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Aeroelastic analysis of bridges using FEM and moving grids

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Abstract. In the recent years flow around bridges are investigated using computer modeling. Selvam (1998), Selvam and Bosch (1999), Frandsen and McRobie (1999) used finite element procedures. Larsen and Walther (1997) used discrete vorticity procedure. The aeroelastic instability is a major criterion to be checked for long span bridges. If the wind speed experienced by a bridge is greater than the critical wind speed for flutter, then the bridge fails due to aeroelastic instability. Larsen and Walther (1997) computed the critical velocity for flutter using discrete vortex method similar to wind tunnel procedures. In this work, the critical velocity for flutter will be calculated directly (free oscillation procedure) similar to the approaches reported by Selvam *et al*. (1998). It is expected that the computational time required to compute the critical velocity using this approach may be much shorter than the traditional approach. The computed critical flutter velocity of 69 m/s is in reasonable comparison with wind tunnel measurement. The no flutter and flutter conditions are illustrated using the bridge response in time.

Key words: computational fluid dynamics; bridge aerodynamics; computational wind engineering; large eddy simulation; flutter analysis; wind loading.

1. Introduction

In the recent years flow around bridges are investigated using computer modeling. Selvam (1998), Selvam and Bosch (1999), Frandsen and McRobie (1999) used finite element procedures. Larsen and Walther (1997) used discrete vorticity procedure. The aeroelastic instability is a major criterion to be checked for long span bridges. If the wind speed experienced by a bridge is greater than the critical wind speed for flutter, then the bridge fails due to aeroelastic instability. Larsen and Walther (1997) computed the critical velocity for flutter using discrete vortex method similar to wind tunnel procedures. In this work, the critical velocity for flutter will be calculated directly similar to the approaches reported by Selvam *et al*. (1998). It is expected that the computational time required to compute the critical velocity using this approach may be much shorter than the traditional approach.

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1.1. Fluid-structure interaction (FSI) modeling

In the fluid-structure interaction (FSI) modeling, the equation of motion of the structure and the fluid must be solved simultaneously. One difficulty in handling FSI problem is that the structural equations are formulated in the Lagrangian co-ordinate system and the fluid equations are formulated in the Eulerian co-ordinate system. Hence to solve both equations, a moving grid is needed at each time step for the fluid portion. Several different approaches are in use at this time. They are arbitrary Lagrangian-Eulerian (ALE) formulation (Nomura and Hughes 1992, Selvam *et al*. 1998 and Tamura *et al*. 1995), co-rotational approach (Murakami and Mochida 1995) and dynamic meshes (De Sampaio *et al*. 1993). The co-rotational approach may be easier to implement by adding extra terms in the Navier-Stokes(NS) equations for movements in one direction. For general problems it will be difficult to apply. In the dynamic mesh approach, for each time step a new mesh is formulated. This needs a very sophisticated grid generator. In the ALE approach, grid can be moved as a whole with constant velocity for each node as reported by Tamura *et al*. (1995) or with different velocity for each node and in some region no movement at all as reported by Selvam *et al*. (1998) and Nomura and Hughes (1992). Moving the grid as a whole is preferred for FSI problem since the structure has rigid body movement. If the structure is very flexible and each node on the structure is moving, then the other grid moving procedure has to be used. Also for this procedure the geometric conservation law has to be satisfied as discussed by Thomas and Lombard (1979) and Ferziger and Peric (1999) if not numerical instability may occur. Moving the grid as a whole may be computationally easy to apply. In the bridge flow modeling the bridge deck is assumed to be rigid and the first approach is used at this time.

1.2. Critical flutter velocity computation for bridges

The critical flutter velocity for bridges is calculated using forced motion and free motion of the bridge cross section as discussed by Enevoldsen *et al*. (1999) and Hansen *et al*. (1999). In the forced motion of the bridge, the aerodynamic derivatives of the bridge cross section are determined. The aerodynamic derivatives and their use in wind-tunnel experiments are reported in Dyrbye and Hansen (1996) and Simiu and Scanlan (1978). This method evolved from wind-tunnel experiment. The bridge cross section is forced to oscillate in pitching or heaving sinusoidal motion with a prescribed frequency and amplitude. The aerodynamic derivatives are calculated from the forces created during the forced motion through a least square minimisation. Larsen and Walther (1997) and Enevoldsen *et al*. (1999) used this procedure.

In the free motion of the bridge cross section, the aeroelastic stability of the cross section is observed directly. Here the cross section is elastically suspended in the flow and the stability of the cross section is observed for various wind speeds. The flow and pressure is computed for the given position of the bridge and then using this pressure the bridge will be moved to a new position due to the dynamic response of the structure. This process is continued in time. The plot of the bridge position in time for various approach wind speeds gives the detail of the aeroelastic stability. The critical flutter velocity may be calculated in few computer runs. The challenge is the accuracy of the numerical procedure. Frandsen and McRobie (1999), Enevoldsen *et al*. (1999), Nomura and Hughes (1992), Mendes and Branco (1995) and Selvam *et al*. (1998) use this procedure. Frandsen and McRobie (1999), and Envoldesn *et al*. (1999) did not give the details of the grid movement procedure. The others used it for different structures.

2. Objective

The objective of this paper is to study the issues involved in the computation of flow around bridges and to compute the critical velocity for flutter in a direct way using a moving grid. In the previous work (1998 and 1999), the finite element grids were refined by using 10,337 nodes to improve the drag coefficient C_d values for the Great Belt East Bridge (GBEB) sections. In this work further improvements in grid refinements using grid generators are reported. The turbulence is modeled using Large Eddy Simulation (LES) and the governing equations are solved by Finite Element Method (FEM).

2.1. Nomenclature

In the following discussion Reynolds number R_e, drag coefficient C_d, lift coefficient C_l and moment coefficient C_m and Strouhal number S_t are defined as :

$$\begin{aligned} R_e &= VB/\nu \\ C_d &= F_x/(0.5\rho V^2 BW) \\ C_l &= F_y/(0.5\rho V^2 BW) \\ C_m &= M/(0.5\rho V^2 B^2 W) \quad \text{and} \\ S_t &= H/(TV) \end{aligned} \tag{1}$$

Where B is the width, H is the height, and W is the length in the z direction of the bridge, V is the reference velocity, ν is the kinematic viscosity, F_x, and F_y are the drag and lift forces, M is the moment, T is the period of oscillation of the lift forces and ρ is the density. For 2D computation, W is considered to be one.

3. Computer modelling using LES

The flow around the bridge is represented using the Navier-Stokes equations. Numerical issues and turbulence modeling issues were discussed in detail in Selvam (1998 and 1999).

3.1. Governing equations for flow

In this work, the LES turbulence model is considered. The two and three-dimensional equations for an incompressible fluid using the LES model in general tensor notation are as follows :

$$\text{Continuity Equation: } U_{i,i} = 0 \tag{2}$$

Momentum Equation:

$$U_{i,t} + (U_j - V_j)U_{i,j} = -(p/\rho + 2k/3)_{,i} + [(\nu + \nu_t)(U_{i,j} + U_{j,i})]_{,j} \tag{3}$$

where : $\nu_t = (C_s h)^2 (S_{ij}^2/2)^{0.5}$,
$S_{ij} = U_{i,j} + U_{j,i}$,
$h = (h_1 h_2 h_3)^{0.333}$ for $3D$,
$h = (h_1 h_2)^{0.5}$ for $2D$,
and $k = (\nu_t/(C_k h))^2$.

Empirical Constants: $C_s = 0.15$ for $2D$ and 0.1 for $3D$, and $C_k = 0.094$

Where U_i, and p are the mean velocity and pressure respectively, V_i is the grid velocity, k is the turbulent kinetic energy, ν_t is the turbulent eddy viscosity, h_1, h_2, and h_3 are control volume spacing in the x, y, and z directions and ρ is the fluid density. Here the area or volume of the element is used for the computation of h. Here a comma represents differentiation, t represents time and $i = 1, 2$ and 3 mean variables in the x, y and z directions. To implement higher order approximation of the convection term (Selvam 1998) the following expression is used in Eq. (3) instead of $U_jU_{i,j}$:

$$(U_j - V_j)U_{i,j} - \theta\,[(U_j - V_j)(U_k - V_k)U_{i,j}]_{,k} / 2 \tag{4}$$

Depending upon the values of θ different procedures can be implemented. For balance tensor diffusivity(BTD) scheme $\theta = \delta t$ is used; where δt is the time step used in the integration. For streamline upwind procedure suggested, θ is considered as :

$$\theta = 1 / \max\,(|U_1| / dx,\ |U_2| / dy,\ |U_3| / dz) \tag{5}$$

Here dx dy and dz are the control volume length and U_1, U_2, and U_3 are the velocities in the x, y and z directions. In this computation $\theta = \delta t$ is used. This has less numerical diffusion as compared to benchmark problems in Selvam (1998). For moving grid the maximum of the BTD or 0.3 times Eq. (5) is considered for better stability of the solution.

3.2. Governing equations for the bridge

Since the flow around the bridge is solved using the non-dimensional NS equations, the structural dynamic equations for the bridge are also solved in a non-dimensional form. The bridge is assumed to have pitching and heaving motion. The structural properties of the GBEB suspended span as reported by Larsen and Walther (1997) are as follows :

Mass moment of inertia $m = 22.7 \times 10^3$ kg/m
Rotational mass moment of inertia $I = 2.47 \times 10^6$ kg.m^2/m
Pitching frequency $\omega_p = 1.709$ rad/s
Heaving frequency $\omega_h = 0.622$ rad/s

The bridge rotates about the shear center and moves vertically from the center of gravity. Since both are along the line of symmetry both are uncoupled. The equations of motion for the pitch, p and heave, h are as follows :

$$\begin{aligned} (p_{,t})_{,t} + \omega_p^2\, p &= C_m(0.5\rho V^2 B^2) / I \\ (h_{,t})_{,t} + \omega_h^2\, h &= C_l(0.5\rho V^2 B) / m \end{aligned} \tag{6}$$

Non-dimensionalizing the length by B and time by B / V, where B is the width of the bridge and V is the reference velocity and simplifying the equations one get

$$\begin{aligned} (p_{,t})_{,t} + (1 / u^*)^2\, p &= 0.5C_m / R_p \\ (h_{,t})_{,t} + (\omega_h / [\omega_p u^*])^2\, h &= 0.5C_l / R_h \end{aligned} \tag{7}$$

Here u^* is the reduced velocity and is equal to $V / (\omega_p B)$, $R_p = I / (\rho B^4)$ and $R_h = m / (\rho B^2)$. In this work $R_p = 2.178$ and $R_h = 19.236$ for the air density $\rho = 1.228$ kg/m^3 are used. The above equations are solved explicitly using the central difference method.

3.3. Finite element scheme to solve NS equations

The NS equations are solved using an implicit method suggested in Selvam (1998). The four-step advancement scheme for Eqs. (2) and (3) is as follows :

Step 1 : Solve for U_i from Eq. (3). The diffusion and higher order convection terms are considered implicitly to be in the current time and the first order convection terms are considered explicitly from the previous time step. The pressure is considered in the right hand side of the equation. This set of equations leads to a symmetric matrix and the preconditioned conjugate gradient (PCG) procedure is used to solve. For simplicity here on p / ρ is considered as p.

Step 2 : Get new velocities as $U_i^* = U_i + \delta t(p_{,i})$ where U_i is not specified

Step 3 : Solve for pressure from $(p_{,i})_{,i} = U_{i,i}^* / \delta t$

Step 4 : Correct the velocity for incompressibility: $U_i = U_i^* - \delta t(p_{,i})$ where U_i is not specified

Step 2 eliminates the checkerboard pressure field when using equal order interpolation for velocity and pressure in the case of FEM. Implicit treatment of the convective and diffusive terms eliminates the numerical stability restrictions. In this work the time step is kept for CFL (Courant-Frederick-Lewis) number less than one. The above NS equations are approximated by FEM procedure. The velocity and pressure are approximated using equal order interpolation. Eight noded brick element is used for 3D and four noded quadrilateral is used for 2D.

The equations are stored in a compact form as discussed in Selvam (1998). The equations are solved by preconditioned conjugate gradient (PCG) procedure. To solve the velocities an underrelaxation factor of 0.7 is used. The iteration is done until the absolute sum of the residue of the equation reduces to 1×10^{-7} times the number of nodes for each time step. Usually the pressure and momentum equations take about 50 and 10 iterations for PCG solution respectively.

3.4. Boundary and initial conditions

The cross section of the GBEB suspension span used for computation is shown in Fig. 1. The

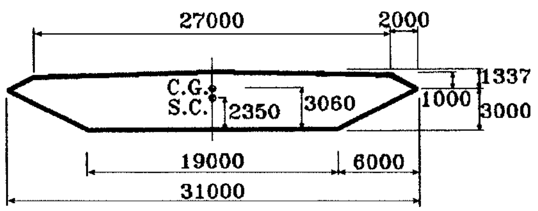

Fig. 1 Cross section of the Great Belt East Bridge suspension span. All dimensions are in mm

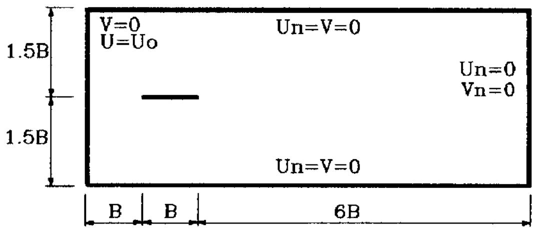

Fig 2. Solution region and boundary conditions

computational region and boundary conditions are shown in Fig. 2 for the fixed grid. The cylinder surface has no slip condition. The upstream boundary has uniform velocity of one in the x direction and zero in the y direction. At the outflow boundary the normal gradient of the velocities are zero and the sides have slip boundaries. Computation is done for Re of 10^5.

4. Results

4.1. Computation for rigid bridge

The FEM grid used for illustration here has 14,805 nodes and 14,570 elements. Around the bridge the grid has 215×63 points as shown in Fig. 3. The minimum grid spacing close to the bridge deck is about 0.0015B. The time step may be around 0.0004 sec. The flow is run for 60 sec.

The computed Cd of 0.062 and Strouhal number St of 0.14 is in good comparison with the wind-

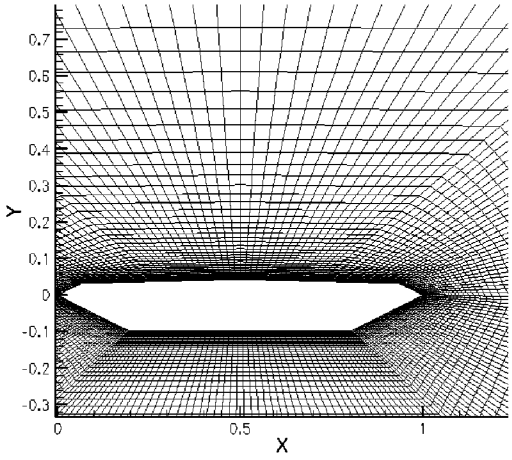

Fig. 3 Finite element grid for GBEB suspension span

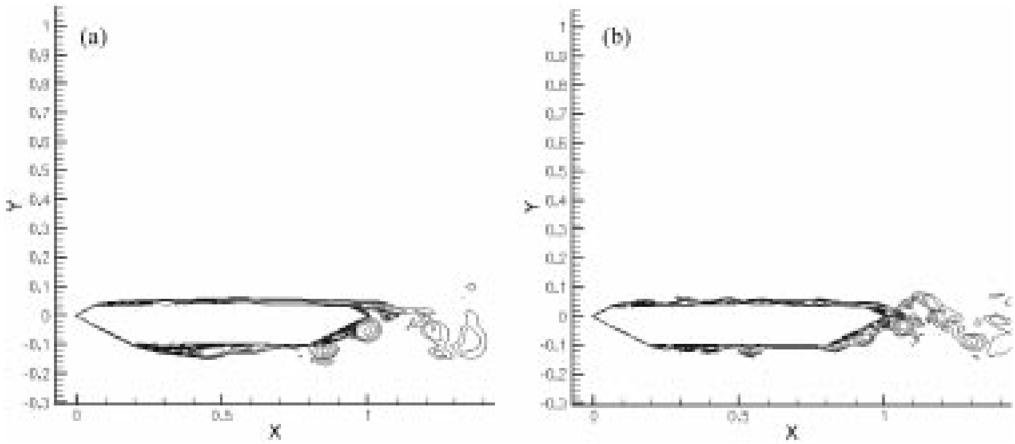

Fig. 4 Vorticity contour diagram using (a) LES model (b) no LES model

tunnel measurements as reported by Larsen and Walther (1997) for the static case at an angle attack of zero degree. The flow features developed were also in reasonable comparison with Larsen and Walther. Previously Selvam and Bosch (1999) could not develop the vortices on the top and bottom decks due to limitation of grid refinements. The flow features on the top and bottom are shown in Fig. 4. The flow is computed either considering turbulence model LES (Fig. 4a) or no LES (Fig. 4b). The second one has much more vortices at the bottom.

4.2. Computation for flexible or moving bridge

The same grid is used for computing the aeroelastic stability. The computed flow using the fixed grid in the previous section is used as the start up solution for computation. This saves computer time when runs are made for many cases. Initially a perturbation of 1.8 degrees is provided and the response of the bridge in time is studied to see if the reduced velocity u^* is above or below the critical flutter velocity. Based on that, u^* values ranging from 0.4 to 1.4 are considered for computation. If the reduced velocity u^* is below the critical velocity for flutter, the oscillations dies down gradually in time as shown in Fig 5. If u^* is above the critical velocity for flutter, the oscillations grows up till the bridge fails (Fig 6). The response of the structure in time are plotted for $u^* = 0.4$ and 1.4 for illustration. The critical velocity for the onset of flutter is determined from the plot of the pitch angle versus time. The aerodynamic damping is positive as long as the pitch angle decreases in time and vice-versa. The plot of the pitch angle versus time is analyzed to study the extent of growth or decay. The rate of growth and decay is found by averaging the change in amplitude values of the last two periods of the pitch angle vs. time plot. These rates are plotted for each u^* value and the point where the plot crosses the zero decay/growth line is found. This point represents the critical value of u^* for the onset of flutter.

It is clearly shown in Fig. 5 that when the velocity ($u^* = 0.4$ and $V = 21.2$ m/s) is less than the critical flutter velocity (70–75 m/s as reported by Enevoldesen *et al.* 1999 from wind tunnel study), the perturbation gradually dies down i.e., no flutter condition is observed. When the velocity ($u^* = 1.4$ and $V = 74.2$ m/s) is higher than or closer to the critical velocity, flutter occurs as shown in

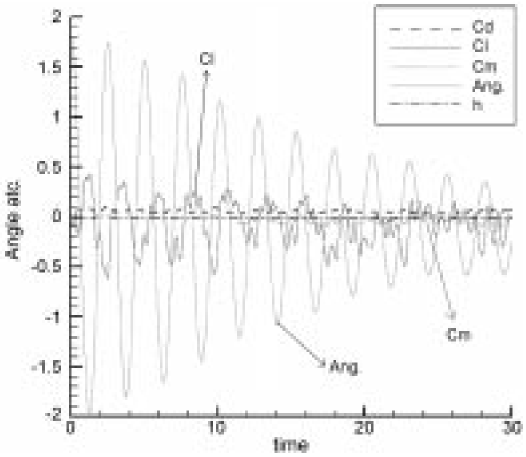

Fig. 5 Bridge response for reduced velocity u^* of 0.4 (no flutter condition)

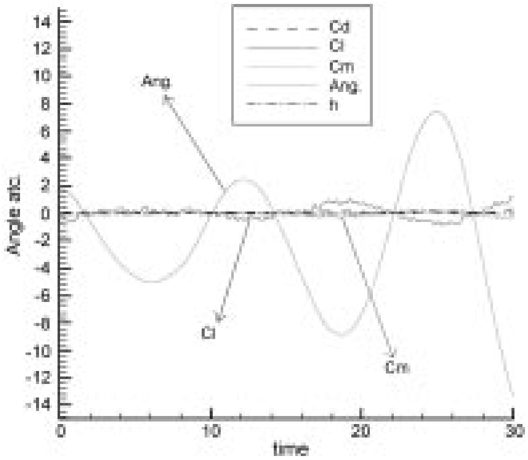

Fig. 6 Bridge response for reduced velocity u^* of 1.4 (flutter condition)

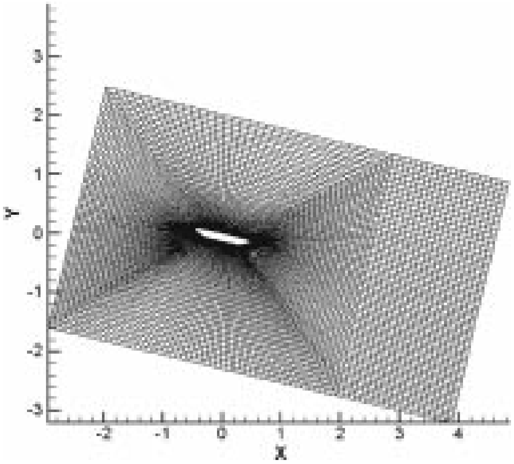

Fig. 7 Bridge and grid position at the end of 30 sec. for $u^* = 1.4$

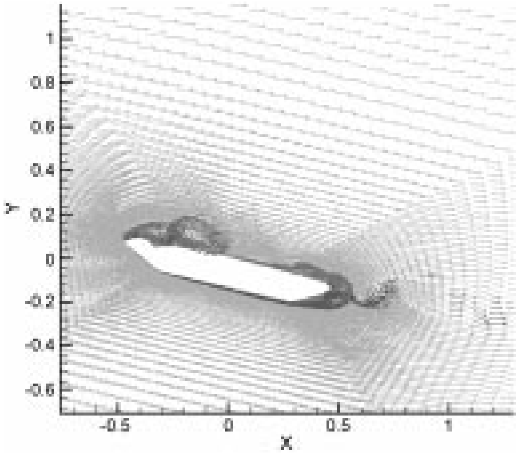

Fig. 8 Close up of the velocity vector diagram for $u^* = 1.4$

Fig. 6. In our case, it is estimated that critical flutter velocity occurs at $u^* = 1.3$ or for a reference velocity of 69 m/s. A time step of 0.001 is used in these computations. Further work is underway to study for longer time duration and to try much accurate solution procedures for the fluid-structure interaction problem.

The grid position at the end of 30 sec. for $u^* = 1.4$ is shown in Fig. 7. The grid is rotated by about 14 degrees from its original position. The velocity vector diagram is also plotted for this case in Fig. 8. One can see the prominent vortices on the top front of the bridge.

5. Conclusions

The flow around the Great Belt East Bridge (GBEB) suspension span is computed using finite element procedure. The turbulence is modelled using large eddy simulation model. The flow is computed for *Re* of 10^5. In this work a reasonably wellrefined grid with 14,805 nodes are considered. The computed *Cd* and *St* are in comparison with wind tunnel measurement.

Numerical procedures related with moving grids are discussed. One of the ALE procedures is selected for the computation of moving grids. In this work the critical flutter velocity is computed using free oscillation technique. The computed critical velocity of 69 m/s is in reasonable comparison with wind tunnel measurement and numerical modelling by Larsen and Walther (1997). The program is verified with proper illustration of no-flutter and flutter condition for $u^* = 0.4$ and 1.4 respectively. Over all the FEM procedure is viable for practical application.

Further work is underway to study for longer time duration and to try much accurate solution procedures for the fluid-structure interaction problem. The computation of critical flutter velocity using forced and free oscillation procedures will be investigated.

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