simulations employing the STKE and KW models.

The hilltop speed-up ratio $S(z)=\bar{u}\,(x=0,\ z)/\bar{u}\,(x=\text{-}25h, z)$, where $z=$elevation above the terrain, $\bar{u}=$mean velocity, is depicted in Fig. 9. The numerical and experimental data are compared with provisions specified in the ASCE 7-98 Standard, ASCE (1998). A very good agreement between the provisions, the experimental data and the results of numerical simulation employing the STKE and KLKE models is seen at elevations $z \le h$. At the elevation of 0.2*h*, the value of the speed-up ratio was 1.49, 1.44, and 1.24, obtained using the STKE, STKL and KW models, respectively. The corresponding values of the speed-up ratio calculated using the experimental data and the ASCE 7-98 Standard were 1.55 and 1.54, respectively.

The above values are in a reasonable agreement with the experimental speed-up ratio of 1.5, reported by Ishihara, *et al.* (1999) at the elevation of 0.125*h* and with value of 1.6 obtained at the elevation of 0.13*h* by Mason and King (1985). The above and other (reference) values of the velocity speed-up ratio are compared in Table 1.

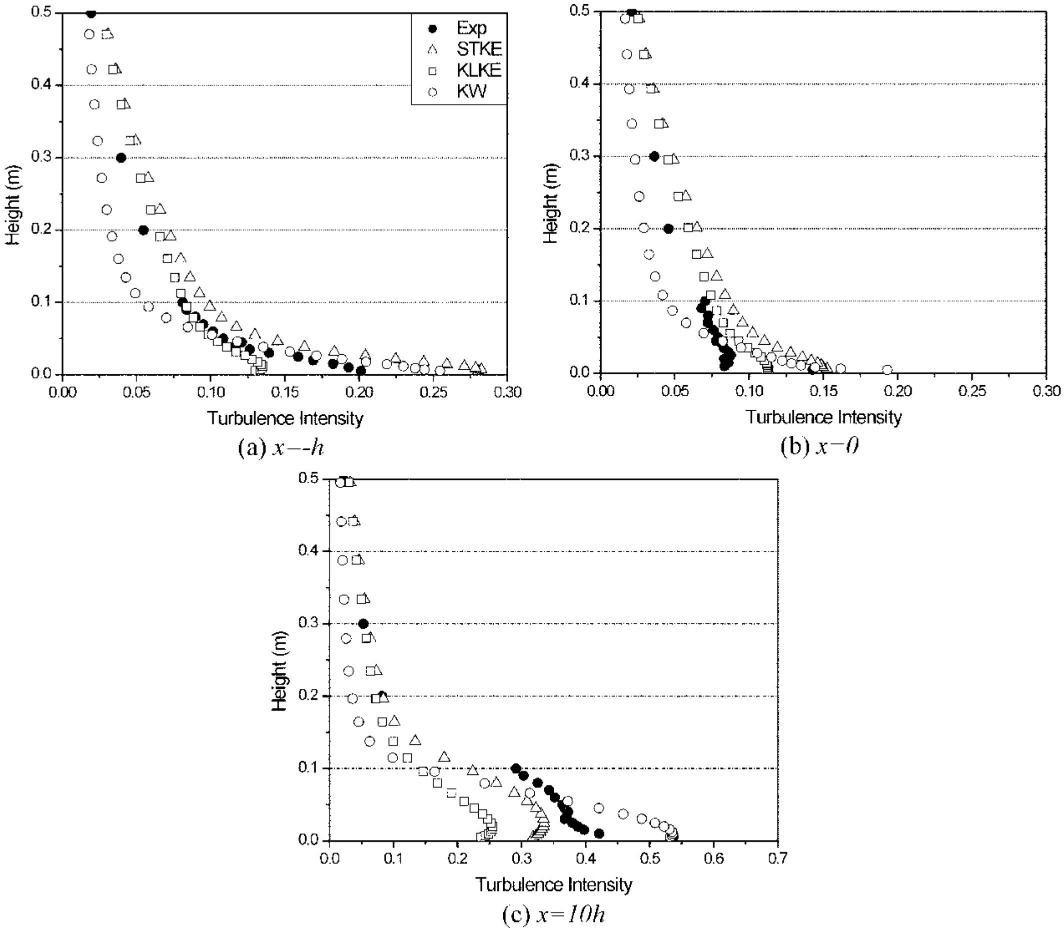


Fig. 8 Numerical and experimental turbulence intensity, 2D hill

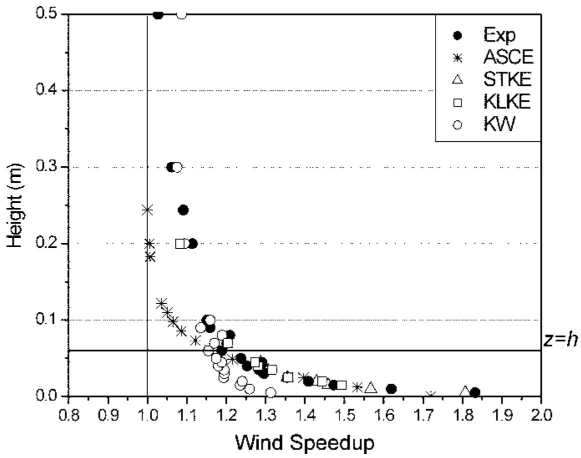


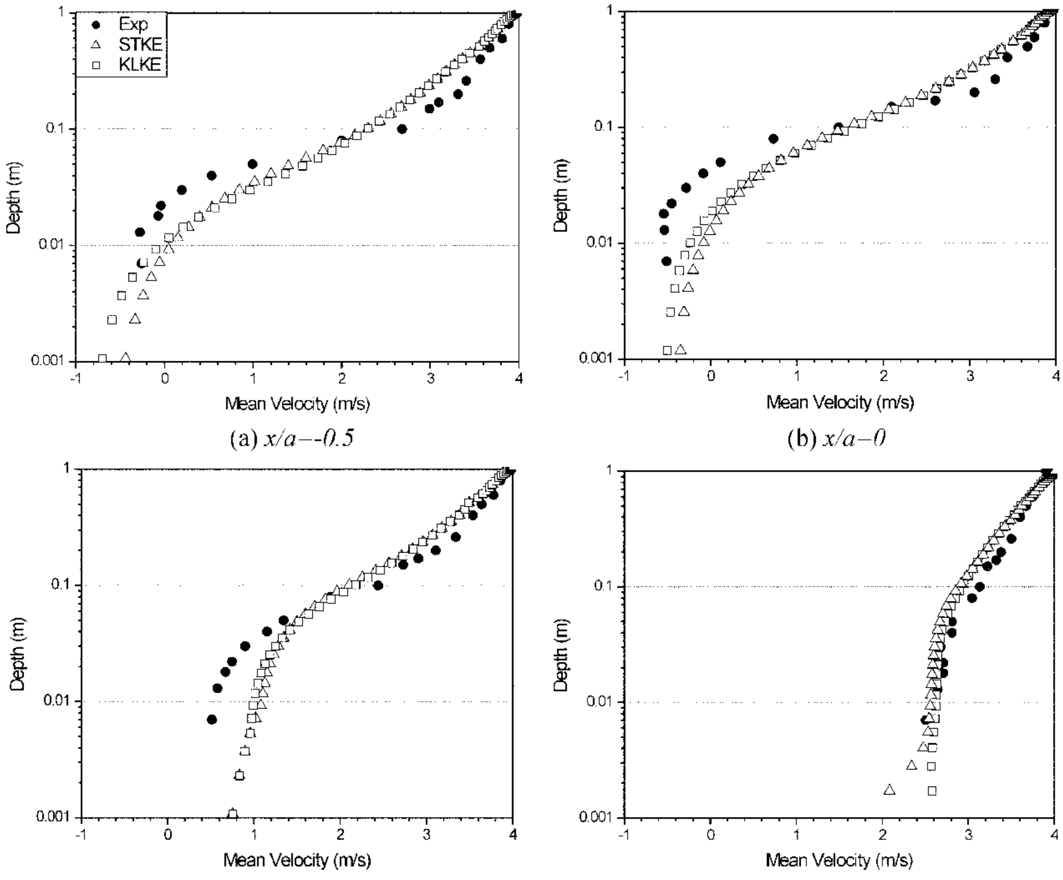
Fig. 9 Comparison of wind speed-up ratio, 2D hill

Table 1 Comparison of wind speed-up ratio

	Wind speed-up ratio	Elevation above hilltop	Hill aspect ratio
STKE model (2D)	1.49	0.20*h*	1:2
KLKE model (2D)	1.44	0.20*h*	1:2
KW model (2D)	1.24	0.20*h*	1:2
Exp. data (2D)	1.55	0.20*h*	1:2
ASCE 7-98 (2D)	1.54	0.20*h*	1:2
Ishihara, *et al*. (3D)	1.50	0.125*h*	1:2.5
Mason and King (3D)	1.60	0.13*h*	1:3

4.2. Flow past valley

Numerical and experimental results for flow past the valley are presented in Figs. 10 through 12. Fig. 10 shows the mean velocity profiles at four measurement locations (x/a = -0.5, x/a = 0, x/a = 0.5,



(a) x/a=-0.5 (b) x/a=0

(c) x/a=0.5 (d) x/a=1.0

Fig. 10 Numerical and experimental mean velocity, 2D valley

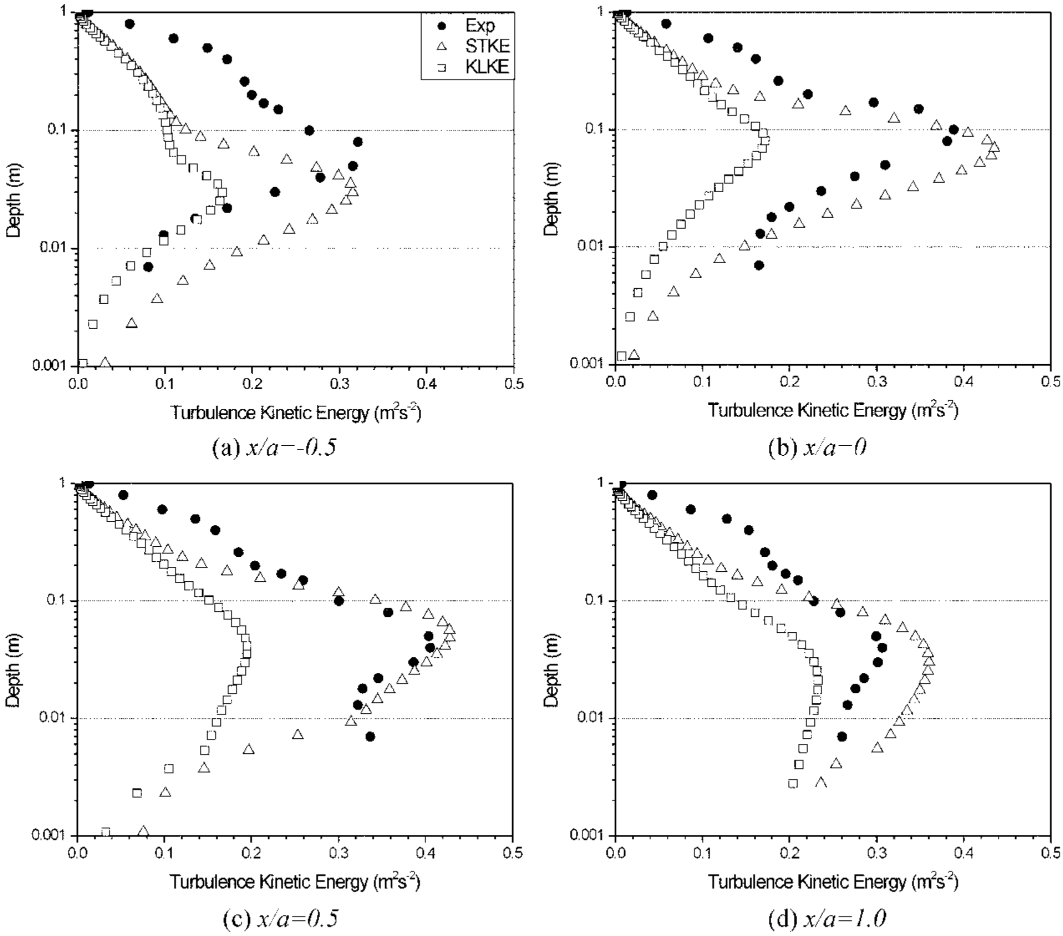


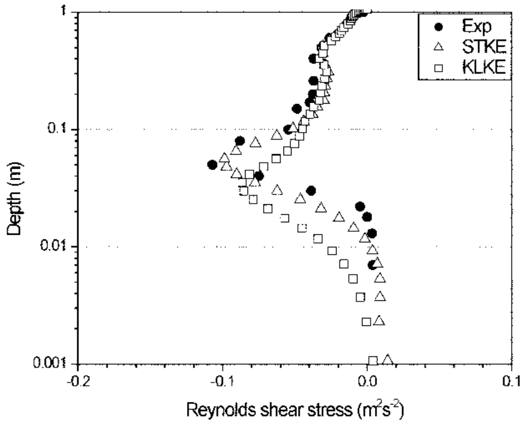
Fig. 11 Numerical and experimental kinetic energy of turbulence, 2D valley

and x/a=1.0). The numerical results obtained using the STKE and KLKE turbulence models are compared with the experimental results published by Khurshudyan, *et al.* (1990).

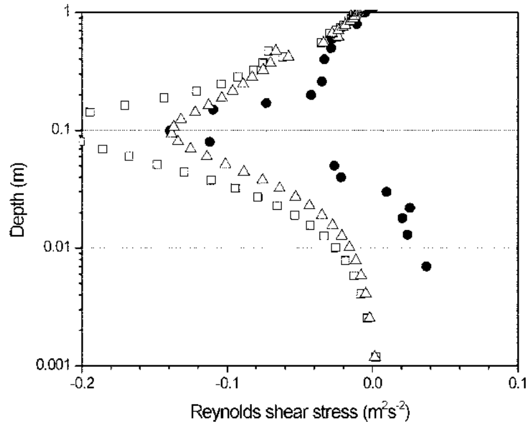
It can be seen in Fig. 10 that the mean velocity profile of the KLKE model is a slightly better agreement with the experimental data. At the downstream edge of valley (x/a=1.0, Fig. 10d), the mean velocity profiles obtained using both the turbulence models are in a very good agreement with the experimental data.

Fig. 11 compares the profiles of the turbulence kinetic energy. Overall, the energy predicted by the KLKE model is substantially lower than the experimental data and the results of simulations employing the STKE model. The STKE simulations are in an overall good agreement with the experimental data, in particular at x/a=0 and x/a=0.5. The lower level of kinetic energy exhibited by the KLKE model can be attributed to the attenuation of the turbulence production term implied by this model.

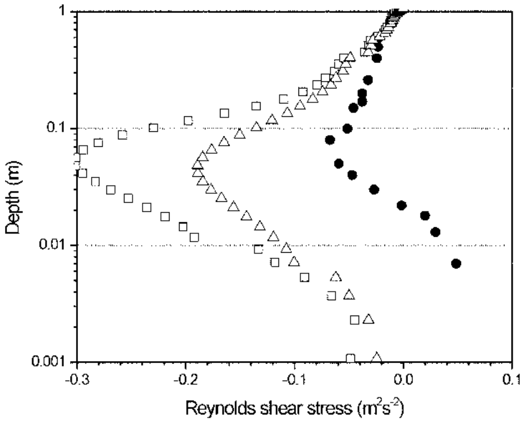
A comparison of the Reynolds shear stress is presented in Fig. 12. It can be seen that at x/a=-0.5 (Fig. 12a) the results of the STKE model are in a very good agreement with the experimental data. Large discrepancies between the compared data occur at x/a=0 (Fig. 12b) and x/a=0.5 (Fig. 12c). At the downstream edge of the valley (Fig. 12d), the disagreement between the numerical and



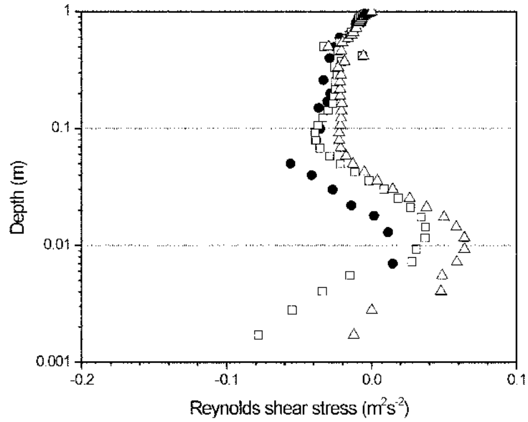
(a) x/a=-0.5



(b) x/a=0



(c) x/a=0.5



(d) x/a=1.0

Fig. 12 Numerical and experimental Reynolds shear stress, 2D valley

experimental data is moderate. For this case, the best agreement is exhibited by the KLKE model.

5. Conclusions

Findings of this study are summarized as follows:

(1) The mean velocity profiles of flow past the hill, obtained using the standard k-ε and modified k-ε models, were in overall good agreement with the experimental data.

(2) The numerical profiles of the turbulence intensity of the flow past the hill were in a worse agreement with the experimental data than the mean velocity profiles. The best agreement was exhibited by the standard k-ε and k-ω models.

(3) The speed-up ratio of the flow past the hill obtained using the standard and modified k-ε models was in a very good agreement with the experimental data and the speed-up ratio implied by the wind provisions of the ASCE 7-98 Standard. For elevations $z>h$, the ASCE7-98 values of the speed-up ratio exhibited disagreement with the numerical and experimental data.

(4) For flow past the valley, the mean velocity profiles resulting from the standard k-ε and

modified k-ε models were in a good agreement with the experimental data, except for locations near the surface of the valley.

(5) The closest agreement between the numerical and experimental profiles of the turbulence kinetic energy and the Reynolds shear stress was obtained using the standard k-ε model.

The above findings indicate that the standard k-ε model appears to be the most suitable turbulence model (out of the three tested models) for numerical simulations of flow past two-dimensional isolated hills of a moderate height and valleys of a moderate depth. Application of this model leads to discrepancy between the numerical and experimental results. The magnitude of this error depends on location relative to the tested hill or valley. Further investigation of the effectiveness of this model in modeling of flow past isolated hills and valleys is desired.

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