

CFD calculations of indicial lift responses for bluff bodies

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Abstract. Two-dimensional formulations for wind forces on elongated bodies, such as bridge decks, are reviewed and links with expressions found in two-dimensional airfoil theory are pointed out. The present research focus on indicial lift responses and admittance functions which are commonly used to improve buffeting analysis of bluff bodies. A computational fluid dynamic (CFD) analysis is used to derive these aerodynamic functions for various sections. The numerical procedure is presented and results are discussed which demonstrate that the particular shapes of these functions are strongly dependent on the evolution of the separated flows around the sections at the early stages.

Key words: indicial lift response; admittance function; buffeting forces; CFD; RNG $k-\varepsilon$.

1. Introduction

In a series of papers, Scanlan and his coworkers (1993, 1999) recalled that “buffeting forces acting on elongated bodies like long-span bridges have been described by theories and analytical formats strongly influenced by analogous expressions found in two-dimensional airfoil theory”. Hence, dimensionless indicial and admittance functions have been extensively employed to improve analytic formulations of buffeting forces for bluff bodies: indicial functions have been used to describe the time history of wind-induced forces associated with a gust of variable velocity and admittance functions to express the frequency dependence of these forces.

In the case of thin airfoils, the mathematical formulation of such typical functions was based on potential flow theory. Wagner (1925) derived the indicial response of an airfoil to a step change in angle of attack and Küssner (1936) the loading response to an airfoil penetrating a sharp edged gust. Using the same mathematical base, Theodorsen (1935) derived the admittance function of an airfoil undergoing complex vertical and torsional oscillatory motions and Sears (1941) developed expression for a thin airfoil penetrating a vertically oscillating gust field.

Bluff bodies, which initiate detached flows, do not have the benefit of such a mathematical base. In this context, airfoil-type functions are unable to depict wind actions and can be viewed only as suggested approximations. However, analogous response functions can be determined for lift associated with step changes in the incoming flow properties, such as a step change in the angle of attack (Wagner-type indicial function) or step change in vertical velocity (Küssner-type indicial function).

The aim of this study is to demonstrate that a computational fluid dynamic (CFD) analysis can be used to derive these functions for elongated bluff bodies, such as bridge decks.

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2. Buffeting analysis

2.1. Time-dependent analysis

Current theories for prediction of wind induced vibrations due to air-turbulence are based upon a quasi-static formulation, which relates the buffeting forces to the incoming wind velocity components. In this format the lift is given by

$$L_b(s) = \frac{1}{2}\rho U^2 B \left[2C_{l_{\alpha_0}} \frac{u_x(s)}{U} + C'_{l_{\alpha_0}} \frac{u_z(s)}{U} \right] \quad (1)$$

in which $s = Ut / B$ is a dimensionless time, $C_{l_{\alpha_0}}$ and $C'_{l_{\alpha_0}}$ the steady lift coefficient and lift slope with angle of attack α_0 , B the structure chord length, ρ air density, U the cross flow velocity, u_x and u_z the along-wind and vertical gust velocity components respectively. Analogous formulations can be obtained for drag and moment expressions.

Following, as a guide method, the thin airfoil theory, these formulations may be improved through the introduction of indicial functions Φ_{l_x} and Φ_{l_z} . The lift may be written as

$$L_b(s) = \frac{1}{2}\rho UB 2C_{l_{\alpha_0}} \int_{-\infty}^s u_x(\sigma) \Phi'_{l_x}(s - \sigma) d\sigma + \frac{1}{2}\rho UBC'_{l_{\alpha_0}} \int_{-\infty}^s u_z(\sigma) \Phi'_{l_z}(s - \sigma) d\sigma \quad (2)$$

With the change of variables $s - \sigma = \tau$, this equation becomes

$$L_b(s) = \frac{1}{2}\rho UB 2C_{l_{\alpha_0}} \int_0^\infty u_x(s - \tau) \Phi'_{l_x}(\tau) d\tau + \frac{1}{2}\rho UBC'_{l_{\alpha_0}} \int_0^\infty u_z(s - \tau) \Phi'_{l_z}(\tau) d\tau \quad (3)$$

In the frequency domain this equation is

$$\hat{L}_b(k) = \frac{1}{2}\rho U^2 B \left[2C_{l_{\alpha_0}} \frac{\hat{u}_x(k)}{U} \hat{\Phi}'_{l_x}(k) + C'_{l_{\alpha_0}} \frac{\hat{u}_z(k)}{U} \hat{\Phi}'_{l_z}(k) \right] \quad (4)$$

in which $\hat{L}_b(k)$ is the Fourier transform of $L_b(s)$ and $k = B\omega / U$ the dimensionless oscillation frequency. In these equations, the indicial functions define the character of the transient lift response to a step change in the characteristics of the incoming flow. In 2D-airfoil transient lift theory, the resulting expressions for these functions were found to have the useful approximate form :

$$\Phi(s') = 1 - ae^{-bs'} - ce^{-ds'} \quad (5)$$

where a, b, c, d are constants and $s' = \{Ut / (B/2)\}$ is a dimensionless time. Considering the indicial response of a thin airfoil to a step change in angle of attack, Wagner (1925) originally found

$$a = 0.165 \quad b = 0.0455 \quad c = 0.335 \quad d = 0.300$$

Considering the loading response to an airfoil penetrating a sharp edged gust, Küssner (1936) found

$$a = c = 0.500 \quad b = 0.130 \quad d = 1.00$$

Both Küssner and Wagner used potential flow theory. For bluff bodies, which initiate detached flows, no general theory exists for deriving indicial functions. In this context, they have been identified through experiment. In a representative case, Scanlan, Beliveau and Budlong (1974) have shown that the indicial function of a bridge deck has the same useful form (5) but that its particular shape, Fig. 4, is strikingly different from the corresponding function for an airfoil.

2.2. Spectral analysis

In wind engineering since the spectral approach, developed by Davenport (1962), remains the most widely accepted method, the spectral form of (4) is used

$$S_{L_b}(k) = \frac{1}{4} \rho^2 U^4 B^2 \left[4C_{l_{\alpha_0}}^2 \frac{S_{u_x}}{U^2} \chi_{l_x}^2 + C_{l_{\alpha_0}}'^2 \frac{S_{u_z}}{U^2} \chi_{l_z}^2 \right] \quad (6)$$

This expression represents a simplification of the complete equation, since the cross power spectral densities of wind turbulence components $S_{u_x u_z}$ and $S_{u_z u_x}$ have been assumed to be negligible (this is a commonly used assumption). In this formulation, $\chi_{l_x}^2(k)$ and $\chi_{l_z}^2(k)$ are so-called “aerodynamic admittance” related to the indicial functions

$$\chi_{l_x}^2 = \hat{\Phi}_{l_x}'(k) \hat{\Phi}_{l_x}^{*}(k) \quad \chi_{l_z}^2 = \hat{\Phi}_{l_z}'(k) \hat{\Phi}_{l_z}^{*}(k) \quad (7)$$

These frequency-based transfer functions relate the spectrum of the incoming flow velocity fluctuations to the spectrum of the force fluctuations experienced by the structure. It should be noted that in Eq. (6), a separate admittance factor is assigned to each component and that the standard quasi-static formulation (1) implies constant unit admittances. Alternate formulation for aerodynamic lift force spectrum, with a single admittance factor, have been also used

$$S_{L_b}(k) = \frac{1}{4} \rho^2 U^4 B^2 \left[4C_{l_{\alpha_0}}^2 \frac{S_{u_x}}{U^2} + C_{l_{\alpha_0}}'^2 \frac{S_{u_z}}{U^2} \right] \chi_l^2 \quad (8)$$

For the vast majority of all structures, $C_{l_{\alpha_0}}$ is significantly less than the slope $C_{l_{\alpha_0}}'$ (for the symmetrical sections, with angle of attack around 0° , this coefficient is very small). Hence the contribution associated with the longitudinal velocity can be assumed to be negligible. Therefore, both Eq. (6) and Eq. (8) reduce to

$$S_{L_b}(k) \approx \frac{1}{4} \rho^2 U^4 B^2 C_{l_{\alpha_0}}'^2 \frac{S_{u_z}}{U^2} \chi_{l_z}^2 \quad (9)$$

This format quantitatively defines $\chi_{l_z}^2$ as a transfer function relating the spectrum of incident vertical gusting velocity to that of associated lift. Noting further that the term $C_{l_{\alpha_0}}' \chi_{l_z}$ can be seen as the term $C_l'(k)$ used by Larose (1997). Therefore the admittance function χ_{l_z} can be seen as the ratio of the unsteady lift slope to the steady case

$$\chi_{l_z} = \frac{C_l'(k)}{C_l'(k=0)}$$

Sears (1941) derived an expression for the admittance of a thin airfoil penetrating a sharp edge

gust with sinusoidal vertical velocity. This expression, known as the Sears' function, has been approximated by Liepmann (1952) :

$$\chi_s^2 = \frac{1}{1 + 2\pi k}$$

The Sears function, usually designated by χ_s and the Küssner function, usually designated by Ψ , are related by a Fourier transform relationship

$$\chi_s = \int_0^\infty \Psi'(\sigma) e^{-ik\sigma} d\sigma = ik \int_0^\infty \Psi(\sigma) e^{-ik\sigma} d\sigma \quad (10)$$

Hence, by using Eq. (5), an expression of the complex Sears function can be obtained. Since the use of admittance function in wind engineering is an extension of its original use in aeronautical context, the Liepman approximation of Sears function was extensively used in buffeting analysis of bluff bodies. But experimental studies have shown that this assumption was not appropriate, even for streamlined bridge deck sections. In this case a CFD acquisition of indicial and/or admittance functions could be a step toward predicting the cross-wind excitation induced by the incident wind.

3. CFD analysis

3.1. Outline of numerical procedure

A finite-element flow solver, CASTEM 2000, has been used to predict indicial lift responses of various sections, such as a NACA0012 airfoil, a rectangular section ($H/C = 0.12$), a bridge deck (Pont de Normandie), penetrating (or enveloped by) a sharp edged gust. Since the present research investigates only indicial responses of along-wind sections of structures sufficiently long in the across-flow direction, a two-dimensional model has been retained for the CFD analysis. The method used has been drawn from the one suggested by Brar, Raul and Scanlan (1996) to calculate flutter derivatives :

1. The wind flow across the section is computed with 0° angle of attack ($u_z = 0$).
2. Next, the vertical velocity of the flow field upstream of the section is changed to a value u_{zg} (the gust represents an instantaneous change in the vertical wind speed).
3. The unsteady-lift response $C_l(s)$ is calculated by integrating the pressure along the boundary of the section as the gust convects with the freestream.
4. The unsteady-lift function is normalized to its steady state value. Hence, the gust-penetration function, which is usually referred to as the Küssner-type indicial function, is defined as

$$\Psi(s) = \frac{C_l(s)}{C_l' \frac{u_{zg}}{U_0}}$$

When the gust propagation speed is the freestream flow velocity U_0 , the indicial function reduces to this commonly used function. Similarly, when the gust propagation speed is infinite, as in Brar *et al.* (1996), the angle of attack over the entire section is changed instantaneously, therefore the unsteady-lift function becomes that for indicial change of angle of attack, which is usually referred to as the Wagner-type indicial function.

As noted earlier, a theoretical link has been established between Küssner indicial function and Sears admittance function. Analogously it has been attempted to determine admittance functions directly from the computed indicial functions by using the Fourier transform relationship (10).

3.2. Governing equations

In the atmospheric boundary layer the wind is an incompressible, unsteady, turbulent flow governed by the Navier-Stokes equations. For solving practical problems in the area of wind engineering, successful calculations have been obtained by solving numerically the Reynolds Average Navier-Stokes(RANS) equations and by taking into account the turbulence effects by a closure model, Rodi (1995). Of the models used, the most popular are the two-equation first-order k - ε models. However the standard k - ε model, developed by Launder and Spalding (1974), is known to product poor results when applied to unsteady separated flows over bluff bodies, Murakami (1997). A variety of modified versions have been proposed to improve the performance of the model among which is the renormalization group(RNG) k - ε model. The use of this model to predict unsteady wind-loading has been validated by the authors in a previous work, Turbelin (2000). Hence, throughout this study, the RNG k - ε turbulence model has been used to computed indicial lift responses of bluff bodies. It should be noted that this model, proposed by Orszag and Yakhot (1993) for high-Reynolds number flows, incorporates modified constants and a new production term in the equation for the dissipation. For the present case, the variables have been rendered nondimensional with regards to the section chord B and the freestream flow velocity U_0 . The governing equations to solve are the ensemble-average Navier-Stokes and the continuity equations

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{1}{Re} + \nu_t \right) \left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] - \frac{\partial p}{\partial x_i} \quad (11)$$

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (12)$$

in which U_i and p are the mean dimensionless velocity and pressure, $Re = U_0 B / \nu$ the Reynolds number and $\nu_t = C_v k^2 / \varepsilon$ the eddy viscosity. The transport equations for the dimensionless turbulent kinetic energy k and dissipation ε are

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\frac{1}{Re} + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \varepsilon \quad (13)$$

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\frac{1}{Re} + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{\varepsilon_1}^* \frac{\varepsilon}{k} P_k - C_{\varepsilon_2} \frac{\varepsilon^2}{k} \quad (14)$$

Where P_k is the turbulent generation term

$$P_k = \nu_t \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

The constants are

$$C_v = 0.0845 \quad C_{\varepsilon_1} = 1.42 \quad C_{\varepsilon_2} = 1.68 \quad \sigma_k = \sigma_\varepsilon = 0.7179$$

$$C_{\varepsilon_1}^* = C_{\varepsilon_1} - \frac{\eta \left(1 - \frac{\eta}{\eta_0}\right)}{1 + \beta \eta^3} \quad \eta = \sqrt{\frac{P_k k}{\nu_t \varepsilon}}$$

in which $\eta_0 = 4.377$ and $\beta = 0.012$.

The flows Reynolds numbers involved in wind engineering are usually quite large, $Re = O(10^7)$ or higher. In this study the Reynolds number across the sections (based on structure chord and average inlet velocity) is taken to be 8.10^6 (experimental results at such Re are found in the literature for the NACA0012 airfoil). It should be noted that at such large Re the flow is fully turbulent.

3.3. Numerical details

The spatial discretization of the model equations has been obtained by a finite element method (Galerkin weighted residual method) using Q_1-P_0 (bilinear velocity, constant pressure) quadrilateral elements stabilized with a macro-element condition, Kechkar and Silvester (1992). The time discretization has been obtained by a semi-implicit first order scheme (implicit for pressure and explicit for velocities and all other unknowns). The time step has been automatically adjusted according to the stability conditions. To suppress propagating oscillations due to the convective terms in Galerkin finite element discretization, the streamline-upwind/Petrov-Galerkin(SUPG) concept with an additional discontinuity-capturing(DC) term has been used, Hughes, Mallet and Mizukami (1986).

4. Calculations

4.1. Computational domain

In all cases treated, the computational domain used was $4B$ long upstream from the leading edge of the section, $12B$ long downstream of the section and $4B$ wide on each side of the section. The calculations have been carried out on a 154×92 grid refined in the vicinity of wall. The grids are plotted in Figs. 1 to 3.

4.2. Initial and boundary conditions

The flow with 0° angle of attack has been first computed with freestream Dirichlet conditions imposed at the inflow boundary. These conditions have been defined by prescribing distribution of velocity, turbulence kinetic energy and dissipation rate. For this study the velocity profile was uniform and the condition for k and ε represented a turbulence intensity, designed by I , of 2%. In dimensionless form, we let

$$U_{in} = 1 \quad k_{in} = 1.5I^2 \quad \varepsilon_{in} = \frac{k_{in}^{3/2}}{l_k/B}$$

in which $l_k = 0.003B$ is the estimated length scale of the energy-containing eddies. Stress free conditions were automatically imposed on the other boundaries (inherently by the finite element formulation) and wall functions have been employed to estimate the wall shear stresses on the solid surfaces.

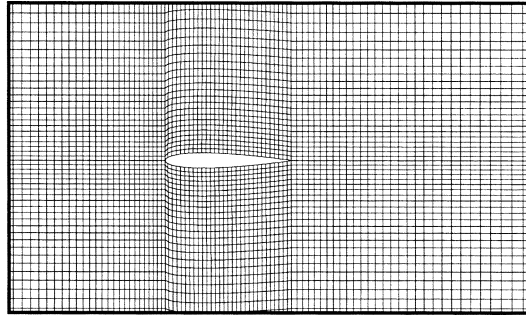


Fig. 1 Finite element mesh in the vicinity of the airfoil

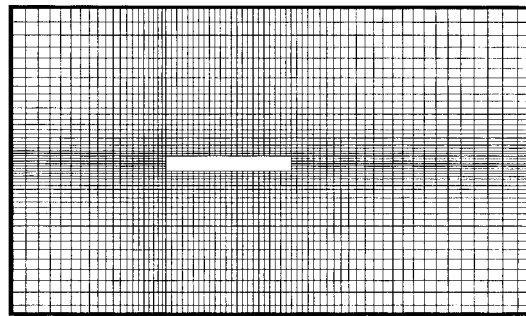


Fig. 2 Finite element mesh in the vicinity of the rectangular section

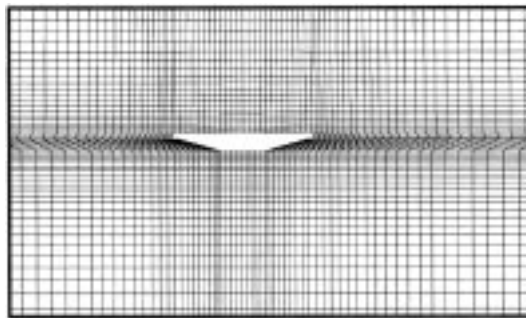


Fig. 3 Finite element mesh in the vicinity of the bridge

Next, the vertical velocity of the flow field upstream of the airfoil was changed to a chosen value $u_{z_0}/U_0 = 0.0875$. This gust of uniform upward velocity corresponds to a change in the direction of the relative air velocity and produces a 5° angle of attack. The indicial lift response was obtained by integrating the pressure along the boundary of the section as the gust convects with the freestream.

For other details of this method the reader is referred to Turbelin (2000). It has been tested for a 12-percent-thick symmetric airfoil and carried out for a rectangular bluff section and for a streamlined bridge deck section (Pont de Normandie).

5. Computed results

5.1. Airfoil case

The method has been first validated by computing the growth of the lift on a NACA0012 airfoil penetrating a stationary sharp-edge gust. The purpose of this computation was to assess the ability of the model to predict indicial and admittance functions. The normalized indicial lift response, as a function of time, is plotted in Fig. 5. It consists of a transient followed by a steady state : after a critical period of time, the lift reaches its quasi-steady value. An examination of the aerodynamic flowfield, Fig. 9, shows that the flow remains attached. Consequently the function obtained is in good agreement with the Küssner solution, given in Eq. (5). Fourier transforming the above curve, admittance function is plotted against $B\omega/U$, Fig. 5. In this case, this function is reduced towards that of a fully attached flow, that is Sears function witch is also plotted for comparison.

5.2. Rectangular section

In the same manner, the indicial lift response of a rectangular section with the same thickness-

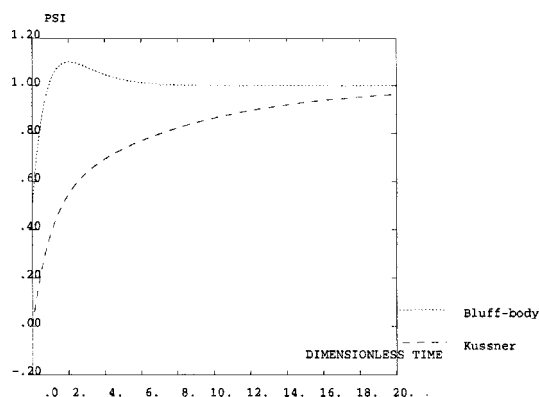


Fig. 4 Normalized indicial lift response of a representative bluff body, Scanlan (1995)

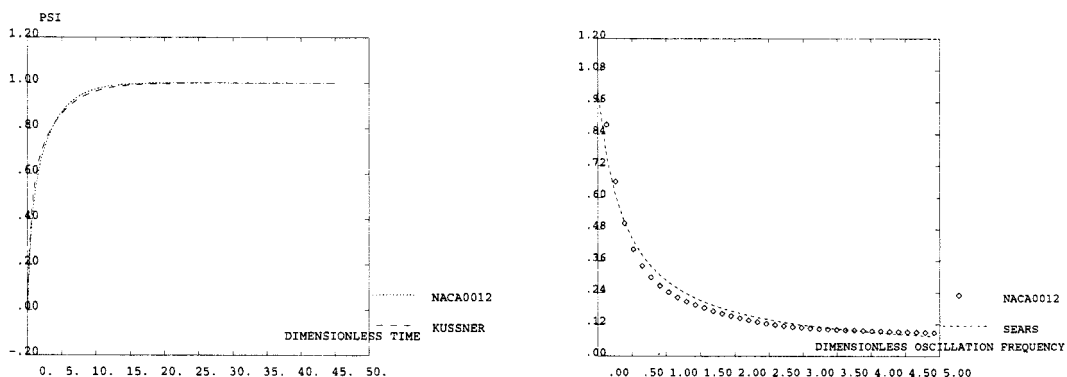


Fig. 5 Normalized indicial lift response and Admittance function, airfoil case

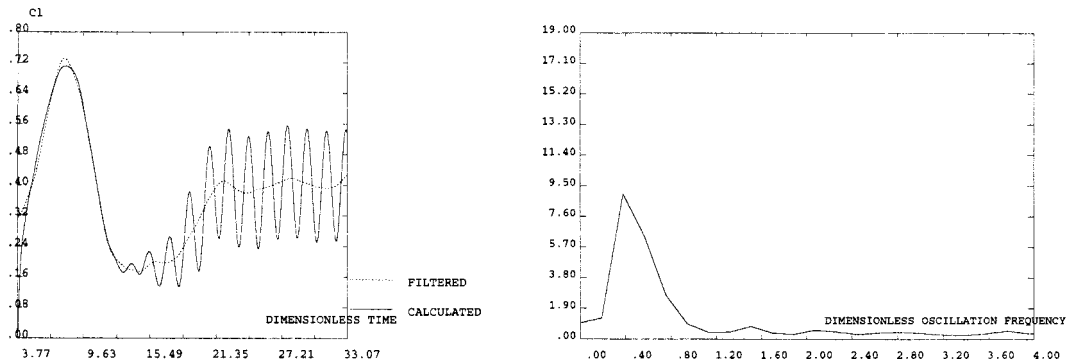


Fig. 6 Indicial lift response and Admittance function, rectangular section

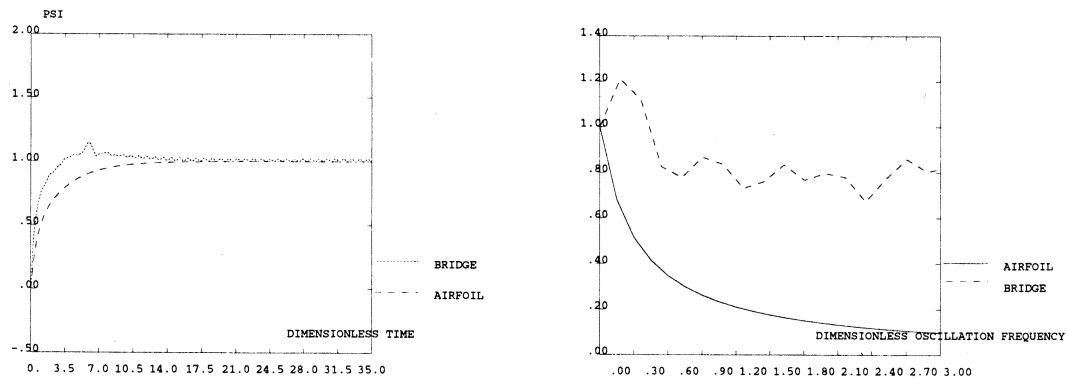


Fig. 7 Normalized indicial lift response and Admittance function, bridge deck section

chord ratio ($H/B = 0.12$) has been obtained. The objective of this computation was to study the particular shapes of the indicial and admittance functions for bluff bodies. It is important to note that the mechanism involved in this case is very different from that for a fully attached flow of an airfoil. In this case, the body-initiated excitation, i.e., the excitation caused by the presence of the body itself, is a major contributor to the buffeting forces. Therefore, the unsteady lift function, plotted in Fig. 6, is quite different from the previous indicial response of an airfoil. The lift displays a significant overshoot, and very distinct oscillations associated with a vortex shedding develop. The expression for the indicial function cannot be evaluated exactly because of these oscillations. Thus the computed function has been filtered by using the discrete wavelet transform properties. The aim of this process is to eliminate the time-scales associated with the periodic vortex shedding. The result shows that the strong overshoot is not associated with these oscillations. An examination of the aerodynamic flowfield, Fig. 8, indicates that this overshoot is linked to the flow separating from the leading edge. As the gust is convected with the freestream, the separated shear layer created at the upstream corner reattaches to the upper side of the section and forms a separation bubble. As time advances, the bubble increases in length. Figs. 8A to 8E show its time development (it can also be observed from the distribution of surface-pressure coefficient). As the separation bubble grows, the lift coefficient increases and overshoots its quasi-steady value. After a critical period of time, the separated flow from the leading edge interacts with the recirculation wake flow region, at the upper

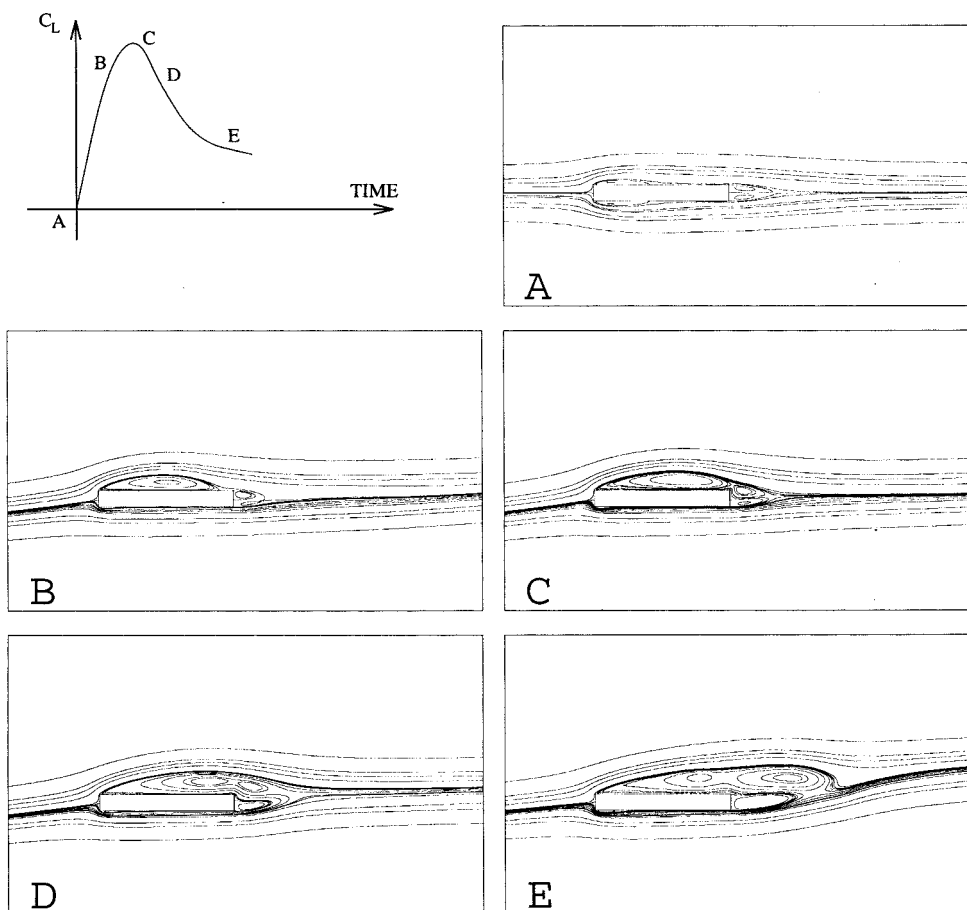


Fig. 8 Time development of the aerodynamic flowfield around the rectangular section

trailing edge corner, to form a vortex. At this time, the lift has already past its maximum level and a new vortex develops in the recirculation zone, at the lower trailing edge corner. As this new vortex grows, the upper vortex is pushed away and is shed into the downstream wake as the lift coefficient decreases. Then a regular vortex street develops in the downstream wake and the lift approaches its quasi-static value.

The Fourier transform of the filtered indicial function is plotted in Fig. 6. It significantly increases with increasing reduced frequency and exceeds unit value in the reduced frequency range of interest, which corresponds to high reduced velocities (for which buffeting forces become important). It should be pointed out that this function reflects the effects of the bubble formed under the reattaching shear layer shed from the leading edge of the section. Therefore it does not conform with the definition of an admittance functions given in Eqs. (6) and (8). However it can be seen as an indicator of the behavior of the actual admittance function. If it increases with reduced frequency, it means that a large proportion of the lift force results from the reattaching shear layer. Thus the lift could be expected not to be as dependent on the overall flow pattern as the lift resulting from a fully attached flow. Therefore the aerodynamic admittance could be expected not to drop with reduced frequency and to exceed unit value. This behavior has been observed with direct measurements

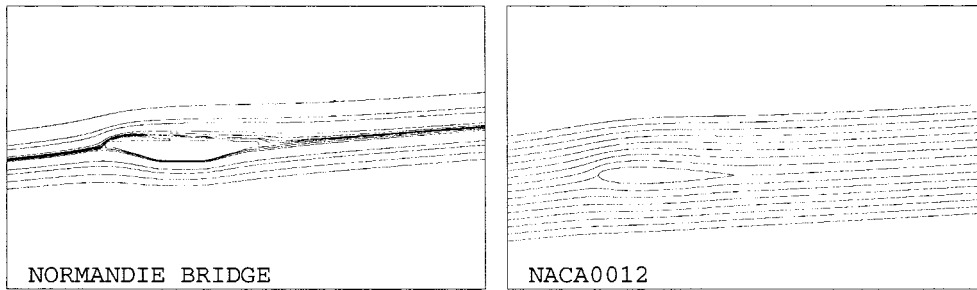


Fig. 9 Aerodynamic flowfields

for a variety of rectangular sections in smooth flow by Jancauskas and Melbourne (1986). Further calculations show the extension of this behavior as the thickness-chord ratio increases, i.e., as the lift generated by the reattaching shear layer increases. This will be the subject of a future paper by the authors. Based on this, these effects are assumed to be minimal as the thickness-chord ratio decreases or for relatively streamlined sections for which only a small proportion of the lift force results from the reattaching shear layer. In this case the aerodynamic functions could be expected to be essentially that of a fully attached flow.

5.3. Bridge deck section

This trend can be observed, at least qualitatively, for the streamlined bridge deck section of the “Pont de Normandie”, a cable-stayed bridge which has a depth-to-width ratio, H/B , of 0.13. In this case, a small overshoot occurs, Fig. 7, and the value of the admittance is close to unity at the lower reduced frequencies and decays with a form not unlike the Sears function as the reduced frequency increases, Fig. 7. However it can be seen that the Sears function underestimates this function. Over this streamlined section only a small proportion of transverse force is generated under the reattaching shear layer, Fig. 9, but the result confirms that the use of Sears function is not appropriate.

6. Conclusions

In this paper, some examples of CFD calculations of gust-penetration functions have been described. These calculations could be a step toward predicting the cross-wind excitation induced by the incident wind and provide an alternative to experimental data for deriving aerodynamic functions useful in buffeting analysis. The functions presented here may be considered to be generalized unsteady lift functions, however further investigations are needed to determine the influence of the incoming flow properties (Reynolds number, turbulence level).

The results of this study show that the particular shapes of the indicial and admittance functions (strong overshoot...) are strongly dependent on the evolution of the separated flow around the section at the early stages. Since most of the works focused on long-term flow development, further details study should be necessary for understanding the influence of these early-stages flow development.

This study also confirms that for bluff sections, and even for streamlined deck sections, the use of classical airfoil functions is not appropriate. Thus, a CFD approach provides a simple yet effective way to know if the body-initiate effects contribute to the buffeting forces and have to be taken into

account.

References

- Brar, P. S., Raul R. and Scanlan R. H. (1996), "Numerical calculation of flutter derivatives via indicial functions", *J. Fluids Struct.*, **10**, 337-351.
- Davenport, A. G. (1962), "The buffeting of a suspension bridge by storm wind", *J. Struct. Div.*, **88**, 233-268.
- Hughes, T. J. R., Mallet, M. and Mizukami, A. (1986), "A New Finite Element Formulation for Computational Fluid Dynamics II. Beyond SUPG", *Comput. Meth. Appl. Mech. Engng.*, **54**, 341-355.
- Jancauskas, E. D. and Melbourne W. H. (1986), "The aerodynamic admittance of two-dimensional rectangular section cylinders in smooth flow", *J. Wind Eng. Ind. Aerod.*, **23**, 395-408.
- Kechkar, N. and Silvester, D. (1992), "Analysis of locally stabilized mixed finite element methods for the Stokes problem", *Math. Comp.*, **58**(197), 1-10.
- Kussner, H. G. (1936), "Zusammenfassender Bericht über den instationären Auftrieb von Flügeln", *Luftfahrt-Forschung*, **13**, 410-424.
- Larose, G. L. and Livesey, F. M. (1997), "Performance of streamlined bridge decks in relation to the aerodynamics of a flat plate", *J. Wind Eng. Ind. Aerod.*, **69-71**, 851-860.
- Launder, B. E. and Spalding, D. B. (1974), "The numerical computation of turbulent flows", *Comput. Meth. Appl. Mech. Eng.*, **3**, 269-289.
- Liepmann, H. W. (1952), "On the application of statistical concepts to the buffeting problem", *J. Aeronaut. Sci.*, **19**, 793-800.
- Murakami, S. (1997), "Overview of turbulence models applied in CWE-97", *Proc. of the 2nd European & African Conf. of Wind Eng., Genova, Italy*, June 22-26.
- Orszag, S. A. and Yakhot, V. (1993), "Renormalization group modeling and turbulence simulation", Near-Wall Turbulent Flows, Edited by So, R. M. C., Speziale, C. G., Launder, B. E., Elsevier.
- Rodi, W. (1995), "Introduction to the numerical simulation approaches in wind engineering", *Wind Climate in Cities, NATO ASI Series, Kluwer Academic Publishers*.
- Scanlan, R. H., Béliveau, J. G. and Budlong, K. (1974), "Indicial aerodynamic functions for bridges decks", *J. Eng. Mech. ASCE*, **100**, 657-672.
- Scanlan, R. H. (1993), "Problematics in formulation of wind-force models for bridges decks", *J. Eng. Mech. ASCE*, **119**, 1353-1375.
- Scanlan, R. H. and Jones, N. P. (1999), "A form of aerodynamic admittance for use in bridge aeroelastic analysis", *J. Fluids Struct.*, **13**(7/8), 1017-1027.
- Sears, W. R. (1941), "Some aspects of non-stationary airfoil theory and its practical application", *J. Aeronautics Sci.*, **8**(3), 104-108.
- Theodorsen, T. (1935), "General theory of aerodynamic instability and the mechanism of flutter", NACA Report 496, U. S. Advisory Committee for Aeronautics, Langley, VA.
- Turbelin, G. (2000), "Modélisation de la turbulence atmosphérique en vue de l'étude du chargement aérodynamique des structures soumises aux effets du vent", Ph. D. Thesis, Université d'Evry, France.
- Wagner, H. (1925), "Über die Entstehung des dynamischen Auftriebes von Tragflügeln", *Zeit. Angew. Math. u. Mech.*, **5**(1), 17-35.