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Translation method: a historical review and its application to simulation of non-Gaussian stationary processes

Hang Choi† and Jun Kanda‡

Institute of Environmental Studies, Graduate School of Frontier Sciences, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

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Abstract. A number of methods based on various ideas have been proposed for simulating the non-Gaussian stationary process. However, these methods have some limitations. This paper reviewed several simulation methods based on the translation method using logarithmic and polynomial functions, which have emerged in the history of statistics and in the field of civil engineering. The applicability of each method is discussed from the viewpoint of the reproducibility of higher order statistics of the object function in the simulated sample functions, and examined using pressure signals measured from wind tunnel experiments for various shapes of buildings. The parameter estimation methods, i.e. the method of moments and quantile plot, are also reviewed, and the useful aspects of each method are discussed. Additionally, a simple worksheet for parameter estimation is derived based on the method of moment for practical application, and the accuracy is discussed comparing with a set of previously proposed formulae.

Keywords: non-Gaussian process; translation method; nonlinear static transformation; simulation; wind pressure.

1. Introduction

Models representing wind characteristics are often assumed to be Gaussian stationary processes due to convenience in analyses and modeling. However, experimental results in recent years have shown that the Gaussian assumption is not appropriate for representing pressure fluctuations in whole pressure fields of bluff structures. For example, surface regions influenced by separated flow experience strong non-Gaussian effects in a pressure distribution characterized by high skewness and kurtosis (Peterka 1983, Okada, *et al*. 1992). The negligence of non-Gaussian properties in application to the pressure fields may lead to greater damages to roof panels and higher fatigue effects on cladding components (Xu 1995).

In order to more accurately estimate loads acting on such components, it is essential for the probability distributions to be more precisely described. For further statistical studies, e.g., extreme value estimation by the Monte Carlo Method, statistical properties such as higher order moments and spectral characteristics should be correctly reproduced through the simulation process. However,

† Associate Professor
‡ Professor

the method based on spectral representation using uniformly distributed phase angles and Discrete Fourier Transform (DFT), in spite of its high performance, does not satisfy these demands. This is because simulated results for any given spectral characteristics are limited to the Gaussian process due to the central limit theorem (Yang 1973, Shinozuka, *et al.* 1991, Grigoriu 1993, Choi, *et al.* 2003). Various methods have been proposed to complement the lack of reproducibility. These methods can be divided into three categories as follows: 1) ARMA class model with non-Gaussian white noise, 2) phase signal correction and DFT and 3) translation, or nonlinear static transformation (NST), from the Gaussian random variables (RV) to non-Gaussian RVs.

Category 1 methods, e.g., EAR(1), TEAR(1) (Gaver, *et al.* 1980) etc., generate time series with specified spectral characteristics and a non-Gaussian probability density function (PDF). However, it is difficult to produce the spiky features in the natural time history signal and impossible to control the higher order moments (Seong, *et al.* 1993). For category 2, Seong, *et al.* (1993, 1997, 1998, 2001) proposed a more reproductive method for the statistical properties obtained from the non-Gaussian observed data, called EARPG (exponential autoregressive peak generation) method, by taking advantage of both the ARMA models and the DFT method. As a similar method, Kumar, *et al.* (1997, 1998, 1999, 2000) proposed a revised method called the EPG (exponential peak generation) method under the assumption that the number of parameters in the EARPG method is inadequate to obtain the fundamental characteristics of a non-Gaussian signal such as skewness and kurtosis. Since the EPG method depends on only one probabilistic parameter, which controls the intensity as well as the frequency of spikes in the phase signal, it may be relatively easy to estimate the required parameter in comparison with the EARPG method. Simulation results show fairly good agreement with experimental results. However, there are some unusual tendencies of the EPG method. First, the skewness and unbiased kurtosis of the simulated sample function do not approach zero (called the normal point) as the probabilistic parameter, say b, increases. This tendency can be interpreted as indicating that the central limit theorem does not hold. Second, as for lognormal distributions, there exists a deterministic mathematical relationship between skewness and kurtosis of the simulated sample functions regardless of the parameter b, which will be discussed in this study. Third, the relationship is almost the same as that of the gamma distribution (Choi, *et al*. 2001), which can be deduced from the exponential distribution based on the regenerative property (Hald 1952, Rubinstein 1981). These facts mean that the EPG method is very restrictive in application. Moreover, since both methods include iteration procedures to fit the required statistical properties, their efficiencies are generally not good.

The NST method, of category 3, is the oldest method for representing skew probability distribution characteristics, dating back more than 100 years. In the field of civil engineering, Grigoriu (1984a, 1984b, 1995, 2002) studied on the application of the translation method, which is called the memoryless transformations, to simulate stationary non-Gaussian stochastic processes and called the transformed processes as the translation processes. These studies can be considered as a milestone in the application of the method to civil engineering problems. However, since considered translation functions in the studies are limited in simple functions such as exponential and cubic functions, detailed information on the applicability was not described. Meanwhile, Ammon (1990) indicated that if the probability distribution differs too much from the Gaussian distribution, the existence conditions of the equivalent spectrum may not be satisfied. However, recent studies by Gioffre, *et al.* (1999, 2000, 2001a, 2001b) have shown that there is no significant violation in the spectral density function with the use of logarithmic transformation, which is the oldest form of NST function, and Kumar, *et al.* (2000) has also indicated the same feature for a polynomial

transformation according to the study by Gurley, *et al.* (1996). If it is true that there is no significant violation with application of NST for a wide range of skewness and kurtosis, the method seems to be most efficient to simulate the non-Gaussian stationary processes with the DFT method.

In this study, the limitations of various NST methods from the viewpoint of reproducibility of higher order statistics up to the fourth order and the preservability of the spectral characteristics are discussed based on the algebraic investigations and by comparison with wind pressure measurement results obtained from the wind tunnel experiments for various shapes of buildings.

2. Applicability of some NST functions

2.1. Evolution of the basic ideas and assumptions

Most NST functions for translation from Gaussian RVs to non-Gaussian RVs were developed from the end of the nineteenth century to the first half of last century. The basic ideas in those transformations were initiated by Galton (1879) and McAlister (1879) in the study whereby medium value among the arithmetic mean and the geometric mean is better. The study introduced an asymmetric distribution, i.e., lognormal distribution. Meanwhile, F.Y. Edgeworth (1892~1924) attempted to generalize the method and named the technique the "Method of Translation". Although there exist some trials to generalize the normal distribution as the distributions of families of transformed normal RVs during this period, they can also be categorized into this method and some discussions on the priority may be helpful to understand the evolution of the method (Pearson 1905, 1906, Pearson 1967 and Stigler 1978). Those early trials can also be found in Hald (1952) except Edgeworth's polynomials, which have been overshadowed by others up to present. Since not all of them are necessary for the present purpose, only two kinds are investigated: logarithmic transformation including Johnson's system and polynomial transformation.

The basic idea of the translation method started from following hypothesis. To begin with, let x and y be the standard normal RV and non-Gaussian RV, and $\phi(x)$, $\Phi(x)$ and $f(y)$, $F(y)$ be their probability density function (PDF) and cumulative distribution function (CDF), respectively. As such, a translation function can be defined as a function of x, which satisfies following relationship (McAlister 1879).

$$\phi(x)dx = f(y)dy \tag{1}$$

In this paper, it is called the probability mass preserving condition. Meanwhile, Eq. (1) can be rewritten by CDF in the following manner. This expression may be more preferable because the assumption of the absolute continuity of CDF is not necessary.

$$\alpha = \Phi(x_\alpha) = F(y_\alpha) \tag{2}$$

in which, α indicates non-exceedence probability, and x_α, y_α are α-quantiles for each distribution. Therefore, the translation function is defined as follows (Grigoriu 1984a, 1984b).

$$y_\alpha = F^{-1} \, o \Phi(x_\alpha) = g(x_\alpha) \tag{3}$$

From Eq. (2), since the CDF is a (right) continuous non-decreasing function mapped on [0,1], i.e.,

if $y \to y_0$, $y \geq y_0$ then $F(y) \to F(y_0)$, $F(y) \geq F(y_0)$ and $F(-\infty) = 0$, $F(+\infty) = 1$, the translation function $g(x_\alpha)$ is also non-decreasing function. Therefore, the translation function should hold the following non-decreasing condition (e.g. Capinski, *et al.* 1999).

$$\frac{dy_\alpha}{dx_\alpha} = \frac{dg(x_\alpha)}{dx_\alpha} \geq 0 \tag{4}$$

If the CDF F is assumed to be an absolutely continuous function, that means PDF is defined for y, the non-decreasing condition of Eq. (4) can also be deduced from Eq. (1) as follows.

$$\frac{\phi(x)}{f(y)} = \frac{dy}{dx} \tag{5}$$

Since $\phi(x)$ is a positive finite function and $f(y)$ is a non-negative finite function by its definition, the left hand of Eq. (5) is defined on $\Re^+ \in (0, \infty)$. Therefore, the translation function should be a monotonic increasing function, i.e., $dy/dx > 0$, regardless of the modality of $f(y)$. If the equality in Eq. (4) is permitted, then there does not exist the one to one relationship between x_α and y_α for any $\alpha \in [0, 1]$.

The assertion on the monotonicity implies two properties related to the measure preserving or the measure invariant transformation. Especially, one of the properties permits the introduction of a non-monotonic translation function, which is described in textbooks on statistics and engineering. The first property is a point isomorphism by Halmos and von Neumann (1942). The definition of point isomorphism is as follows. Let $X_1(\{X_1\}, m_1)$ and $X_2(\{X_2\}, m_2)$ be measure space and, { } and m indicates measurable set and a measure in each measure space, respectively. Then, a point isomorphism between X_1 and X_2 is a one to one mapping from almost all of X_1 on almost all of X_2 such that $x_1 \in X_1$ if and only if $x_2 = g(x_1) \in X_2$, and then $m_1(x_1) = m_2(x_2)$. If such a mapping g exists, X_1 and X_2 are point isomorphic. This definition is no more and no less than to rewrite Eq. (2), Eq. (3) and the assertion from Eq. (5) in the measure theoretic expressions. Moreover, since the isomorphism implies the existence of its inverse one to one mapping, the function g should be an increasing (or decreasing) monotonic function. Meanwhile, a necessary and sufficient condition that a measure space of total measure one be a point isomorphic to the unit interval is that the measure space is complete, properly separable and non-atomic. From the property of the properly separable space or simply of the properly decomposed non-overlapped space, the non-monotonic translation function can be introduced (e.g., Doob 1994). For this case, it may be helpful to take a practical example such as $y = x^2$, which is a most famous non-monotonic translation function and the function leads the standard normal distribution to the central chi-square distribution with single degree of freedom. As it is known, the measure space for x is firstly divided into two spaces for $x > 0$ and $x < 0$, and then the translation function maps from almost all $x \in X \in \Re$ on almost all of $y \in Y \in \Re^+$ with an isomorphic manner in each decomposed space. Meanwhile, this example also shows a most important condition to decide whether a non-monotonic translation function is applicable. That is, if a set $K_{x'}$ is defined as $K_{x'} = Ker(dy/dx)$, then the PDF for $y_K = g(x \in K_{x'})$ becomes infinity and it contradicts the definition of PDF. Therefore, it can be asserted that the applicability of non-monotonic translation function fully depends on the property of the set $K_{x'}$, whether the problem requires the existence of PDF or the absolute continuity of CDF and on the definition range of PDF. If the cardinal number of $K_{x'}$ is greater than two, the condition

for the existence of PDF seldom holds, but some special cases such as $y = a \cdot \cos \omega x$ or $y = a \cdot \sin \omega x$ for which the range of y is restricted on $(-a, a)$, and the range of y is also mapped on a discontinuous real space in an unusual manner.

On the other hand, if the CDF of y is assumed to be an absolutely continuous function, then it can be said that $F(y)$ is absolutely continuous with respect to $\Phi(x)$ because there exists an isomorphism. As such, the Radon-Nikodym derivatives $dF(y)/d\Phi(x)$ should be positive, i.e., $dF(y)/d\Phi(x)>0$ (e.g., Maharam 1966, 1969). If we put the relationship of $dF(y)=f(y)dy$ and $d\Phi(x)=\phi(x)dx$ into the derivatives, the condition for the translation function $dy/dx>0$ is acquired by the analogy for Eq. (5). This is the second property.

Based on preceeded discussions, the translation function is restricted as a monotonically increasing continuous function in this paper. The same condition can also be found in Johnson's conditions (Johnson 1949) and they are; (1) It should be a monotonic function of x. (2) It should be simple in form and easy to calculate. (3) The range of values of the NST function corresponding to the actual range of possible values of RV y should be from $-\infty$ to $+\infty$. Although a good approximation may sometimes be obtained even when this requirement is ignored, it is highly desirable that it should be satisfied, since RV x is assumed to vary from $-\infty$ to $+\infty$. (4) The resulting distribution systems of y (and so of x) should include distributions of most, if not all, of the kinds encountered in collected data.

Additionally, it is also required for this study that the higher order moments and their spectral characteristics should be preserved after transformation.

For parameter estimation in the NST function, the following methods are available: (1) the method of probability plotting, (2) the method of moments, and (3) the method of maximum likelihood. Since method (3) is rather difficult to apply to a translation system (Johnson 1949), the first two methods are adopted in the study. The availability of NST functions is examined on the basis of comparison with wind pressure data in wind tunnel experiments and ease of parameter estimation. For convenience, both Gaussian RV, x, and non-Gaussian RV denoted by y, are assumed to be normalized with their mean and standard deviation, i.e., $E(x)=E(y)=0$, $Var(x)=Var(y)=1$.

2.2. The logarithmic transformation

In the nineteenth century, the normal distribution was commonly used. It was assumed that observations of a single phenomenon, homogeneous with respect to all but random, individually insignificant factors, would follow the normal distribution. In the late nineteenth century, a path breaking work by Francis Galton (1879) was directed at the separation of these normal worlds into non-normal worlds (Stigler 1986). Galton had introduced a powerful way of dealing with asymmetry and enlarged the scope of applications for normal distribution with his suggestion that the logarithms of the observed data should be analyzed. He asked Donald McAlister to carry out a mathematical investigation (McAlister 1879). McAlister thus appears to have been the first person to set down explicitly and in some detail a theory for the lognormal distribution, which is termed by Gaddum (1945). A more detailed historical background can be found in Aitchison, *et al.* (1957).

The most generalized form of the logarithmic transformation, which Johnson (1949) termed S_L transformation, can be expressed as:

$$\frac{x-\gamma}{\delta} = \log \frac{y-\xi}{\lambda} = \log y' \tag{6}$$

$$y = g(x) = \xi + \lambda \exp\left(\frac{x-\gamma}{\delta}\right) \tag{7}$$

$$y' = \exp\left(\frac{x-\gamma}{\delta}\right) \tag{8}$$

The shifted lognormal transformation by Grigoriu (1995) is a special case for which $\lambda = 1$. Another three-parameter lognormal distribution for a negatively skewed distribution by Fisher (1936) is also categorized into this as a special case.

Since the r-th moment of y' can be obtained from

$$\mu_r(y') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{\frac{r(x-\gamma)}{\delta} - \frac{x^2}{2}\right\} dx = \exp\left\{\frac{r^2\delta^{-2}}{2} - r\gamma\delta^{-1}\right\} \tag{9}$$

the first four statistics are:

$$\begin{aligned} &\omega = \exp(\delta^{-2}),\ \rho = \exp(-\gamma/\delta) \\ &E(y') = \sqrt{\omega}\rho,\ Var(y') = \omega\rho^2(\omega-1) \\ &\gamma_1^2 = \beta_1 = (\omega-1)(\omega+2)^2 \\ &\gamma_2 = \beta_2 - 3 = \omega^4 + 2\omega^3 + 3\omega^2 - 6 \end{aligned} \tag{10}$$

in which, γ_1 and γ_2 are skewness and unbiased kurtosis coefficients, respectively. Points (γ_1, γ_2) for lognormal distributions therefore lie on a curve of approximately $\gamma_2 \cong 1.85\gamma_1^2$, called a lognormal line defined by the parametric Eq. (10). It is worth noting that the approximation is only valid for δ less than about 0.5, at which coefficients of skewness and unbiased kurtosis are 1.75 and 5.9, respectively. For δ greater than 0.5, the approximation is not valid because discrepancies between Eq. (10) and the approximation grow rapidly. However, the coefficients range for practical purposes may not greatly exceed this range as described by Pearson (1901) and Edgeworth (1914).

The approximation is shown in Fig. 1 with some wind tunnel experiment results plotted in (γ_1, γ_2) plane. In the figure, the legend for "experiment" indicates the pressure data from 240 points located over the surface of a square section prism with aspect ratio 4 under suburban flow conditions, i.e., a power law exponent=0.22. The data is sampled with a sampling frequency of 400 Hz after passing through low pass filter with cut-off frequency of 200 Hz. The sampled data is averaged by one second moving average window in full scale. Coefficients of skewness and unbiased kurtosis plotted in Fig. 1 are for a wind direction of 45 degrees. As shown in the figure, the lognormal line cannot cover all the data, which correspond to the skewness and unbiased kurtosis coefficients for measured wind pressure data from the wind tunnel experiments for various shapes of buildings. This restriction of the locus of (γ_1, γ_2) is easily expected since the skewness and the kurtosis in Eq. (10) are determined by only one parameter ω. Consequently, application of the shifted lognormal transformation to pressure data as shown in Gioffre, *et al.* (1999, 2000, 2001a, 2001b) and of three parameter lognormal distributions as shown in Calderone, *et al.* (1993, 1994), Li, *et al.* (1999) and in Matui, *et al.* (1982) are not appropriate for various conditions of higher

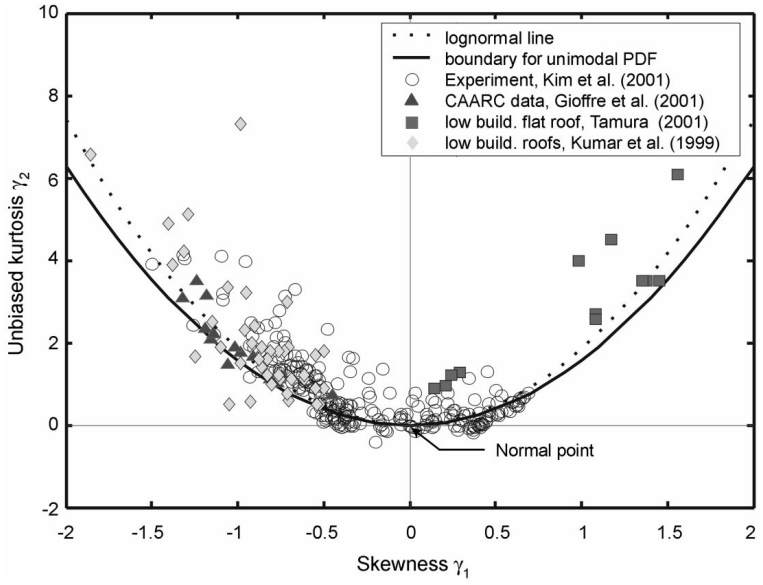


Fig. 1 Comparison of lognormal line and boundary for unimodal PDF of third order polynomial NST with some experiment results

statistics. As a reference, the locus of (γ_1, γ_2) of the gamma distributions is also restricted on a curve that can be expressed by $\gamma_2 = 1.5\gamma_1^2$. Therefore, if the locus of (γ_1, γ_2) from the EPG method is restricted as shown in Kumar, *et al.* (1999), which can be approximated by $\gamma_2 \cong 1.5\gamma_1^2 + 0.8$ for $|\gamma_1| \geq 0.4$, it may be asserted that the applicability of the EPG method would also be restricted by the similar manner of gamma distributions.

2.3. Extension of logarithmic transformation

Despite its successful application in a number of cases to engineering problems, the logarithmic transformation has restricted flexibility as shown above. It thus seems reasonable to assume that useful extensions of the NST function might be obtained by using a different function. The S_U function by Johnson (1949) is one of most flexible functions and is defined as follows.

$$\frac{x-\gamma}{\delta} = \sinh^{-1}\left(\frac{y-\xi}{\lambda}\right) \tag{11}$$

or

$$y' = \frac{y-\xi}{\lambda} = \sinh\left(\frac{x-\gamma}{\delta}\right) = \frac{1}{2}\{e^{(x-\gamma)/\delta} - e^{-(x-\gamma)/\delta}\} \tag{12}$$

From these, the PDF of y' is:

$$p(y') = \frac{\delta}{\sqrt{2\pi}}\frac{1}{\sqrt{y'^2+1}}\exp\left\{-\frac{1}{2}\{\gamma + \delta\log(y' + \sqrt{y'^2+1})\}^2\right\} \tag{13}$$

and four parameters, i.e., γ, δ, ξ and λ, in Eq. (11) or Eq. (12) can be estimated from the first four moments of observed data. Conversely, the moments can also be calculated from the four given parameters. For the former case, Leslie (1959) obtained a set of formulae for the estimation of skewness and unbiased kurtosis based on the formulae derived by Johnson (1949), and later, it is modified by Johnson (1965) as follows.

$$E(y') = -\sqrt{\omega}\sinh\Omega,\ Var(y') = 1/2(\omega-1)(\omega\cosh 2\Omega + 1)$$

$$\gamma_1^2 = \frac{m(\omega-1)[4(\omega+2)m + 3(\omega+1)^2]^2}{2(2m+\omega+1)^3},\quad \gamma_2 = \frac{(\omega-1)[A_2(\omega)m^2 + A_1(\omega)m + A_0(\omega)]}{2(2m+\omega+1)^2} \tag{14}$$

in which, $\omega = \exp(\delta^{-2})$, $\Omega = \gamma/\delta$ and

$$m = \omega\sinh^2\Omega = \{E(y')\}^2,\ A_2(\omega) = 8(\omega^3 + 3\omega^2 + 6\omega + 6)$$

$$A_1(\omega) = 8(\omega^4 + 3\omega^3 + 6\omega^2 + 7\omega + 3),\ A_0(\omega) = \omega^5 + 3\omega^4 + 6\omega^3 + 10\omega^2 + 9\omega + 3 \tag{15}$$

Once ω and Ω are determined from Eq. (14) and Eq. (15), ξ and λ can be determined from the following relationship.

$$\lambda^2 = Var(y')/Var(y),\ \xi = E(y) - \lambda E(y') \tag{16}$$

in which, $E(y')$ and $Var(y')$ are given by Eq. (14). However, it is the most difficult step to use the S_U function, and because of the absence of an analytical solution, a numerical iteration is needed to obtain the parameters. There have been some valuable studies on estimation of the parameters. For instance, Hill, *et al.* (1976) described an iteration algorithm for the method of moments for Johnson's system in ISO Fortran, and Wheeler (1980) proposed an estimation procedure by means of quantile estimators. However, the results are generally not good for modest sample sizes. Furthermore, Johnson (1949) gave an abac for given δ and Ω, and a table for given γ and δ in terms of γ_1^2 and $\gamma_2 + 3$ (Johnson, *et al.* 1965). The applicable range of the S_U transformation is upward of the lognormal line in Fig. 1. However, there are both difficulties and irrational assumptions in the estimation of the parameters near the lognormal line. As described by Johnson, when γ increases from zero to infinity, (γ_1, γ_2) varies from $(0, 0.5\omega^4 + \omega^2 - 1.5)$ to a point on the lognormal line. Meanwhile, as δ moves from infinity to zero, ω varies from unity to infinity. Therefore, if (γ_1, γ_2) is located near or on the lognormal line including the normal point, an iteration procedure does not converge, depending on the convergence criteria. Moreover, Johnson's system is based on the assumption that if a given (γ_1, γ_2) is located on or below the lognormal line, the PDF for that region should have the form of one end or both ends bounded. However, there is no evidence that this assumption is correct at any time. From the viewpoint of practical application, the existence of RV's bound becomes a weak point because many of the pressure data located in this region as shown in Fig. 1 and in the tails of PDF seems to have significant meaning in engineering problems.

2.4. Polynomial transformation

As aforementioned, toward generalizing the normal distribution, which is to consider the distributions of families of transformed normal RVs, Edgeworth (1892) set forth a polynomial transformation. The idea was named the "method of translation" in later study (Edgeworth 1898, Stigler 1978). As the first trial, he examined a quadratic function and its generalized form can be expressed as follows.

$$y = a_q(x + b_q)^2 + c_q \tag{17}$$

in which, a_q, b_q and c_q are parameters. It is well known that y follows a non-central chi-square distribution with single degree of freedom. Furthermore, Edgeworth (1898) made another assumption that the weight of a man is proportional to the cube of his height and that the frequency of men's height is normal, and examined the third order polynomial. That is,

$$y = k(x + g)^3 = kg^3\left\{1 + \frac{3x}{g}\left(1 + \frac{x}{g} + \frac{x^2}{3g^2}\right)\right\} \tag{18}$$

Eq. (18) is generalized as follows (Edgeworth 1924).

$$y=a+bx+cx^2+dx^3 \tag{19}$$

in which, a, b, c and d are parameters. It is easy to understand that Eq. (17) is a special case of Eq. (19), if we put $d=0$ in Eq. (19).

Meanwhile, Pearson divided asymmetrical distributions into two distinct classes: a heterogeneous one composed by two or more homogeneous events, and a homogeneous one in which the tendency to deviate to one side of the mean is not equal to the tendency to deviate to the other side. For the heterogeneous class, he suggested a method using a mixture of two normal RVs (Pearson 1894), which is now called α stable RVs (e.g., Grigoriu 1995) if the mixture holds the Levy's hypothetical lemma (LeCam 1986). For the homogeneous class, he derived a generalized form of asymmetrical PDF based on the hypergeometrical series, which is expressed in the following form (Pearson 1895).

$$\frac{1}{f(y)}\frac{df(y)}{dy} = -\frac{y}{c_0 + c_1y + c_2y^2} \tag{20}$$

The right term of the equation is closely related to Edgeworth's polynomial function (Pearson 1905, 1906) and is slightly different from the form referred at present. Based on the probability mass preserving transformation,

$$f(y)dy=\phi(x)dx \tag{21}$$

Taking logarithmic differentials,

$$\frac{1}{f(y)}\frac{df(y)}{dy} = \frac{1}{\phi x}\frac{d\phi(x)}{dx}\frac{dx}{dy} + \frac{d^2x}{dy^2} \Big/ \frac{dx}{dy} \tag{22}$$

Since x follows the standard normal distribution as assumed in this study, the following relationship is established.

$$\frac{1}{\phi(x)}\frac{d\phi(x)}{dx} = -x \tag{23}$$

Eq. (23) describes Pearson's main idea and the idea is closely related with the standard normal distribution (Pearson 1895). Let $x = \tilde{f}(y)$ and substituting it into Eq. (23):

$$\frac{1}{f(y)}\frac{df(y)}{dy} = -\frac{y}{\tilde{F}(y)} \tag{24}$$

in which,

$$\tilde{F}(y) = \frac{y \cdot d\tilde{f}(y)/dx}{\tilde{f}(y)(d\tilde{f}(y)/dx)^2 - d^2\tilde{f}(y)/dx^2} \tag{25}$$

Since $\tilde{f}(y)$ is an arbitrary function as written by Edgeworth, $\tilde{F}(y)$ can also be selected arbitrarily regardless of Eq. (25). As an alternative to $\tilde{F}(y)$, by taking up to the second order terms of the Maclaurin series expansion of $\tilde{F}(y)$, the result coincides with Eq. (20). If an analytical function, e.g., $f(y)=\log y$ or Kapteyn's polynomial form $f(y)=(y+a)^c$ (Kapteyn 1903), is assumed, each result from Eq. (21) and Eq. (24) should be exactly the same through the relation of Eq. (25). Therefore, it can be said that the two methods are intrinsically the same, except for the difference of whether the normal distribution is introduced explicitly, even though Pearson claimed a lack of physical meaning in the relation $x = \tilde{f}(y)$ (Pearson 1905). However, explicit introduction of the normal RV would help complement the simulation method based on spectral representation with uniformly distributed phase angles and DFT for non-Gaussian stationary processes, if no excessive distortion remains in the spectral characteristics of the translated stochastic processes. The relative advantages of these two systems are summarized and discussed by Edgeworth (1916, 1917).

The reason why Edgeworth and Pearson take only finite terms in their arbitrary functions can be summarized as follows (Pearson 1904): 1) an empirical reason: fairly wide experience shows that polynomials of the second and third order suffice as rules to describe the skewness of the regression line between x and y; 2) a mathematical reason: the higher powers involve moments of the fifth and higher orders and their probable errors are very large, which is called a statistical fluctuation (e.g., Kendall, *et al.* 1977). These two reasons are still valuable and useful for many practical problems studied at present because many of events are modeled based on the information from observations and experiments, i.e. an empirical insight, rather than a theoretical insight.

In a biographical essay on Edgeworth, Stigler (1978) indicates that Edgeworth has remained one of the least known major figures in the history of mathematical statistics. The method of translation is not an exception. Especially, the validity of Eq. (19) has been buried in so many achievements in modern mathematical statistics until the date of its resurrection in 1978, but with different names and its inventor. In 1978, A.I. Fleishman proposed a method, termed the power method, for simulation of non-normal RVs from normal RV by using two kinds of polynomial functions of x. These polynomial functions are exactly the same as Eq. (17) and Eq. (19), respectively. He also computed the parameters in Eq. (19) based on the method of moments and tabulated them with

respect to skewness and unbiased kurtosis coefficients. However, the table and derivation method based on the given first four moments were shown by Edgeworth (1914, 1924), and an enlarged table was accommodated by Bowley (1928) based on a slightly different form of the PDF of x for y-a in Eq. (19). Edgeworth and Bowley used the PDF with standard deviation of $(\sqrt{2})^{-1}$, whereas Fleishman used the standard normal PDF. The even order moments for each case are as follows. In this study, the latter is used.

$$\mu_{2r}^{*} = \int_{-\infty}^{\infty} x_{*}^{2r} \frac{\exp(-x_{*}^{2})}{\sqrt{\pi}} dx_{*} = \frac{(2r)!}{2^{r} r!} \left(\frac{1}{2}\right)^{r} \tag{26}$$

$$\mu_{2r} = \int_{-\infty}^{\infty} x^{2r} \frac{\exp(-x^{2}/2)}{\sqrt{2\pi}} dx = \frac{(2r)!}{2^{r} r!} \tag{27}$$

The power method is more celebrated than the translation method. For instance, Kotz, *et al.* (2000) introduced Fleishman's method with some references for various application examples including the method of generating bivariate non-normal RVs. And also, Johnson, *et al.* (1994) described the translation method in only four lines as "not often used method at present". However, the translation method for non-normal bivariate had already been shown and discussed by Edgeworth in 1914. On this aspect, Kendall (1968) also has indicated that the method of translation to bivariate surfaces never seems to have been adequately followed up after Edgeworth. It is interesting to note that the description of the translation method by Johnson, *et al*. contrasts with the relatively long and detailed description of the Johnson's system and may be corrected by the description by Pearson (1967), in which Pearson briefly mentioned the contributions of the translation method to later studies, i.e. the Johnson's system. A wrong eponym may not be a rare event at this time, because there exists a law called Stigler's Law of Eponymy (e.g., Stigler 1999), which in its simplest form states that no scientific discovery is named after its original discoverer.

2.5. Theoretical infinite polynomials and some truncated forms

The proofs of the central limit theorem show that the distribution of the sum of n independent RVs with finite moments may be expressed in a series form (e.g., Hald 1998). The Gram-Charlier and the Edgeworth series, which are substantially the same, are the most well known series. These series are generally used with finite terms, e.g., three terms for the Gram-Charlier series and four terms for the Edgeworth series approximations, because the series expansions with many terms do not always provide more accurate approximations. However, the truncated series have significant defects such that negative PDF appears (e.g., Yim, *et al*. 1999) and the approximation breaks down not very far from the center of the probability distributions for relatively large skewness and kurtosis. For this reason, Edgeworth recommended the method of translation rather than the series form (Edgeworth 1917, 1926). Furthermore, Draper, *et al*. (1972) shows the regions for unimodal PDF and for positive definite of the Gram-Charlier and the Edgeworth series approximations. On the other hand, if all order of moments are available for a distribution such as theoretical non-Gaussian distribution, the infinite series expansion based on the Hermite-Chebyshev polynomials would be orthogonal decomposition of the non-Gaussian PDF with weight function of $\phi(x)$ because the Hermite-Chebyshev polynomials constitute the orthonormal system of the Hilbert space $L^{2}(\Re, d\Phi)$. However, this fact does not mean that the series expansion can approximate any form

of PDF. In order that the series converges to $f(y)$, some conditions should be satisfied (Kendall, *et al.* 1977).

Meanwhile, Cornish and Fisher (1937) and Fisher and Cornish (1960) used the Edgeworth series to find an asymptotic expansion of the quantiles of the distribution of y with known cumulants in terms of the corresponding normal deviate. There are two types of expressions: 1) finite form and 2) infinite form. The finite form generally assumes the existence of the cumulants up to the eighth or lower order (Koning, *et al.* 1988). Even in infinite form, cumulants higher than sixteenth order are seldom used (Lee, *et al.* 1992).

Using the cumulants up to the fifth order, it can be expressed as:

$$y_\alpha = x_\alpha + \sum_j B_j(x_\alpha) \tag{28}$$

in which, $j=1,2,3\ \ldots$ and

$$B_1(x_\alpha) = \frac{\kappa_3}{6}(x_\alpha^2 - 1)$$

$$B_2(x_\alpha) = \frac{\kappa_4}{24}(x_\alpha^3 - 3x_\alpha) - \frac{\kappa_3^2}{36}(2x_\alpha^3 - 5x_\alpha)$$

$$B_3(x_\alpha) = \frac{\kappa_5}{120}(x_\alpha^4 - 6x_\alpha^2 + 3) - \frac{\kappa_3\kappa_4}{24}(x_\alpha^4 - 5x_\alpha^2 + 2) + \frac{\kappa_3^3}{324}(12x_\alpha^4 - 53x_\alpha^2 + 17)$$

In the equation, κ_j is the j-th order cumulant.

Since it is unusual to use moments higher than the fourth order for measured data, and many terms are needed to acquire good approximation, e.g. more than six terms for chi-square distributed RVs (Zar 1978) and more than ten terms for Weibull and gamma distributed RVs (Choi, *et al.* 2002), the Cornish-Fisher series expansion is generally used to approximate a theoretically distributed RV. Although the higher order Cornish-Fisher series expansion provides an excellent approximation, derivation of the inverse function of the series is quite difficult, which is necessary to ensure the existence of PDF and to correct the distortion of correlation functions by an algebraic method, which will be discussed in § 4. On the contrary, if the series is truncated to avoid a statistical fluctuation or to obtain analytic inverse function, i.e., using the first three terms, a good approximation cannot be expected. For these reasons, Eq. (19) may be the most probable approximation of Eq. (28) under the constraint for available moments.

Meanwhile, the same invention can also be found in the field of civil engineering. Winterstein (1988) proposed a translation method to approximate non-normal responses based on a series approximation of PDF (Crandall 1980), which is based on the Gram-Charlier series and semi-invariants, i.e., cumulants. That is,

$$y = h_2\{x + h_3(x^2 - 1) + h_4(x^3 - 3x)\} \tag{29}$$

or arranging terms with respect to each order of x, then

$$y = -h_2h_3 + (h_2 - 3h_2h_4)x + h_2h_3x^2 + h_2h_4x^3 \tag{30}$$

in which, h_2 is a scaling factor to ensure unit variance of y. Comparing Eq. (29) with Eq. (28), it is easily seen that Eq. (29) is a truncated form of Eq. (28) at the first term of B_2, and Eq. (30) has the same form as Eq. (19). He also showed approximated parameter estimators for Eq. (29) or Eq. (30) to be applicable to most possible values of skewness and unbiased kurtosis greater than zero ensuring unimodal PDF. The estimators are as follows:

$$h_2 = (1 + 2h_3^2 + 6h_4^2)^{-1/2},\ h_3 = \frac{\gamma_1}{4 + 2\sqrt{1 + 1.5\gamma_2}},\ h_4 = \frac{\sqrt{1 + 1.5\gamma_2} - 1}{18} \tag{31}$$

However, in order to ensure the unimodal PDF, the following relationship should be satisfied as written by Winterstein, but the accuracy and the availability of Eq. (31) were not fully verified in the stated region of (γ_1, γ_2).

$$\frac{dy}{dx} > 0 \tag{32}$$

On the other hand, Fleishman (1978) showed that the numerically solvable region of Eq. (19) regardless of the unimodality of PDF or the monotonicity of the translation function, which can be approximated by $\gamma_2 \geq 1.59\gamma_1^2 - 1.14$, is narrower than the limit of possible skewness and unbiased kurtosis for all distributions, i.e., $\gamma_2 \geq \gamma_1^2 - 2$. Therefore, the unimodal definite region of Eq. (19) or Eq. (29,30) may be narrower than the solvable region. This means that Winterstein's assertion on the applicable region may not be correct, and this point will be discussed in following section.

This method is also referred to as moment-based Hermite transformation by Tognarelli, *et al.* (1997a). Moreover, Toganarelli, *et al.* (1997b) developed an iterative procedure for the estimation of the polynomial coefficients and showed that Eq. (31) provides relatively good solution. The method is also used by Gurley, *et al.* (1997a, 1997b, 1998) to develop a procedure for the correction of distorted correlation function.

One of the remarkable models for the translation function has been studied by Puig, *et al.* (2002). The basic idea is not different from the orthogonal decomposition of PDF of y with respect to that of the standard normal PDF, but that the orthogonal decomposition is applied to the translation function itself with respect to x. The model assumed that the translation function is an element of the Hilbert space $L^2(\Re, d\Phi)$. If the assumption is true, then the translation function can be expressed as a linear combination of the Hermite-Chebyshev polynomials: there exists a real sequence of coefficients $(c_n)_n$ such that

$$g(x) = F^{-1} \circ \Phi(x) = \sum_{n=0}^{\infty} c_n H_n(x) \tag{33}$$

in which, $H_n(x)$ means the Hermite-Chebyshev polynomials,

$$H_0(x) = 1,\ H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{(-x^2/2)} \tag{34}$$

$$c_n = (n!)^{-1} \int_{\Re} g(x) H_n(x) \phi(x) dx \tag{35}$$

the series being convergent in $L^2(\Re, d\Phi)$.

In order to guarantee the convergence of the right hand of Eq. (33) to $g(x)$ for any $x \in \Re$, the following condition should be satisfied because $\{H_n(x)\}_{n \in N}$ is complete for the class of functions which holds the following condition (e.g., Kaneko, *et al.* 1984).

$$\int_{\Re} \{g(x)\}^2 \phi(x) dx < \infty \tag{36}$$

As the condition holds, the coefficients of the Hermite-Chebyshev polynomials c_n tends to zero as $n \to \infty$. Consequently, $g(x)$ is approximated by the polynomials having infinite (or finite) terms in the sense of quadratic mean. That is,

$$g(x) = \sum_{k=1}^{n} a_k x^k \tag{37}$$

If the range of x restricted in a compact finite interval of $\Re$, Eq. (37) converges to $g(x)$ in the meaning of L^{∞} approximation by the Weierstrass' polynomial approximation theorem (e.g., Yajima 2002). Eq. (37) and the estimation method of coefficients can also be found in a series of studies on the development of a practical mathematics theory of stochastic processes by Kuznetsov *et al.* (1965).

However, the difficulty in the derivation of inverse function as discussed on the higher order Cornish-Fisher series expansion still remains unsolved. On the other hand, even though the accuracy of approximation is important, the ease of use also cannot be ignored for practical purpose as described by Johnson (1949). For such reasons, this paper sheds light on the applicability of the third order polynomial of Eq. (19).

3. Parameter estimation in polynomial NST function

As aforementioned, there are two practical methods for estimating parameters in NST functions: 1) the method of probability plotting and 2) the method of moments. In a number of situations, the method of moments is quite adequate, although it cannot be regarded as giving the best solution in any sense. However, it is not always a desirable procedure. In particular, if the moments are subjected to sampling error due to extraordinary outliers, the results may be strongly affected and the former method may be more desirable. Consequently, these two methods are complementary to each other.

3.1. General conditions for parameters

In order to satisfy the monotonicity of translation function, the first order derivative of Eq. (19) should be greater than zero. It follows that

$$\frac{dy}{dx} = 3dx^2 + 2cx + b > 0 \tag{38}$$

In order to satisfy Eq. (38), it is necessary that $c^2 - 3bd < 0$. If the condition holds, the unimodality also holds as stated by Winterstein (1988). The boundary on the (γ_1, γ_2) plane for the monotonicity and unimodality can be estimated from the following condition and the estimated

boundary indicates the lower bound of (γ_1, γ_2) for application, but no upper bound exists.

$$c^2 - 3bd = 0 \tag{39}$$

3.2. Method of moments

For Eq. (19), the method of moments has already been developed by Fleishman (1978) based on Eq. (27). The parameters can be estimated from the following parametric equations.

$$E(y) = a + c = 0 \tag{40a}$$

$$Var(y) = b^2 + 6bd + 2c^2 + 15d^2 = 1 \tag{40b}$$

$$\gamma_1(y) = 2c(b^2 + 24bd + 105d^2 + 2) \tag{40c}$$

$$\gamma_2(y) = 24\{bd + c^2(1 + b^2 + 28bd) + d^2(12 + 48bd + 141c^2 + 225d^2)\} \tag{40d}$$

In order to estimate the parameters, Eq. (40b~d) must be solved simultaneously, and for this purpose an algorithm like the least squares method for nonlinear parameters is needed. Meanwhile, for computational efficiency, it would be desirable to reduce the number of equations to be solved simultaneously. An available form was introduced by Edgeworth, even though it was based on Eq. (26) (Edgeworth 1914). It is relatively easy to modify the form based on Eq. (27) as shown below.

Since parameter a is a location parameter, subtracting it from both sides of Eq. (19) and denoting it by y', the equation can be expressed as follows.

$$y' = y - a = b\{x + c'x^2 + d'x^3\} \tag{41}$$

Using Eq. (27) instead of Eq. (26) for Eq. (41), the parametric equations corresponding to Eq. (40a~d) can be expressed as

$$E(y') = bc' \tag{42a}$$

$$Var(y) = b^2\{1 + 2c^* + 6d' + 15d'^2\} \tag{42b}$$

$$\gamma_1^2(y) = \frac{c^*(6 + 8c^* + 72d' + 270d'^2)^2}{(1 + 2c^* + 6d' + 15d'^2)^3} \tag{42c}$$

$$\gamma_2(y) = \frac{24\{2c^* + 2c^{*2} + 36c^*d'(1 + 5d') + d' + 18d'^2 + 135d'^3 + 405d'^4\}}{(1 + 2c^* + 6d' + 15d'^2)^2} \tag{42d}$$

in which, $c^* = c'^2$, $c' = c/b$, $d' = d/b$.

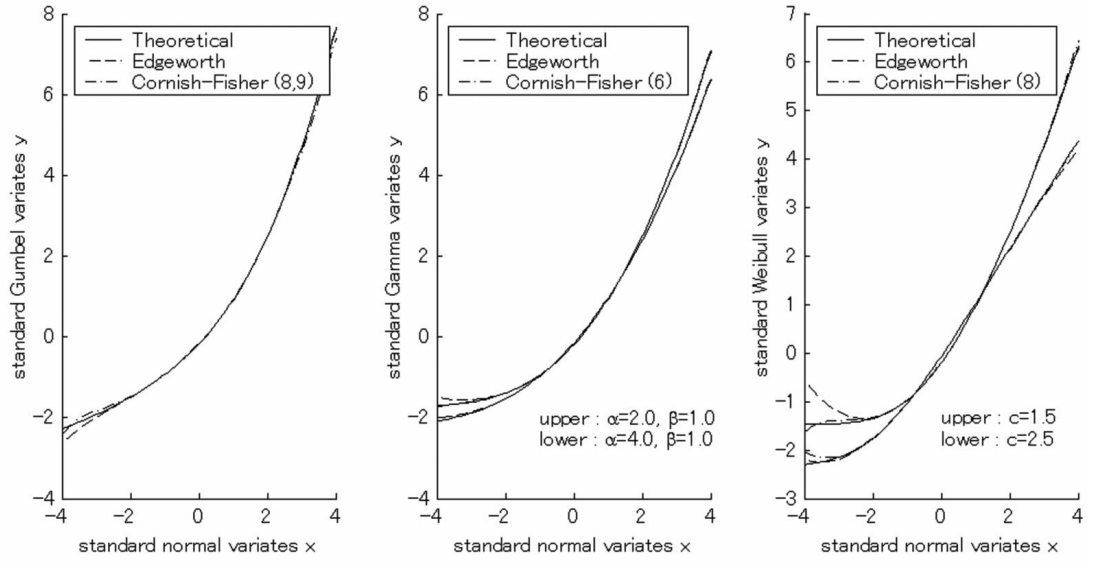
When Eq. (42c) and Eq. (42d) are solved with respect to c^* and d' for given (γ_1, γ_2), parameter b can be estimated from Eq. (42b). Then c and d are straightforwardly obtained. Finally, a is estimated from the relation $E(y) - a = E(y') = c$, i.e., Eq. (40a). It is worth noting that the sign of parameter c follows that of skewness.

In order to solve the parametric equations, an algorithm for the least squares method for nonlinear parameters is used, and it is confirmed that the accuracy of the results is sufficiently high. Through the solving processes, the lower bound is estimated and it can be approximated by

$$\gamma_2 = 0.0372|\gamma_1|^3 + 1.4489|\gamma_1|^2 + 0.0218|\gamma_1| \tag{43}$$

Modifying it to the form of the lognormal line equation, it can be expressed as $\gamma_2 \cong 1.57\gamma_1^2$. Thus, the applicable range of the third order NST function becomes $\gamma_2 \geq 1.57\gamma_1^2$ and the range is wider than that of the S_U function, but narrower than the limit of all distributions, i.e., $\gamma_2 \geq \gamma_1^2 - 2$. The results are compared in Fig. 1. In the region above the boundary line, no insoluble region is found, but in the region below the boundary line, in which some experimental data are located, the method may not be appropriate because Eq. (38) cannot be satisfied, and absolutely continuous PDF does not exist in a strict sense. However, if two roots of $dy/dx = 0$ are sufficiently separated such that $\Phi(x_U) - \Phi(x_L) \approx 1$ for the two roots $-\infty < x_L < 0 < x_U < \infty$, the translation function may be utilized for the range between two roots. And also, if $\Phi(x_U) \cong 0$ for positively skewed distribution or $\Phi(x_L) \cong 1$ for negatively skewed distribution, the approximation by Eq. (19) may be available with the restriction of the range of y in $y > g(x_U)$ or $y < g(x_L)$, respectively. Whereas, the PDF of y does not exist in any sense when two roots are close to each other. Therefore, before extending the applicable range below the boundary line, the range of y and the locus of roots should be carefully considered.

Some examples related to above discussions are shown in Fig. 2 for theoretical distributions whose (γ_1, γ_2) are located above and below the boundary line. The considered theoretical



(a) Standard Gumbel (type 1) distribution (b) Standard gamma distribution (c) Standard Weibull distribution

Fig. 2 Accuracies of the Edgeworth's third order polynomial translation function (Eq. (19)) with moment method and the Cornish-Fisher series expansion compared with some theoretical distributions (Numbers in the brackets indicates the maximum order of cumulant)

distributions are as follows. Parameters in the distributions are adjusted, such as the mean and the variance of each distribution be zero and one, respectively.

(a) standard Gumbel distribution: $\gamma_2 > 1.57\gamma_1^2$

$$f(y) = \sigma^{-1} e^{-(y-\mu)/\sigma} \exp[-e^{-(y-\mu)/\sigma}], \quad \gamma_1 \cong 1.14, \quad \gamma_2 = 2.40$$

(b) standard gamma distribution: $\gamma_2 < 1.57\gamma_1^2$ $(\gamma_2 = 1.5\gamma_1^2)$

$$f(y) = \frac{(y-\xi_g)^{\alpha-1}\exp\{-(y-\xi_g)/\beta\}}{\beta^{\alpha}\Gamma(\alpha)}, \quad \alpha > 0, \ \beta > 0; \ y > \xi_g, \ \gamma_1 = 2\alpha^{-1/2}, \ \gamma_2 = 6\alpha^{-1}$$

(c) standard Weibull distribution: $\gamma_2 < 1.57\gamma_1^2$

$$f(y) = c\{(y-\xi_w)/\lambda\}^{c-1}\exp[-\{(y-\xi_w)/\lambda\}^{c}], \ y > \xi_w$$

$$\gamma_1 = (\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1)/\kappa_2^{3/2}, \ \gamma_2 = (\mu'_4 - 4\mu'_3\mu'_1 - 3\mu'^2_2 + 12\mu'_2\mu'^2_1 - 6\mu'^4_1)/\kappa_2^2$$

$$\mu'_r = \Gamma(r/c + 1), \quad \kappa_2 = \mu'_2 - \mu'^2_1$$

In each case, although there exist some departures compared with given theoretical functions in lower and upper tail region, the accuracy of the Cornish - Fisher series expansion with high order cumulants, which is computed by using the algorithm by Lee, *et al.* (1992), is sufficiently good regardless of the locus of (γ_1, γ_2) and the accuracy of Eq. (19) for the case of (a) is better than the series expansion in upper tail. The numbers in the brackets for the Cornish-Fisher series expansion indicate the maximum order of cumulants. It is worth noting that the maximum order of cumulant, which provides the best approximation, for the standard Gumbel distribution is different for $x<0$, i.e., the eighth order, and for $x>0$, i.e., the ninth order. Meanwhile, the accuracy of Eq. (19) with the method of moments depends on the locus of (γ_1, γ_2) in the outside of the boundary. For the case of (b), since x_U is less than - 3.0, the accuracy in the upper tail is shown as sufficiently good. However, for the case of (c) with the parameter of $c=2.5$, some discrepancies are shown in lower and upper tails because the two roots of x_L and x_U are close to each other. And also, for the case of $c=1.5$, the good approximation in upper tail breaks down over the figured range of x. From the figures, it can be indicated that Eq. (19) may be applicable in accordance with the purpose of application, even though the locus of (γ_1, γ_2) is located below the boundary line and then the PDF can not be defined in a strict sense.

The parameters in Eq. (40) or Eq. (42) can be approximately estimated from Table 1 in the following manner.

$$\{b, c, d\} = \{T_j\}^T[\{b_j\}\{c_j\}\{d_j\}] \tag{44}$$

in which, T_j indicates the second column in the table, and $[\{b_j\}\{c_j\}\{d_j\}]$ means the coefficients column vectors in the table, which are estimated from the results of Eq. (40) by means of the least squares method.

Table 1 Coefficients in Eq. (44) for the parameters of Eq. (19)

j	T_j	b_j		c_j		d_j	
		$\gamma_2<1.5$	$\gamma_2\geq1.5$	$\gamma_2<1.5$	$\gamma_2\geq1.5$	$\gamma_2<1.5$	$\gamma_2\geq1.5$
1	1	9.9996E-1	9.6982E-1	1.5632E-6	1.1607E-3	1.4790E-5	1.1225E-2
2	γ_1	-1.3648E-3	-3.0509E-2	1.6683E-1	1.5664E-1	7.3214E-4	1.2924E-2
3	γ_2	-1.2383E-1	-7.6541E-2	-6.5683E-6	-8.7296E-4	4.1217E-2	2.3597E-2
4	γ_1^2	1.2237E-1	5.5809E-2	1.9494E-3	-2.3561E-3	-4.6869E-2	-1.7687E-2
5	γ_2^2	3.5256E-2	5.3702E-3	-1.3070E-5	2.3911E-4	-1.3118E-2	-1.7871E-3
6	γ_1^3	-4.9116E-2	-3.4754E-2	6.5320E-2	4.6615E-2	2.5750E-2	2.1619E-2
7	γ_2^3	-8.5046E-3	-1.9598E-4	8.0788E-5	-1.6859E-5	3.2899E-3	6.4808E-5
8	$\gamma_1\gamma_2$	2.7301E-3	1.8146E-2	-3.9729E-2	-1.5542E-2	-9.2631E-4	-6.0632E-3
9	$\gamma_1^2\gamma_2$	-7.6755E-2	-1.2970E-2	1.7815E-2	2.3649E-2	3.1404E-2	8.7385E-3
10	$\gamma_1\gamma_2^2$	-7.5293E-3	-4.1057E-3	1.8298E-2	2.6161E-3	2.0800E-3	1.5638E-3
11	$\gamma_1^3\gamma_2$	1.3432E-2	2.9246E-3	-6.8372E-3	2.3482E-3	-1.0848E-2	-9.0710E-4
12	$\gamma_1\gamma_2^3$	7.0720E-4	2.6559E-4	-1.7732E-3	-1.8345E-4	1.0438E-3	-1.1160E-4
13	$\gamma_1^2\gamma_2^2$	-1.0054E-2	-1.9448E-4	-7.0658E-3	-2.8937E-3	1.7648E-3	-5.0330E-4
14	$\gamma_1^2\gamma_2^3$	1.0331E-2	1.1388E-4	1.3560E-2	5.2661E-5	2.1617E-4	-2.7428E-5
15	$\gamma_1^3\gamma_2^2$	-3.2158E-2	-1.7751E-4	-1.6720E-2	-1.1379E-3	1.6541E-2	-1.1518E-4
16	$\gamma_1^3\gamma_2^3$	1.2655E-2	2.1626E-5	2.0708E-2	4.7728E-5	-3.2886E-3	-1.2981E-7

Parameters: $b = \sum_{j=1}^{16} T_j b_j \quad c = \sum_{j=1}^{16} T_j c_j \quad d = \sum_{j=1}^{16} T_j d_j$

Table 2 Comparison between given (γ_1, γ_2) and values calculated from Table 1 and Eq. (40) in ( ) and from Eqs. (30,31) and Eq. (40) in [ ].

Given (γ_1, γ_2)	Calculated from Table 1 and Eq. (30,31)	Given (γ_1, γ_2)	Calculated from Table 1 and Eq. (30,31)
(0.0 , 0.0)	(9.38E-6 , 3.55E-4) [0.00, 0.00]	(-0.3 , 0.2)	(-3.00E-1 , 2.00E-1) [-2.99E-1, 3.18E-1]
(0.0 , 0.1)	(5.26E-6 , 9.98E-2) [0.00, 1.00E-1]	(-0.5 , 0.4)	(-5.00E-1 , 4.00E-1) [-4.96E-1, 7.21E-1]
(0.0 , 0.3)	(3.83E-6 , 3.00E-1) [0.00, 3.02E-1]	(-0.8 , 1.0)	(-7.99E-1 , 1.00E+0) [-7.87E-1, 1.80E+0]
(0.0 , 0.5)	(3.40E-5 , 5.01E-1) [0.00, 5.06E-1]	(-0.8 , 4.0)	(-7.89E-1 , 4.04E+0) [-8.03E-1, 5.38E+0]
(0.0 , 1.0)	(4.51E-4 , 1.00E+0) [0.00, 1.03E+0]	(-1.0 , 1.6)	(-9.99E-1 , 1.66E+0) [-9.79E-1, 2.84E+0]
(0.0 , 3.0)	(2.08E-3 , 2.99E+0) [0.00, 3.41E+0]	(-1.5 , 3.5)	(-1.49E+0 , 3.51E+0) [-1.45E+0, 6.25E+0]
(0.0 , 5.0)	(6.58E-3 , 5.06E+0) [0.00, 6.12E+0]	(-1.5 , 6.0)	(-1.51E+0 , 5.99E+0) [-1.46E+0, 9.48E+0]
(0.0 , 9.0)	(4.38E-3 , 9.00E+0) [0.00, 1.21E+1]	(-2.0 , 6.5)	(-1.98E+0 , 6.53E+0) [-1.89E+0, 1.14E+1]

Some calculation results are also shown in Table 2 with the results from Eq. (31). For the result from Table 1, each maximum difference is less than 0.06 for a given unbiased kurtosis and is sufficiently small for a given skewness, respectively. It is worth noting that Table 1 is for negative

and zero skewness. Therefore, if the coefficients for positive skewness are needed, it can be obtained by following manner: 1) calculate for negative skewness and 2) change the sign of c. Additionally, Eq. (44) is available for the region below the boundary line.

Meanwhile, the results from Eq. (31) are shown relatively good agreement compared with the given values for small skewness and unbiased kurtosis as discussed by Tognarelli, *et al.* (1997b). However, the accuracy of Eq. (31) becomes worse with increase in skewness and kurtosis, and especially, the estimated unbiased kurtosis are significantly departed from the given values. This tendency becomes distinct with increase in skewness. Such discrepancy may be caused by over simplified form of Eq. (31).

3.3. The method of probability plotting

As an alternative method for the estimation of the polynomial coefficients, the method of probability plotting is partly more flexible than the method of moments. This method is fundamentally based on an empirical cumulative distribution function (ECDF) of two RVs, x and y. There are basically two kinds of probability plots: quantile versus quantile plots (Q-Q plots) and percent versus percent plots (P-P plots). An elementary property of Q-Q plots is that the relationship between x and y directly appears in the corresponding Q-Q plot. If y is a nonlinear function of x, then the corresponding Q-Q plot will be nonlinear. This invariance property has made the use of Q-Q plots appealing and valuable. Moreover, where the variables have long tails, the Q-Q plot tends to emphasize the comparative structure in the tails and to blur the distinctions in the middle where the densities are high. However, the P-P plot is usually sensitive to discrepancies in the middle of a distribution rather than in the tails (Wilk *et al.* 1968). Thus, the former method is more appropriate for this study.

If the standard normal RV is used as reference quantiles, the Q-Q plot will show "skew correlation" between normal RV and non-normal RV. Therefore, if the ECDF of each RV is available, the skew correlation can be estimated. As an appropriate ECDF, Hazen's plot, i.e., a plot of the i-th ordered value as ordinate against $(i-1/2)/N$ as abscissa, is adopted in this study. The estimation procedure is as follows: (1) sort observed data y in ascending order, (2) estimate ECDF (α) for each y_i, (3) estimate α-quantiles of $x(x_\alpha)$ from unit normal distribution, (4) plot (x_α, y_α), and (5) approximate the function $y_\alpha=g(x_\alpha)$. If the number of each observed data, say N, is the same, the results of processes (2) and (3) can be used repeatedly. For the procedure of (5), the function can be approximated by the least squares method. If the squared mean error between y_i and $g(x_i)$ is denoted by S_e, parameters could be obtained by the minimization of S_e with respect to each parameter.

$$S_e = \frac{1}{N}\sum_{i=1}^{N}\{y_i - (a + bx_i + cx_i^2 + dx_i^3)\}^2 \tag{45}$$

From the condition $\{\partial/\partial a, \partial/\partial b, \partial/\partial c, \partial/\partial d\}S_e = \{0\}$, each parameter is estimated more directly as follows.

$$a = -c,\ b = \frac{5K_1 - K_3}{2},\ c = \frac{K_2}{2},\ d = \frac{K_3 - 3K_1}{6} \tag{46}$$

in which, $K_j = E[x^j y]$. The reproducibility of higher order statistics can be confirmed by substituting parameters into Eq. (40) or Eq. (42).

3.4. Comparison with experiment data

Fig. 3 compares the results of Eq. (19) using two kinds of parameter estimation methods and experimental data. The S_U function is also shown as a reference.

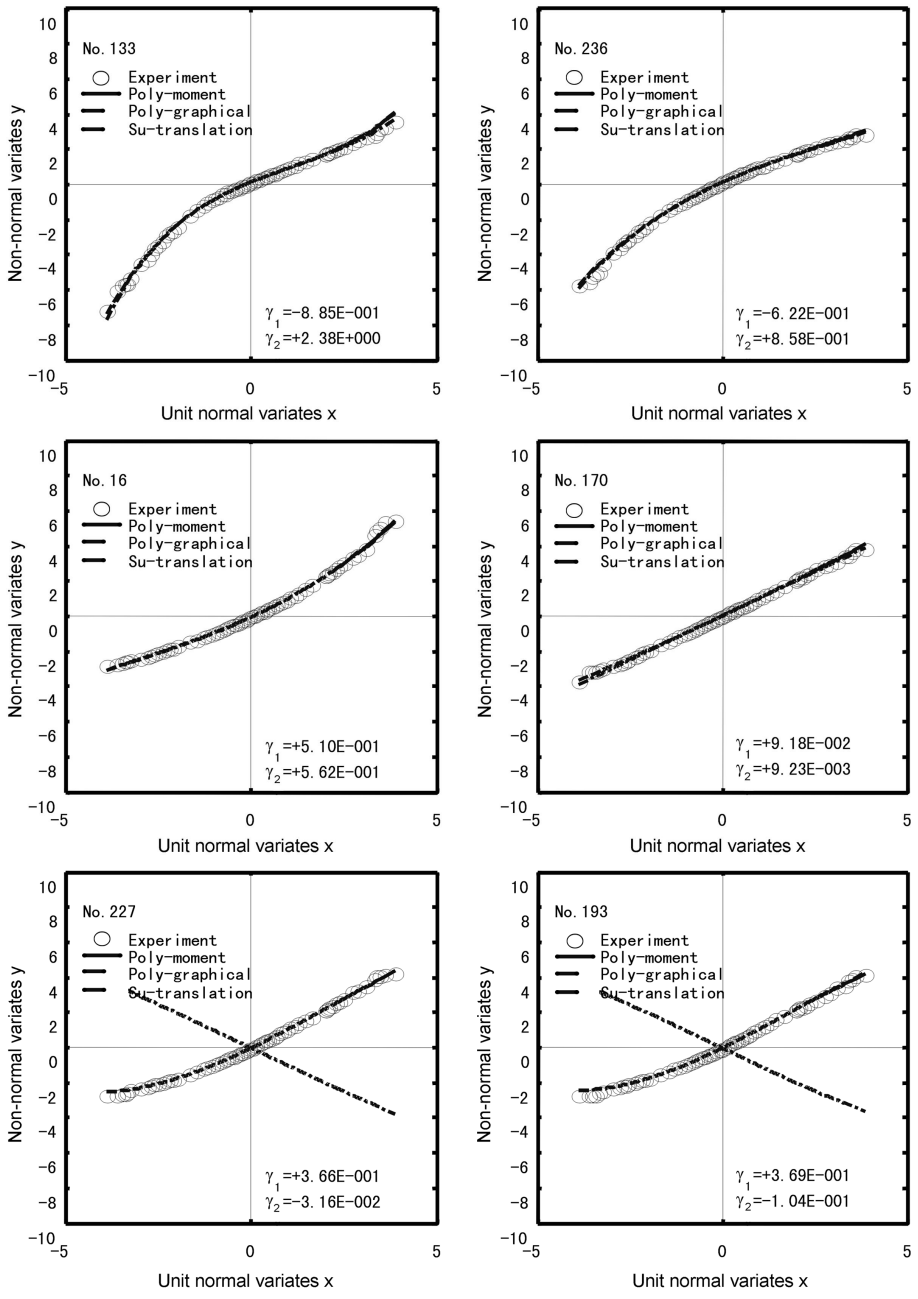


Fig. 3 Q-Q plots of experimental results with approximations by third order polynomial function and the S_U function

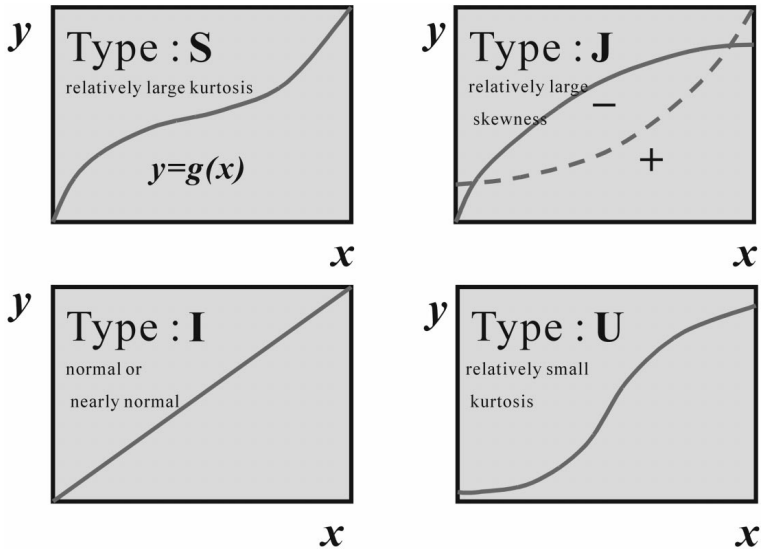


Fig. 4 Types of Q-Q plots for pressure signals measured from wind tunnel experiments

From the result, it is clear that Eq. (19) has almost the same degree of applicability as the S_U function, except in the region near the normal point and near the lognormal line. The higher order statistics reproducibility of Eq. (46) based on the Q-Q plot is fairly good compared with that from the method of moments, and their differences are less than 5% in most cases. Such differences may be reduced by the exclusion of some extremes in the observed data. It is worth noting that the tail features do not sufficiently appear in detail with a ten thousand of data and some difficulties always exist to decide whether the applied translation function is appropriate.

Tendencies of Q-Q plots can be briefly divided into four types as summarized in Fig. 4. These are 1) Type S (e.g., No.133 in Fig. 3), which has large kurtosis rather than skewness; PDF's tails elongate in both positive and negative directions, 2) Type J (e.g., No.16 and No.236), which has relatively large skewness rather than kurtosis; PDF's tail elongates in one direction, 3) Type I (e.g., No.170), similar to normal PDF and 4) Type U (e.g., No.193 and No.227), PDF's tails are bounded in both direction. Types S and I, which appear in the region of $\gamma_2 > 1.57\gamma_1^2$, can be approximated by Eq. (19) with high accuracy and with excellent reproducibility of higher order statistics. Meanwhile, for Type *J*, which appears below the lognormal line, Eq. (19) may not be an appropriate transformation in a strict sense because the bound in either tail cannot be perfectly approximated, cf. the standard gamma distribution shown in Fig. 2(b). This kind of limitation cannot be avoided in the application of polynomial function regardless of the maximum order, if the range of x is not restricted in a compact finite interval of $\Re$ or the approximated translation function is not ensured its monotonicity. Furthermore, Eq. (19) cannot be applied to Type *U*, which appears below the boundary line and it may be more preferable to select a different form of translation function. However, in most practical problems, even though we vaguely know the existence of bounds, to set a strict bound of *RV* is difficult and it generally depends on an engineering judgment. In this meaning, Type *U* distribution may be less important and an unbounded polynomial NST approximation including Eq. (19) is useful, even though it may lead to an overestimation of extremes. From the those reasons, Eq. (19) with the method of Q-Q plots could be a good choice as

shown in Fig. 3. It is worth noting that the loss of the reproducibility of given statistics does not always lead to a loss of approximation accuracy over the whole range of RVs, because higher order statistics are often strongly affected by a local fluctuation of observed data in the tails.

4. PDF and correlation distortion

4.1. Probability density function of y

Since $y=g(x)$ is a monotonic function, there is only one root. Therefore, the PDF of y can be deduced by using an inverse function $x=g^{-1}(y)$ as follows.

$$f_Y(y) = \frac{1}{\left| g'(g^{-1}(y)) \right|} \phi(g^{-1}(y)) \tag{47}$$

From a formula for the root of the third order polynomial, the inverse function $g^{-1}(y)$ can be expressed as

$$y' = g^{-1}(y) = T_1 + T_2 - \frac{c}{3d}, \quad g'(x) = 3dx^2 + 2cx + b \tag{48}$$

$$T_1 = \sqrt[3]{T_4 + \sqrt{T_3^3 + T_4^2}}, \quad T_2 = \sqrt[3]{T_4 - \sqrt{T_3^3 + T_4^2}}$$

$$T_3 = \frac{1}{9d^2}(3bd - c^2), \quad T_4 = \frac{1}{54d^3}\{9bcd - 27d^2(a-y) - 2c^3\}$$

in which $T_3 \geq 0$ might be guaranteed if the condition of monotonicity in Eq. (38) holds. Consequently, the PDF of y becomes

$$f_Y(y) = \frac{1}{\sqrt{2\pi(3dy'^2 + 2cy' + b)^2}} \exp\left\{ -\frac{y'^2}{2} \right\} \tag{49}$$

4.2. Correlation-distortion and its correction

In order to correct the distortion of the given spectral characteristics due to nonlinear transformation, two kinds of methods are available at present: 1) an iterative method based on the spectral density function by Yamazaki and Shinozuka (1988), and 2) an algebraic method for the correlation function. For the former method, some studies have been presented and it has been indicated that the method cannot be applied for the case of excessive skewness and kurtosis (Popescu, *et al.* 1998, Deodatis, *et al.* 2001). Meanwhile, the latter method dates from the era of Edgeworth. The basic concept can be formulated as follows.

Let R_T and R_{xx}^1 be a given auto-correlation function and it's a realization by simulation, respectively. Then, R_{xx}^1 is changed to R_{yy}^1 through the translation, denoted as Ψ_R. If the inverse function of Ψ_R exists, the distorted auto-correlation function R_{yy}^1 can be algebraically corrected by using the inverse function Ψ_R^{-1} as follows, and the corrected spectral density function can be obtained by applying the Wiener-Khintchine relationship to the corrected auto-correlation function R_{yy}^2.

$$R_{yy}^1 = \Psi_R(R_{xx}^1) \Leftrightarrow R_{xx}^1 = \Psi_R^{-1}(R_{yy}^1)$$

$$R_{xx}^2 = \Psi_R^{-1}(R_{xx}^1) \Rightarrow R_{yy}^2 = \Psi_R(R_{xx}^2) = \Psi_R\{\Psi_R^{-1}(R_{xx}^1)\} = R_{xx}^1 \approx R_T \tag{50}$$

For Eq. (19), according to the definition, the auto-correlation function of $y = a + bx + cx^2 + dx^3$ is obtained from

$$R_{yy}(\tau) = \sum_{j=0}^{3} a_j^2 E[x^j(t)x^j(t+\tau)] + \sum_{j=0}^{3}\sum_{k>j}^{3} a_j a_k E[x^j(t)x^k(t+\tau)] \tag{51}$$

in which, $a_0=a$, $a_1=b$, $a_2=c$, $a_3=d$. And, let $R_{jk}(\tau)=E[x^j(t)x^k(t+\tau)]$, $x^j(t)=x_1^j$, $x^k(t+\tau)=x_2^k$, $R_{x_1x_2}(\tau)=\rho$. Then, from the property of the standard normal distribution $R_{11}=\rho$, $R_{13}=R_{31}=3\rho$, $R_{22}=1+2\rho^2$, $R_{33}=6\rho^3+9\rho$. Consequently, $R_{yy}(\tau)$ can be expressed as follows (Edgeworth 1914):

$$R_{yy}(\tau)=6d^2R_{xx}^3(\tau)+2c^2R_{xx}^2(\tau)+(b^2+6bd+9d^2)R_{xx}(\tau)+(a+c)^2 \tag{52}$$

However, since $a+c=0$, i.e., Eq. (40a), $R_{yy}(\tau)$ can be rewritten in the following form. The inverse function Ψ_R^{-1} is derived from this equation.

$$R_{yy}(\tau) = \{a' + b'R_{xx}(\tau) + c'R_{xx}^2(\tau)\}R_{xx}(\tau) \tag{53}$$

in which, $a'=b^2+6bd+9d^2$, $b'=2c^2$, and $c'=6d^2$. In Eq. (53), $a'+b'+c'$ should be one from the condition of $Var(y)=1$, and it will be satisfied by the high reproducibility of the method. Therefore, $R_{yy}(\tau)$ has the same value as $R_{xx}(\tau)$ for $R_{xx}(\tau)=0$ and 1. However, for other values of $R_{xx}(\tau)$, $R_{yy}(\tau)$, is distorted as indicated by Ammon (1990). The maximum difference appears at $R_{xx}(\tau)=-1$ by as much as $2b'$. In Fig. 5, contour lines are drawn with respect to $2b'-1$=-0.95, -0.90, -0.85 and

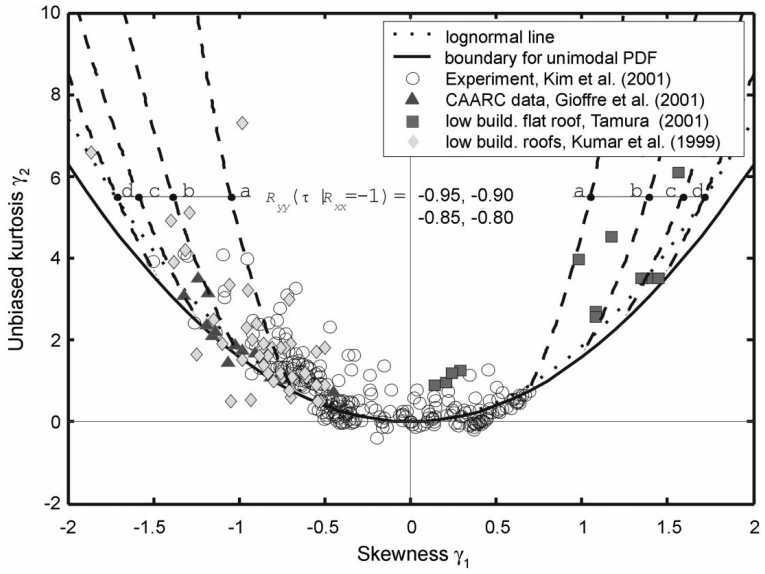


Fig. 5 Degree of correlation distortion with respect to skewness and unbiased kurtosis

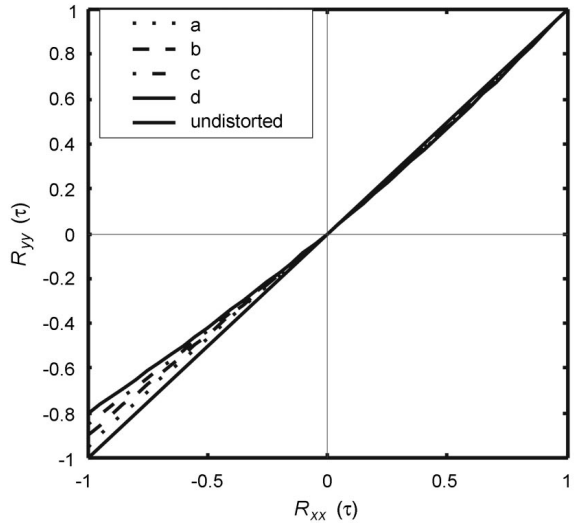


Fig. 6 Relation between distorted and undistorted autocorrelation function (*a, b, c* and *d* indicate contour lines shown in Fig. 5)

-0.80 with experimental data. In Fig. 6, $R_{yy}(\tau)$ for each $2b'-1$ value are shown with respect to $R_{xx}(\tau)$. The necessity of correction strongly depends on the spectral characteristics and given higher order statistics. For example, if the auto-correlation function is a positive definite function, e.g. $R_T=\exp(-\alpha|\tau|)$, $\alpha>0$, the isomorphic inverse function always exists and the distortion is not serious as discussed by Gioffre, *et al.* (2001a, 2001b) and Sakamoto, *et al.* (2002). However, for the case of $R_T = \exp(-\alpha|\tau|)\cos(\omega_0\tau)$, this means that the spectral density function has a spectral peak centered at ω_0, the necessity of correction strongly depends on the minimum auto-correlation coefficients and given statistics. If the minimum auto-correlation coefficient is too small and the given distribution is highly skewed such that (γ_1, γ_2) located below the boundary line, there exists such case that the inverse function has no root. Therefore, it can be said that the applicability of this method also has a limitation.

In the use of this procedure, it is also worth noting that Ψ_R is not always isomorphic even in the boundary for the monotonicity. The region on the (γ_1, γ_2) plane in which Ψ_R is not isomorphic, can be expressed approximately by $\gamma_2 < -0.03|\gamma_1^3| + 1.73|\gamma_1^2|$ or simply $\gamma_2 < 1.63\gamma_1^2$. Therefore, if given (γ_1, γ_2) is located in this region, the most similar value of $R_{xx}^2(\tau)$ compared with $R_{xx}^1(\tau)$ or R_T should be chosen from the three roots estimated by following equations.

$$R_{xx,j}(\tau) = 2\sqrt{Q'}\cos\left(\frac{\theta}{3}+(j-1)\frac{2}{3}\pi\right)-\frac{b'}{3c'}, j = 1, 2, 3 \quad (54)$$

$$Q' = \frac{1}{9c'^2}(b'^2-3a'c'),\ R' = \frac{1}{54c'^3}\{9a'b'c'+27c'^2R_{yy}(\tau)-2b'^3\}$$

$$\theta = \cos^{-1}(R'/\sqrt{Q'^3})$$

4.3. Numerical example

As an example for the preceded discussions, following conditions are considered.

1) Assumed one-sided power spectral density function and its auto-correlation function are following exponential cosine type functions. As required parameters, a=0.7 and f_0=1.0 are used.

$$G_T(f) = 2a\left\{\frac{1}{a^2+4\pi^2(f+f_0)^2}+\frac{1}{a^2+4\pi^2(f-f_0)^2}\right\},$$

$$R_T(\tau) = \exp(-a|\tau|)\cdot\cos(2\pi f_0\tau) \tag{55}$$

2) Desired probability density function of RV y is assumed as following exponential gamma distribution.

$$f(y) = \frac{1}{\Gamma(1/2)}\exp\left\{\frac{y}{2}-\exp(y)\right\} \tag{56}$$

For Eq. (56), first four statistics to be reproduced are as follows.

$$E\{y\} = 0, Var\{y\} = 1, \gamma_1 = -1.54, \gamma_2 = 4.0$$

3) From the given statistics, the third order polynomial function, i.e., Eq. (19), becomes as follows from Table 1.

$$y = g(x) = 0.2237 + 0.8559x - 0.2237x^2 + 0.0298x^3 \tag{57}$$

As such, the theoretically reproduced first four statistics are [0.0, 0.9993, -1.5387, 3.990].

4) In order to simulate Gaussian process, following spectral representation method is used and the number of frequency components is assumed as 2^{14} to ensure the normality and the cut-off frequency is set as 25 f_0.

$$y(t) = \sqrt{2}\sum_{k=1}^{m}\sqrt{G(k\Delta\omega)\Delta\omega}\cos(k\Delta\omega t+\theta_k) \tag{58}$$

The results are shown in Fig. 7 for a sample function. It is easy to find that the given spectral density function is distorted by translation through Eq. (57) as shown in Fig. 7 (2). Figures from (3a) to (6a) show the correction procedure based on Eq. (50). In this procedure, AR method with Levinson algorithm, denoted as AR-ACC, is used to estimate the auto-correlation function and MEM is adopted for spectral analysis for figurative simplicity. The correction results shown in Fig. (3a) and (4a) are quite satisfactory and the probability density distribution of RV y is well coincided with given theoretical function. In figures from (3b) to (6b), the results from the application of Yamazaki-Shinozuka's iterative correction algorithm, denoted as YSA, are shown. In Figs. (3b) and (4b), an incompleteness of the correction can be found on the right side of the peak and around the minimum auto-correlation coefficient. This incompleteness was not improved by increase of iteration count. However, it can be said that both methods are almost equivalent in the applicability under the assumption that the translation function is monotonic.

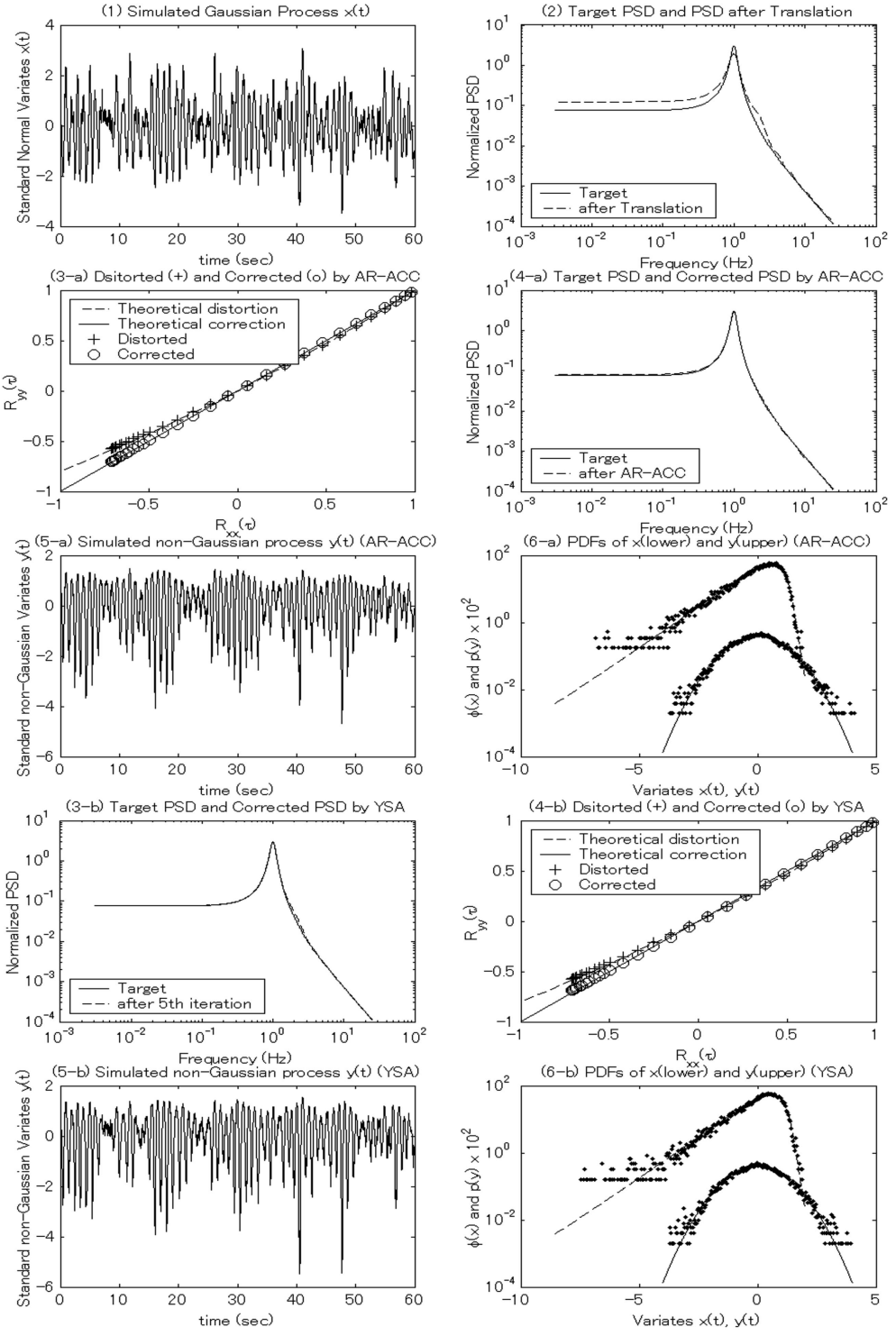


Fig. 7 Numerical example for the translation method with Eq. (19) and for the correction of the distorted spectral density function due to the translation by two different algorithms AR-ACC and YSA

5. Conclusions

From the end of the nineteenth century, many ideas have been examined for expressing the statistical characteristics of non-Gaussian events, and these efforts are still continuing. Some of these ideas, e.g. lognormal models, have achieved great success and are often utilized. However, since the characteristics of events happening in nature are so complex, no single specified model can be applied to all kinds of events. The lognormal model also has a significant restriction in application to wind pressure signals, which show a wide variation of skewness and kurtosis. Thus, based on a survey of previous studies, we have attempted to find a method without such restrictions to simulate non-Gaussian stationary stochastic processes including fluctuating pressure signals in separated shear layers. This study focused on the method of translation with third order polynomial function proposed by F.Y. Edgeworth. Even though there is a region of skewness and kurtosis in which the method cannot guarantee good reproducibility of higher order statistics and good approximation of quantile-quantile relationships, it shows very high applicability to non-Gaussian properties of pressure signals. It is also found that the applicability may be improved by an appropriate parameter estimation method that enables more direct parameter estimation with sufficient accuracy compared with a numerical iteration algorithm and the previously proposed formulae. An approximated parameter estimation worksheet may provide a convenience in practical applications. Furthermore, a method to correct a distortion in the auto-correlation function due to nonlinear transformation is reviewed and discussed on its limitation. On the other hand, some problems are still remained in unsolved. For example, more detailed study is needed to clarify the limitation of the iterative correction method indicated in some previous studies comparing the algebraic method discussed in this paper. And also, the accuracy of the approximations for theoretical translation function should also be investigated to apply the method to the problem related with extremes.

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