Wind and Structures, Vol. 6, No. 6 (2003) 437-450
DOI: http://dx.doi.org/10.12989/was.2003.6.6.437

The random structural response due to a turbulent boundary layer excitation

S. De Rosa† and F. Franco‡

Department of Aeronautical Engineering, Università degli Studi di Napoli "Federico II", Naples, Italy

G. Romano‡† and F. Scaramuzzino‡‡

Department of Aerospace and Mechanical Engineering, Seconda Università degli Studi di Napoli, Aversa (CE), Italy

(Received October 18, 2002, Accepted October 15, 2003)

Abstract. In this paper, the structural random response due to the turbulent boundary layer excitation is investigated. Using the mode shapes and natural frequencies of an undamped structural operator, a fully analytical model has been assembled. The auto and cross-spectral densities of kinematic quantities are so determined through exact analytical expansions. In order to reduce the computational costs associated with the needed number of modes, it has been tested an innovative methodology based on a scaling procedure. In fact, by using a reduced spatial domain and defining accordingly an augmented artificial damping, it is possible to get the same energy response with reduced computational costs. The item to be checked was the power spectral density of the displacement response for a flexural simply supported beam; the very simple structure was selected just to highlight the main characteristics of the technique. In principle, it can be applied successfully to any quantity derived from the modal operators. The criterion and the rule of scaling the domain are also presented, investigated and discussed. The obtained results are encouraging and they allow thinking successfully to the definition of procedure that could represent a bridge between modal and energy methods.

Keywords: stuctural random response; turbulent boundary layer; modal methods; energy methods.

1. Introduction

In the recent years, internal cabin noise has been one the major concerns of the public transport industry. The problem became more evident when the cruise speed has been increased not only for the aircraft, but also for ships and trains. Furthermore, the environmental problems enforced the scientific community in better analysing the overall problem. All the structural and aerodynamic parameters which strongly affect the emitted and the in cabin transmitted noise would be included in the design phase: this is not completely possible at this stage of the research application. The

† Associate Professor
‡ Assistant Professor
‡† Graduate Student
‡‡ Full Professor

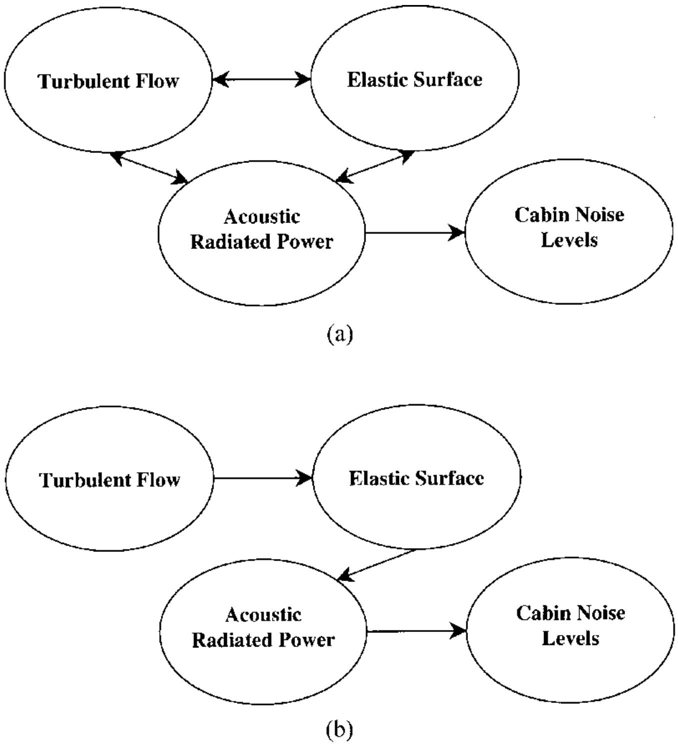

Fig. 1 The aero-acousto-elastic problem (a) complete approach, (b) simplified approach

aerodynamic noise sources have been specifically and carefully treated in the aerospace field, but the cited increase of speed has evidenced the problem also in other transports. Among these sources, the most important is the turbulent boundary layer (TBL).

The TBL is one of the common features of the vehicles, moving in air, water, on rail and ground. It is a source of external pollution and further, it influences sensibly the internal noise levels. The main characteristic of the TBL is such that it produces on the wetted surface a random pressure distribution. Some models are available in terms of stochastic distribution, but the predictive problem is complicated since the needed degree of accuracy can be computer time consuming, and further experimental measurements can be really complicated.

The present work deals with the problem of the predictive response of a simplified structural operator when loaded with a one dimensional TBL pressure distribution. Standard and innovative modal expansions are presented and discussed. Particular attention will be devoted to a scaling procedure that will greatly reduce the computational costs associated with the modal technique. The procedure herein developed contains all the relevant parameter for ensuring the largest possible generalisation. The present predictive problem is clearly multidisciplinary: it involves the fluid dynamics, the structural dynamics and the acoustic propagation. Fig. 1 presents two views of the problems. In the first, all the possible couplings are considered: it is a full *aeroacoustoelastic* problem; in the second, the standard simplifications based on both experimental and theoretical evidences are used: the TBL is frozen, the structural response is evaluated “in vacuum”, and the acoustic propagation does not influence both the structural response and the random pressure distributions. Some remarks will be given about the suitability of this simplified approach and some minor modifications to the procedure for taking into account some topics of the more general problems. The results herein presented are encouraging, and even if energy methods will always give the same quality of the predictive responses at lowest computation costs, the innovative modal expansion could be well considered as a hybrid method between standard modal and energy approaches.

2. Preliminary remarks

The excitation field generated by the TBL is random and convective.

The randomness is associated with the pressure fluctuations, which can be only represented by using stochastic representation. The convective character is associated with the undisturbed flow speed: its increase leads to an increase of the absolute levels of excitation and to a different frequency spectrum. Models for the random pressure fields as simple as possible are necessary for the quantitative response of the vibration field due to the TBL. Leaving the details of the theoretical and experimental problems to the specific references (Bull 1996), it has to be here recalled the commonly used model for the random load distribution, (Corcos 1963, 1967).

For a surface belonging to an *xz*-plane, *x* is the stream wise axis and *z* is the cross-stream wise one, Corcos assumes this function for the (auto and cross) pressure spectral density between two points $P(x_1, z_1)$ and $Q(x_2, z_2)$:

$$S_q(x_1, x_2, z_1, z_2, \omega) = \Phi_q(\omega)\exp\left(-\alpha_x \frac{\omega|\xi_x|}{U_c}\right)\exp\left(-\alpha_z \frac{\omega|\xi_z|}{U_c}\right)\exp\left(\frac{i\omega\xi_x}{U_c}\right) \tag{1}$$

where $\xi_x = x_2 - x_1$; $\xi_z = z_2 - z_1$; i is the imaginary unit; ω is the radian excitation frequency; α_x and α_z

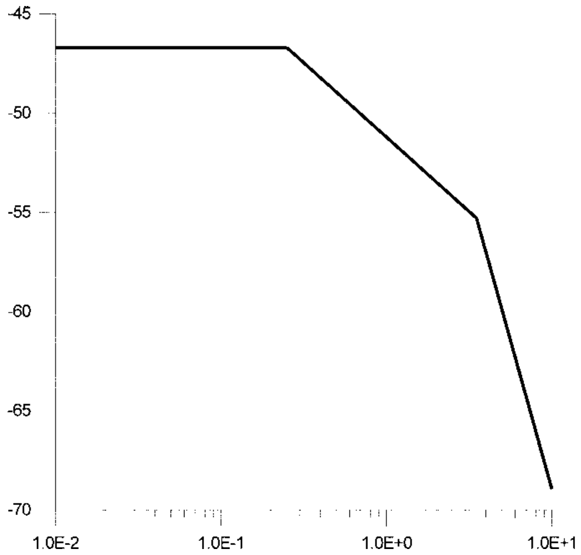

Fig. 2 The nondimensional Corcos spectrum $\{\Phi_q(\omega)U_\infty / q^2\delta^*\}$ vs. nondimensional freq. $\{\omega\delta^*/U_\infty\}$ ($q=1/2\rho U_\infty^2$ is the dynamic pressure and δ^* is the displacement thickness)

are two constants based on experimental measurements fits: they represent a metric for the correlation lengths. $\Phi_p(\omega)$ denotes the power spectral density of the wall pressure fluctuations and it is normally expressed by using nondimensional groups, Fig. 2, as result of the Fourier transform of the Eq. (1), (Cousin 1999).

$\Phi_p(\omega)$ is the member of the function that gives the units to the whole group: it is a pressure spectral density. U_c denotes the convective speed: the whole TBL can be thought as a rigid body moving at such speed, the flow characteristics are invariant for an observer moving at U_c; it is often assumed simply that $U_c = KU_\infty$ ($K = 0.6 \div 0.85$), where U_∞ is the undisturbed flow speed.

3. Beam response

The random response of a 1D flexural beam operator represented by a modal expansion can be given in terms of the cross spectral density function of the displacement for a 1D operator, $S_w(x_1, x_2, \omega)$, as follows, (Elishakoff 1983):

$$S_w(x_1, x_2, \omega) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left\{\begin{array}{l} H_j^*(\omega)H_k(\omega)\psi_j(x_1)\psi_k(x_2)\nu_j^{-2}\nu_k^{-2}. \\ \cdot\int_0^L\int_0^L S_q(x_1, x_2, \omega)\psi_j(x_1)\psi_k(x_2)dx_1dx_2 \end{array}\right\} \quad (2)$$

where the modal expansion of the distributed random load, $S_{q_jq_k}(\omega)$, is evident:

$$S_{q_jq_k}(\omega) = \nu_j^{-2}\nu_k^{-2}\int\int_{[0,L]^2} S_q(x_1, x_2, \omega)\psi_j(x_1)\psi_k(x_2)dx_1dx_2 \quad (3)$$

with

$$H_j(\omega) = L_j(\omega)^{-1}; L_j(\omega) = \{(\omega_j^2 - \omega^2) + i\eta\omega_j^2\}\rho A \quad (4)$$

$$\psi_j(x) = \sin\left(\frac{j\pi x}{L}\right); \quad \omega_j^2 = \frac{EI}{\rho A}\left(\frac{j\pi}{L}\right)^4 \quad (5)$$

$$\nu_j^2 = \int_0^L \psi_j^2(x)dx = \frac{L}{2} \quad (6)$$

The symbols denote respectively: x (or x_1, or x_2) is the beam abscissa; L is the beam length; A is the beam section area; E is the Young module; ρ is the material density; ω_j is the j-th natural radian frequency; ω is the excitation radian frequency; η is the structural damping; j is the mode shape index; ψ is the j-th mode. In the Eq. (4), for sake of simplicity, a constant value of the structural damping has been used. It has to be further noted, that Eq. (2) can be applied to any structural operator, when the proper modal base is provided. Commonly, the random response is defined through the acceptance function:

$$A_{q_j q_k}(\omega) = \int_0^L\int_0^L \frac{S_q(x_1, x_2, \omega)}{\Phi_q(\omega)L^2}\psi_j(x_1)\psi_k(x_2)dx_1 dx_2 \tag{7}$$

where $\Phi_q(\omega)=S_q(0,0,\omega)$ is the autopower spectral density of the load, $\xi_x=0$.

The acceptance function is named as joint acceptance, if $j=k$, or cross acceptance otherwise. Further, it has to be noted that by using Corcos-like model, $A_{q_j q_k}(\omega) = A^*_{q_k q_j}(\omega)$.

In short form, the 1D response (cross spectral density or cross spectrum) is given by:

$$S_w(x_1, x_2, \omega) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{\psi_j(x_1)\psi_k(x_2)}{L_j^*(\omega)L_k(\omega)}\right)\left(\frac{\Phi_q(\omega)L^2}{\nu_j^2\nu_k^2}\right)A_{q_j q_k}(\omega) \tag{8}$$

The autopower spectral density, $x=x_1=x_2$, is given by:

$$S_w(x, \omega) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{\psi_j(x)\psi_k(x)}{L_j^*(\omega)L_k(\omega)}\right)\left(\frac{\Phi_q(\omega)L^2}{\nu_j^2\nu_k^2}\right)A_{q_j q_k}(\omega) \tag{9}$$

Often, it is preferred to express the last equation response in terms of two separate summations, the first containing the joint and the second the cross-acceptances:

$$S_w(x, \omega) = \sum_{j=1}^{\infty}\frac{\psi_j^2(x)\Phi_q(\omega)L^2}{|L_j(\omega)|^2\nu_j^4}A_{q_j q_j}(\omega) + \sum_{j=1}^{\infty}\sum_{\substack{k=1\\k\neq j}}^{\infty}\left(\frac{\psi_j(x)\psi_k(x)}{L_j^*(\omega)L_k(\omega)}\right)\left(\frac{\Phi_q(\omega)L^2}{\nu_j^2\nu_k^2}\right)A_{q_j q_k}(\omega) \tag{10}$$

It is simple to demonstrate that the Eq. (10) always returns a real value, as expected.

By using the Corcos model, the acceptance becomes:

$$A_{q_j q_k}(\omega) = \frac{1}{L^2}\int_0^L\int_0^L \exp\left(-\alpha_k\left|\frac{\omega(x_2-x_1)}{U_c}\right|\right)\exp\left[i\frac{(\omega(x_2-x_1))}{U_c}\right]\psi_j(x_1)\psi_k(x_2)dx_1 dx_2 \tag{11}$$

In order to use the beam model, the cross-wise variation of the load has been neglected.

The numerical response in term of the auto power spectral density could be computational expensive. It could be accepted to get a simplified response by looking for an average of the displacement, for example, for examining the behaviour of the structural operator. This is also computationally efficient since for increasing values of the modal overall factor ($m>1$), this mean value is able to evidence the overall behaviour.

It is needed here to recall the definition of the modal overlap factor, m:

$$m(f)=n(f)\eta(f)f \tag{12}$$

where f is the excitation frequency; $n(f)$ is the modal density and $\eta(f)$ is the damping loss factor. The predictive modal methods work well for $m \ll 1$, while the predictive energy methods are useful for $m \gg 1$, where the mean value of the response is a good representation of the system. Here, the mean response is defined as an average taken over the spatial co-ordinates.

For the well-known property of orthogonality of the mode shapes, the predictive modal expansion for the mean response has a simple expression. In fact, having defined the displacement mean response as follows,

$$\bar{S}_w(\omega) = \frac{1}{L}\int_0^L S_w(x, \omega)dx \tag{13}$$

one has:

$$\bar{S}_w(\omega) = \frac{1}{L}\sum_{j=1}^{\infty}\frac{\Phi_q(\omega)L^2}{|L_j(\omega)|^2 \nu_j^2}A_{q_jq_j}(\omega) = 2\sum_{j=1}^{\infty}\frac{\Phi_q(\omega)}{|L_j(\omega)|^2}A_{q_jq_j}(\omega) \tag{14}$$

The double summation of Eq. (10) including the evaluation of the cross acceptance disappears. It has to be noted that the mean response Eq. (14) can be obtained by using only the 5% of the computational time used for the local response Eq. (10).

A FORTRAN code was written for analysing the random beam response. It has to be noted that for the beam (and plate) random response with a TBL model with separable variable, analytical solutions for the acceptance are available (Elishakoff 1983, Skudrzyk 1968); in order to generalise the possibility of the predictive code, the acceptance was solved numerically, through standard quadrature routine. The number of modes to be included in the summations was selected automatically by doubling the frequency range to be analysed that is considering all those contained in a frequency range twice the selected one.

All the results are presented in a nondimensional form through a specific response metric, S:

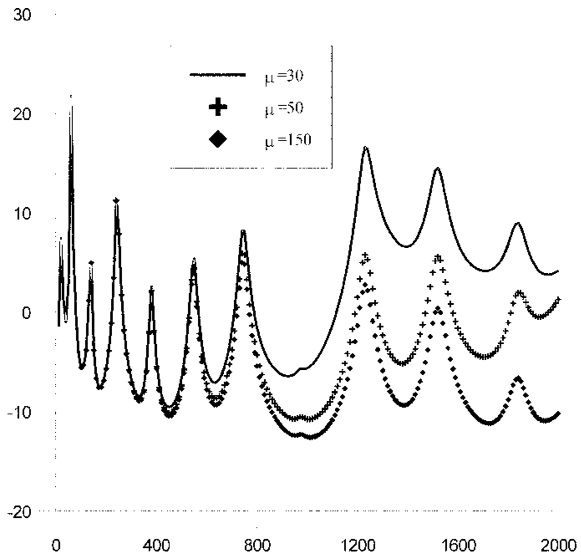

Fig. 3 Metric S - local response [dB] vs. freq. [Hz] (U=115 m./s.; x/L=0.373, η=0.04, Δf=10 Hz)

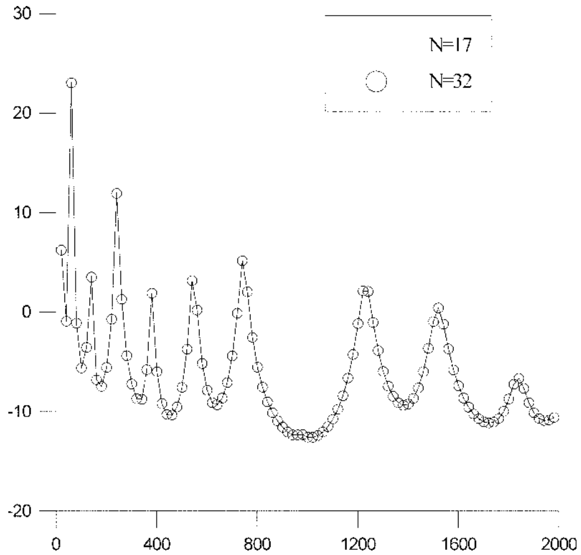

Fig. 4 Metric S - local response [dB] vs. freq. [Hz] (U=115 m./s.; x/L=0.373, η =0.04, Δf=10 Hz)

$$S = 10\log_{10}\left[B\frac{S_W(x, \omega)}{\Phi(0, 0, \omega)}\right] \text{ [dB]} \tag{15}$$

with $B=I\rho^2\omega^4$.

This is the list of parameters defining the numerical test: E=7.10 10^{10} Nm^{-2}; I=3.33 10^{-8} m^4; A= 10^{-3} m^2 ; K=0.8; L=1.75 m. ; α_x=0.116.

Fig. 3 and Fig. 4 present the influence of some parameters for the local response.

In particular, Fig. 3 presents a convergence analysis with a quadrature index named μ: it is a measure of the integration steps; Fig. 4 shows the convergence with the number of modes. They both refer to local response.

4. Scaling procedure and results

Leaving the specific details to the references, (De Rosa 1997, 2003, Franco 1997), here the fundamental topic of the scaling procedure are briefly recalled.

All the parameters not involved in the energy transmission are multiplied by a scaling coefficient, α. In the present problem, the scaled length will be: $L_S=\alpha L$. All the mode shapes and natural frequency will move to higher frequency, for $\alpha<1$; for obtaining the same energy content, the original damping has to be accordingly modified: $\eta_S=\eta/\alpha$. This simple transformation generates a scaled system in which the energy representation (mean response) will be the same as the original system.

It is difficult to find a rigorous mathematical demonstration of the applicability of the scaling procedure. It can be explained in elegant and simple way by using the statistical energy analysis, the more diffused technique for vibration and noise prediction at high values of the modal overlap factors, that is at increasing excitation frequency (Lyon 1995).

For sake of precision and completeness, some details are herein given. For a generic beam, excited by a mechanical force, the mean square velocity can be obtained as follows:

$$\hat{v}_I^2(\omega) = \frac{|F_I|^2}{4\rho_I^2 A_I^2 c_B(\omega)\eta_I \omega L_I} \tag{16}$$

The flexural wave speed has been indicated by the symbol c_B, and F denotes the mechanical force. It has been supposed a constant value of structural damping, η. The mean square velocity, $\hat{v}_I^2(\omega)$, is an average value taken over the spatial positions of the response and the excitation. A second beam is now considered: it has different length and damping; all the remaining parameters are left unaltered:

$$\hat{v}_{II}^2(\omega) = \frac{|F_I|^2}{4\rho_I^2 A_I^2 c_B(\omega)\eta_{II} \omega L_{II}} \tag{17}$$

It is simple to check that if $L_{II}=\alpha L_I$ then

$$\text{for } \eta_{II} = \frac{\eta_{II}}{\alpha} \Rightarrow \hat{v}_{II}^2(\omega) = \hat{v}_I^2(\omega) \tag{18}$$

The beam of reduced length ($\alpha<1$) will reproduce the same energy content by using an artificial damping obtained by accordingly increasing the original one.

Further considerations are possible. In fact, it is useful to analyse the modal overlap factors for the two flexural beams:

$$m_I(\omega) = \eta_I \omega n_I(\omega) = \frac{\eta_I \omega L_I}{2\pi c_B(\omega)} \tag{19}$$

$$m_{II}(\omega) = \eta_{II}\omega n_{II}(\omega) = \frac{\eta_{II}\omega L_{II}}{2\pi c_B(\omega)} = \frac{\frac{\eta_I}{\alpha}\omega\alpha L_I}{2\pi c_B(\omega)} = m_I(\omega) \tag{20}$$

For the second beam: the modal density is reduced, the damping is augmented so that the modal overlap factor remains unchanged. It is straigthful to consider that the (modal) deterministic simulation of the second beam will cost less than the first one. In fact, the flexural wavelength remains the same, but the domain is reduced: for example, the number of needed finite element points will be reduced.

Any modal formulation could be applied on a reduced spatial domain, keeping the parameters involved in the energy transmission. It has to be well highlighted that the scaled models will be able only to represent the energy content (mean square values, as Eq. (14)) while the local information is completely lost.

The number of needed modes could be also kept or scaled too. It is simple to check that if N are the modes needed for the original response in an assigned frequency band, the needed minimun number for the scaled response is $N_S=N\alpha$.

The scaling coefficient, α, belongs to the range [0,1]: for $\alpha=1$ the scaled system equals the original. Clearly, inferior limits exist and the will depend on several factors.

From a physical point view, the lowest value to be assigned to α should preserve the energy content of the original system. Several criterions have been tested, and it is herein presented the only one that gave stable and reliable results. It has been based on the ratio between the first natural frequencies of the system, damped and undamped. For the original and scaled system, it is the following:

$$\gamma = \frac{\omega_{1,\,DAMPED}}{\omega_{1,\,UNDAMPED}} = \sqrt{1 - 2\left(\frac{\eta}{2}\right)^2} \;,\; \gamma_S = \sqrt{1 - 2\left(\frac{\eta}{2\alpha}\right)^2} \tag{21}$$

At this point an error function can be simply defined by accordingly comparing the last two expressions: $E = 1 - \gamma_S/\gamma$. This error function depends on the original damping, as expected. For decreasing values of α, the error function increase monotonically up to values that lead to unacceptable scaled representations. Fig. 5 presents the error function in percentage scale. It is useful to fix the error (left scale) around 1%. The figure allows using for a damping value of η=0.02, $\alpha_{MIN} \sim 0.12$. By using values lower than this means that the scaled solution will spread the same energy content over a too wide frequency range. A further analysis of Fig. 5, shows that keeping the 1% as error limit, for increasing values of the original damping, the possibility of scaling are increasingly reduced: for example, the figure allows using for a damping value of η=0.12, $\alpha_{MIN} \sim 0.55$. This was expected since the scaling procedure is based on the original damping and modal overlap factor values: if the original model is highly damped, the possibility of scaling are reduced.

It has to be clearly stated that Eq. (21) refers to a 1D propagation. For a 2D propagation, such as in plates, the situation will be slightly, different (Franco 1997): the error function has to be accordingly defined.

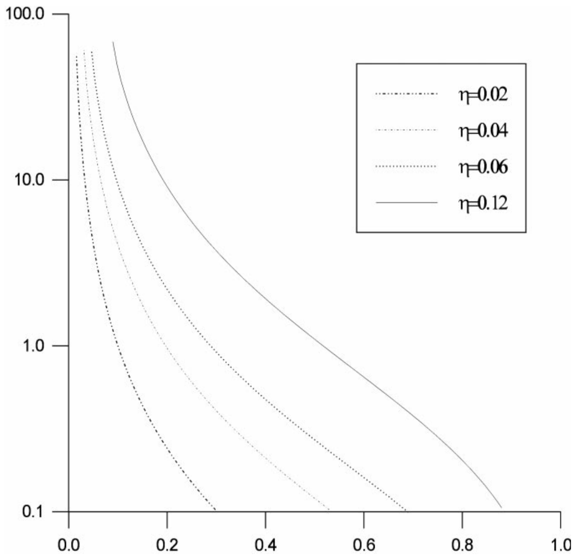

Fig. 5 Analysis of the scaling criterion, error function (%) vs. scaling coefficient

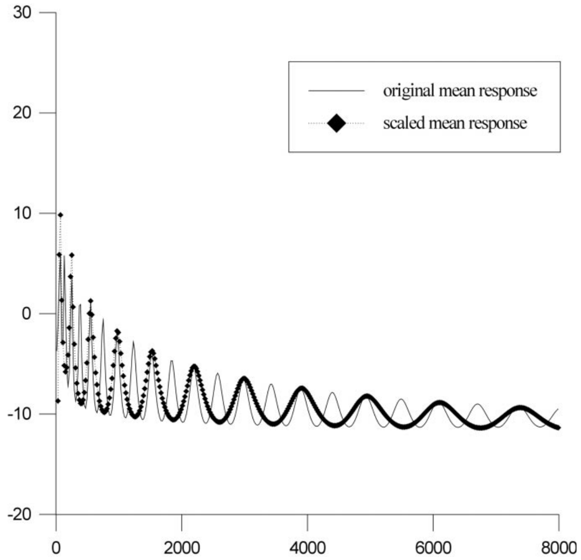

Fig. 6 Metric S [dB] vs. freq. [Hz] (U=115 m./s.; η=0.06; Δf=20 Hz, N=32, N_S=16, α=0.5)

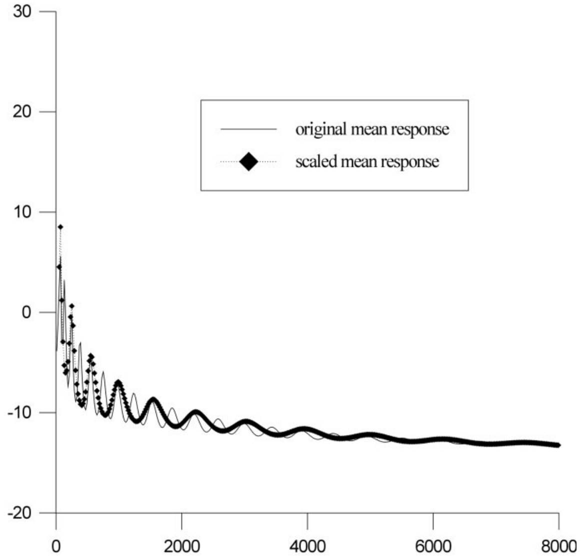

Fig. 7 Metric S [dB] vs. freq. [Hz] (U=115 m./s.; η=0.12; Δf=20 Hz, N=32, N_S=16, α=0.5)

Fig. 6 and Fig. 7 present the application of the scaling procedure. It is evident that the scaled mean responses will not reproduce exactly the original ones, but in average they are an acceptable representation, well within the engineering confidence. Fig. 8 contains a variation of the mean response with the scaling coefficient; the way of working of the scaling procedure is evident, since it uses the same modal base toward higher frequencies at increased damping.

For sake of precision, it has to be highlighted also that the scaled response could reproduce the

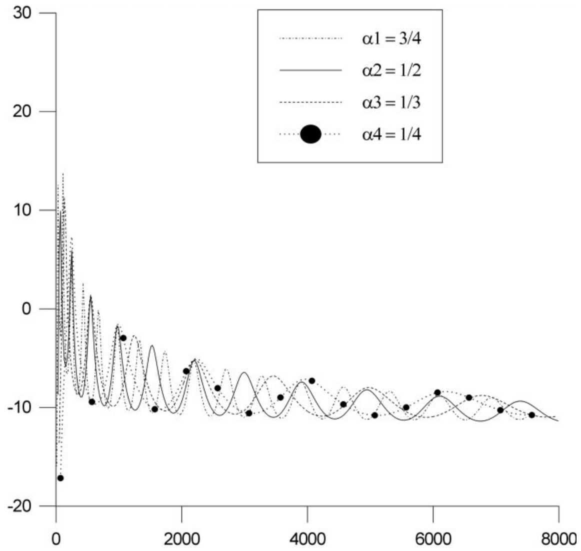

Fig. 8 Metric *S* [dB] vs. freq.[Hz] - scaled mean responses (U=115 m./s.; η=0.06; Δf=20 Hz)

original response by multiplying only the final result for some power of scaling coefficient α. In the present work a simplified approach has been used. In fact, the predictive code has been used unaltered for the scaled response: only the input was modified, since the modal base (natural frequency and mode shapes) and the damping were those associated with the scaled model of the beam.

The numerical results from both the models allowed to establish in the present case that:

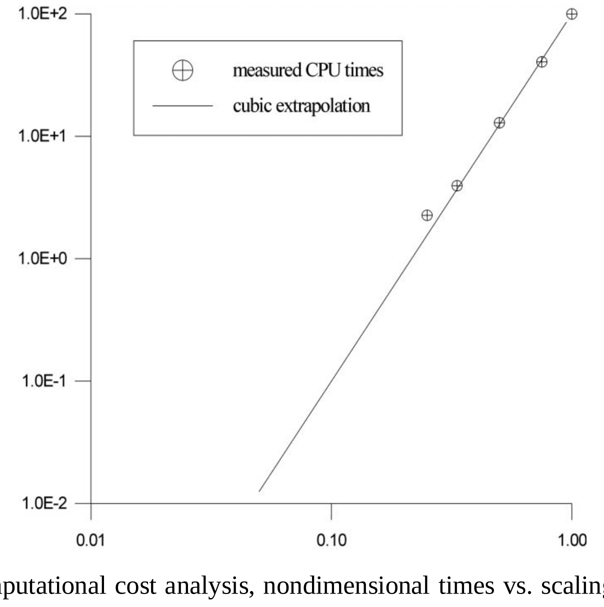

Fig. 9 Computational cost analysis, nondimensional times vs. scaling coefficient

$$\bar{S}_{w,\,ORIGINAL}(\omega) \;=\; \frac{\bar{S}_{w,\,SCALED}(\omega)}{\alpha} \tag{22}$$

the scaled response has to be divided by α to recover the original response.

For load functions simpler than the present turbulent boundary layer wall pressure distribution, it is possible determining "*a priori*" the relation between the original and scaled responses.

An analysis of the computational costs for the mean responses is finally presented in Fig. 9. The left axis contains the ratio between the scaled and original model CPU times, in percentage: it is evident the reduction of the computational time associated with the scaled model.

5. Extension of the method

This work has been partially developed under the early stages of the project "Environmental Noise Associated with turbulent Boundary Layer Excitation", ENABLE, EU Research Programme FP5, and Contract No. G4RD-CT-2000-00223, Apr 2000-Mar 2003, Prime Contractor: Dassault Aviation, France.

In the cited project, one of the main targets was to validate the structural and acoustic predictive methodologies by using available theoretical solutions and experimental measurements for several test articles: plane and stiffened plates.

The scaling procedure is there applied to the finite element model of one of the test plate (an aluminium plane plate: 0.768 m.×0.328 m., thickness=16 mm.; U=115 m/s.; η=0.02. The original model was assembled by using a mesh of 101×43 nodes; the MSC/NASTRAN has been used for generate the modal base in the 0 to 8000 Hz frequency range. Further, a scaled model of the same plate by using α=0.5, and a mesh of 51×41 nodes has been used.

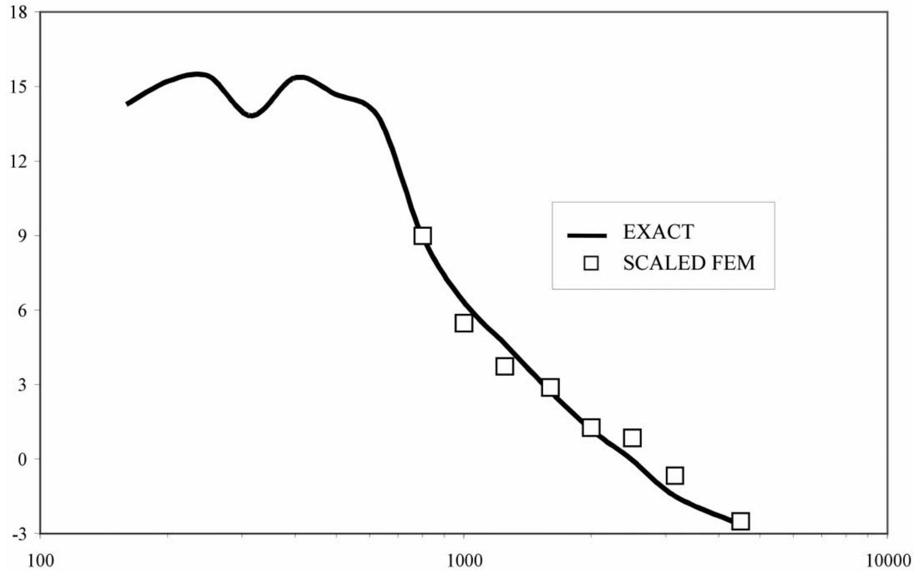

Fig. 10 Simply supported plate, mean responses, metric S [dB] vs. freq. [Hz]

The modal bases represented the input for TUBULAR, a suite of FORTRAN codes for the evaluation of the structural and acoustic response of any planar structural operator. In fact, since the beginning of the analysis in this challenging research argument was clear that having the random structural response, the associated radiated acoustic power could be easily evaluated by using the Rayleigh integral, (Davies 1971).

Fig. 10 reports one of the first and relevant results: it is just related to the scaled finite element model. It is evident the high predictive quality even at very high values of the excitation frequency.

For sake of precision, the original modal base in the $0 \div 8000$ presents about 190 modes; for obtaining the scaled response, only 90 scaled modes have been used.

Finally, it is has to be highlighted that the CPU time needed by the scaled F.E.M. for generating the plate response for each excitation frequency is 1 second: the standard FEM would run in 76 seconds.

Other results obtained in the same project and not reported here have also demonstrated the applicability of the same scaling procedure in predicting the radiated power. The use of the original modal base over a frequency band wider than the original one at increased damping levels does not influence the quality of the acoustic predictive response: the scaled models are able to preserve both of the acoustic and vibrational energies, (De Rosa 2003).

6. Conclusions

The predictive response for a simply supported beam under a turbulent boundary layer excitation has been herein presented and discussed. The modal expansion has been used to obtain such a response, since this method well generalises the approach for more complicated structure and reproduce the standard finite element approach. The choice of the Corcos model for the TBL distribution has been done only for keeping the simplicity of representation, even if this includes all the most important parameter of the TBL. It has been evidenced that the modal approach could represent a solution to the predictive response even if the associated computational cost will assume unacceptable values.

A scaling procedure has been so applied in order to overcome this problem. A computational domain has been defined by simply reducing the dimensions not involved in the energy transmission. The associated damping has been accordingly increased, in order to keep the same energy representation. The results are quite satisfactory, and moreover a useful criterion for the scaling phase has been also defined and tested.

All the themes herein contained can be successfully applied also to discrete coordinate solvers such finite element based codes as demonstrated by a sample result.

Acknowledgements

The authors wish to thank all the ENABLE partners that allowed the publication of the results. The first two authors in collaboration with ing. D. Melluso (ALENIA Aeronautics) obtained the results presented in Fig. 10; they refer specifically to the application of the methodology to the plate responses.

References

Bull, M.K. (1996), “Wall pressure fluctuations beneath turbulent boundary layers: some reflections on forty years of research”, *J. Sound Vib.*, **190**(3), 299-315.

Cousin, G. (1999), “Sound from T.B.L. induced vibrations”, Royal Institute of Technology, Dept. of Vehicle Engineering, The Marcus Wallenberg Laboratory for Sound and Vibration Research, ISSN 1103-470X, ISRN KTH/FKT/L-99/35-SE.

Corcos, G.M. (1963), “The structure of the turbulent pressure field in boundary layer flows”, *J. Fluid Mech.*, **18**, 353-378.

Corcos, G.M. (1967), “The resolution of turbulent pressure at the wall of a boundary layer”, *J. Sound Vib.*, **6**, 59-70.

Davies, H.G. (1971), “Sound from turbulent boundary layer excited panels”, *J. Acoustical Society of America*, **49**(3), Part II, 878-889.

De Rosa, S., *et alii* (1997), “First assessment of the energy based similitude for the evaluation of the damped structural response”, *J. Sound Vib.*, **204**(3), 540-548.

De Rosa, S., *et alii* (2003), “Analysis of radiated power from a plate in a turbulent boundary layer”, *Invited Paper at Euronoise Conference*, Naples, Italy, 19-21 May, Paper #47.

Elishakoff, I. (1983), *Probabilistic Methods in the Theory of Structures*, John Wiley & Sons, ISBN 0471875724.

Franco, F., *et alii* (1997), “The energy based similitude for the energetic vibration response prediction of 2D systems”, *C.E.A.S. Intern. Forum on Structural Dynamics and Aeroelasticity*, **III**, 223-229, Rome.

Lyon, R.H. and Dejong, R.G. (1995), *Theory and Application of Statistical Energy Analysis*, RH Lyon Corp; 2nd edition, ISBN 0750691115

Skudrzyk, E. (1968), *Simple and Complex Vibratory Systems*, The Pennsylvania State University Press, University Park and London, Catalog Card 66-18222.