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An analytical solution for free vibration of functionally graded beam using a simple first-order shear deformation theory

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Abstract. In this paper, a simple first-order shear deformation theory is presented for dynamic behavior of functionally graded beams. Unlike the existing first-order shear deformation theory, the present one contains only three unknowns and has strong similarities with the classical beam theory in many aspects such as equations of motion, boundary conditions, and stress resultant expressions. Equations of motion and boundary conditions are derived from Hamilton's principle. Analytical solutions of simply supported FG beam are obtained and the results are compared with Euler-Bernoulli beam and the other shear deformation beam theory results. Comparison studies show that this new first-order shear deformation theory can achieve the same accuracy of the existing first-order shear deformation theory.

Keywords: free vibration; functionally graded materials; boundary conditions; shear deformation theories; Hamilton's principle

1. Introduction

In recent years functionally graded materials (FGMs) have gained considerable importance as materials to be used in extremely high temperature environments such as nuclear reactors and high-speed spacecraft industries (Yamanouchi *et al.* 1990). FGMs were first introduced by a group of scientists in Sendai Japan in 1984 (Koizumi 1997). FGMs are new inhomogeneous materials, in which the mechanical properties vary smoothly and continuously from one surface to the other. This is achieved by gradually varying the volume fraction of the constituent materials. This continuous change in composition results in the graded properties of FGMs (Reddy 2001). Typically these materials are made from a mixture of ceramic and metal or from a combination of different materials. The ceramic constituent of the material provides the high-temperature resistance due to its low thermal conductivity. The ductile metal constituent on the other hand, prevents fracture caused by stresses due to the high temperature gradient in a very short period of time. Furthermore a mixture of ceramic and metal with a continuously varying volume fraction can be easily manufactured (Fukui, 1991).

Dynamic analyses of FGM structures have attracted increasing research effort in the last decade because of the wide application areas of FGMs. For instance, Sankar *et al*. (2001) gave an elasticity solution based on the Euler-Bernoulli beam theory for functionally graded beam subjected to static transverse loads by assuming that Young's modulus of the beam vary exponentially through the thickness. Aydogdu and Taskin (2007) investigated the free vibration behavior of a simply supported FG beam by using Euler-Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation theory. Zhong and Yu (2007) presented an analytical solution of a cantilever FG beam with arbitrary graded variations of material property distribution based on two-dimensional elasticity theory. Taj *et al.* (2013) conducted static analysis of FG plates using higher order shear deformation theory. Bourada *et al.* (2015) used a new simple shear and normal deformations theory for functionally graded beams. Hebali *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Bennoun *et al.* (2016) analyzed the vibration of functionally graded sandwich plates using a novel five variable refined plate theory. Bousahla *et al*. (2014) investigated a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Draiche *et al.* (2016) used a refined theory with stretching effect for the flexure analysis of laminated composite plates. Hamidi *et al*. (2015) proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Belabed *et al*. (2014) used an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Bessaim *et al*. (2013) investigated a new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets. Bouafia *et al.*

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(2017), used a nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams. Abualnour *et al*. (2018) analyze the free vibration of advanced composite plates using a novel quasi-3D trigonometric plate theory. Abdelaziz *et al.* (2017) studied the bending, buckling and free vibration of FGM sandwich plates with various boundary conditions using an efficient hyperbolic shear deformation theory. Amar Meziane *et al.* (2014) proposed an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Bouderba *et al.* (2016) studied the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Bellifa *et al.* (2016) studied the bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position. Bousahla *et al.* (2016) investigated the thermal stability of plates with functionally graded coefficient of thermal expansion. Beldjelili *et al*. (2016) studied the hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Bouderba *et al*. (2016) analyze the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Zidi *et al*. (2014) analyse the bending of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. El-Haina *et al.* (2017) used a simple analytical approach for thermal buckling of thick functionally graded sandwich plates. Menasria *et al.* (2017) analyze the thermal stability of FG sandwich plates using a new and simple HSDT. Chikh *et al.* (2017) studied the thermal buckling analysis of cross-ply laminated plates using a simplified HSDT. Tounsi *et al.* (2013) use a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Mouffoki *et al.* (2017) studied the vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory. Khetir *et al.* (2017) developed a new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates. Hamidi *et al*. (2015) proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Attia *et al.* (2015) developed the free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories. Karami *et al.* (2017) studied the effects of triaxial magnetic field on the anisotropic nanoplates. Zemri *et al*. (2015) proposed an assessment of a refined nonlocal shear deformation theory beam theory for a mechanical response of functionally graded nanoscale beam. Bellifa *et al.* (2017a) used a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Bellifa *et al*. (2017b) used An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates. Bounouara *et al.* (2014) studied the free vibration of functionally graded nanoscale plates resting on elastic foundation using a nonlocal zeroth-order shear deformation theory. Ahouel *et al.* (2016) investigated a size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Saidi *et al.* (2016) investigated a simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations. Belkorissat *et al.* (2015) developed a new nonlocal refined four variable model for the vibration properties of functionally graded nano-plate. Larbi Chaht *et al.* (2015) studied the bending and buckling of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Al-Basyouni *et al.* (2015) investigated size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Boukhari *et al*. (2016) used an efficient shear deformation theory for wave propagation of functionally graded material plates. Houari *et al.* (2016) used a new simple three-unknown sinusoidal shear deformation theory for functionally graded plates. Ait Yahia *et al.* (2015) analyzed the wave propagation in functionally graded plates with porosities. Benadouda *et al*. (2017) developed an efficient shear deformation theory for wave propagation in functionally graded material beams with porosities. Hanifi *et al.* (2017) investigated the size-dependent behavior of functionally graded micro-beams with porosities. Ghorbanpour *et al.* (2016) studied the dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory. Ait Atmane *et al.* (2016) studied the effect of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory. Mahmoud *et al.* (2016) analyze the buckling of functionally graded sandwich plates with stretching effect using a new shear deformation plate theory. Recently, Besseghier *et al.* (2017) developed the free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory. Attia *et al.* (2018) used a refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations. Kaci *et al.* (2018) studied the post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory. Zine *et al.* (2018) studied the bending and free vibration analysis of isotropic and multilayered plates and shells the using a novel higher-order shear deformation theory. Youcef *et al.* (2018) analyze the dynamic of nanoscale beams including surface stress effects. Benchohra *et al.* (2018) used a new quasi-3D sinusoidal shear deformation theory for functionally graded plates. Belabed *et al*. (2018) developed a new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate.

In this paper, a simple FSDT which was recently developed by Thai and Choi (2013) for functionally graded plates is evaluated for FG beams. Unlike the existing FSDT, the one presented by Thai and Choi contains only three unknowns and has strong similarities with the CPT in many aspects such as equations of motion, boundary conditions, and stress resultant expressions. The partition of the transverse displacement into the bending and shear parts makes the theory simple to use. Equations of motion and boundary conditions are derived from Hamilton's principle. Closed-form solutions of simply supported FG beam are obtained and the results are compared with the existing solutions.

2. Theoretical formulation

Consider a functionally graded beam with length L and rectangular cross section $b \times h$, with b being the width and h being the height as shown in Fig. 1. The beam is made of isotropic material with material properties varying smoothly in the thickness direction.

2.1 Material properties

The properties of FGM vary continuously due to the gradually changing volume fraction of the constituent materials (ceramic and metal), usually in the thickness direction only. The power-law function is commonly used to describe these variations of materials properties. The expression given below represents the profile for the volume fraction.

$$V_C = \left(\frac{z}{h} + \frac{1}{2} \right)^k \tag{1a}$$

k is a parameter that dictates material variation profile through the thickness. The value of k equal to zero represents a fully ceramic beam, whereas infinite k indicates a fully metallic beam, and for different values of k one can obtain different volume fractions of metal.

The material properties of FG beams are assumed to vary continuously through the depth of the beam by the rule of mixture (Marur 1999) as

$$P(z) = (P_t - P_b)\mathrm{V_C} + P_b \tag{1b}$$

where P denotes a generic material property like modulus, P_t and P_b denotes the property of the top and bottom faces of the beam respectively, Here, it is assumed that modules E, G and ν vary according to the Eq. (1(b)). However, for simplicity, Poisson's ratio of beam is assumed to be constant in this study for that the effect of Poisson's ratio ν on deformation is much less than that of Young's modulus (Delale and Erdogan 1983, Benachour *et al.* 2011).

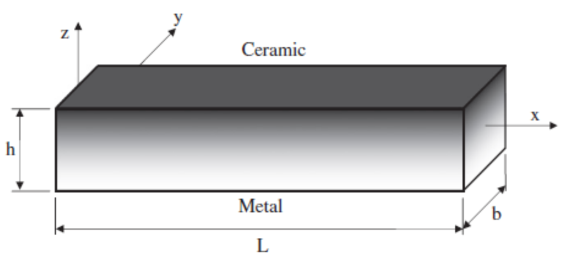

Fig. 1 Geometry of Rectangular FG Plate and Coordinates

2.2 Kinematics and constitutive equations

In this study, further simplifying assumptions are made to the existing FSDBT. The displacement field of the existing FSDT is given by

$$\begin{aligned} u(x,z,t) &= u_0(x,t) - z\varphi_x \\ w(x,z,t) &= w_0(x,t) \end{aligned} \tag{2}$$

where u_0, w_0 and φ_x are three unknown displacement functions of the midplane of the beam; and h is the thickness of the beam. By deviding the transverse displacement w into bending and shear parts (i.e., $w = w_b + w_s$) and making further assumptions given by $\varphi_x = -\partial w_b / \partial x$, the displacement field of the new theory can be rewritten in a simpler form as

$$\begin{aligned} u(x,z,t) &= u_0(x,t) - z\frac{\partial w_b}{\partial x} \\ w(x,z,t) &= w_b(x,t) + w_s(x,t) \end{aligned} \tag{3}$$

Clearly, the displacement field in Eq. (3) contains only three unknowns (u_0, w_b and w_s). In fact, the idea of partitioning the transverse displacements into the bending and shear components is first proposed by Huffington et al. (1963), and recently by Thai and his colleagues (2012).

The nonzero strains associated with the displacement field in Eq. (3) are

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} \tag{4a}$$

$$\gamma_{xz} = \frac{\partial w_s}{\partial x} \tag{4b}$$

By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become

$$\sigma_x = Q_{11}(z)\,\varepsilon_x \quad \text{and} \quad \tau_{xz} = k_S Q_{55}(z)\,\gamma_{xz} \tag{5a}$$

k_S is a shear correction factor which is analogous to shear correction factor proposed by Mindlin (1951). Using the material properties defined in Eq. (1(b)), stiffness coefficients, Q_{ij} can be expressed as

$$Q_{11}(z) = \frac{E(z)}{1-\nu^2} \quad \text{and} \quad Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \tag{5b}$$

2.3 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as Reddy (2002)

$$\delta\int_{t_1}^{t_2}(U-T)dt=0 \tag{6}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; and δT is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned}\delta U &= \int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_x\delta\,\varepsilon_x+\tau_{xz}\delta\,\gamma_{xz})dzdx \\ &= \int_0^L\left(N_x\frac{d\delta\,u_0}{dx}-M_x\frac{d^2\delta\,w_b}{dx^2}+Q_{xz}\frac{d\delta w_s}{dx}\right)dx\end{aligned} \tag{7}$$

where N, M and Q are the stress resultants defined as

$$(N_x,M_x)=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z)\,\sigma_x dz_{ns} \text{ and } Q_{xz}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\tau_{xz}dz \tag{8}$$

The variation of the kinetic energy can be expressed as

$$\begin{aligned}\delta T &= \int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho(z)[\dot{u}\delta\,\dot{u}+\dot{w}\delta\,\dot{w}]dzdx \\ &= \int_0^L\left\{I_0[\dot{u}_0\delta\dot{u}_0+(\dot{w}_b+\dot{w}_s)\delta(\dot{w}_b+\dot{w}_s)]+I_2\left(\frac{\partial\,\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x}\right)\right\}dx\end{aligned} \tag{9}$$

dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_2) are the mass inertias defined as

$$(I_0,I_2)=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z^2)\rho(z)dz \tag{10}$$

Substituting the expressions for δU and δT from Eqs. (7) and (9) into Eq.(6) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s, the following equations of motion of the functionally graded beam are obtained

$$\delta\,u_0:\ \frac{\partial N_x}{\partial x}=I_0\frac{\partial u_0}{\partial t^2} \tag{11a}$$

$$\delta\,w_b:\ \frac{d^2M_x}{dx^2}=I_0\left(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2}\right)-I_2\frac{\partial^4 w_b}{\partial x^2\partial t^2} \tag{11b}$$

$$\delta\,w_s:\ \frac{\partial Q_{xz}}{\partial x}=I_0\left(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2}\right) \tag{11c}$$

Eqs. (11) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (4), (5), and (8) as follows

$$A_{11}d_{11}u_0-B_{11}d_{111}w_b=I_0\ddot{u}_0 \tag{12a}$$

$$B_{11}d_{111}u_0-D_{11}d_{1111}w_b=I_0(\ddot{w}_s+\ddot{w}_s)-I_2d_{11}\ddot{w}_b \tag{12b}$$

$$A_{55}^S d_{11}w_s=I_0(\ddot{w}_s+\ddot{w}_s) \tag{12c}$$

where A_{11}, B_{11}, D_{11}, etc., are the beam stiffness, defined by

$$(A_{11},B_{11},D_{11})=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{11}(1,z,z^2)dz \tag{13a}$$

and

$$A_{55}^S d_{11}w_s=I_0(\ddot{w}_s+\ddot{w}_s) \tag{13b}$$

3. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_b, w_s can be written by assuming the following variations

$$\begin{Bmatrix}u_0\\ w_b\\ w_s\end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix}U_m\cos(\lambda\,x)\,e^{i\,\omega\,t}\\ W_{bm}\sin(\lambda\,x)\,e^{i\,\omega\,t}\\ W_{sm}\sin(\lambda\,x)\,e^{i\,\omega\,t}\end{Bmatrix} \tag{14}$$

where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with m th eigenmode, and $\lambda=m\pi/L$.

Substituting Eq. (14) into Eqs. (11(a)- 11(c)), the closed form solutions can be obtained from

$$([C]-\omega^2[M])\{\Delta\}=0 \tag{15}$$

where $\{\Delta\}=\{U_m,\psi_m,W_m\}^t$, and $[C]$ and $[M]$ are the symmetric matrixes given by

$$[C]=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\ a_{12}&a_{22}&a_{23}\\ a_{13}&a_{23}&a_{33}\end{bmatrix},\ [M]=\begin{bmatrix}m_{11}&a_{12}&a_{13}\\ a_{12}&a_{22}&a_{23}\\ a_{13}&a_{23}&a_{33}\end{bmatrix} \tag{16}$$

Table 1 Non-dimensional natural frequencies of simply supported homogenous beam versus thickness-to-length ratio $(k=0)$. $\bar{\omega}=\frac{\omega L^2}{h}\sqrt{\frac{\rho_c}{E_c}}$

h/L	Euler-Bernoulli (Reddy 1999)	NFSDBT (Hadji 2016)	FSDBT (Koochaki 2011)	Present FSDBT
0.01	2.985526	2.9861309	2.986137	2.986134
0.0125	2.985232	2.9858301	2.985827	2.985828
0.0142	2.984340	2.9855685	2.985556	2.985582
0.0166	2.984865	2.9851691	2.985155	2.985180
0.02	2.983701	2.9845053	2.984505	2.984505
0.025	2.982588	2.9832857	2.983285	2.983285
0.033	2.979668	2.9806569	2.980657	2.980776
0.04	2.976570	2.9780219	2.978020	2.978021
0.05	2.971688	2.9731933	2.973193	2.973193
0.066	2.961235	2.9628589	2.962858	2.963326
0.1	2.931568	2.9340444	2.934044	2.934044

where

$$a_{11}=-A_{11}\lambda^2,\ a_{12}=0,\ a_{13}=0$$
$$a_{22}=-D_{11}\lambda^4,\ a_{23}=0,\ a_{33}=-A_{55}^S\lambda^2 \tag{17a}$$

$$m_{11}=-I_0,\ m_{12}=0,\ m_{13}=0$$
$$m_{22}=-(I_0+I_2\lambda^2),\ m_{23}=-I_0,\ m_{33}=-I_0 \tag{17b}$$

4. Numerical examples

In this section, various numerical examples are presented and discussed to verify the accuracy of present theories in predicting the free vibration response of simply supported FG beams. The FG beam is taken to be made of aluminum and alumina with the following material properties :

Ceramic (P_C : Alumina, Al_2O_3): $E_c=380$ GPa; $\rho_c=3800 kg/m^3$; $\nu=0.3$;

Metal (P_M : Aluminium, Al): $E_m=70$ GPa; $\rho_m=2707 kg/m^3$; $\nu=0.3$;

And their properties change through the thickness of the beam according to power-law. The bottom surfaces of the FG beams are aluminium rich, whereas the top surfaces of the FG beams are alumina rich.

For convenience, the following dimensionless form is used

$$\bar{\omega}=\frac{\omega L^2}{h}\sqrt{\frac{\rho_c}{E_c}} \tag{18}$$

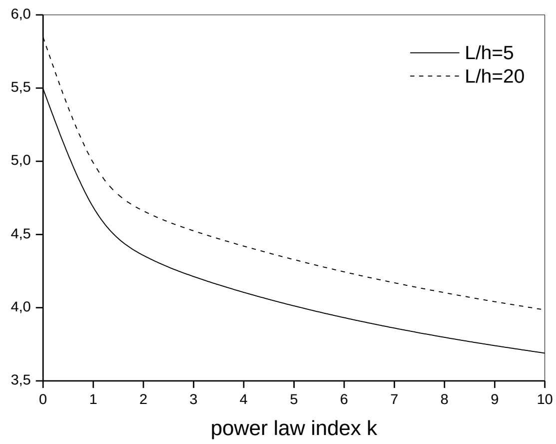

Fig. 2 Variation of the fundamental frequency $\bar{\omega}=\frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$ of FG beam with power-law index k

Table 1 show the nondimensional fundamental frequencies $\bar{\omega}$ of FG beams for homogenous beam $(k=0)$ for different values of span-to-depth ratio h/L using a simple first-order shear deformation beam theory FSDBT and are compared with Euler-Bernoulli beam theory results (Reddy 1999), the first order shear deformation (Koochaki 2011) and the new first-order shear deformation theory (Hadji 2016).

As can be seen the results of the simple first shear deformation beam theory is in good agreement with the Euler-Bernoulli beam and other shear deformation beam theory results. Also, the frequencies predicted by the three shear deformation theories are very close to each other.

Fig. 2 shows the non-dimensional fundamental natural frequency $\bar{\omega}$ versus the power law index k for different values of span-to-depth ratio L/h using the present theory. It is observed that an increase in the value of the power law index leads to a reduction of frequency. The highest frequency values are obtained for full ceramic beams (p = 0) while the lowest frequency values are obtained for full metal beams ($p \to \infty$). This is due to the fact that an increase in the value of the power law index results in a decrease in the value of elasticity modulus. In other words, the beam becomes flexible as the power law index increases, thus decreasing the frequency values. It can be also seen that the span-to-depth ratio L/h has a considerable effect on the non-dimensional fundamental natural frequency $\bar{\omega}$ where this latter is reduced with decreasing L/h. This dependence is related to the effect of shear deformation.

5 Conclusions

A simple FSDT was proposed for free vibration analysis of FG beam. Equations of motion derived from Hamilton's principle are analytically solved for simply supported FG beam. The effects of volume fraction ratio and thickness-to-length ratio on fundamental frequencies are investigated. The accuracy of the present theory is verified by comparing the obtained results with those reported in the literature. Finally, it can be concluded that the simple first-order shear deformation beam theory FSDBT is not only accurate but also simple in predicting the dynamic behavior of FG beam.

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