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Alternative robust estimation methods for parameters of Gumbel distribution: an application to wind speed data with outliers

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Abstract. An accurate determination of wind speed distribution is the basis for an evaluation of the wind energy potential required to design a wind turbine, so it is important to estimate unknown parameters of wind speed distribution. In this paper, Gumbel distribution is used in modelling wind speed data, and alternative robust estimation methods to estimate its parameters are considered. The methodologies used to obtain the estimators of the parameters are least absolute deviation, weighted least absolute deviation, median/MAD and least median of squares. The performances of the estimators are compared with traditional estimation methods (i.e., maximum likelihood and least squares) according to bias, mean square deviation and total mean square deviation criteria using a Monte-Carlo simulation study for the data with and without outliers. The simulation results show that least median of squares and median/MAD estimators are more efficient than others for data with outliers in many cases. However, median/MAD estimator is not consistent for location parameter of Gumbel distribution in all cases. In real data application, it is firstly demonstrated that Gumbel distribution fits the daily mean wind speed data well and is also better one to model the data than Weibull distribution with respect to the root mean square error and coefficient of determination criteria. Next, the wind data modified by outliers is analysed to show the performance of the proposed estimators by using numerical and graphical methods.

Keywords: Gumbel distribution; least absolute deviation; median/MAD estimator; Monte-Carlo simulation; wind speed data

1. Introduction

Energy consumption is increasing parallel to global population growth and technological mprovements (Akgul et al. 2016). Fossil fuels are widely-used energy resources, but they will run out one day. Therefore, alternative renewable energy resources will play an increasingly vital role in the future of energy production. In recent decades, wind energy, a renewable energy resource, is environmentally friendly and has become more popular throughout the world. Thus, wind speed distribution is a most important factor in evaluating the wind energy potential needed to design wind farms.

Gumbel distribution, which is a special case of generalized extreme value distribution, has a wide application area, especially in hydrology; see Gumbel (1941), Jenkinson (1955). It plays an important role in modelling extreme events data; see also Simiua *et al.* (2001), Koutsoyiannis (2004), Graybeal and Leathers (2006), Ercelebi and Toros (2009), Lee *et al.* (2012), Hong and Mara (2013), Aydin and Senoglu (2015) and Aydin (2018). Furthermore, this distribution has been used to evaluate wind energy potential for the determination of wind turbine class in the wind power industry (Kang *et al.* 2015). However, this study proposes that Gumbel distribution can be used in modelling mean wind speed.

Much research has considered the problem of estimation of the parameters of Gumbel distribution. Traditional estimation methods, such as method of moments, method of maximum likelihood (*ML*), and method of least squares (*LS*) are commonly-used in estimates of the parameters of Gumbel distribution. Landwehr *et al.* (1979) proposed a method of probability weighted moment (*PWM*) which is unbiased even for a small sample. Its efficiency is compared with efficiencies of certain classical estimation methods. Raynal and Salas (1986) and Phien (1987) showed that *PWM* is the best of estimation methods in terms of bias and mean square error. Corsini *et al.* (1995) considered the *ML* and Cramer-Rao bounds for parameters of the Gumbel distribution. Mousa *et al*. (2002) obtained bayesian estimators for the two parameters of the Gumbel distribution, based on records. Rasmussen and Gautam (2003) described the estimator alternative *PWM*, and show that the new estimator is slightly better than the classical *PWM* method. Aydin and Senoglu (2015) analysed the performances of different estimators, such as modified maximum likelihood, percentile and probability weighted moments of parameters with numerical simulations. Aydin (2018) obtained the various estimators of the lower and the upper quantiles of the Gumbel distribution and then analysed wind speed data modelled by Gumbel distribution to show the performances of them.

However, in practice, it has been observed that data including outliers is quite prevalent. It is well-known that commonly-used estimation techniques such as *ML* and *LS* are not resistant to outliers. In other words, efficiencies of non-robust estimators significantly change in the presence

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of outliers. Therefore, in the literature, many robust estimation methods have been suggested to deal with the problem. In this context, robust estimation methods have not been used as often as traditional estimation methods to estimate Gumbel parameters. However, Trzpiot (2009) analysed a number of methodological aspects to show how robust methods can improve the quality of extreme value theory data analysis by providing information on influential observations. Fischer *et al.* (2015) compared efficiencies of the maximum-likelihood estimator and *l*-moments with robust estimators, trimmed *l*-moments and minimum distances via a simulation study.

The purpose of this study is to consider certain alternative robust estimators of location and scale parameters of Gumbel distribution. These are least absolute deviations (*LAD*), weighted least absolute deviations (*WLAD*), Median/*MAD* and least median of squares (*LMS*). We provide a simulation study to compare the performances of these methods with the widely-used estimation techniques (the least squares based on percentile and *ML*) for a data set with and without outliers. Furthermore, bias, mean square deviation (*MSD*) and total mean square deviation (*TMSD*) criteria are used to evaluate the performances of proposed estimators.

The remainder of the paper is organized as follows: Section 2 presents Gumbel distribution and its properties. In Section 3, we briefly introduce estimation methods. The best estimation method has been determined with respect to different comparison criteria using a Monte-Carlo simulation in Section 4. A wind speed data set is analysed as an example in Section 5. We present conclusions in Section 6.

2. Gumbel distribution

Let random variable X has Gumbel distribution with a location parameter $\phi \in \mathbb{R}$ and a scale parameter $\psi > 0$. The corresponding probability density function (*pdf*) and cumulative distribution function (*cdf*) are given by

$$f(x;\phi,\psi) = \frac{1}{\psi} exp\left(-\frac{x-\phi}{\psi} - exp\left(-\frac{x-\phi}{\psi}\right)\right), x \in \mathbb{R} \quad (1)$$

and

$$F(x;\phi,\psi) = exp\left(-exp\left(-\frac{x-\phi}{\psi}\right)\right) \quad (2)$$

respectively. The mean, variance, skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) values of Gumbel distribution are given by

$$E(X) = \phi + \psi\gamma,\ Var(X) = \frac{\pi^2}{6}\psi^2,$$

$$\sqrt{\beta_1} = \frac{12\sqrt{6}}{\pi^3}\varsigma(3) \text{ and } \beta_2 = \frac{27}{5},$$

where γ is Euler's constant defined by

$$\gamma = -\int_0^\infty \ln x\, exp(-x)dx \quad (3)$$

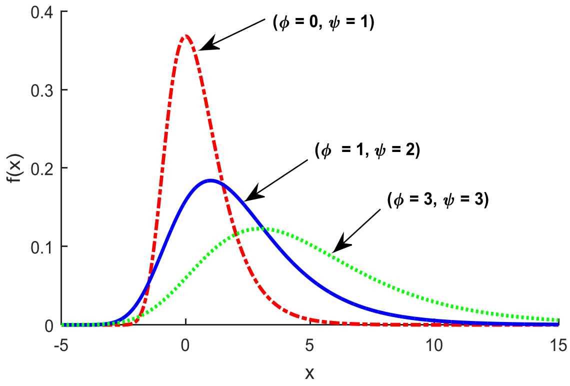


Fig. 1 Different density functions of Gumbel distribution for certain selected values of ϕ and ψ

and $\varsigma(3)$ is Apéry's constant (Apéry 1979). Before introducing a few of its characteristics, let us first look at the density function of Gumbel distribution for different values of the location parameter ϕ and scale parameter ψ in Fig. 1. As can be seen in Fig. 1, the Gumbel distribution is unimodal and positively skewed.

Now, we present a number of the characteristics of Gumbel distribution. From Eq. (1), the moment generating function is obtained as

$$M(t) = exp(\phi t)\Gamma(1-\psi t),\ t < 1/\psi \quad (4)$$

The corresponding reliability function and hazard function have the following forms

$$R(t) = 1 - exp\left(-exp\left(-\frac{t-\phi}{\psi}\right)\right),\ -\infty \le t \le \infty \quad (5)$$

and

$$h(t) = \frac{1}{\psi}\frac{exp\left(-\frac{t-\phi}{\psi} - exp\left(-\frac{t-\phi}{\psi}\right)\right)}{1-exp\left(-exp\left(-\frac{t-\phi}{\psi}\right)\right)},\ -\infty \le t \le \infty \quad (6)$$

respectively; see Wolstenholme (1999).

A doubly truncated *cdf* is defined by

$$G(x) = \frac{F(x)-F(t)}{F(T)-F(t)},\ t \le x \le T \quad (7)$$

Here, $F(x)$ is *cdf* of the random variable X, t is the lower truncation point and T is the upper truncation point. $G(x)$ in Eq. (7) contains the same location parameter ϕ and scale parameter ψ based on the characteristics of the usual Gumbel distribution. If $F(x)$ is taken as the Gumbel distribution given in Eq. (2), the *cdf* of a doubly-truncated Gumbel distribution can be obtained. When $t \to -\infty$ and $T \to \infty$, the doubly-truncated distribution becomes a two parameter Gumbel distribution. When $t \to -\infty$, it is the upper truncated Gumbel distribution and when $T \to \infty$, it is the lower truncated Gumbel distribution.

If X has a Gumbel distribution with location parameter ϕ and scale parameter ψ, then the transformation $Y = exp(-X)$ has Weibull distribution with *pdf*

$$f(y;\alpha,\delta) = \frac{\alpha}{\delta}\left(\frac{y}{\delta}\right)^{\alpha-1} exp\left(-\left(\frac{y}{\delta}\right)^{\alpha}\right),\ y > 0 \tag{8}$$

where the shape parameter $\alpha = 1/\psi$ and scale parameter $\delta = exp(-\phi)$.

3. Estimation methods for ϕ and ψ

In this section, the estimation techniques used in this study are briefly described for Gumbel distribution. We also show how to obtain estimates of unknown parameters using each of the mentioned techniques.

3.1 Least absolute deviation method for ϕ and ψ

Let $\{X_1, X_2, \cdots, X_n\}$ be a random sample of size n from the two parameter Gumbel distribution and $X_{(i)}$ be the *i*-th order statistic obtained by arranging X_i $(i = 1,2,\cdots,n)$ in ascending order of magnitude, i.e., $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$. The *cdf* given in Eq. (2) will be transformed to a linear function by taking the logarithm twice on both sides. Then we obtain the following

$$\ln\left(-\ln F\left(x_{(i)}\right)\right) = -\frac{x_{(i)} - \phi}{\psi} \tag{9}$$

The least absolute deviation (*LAD*) estimators of ϕ and ψ are obtained by minimizing the following function with respect to the parameters of interest

$$G_{LAD}(\phi,\psi) = \sum_{i=1}^{n} \left|\frac{x_{(i)} - \phi}{\psi} + \ln\left(-\ln \hat{F}\left(x_{(i)}\right)\right)\right| \tag{10}$$

Here, $\hat{F}\left(x_{(i)}\right)$, the empirical estimate of $F\left(x_{(i)}\right)$, is called a rank estimator. In the literature, several rank estimators have been proposed to estimate $F\left(x_{(i)}\right)$; for example, the Herd-Johnson mean rank estimator (Herd 1960, Johnson 1964), the Bernard median rank estimator (Bernard and Bosi-Levenbach 1953) and the Blom median rank estimator (Blom 1958)

$$\hat{F}\left(x_{(i)}\right) = \frac{i}{n+1},\ \hat{F}\left(x_{(i)}\right) = \frac{i-0.3}{n+0.4} \text{ and } \hat{F}\left(x_{(i)}\right) = \frac{i-0.375}{n+0.25}$$

respectively.

In this study, we use the Bernard median rank estimator, $\hat{F}\left(x_{(i)}\right) = \frac{i-0.3}{n+0.4}$, as an estimator of $F\left(x_{(i)}\right)$ since it shows good performance on a complete data set (Tiryakioglu and Hudak 2007, Yavuz 2012, Zyl and Schall 2012).

Least absolute deviation (*LAD*) estimators are less sensitive to outliers than ordinary least square estimators in the case of heavy-tailed distributions or outliers (Bai 1995); however, they are sensitive to leverage points (Croux *et al.* 2003). Furthermore, *LAD* is an asymptotically normal distribution without assuming the distribution of errors; see Pollard (1991), Dunsmuir and Spencer (1991) and Davis and Dunsmuir (1997).

3.2 Weighted least absolute deviation method for ϕ and ψ

Using the weighted least absolute deviation (*WLAD*) method, estimates of ϕ and ψ are found by minimizing the following function with respect to the parameters ϕ and ψ:

$$G_{WLAD}(\phi,\psi) = \sum_{i=1}^{n} w_i \left|\frac{x_{(i)} - \phi}{\psi} + \ln\left(-\ln \hat{F}\left(x_{(i)}\right)\right)\right| \tag{11}$$

Here, the major difficulty in applying the *WLAD* method is the determination of weight function w_i in the $G_{WLAD}(\phi,\psi)$ function. Zyl and Schall (2012) report that the weighted least square, which has a weight function obtained by the delta method, shows a performance almost as well as the maximum likelihood estimation does. Therefore, we use

$$w_i = \frac{n\hat{F}\left(x_{(i)}\right)\left(\ln \hat{F}\left(x_{(i)}\right)\right)^2}{1 - \hat{F}\left(x_{(i)}\right)} \tag{12}$$

as a weight function in this study.

WLAD is also a robust method, and has asymptotically normal distribution if the density and its derivative are uniformly bound; see Ling (2005) and Pan *et al.* (2007).

3.3 Median/MAD method for ϕ and ψ

The Median/*MAD* estimates of ϕ and ψ are given by:

$$\hat{\phi} = Med_i(x_i) - \ln(\ln 2)\hat{\psi} \tag{13}$$

and

$$\hat{\psi} = 1.3037MAD(X) \tag{14}$$

where

$$MAD(X) = Med_j\left(\left|x_j - Med_i(x_i)\right|\right)$$

The median/*MAD* is an explicit estimator, and was considered by Olive (2006) for transformed location scale families. Boudt *et al.* (2011) also examine its robustness properties of shape and scale parameters for Weibull distribution.

3.4 Least median of squares method for ϕ and ψ

The least median of squares (*LMS*) estimates for location and scale parameters are obtained by minimizing the following function with respect to the unknown parameters

$$G_{LMS}(\phi,\psi) = Med_i\left(\frac{x_{(i)} - \phi}{\psi} + \ln\left(-\ln \hat{F}\left(x_{(i)}\right)\right)\right)^2 \tag{15}$$

The *LMS* method proposed by Rousseeuw (1984) is robust, which signifies that it is insensitive to outliers or deviations from the model assumptions; see Rousseeuw and Leroy (1987).

3.5 Least squares method based on percentile for ϕ and ψ

Using the least squares method based on percentile, estimates of ϕ and ψ are found by minimizing the following function with respect to the parameters of interest

$$G_{LS}(\phi,\psi)=\sum_{i=1}^{n}\left(\frac{x_{(i)}-\phi}{\psi}+\ln\left(-\ln\hat{F}\left(x_{(i)}\right)\right)\right)^2 \qquad (16)$$

These estimators are known as percentile estimators (PE). This was originally explored by Kao (1958, 1959) to estimate the parameters of Weibull distributions. In addition, the PE is not robust, even to a single outlier, as an ordinary least squares estimator.

3.6 Maximum likelihood method for ϕ and ψ

Suppose that $\{X_1, X_2, \cdots, X_n\}$ is a random sample of size n from two parameter Gumbel distribution. The *ML* estimates of the unknown parameters are the solutions of the following likelihood equations:

$$\frac{\partial \ln L(\phi,\psi)}{\partial\phi}=\sum_{i=1}^{n}\frac{1}{\psi}-\frac{1}{\psi}\sum_{i=1}^{n}g(z_i)=0 \qquad (17)$$

and

$$\frac{\partial \ln L(\phi,\psi)}{\partial\psi}=-\frac{n}{\psi}+\frac{1}{\psi}\sum_{i=1}^{n}z_i-\frac{1}{\psi}\sum_{i=1}^{n}z_i g(z_i)=0 \qquad (18)$$

where

$$\ln L(\phi,\psi)=-n\ln\psi-\sum_{i=1}^{n}z_i-\sum_{i=1}^{n}g(z_i),\ g(z_i)=e^{-z_i}$$
$$\text{and } z_i=(x_i-\phi)/\psi$$

It is obvious that likelihood Eqs. (17) and (18) do not have explicit analytical solutions since g(.) is a nonlinear function.

4. Simulation study

In this section, for data with and without outliers, efficiencies of the different estimators proposed in the previous sections are compared via a Monte Carlo simulation study. To compare their efficiencies, we use bias, *MSD* and *TMSD* comparison criteria. The *TMSD* is a measure of the joint efficiency of the pair ($\hat{\phi}$, $\hat{\psi}$). It is calculated by the formula

$$TMSD(\hat{\phi},\hat{\psi})=MSD(\hat{\phi})+MSD(\hat{\psi})$$

where

$$MSD(\hat{\phi})=\frac{1}{n}\sum_{i=1}^{r}(\phi-\hat{\phi}_i)^2$$

Here, $\hat{\phi}_i$ is an estimate of ϕ in the *i*-th replication. To generate random numbers from Gumbel distribution, we use the inverse transformation method

$$z=F_Z^{-1}(u)=-\ln(-\ln u), \qquad 0<u<1$$

Without any loss of generality, we can assume that $\phi=0$ and $\psi=1$ because all the estimators are invariant under the linear transformations of the data (Aydin and Senoglu 2015). In the literature, several models for outliers have been proposed; see Barnett and Lewis (1994). However, the most commonly-used outlier model is the one given by Dixon (1950). In this study, to examine the robustness of estimators, we generate our data with outliers using the Dixon outlier model. In this model, $(n-r)$ observations come from $G(\phi,\psi)$ and r of them come from $G(\phi,4\psi)$ for $r=1,2$.

The bias, *MSD* and *TMSD* values of the estimates are computed by generating 5000 replications of samples of size $n=10,20,30,40,50$ and 100. Next, the results of numerical experiments are separately reported in Tables 1-3 for different sample sizes and the data with and without outliers.

In Table 1, simulated bias, *MSD* and *TMSD* values of location parameter ϕ and scale parameter ψ are given for different sample sizes in the case without outliers. The results shown in Table 1 indicate that the following conclusions can be reached: (*i)* All estimators of ϕ are unbiased; however, we realize that the bias of *MMAD* increases when the sample size n increases; *(ii)* For location parameter ϕ, the *LMS* estimator shows the best performance among the others for all sample sizes in terms of *MSD*; *(iii)* All considered estimators of scale parameter ψ have small bias for $n=5$, but their biases decrease when the sample size n increases; *(iv)* For scale parameter ψ, the *ML* with the smallest *MSD* is the best estimator of all the considered estimators; *(v)* In terms of the *TMSD* criterion, *ML* outperforms other estimation methods for all sample sizes as expected.

When the data set contains one outlier, Table 2 presents the simulation results of Gumbel parameters for different sample sizes. We can obtain the following conclusions from the results in Table 2: (*i)* In terms of *bias*, the *WLAD* estimate of ϕ shows better performance than other robust estimation methods apart from $n=5$. Additionally, the *MMAD* estimator shows poor performance as sample size increases; (*ii)* Since the *LMS* estimator of parameter ϕ has the smallest *MSD*, the *LMS* is the most efficient estimator among the others for all sample sizes. Moreover, we observe that *ML* shows the worst performance with the greatest *MSD* value for $n=5$; (*iii)* For parameter ψ, the *MMAD* estimate has the smallest bias value for all sample sizes; *(iv)* In terms of *MSD*, the *WLAD* estimator of parameter ψ shows worse performance than the *PE* estimator of parameter ψ for $n=5$. However, its performance dramatically increases as sample size *n* gets larger and it shows the best result with the smallest *MSD* values of all considered estimation methods for $n\geq 30$; *(v)* The *LMS* outperforms other estimation methods, except for $n=5$, based on the *TMSD* comparison criterion.

Table 3 shows the Bias, *MSD* and *TMSD* values of the estimates of the ϕ and ψ parameters for different sample sizes in the presence of the two outliers in the data set. The following conclusions can be made based on the results given in Table 3: (*i)* For parameter ϕ, the bias of the *WLAD* estimate is smaller than the others for almost all sample sizes; (*ii)* The *LMS* estimation of parameter ϕ has

Table 1 Bias, *MSD* and *TMSD* values of estimates for data without outliers

		$\hat{\phi}$		$\hat{\psi}$		
n		Bias	*MSD*	Bias	*MSD*	*TMSD*
5	*PE*	0.0151	0.3522	-0.2082	0.6751	1.0273
	ML	-0.0829	0.2409	0.1574	0.1414	0.3823
	LAD	0.0370	0.3249	-0.2925	1.3632	1.6881
	WLAD	-0.0508	0.2508	-0.3295	1.7045	1.9552
	MMAD	-0.0279	0.3578	0.1618	0.2878	0.6456
	LMS	0.0351	0.0940	-0.2060	0.4049	0.4989
10	*PE*	0.0164	0.1214	-0.1342	0.6751	1.0273
	ML	-0.0411	0.1202	0.0795	0.1414	0.3823
	LAD	0.0130	0.1163	-0.1656	1.3632	1.6881
	WLAD	-0.0286	0.1239	-0.1512	1.7045	1.9552
	MMAD	0.0327	0.1747	0.0798	0.2878	0.6456
	LMS	0.0140	0.0656	-0.1077	0.4049	0.4989
20	*PE*	0.0184	0.0571	-0.0983	0.1472	0.2686
	ML	-0.0217	0.0554	0.0401	0.0646	0.1849
	LAD	0.0043	0.0574	-0.0828	0.1785	0.2948
	WLAD	-0.0158	0.0593	-0.0728	0.1449	0.2688
	MMAD	0.0757	0.0902	0.0431	0.1336	0.3083
	LMS	0.0101	0.0295	-0.0763	0.1334	0.1990
30	*PE*	0.0203	0.0385	-0.0764	0.0786	0.1357
	ML	-0.0111	0.0359	0.0270	0.0319	0.0873
	LAD	0.0058	0.0375	-0.0543	0.0629	0.1203
	WLAD	-0.0077	0.0386	-0.0476	0.0560	0.1153
	MMAD	0.0899	0.0634	0.0280	0.0723	0.1625
	LMS	0.0116	0.0185	-0.0623	0.0599	0.0894
40	*PE*	0.0176	0.0294	-0.0657	0.0476	0.0861
	ML	-0.0092	0.0273	0.0200	0.0204	0.0563
	LAD	0.0040	0.0289	-0.0422	0.0339	0.0714
	WLAD	-0.0059	0.0292	-0.0356	0.0334	0.0719
	MMAD	0.0968	0.0527	0.0225	0.0474	0.1108
	LMS	0.0120	0.0136	-0.0556	0.0401	0.0586
50	*PE*	0.0101	0.0234	-0.0538	0.0366	0.0660
	ML	-0.0115	0.0223	0.0151	0.0159	0.0432
	LAD	-0.0018	0.0238	-0.0344	0.0249	0.0538
	WLAD	-0.0100	0.0245	-0.0313	0.0246	0.0539
	MMAD	0.0919	0.0449	0.0165	0.0364	0.0891
	LMS	0.0062	0.0108	-0.0473	0.0318	0.0454
100	*PE*	0.0093	0.0118	-0.0334	0.0279	0.0513
	ML	-0.0043	0.0110	0.0078	0.0124	0.0347
	LAD	0.0006	0.0117	-0.0181	0.0190	0.0428
	WLAD	-0.0032	0.0119	-0.0165	0.0190	0.0435
	MMAD	0.1038	0.0289	0.0058	0.0291	0.0740
	LMS	0.0049	0.0052	-0.0294	0.0252	0.0360

Table 2 Bias, *MSD* and *TMSD* values of estimates for data with one outlier

		$\hat{\phi}$		$\hat{\psi}$		
n		Bias	*MSD*	Bias	*MSD*	*TMSD*
5	*PE*	0.1976	0.7658	-1.3327	4.8560	5.6218
	ML	-0.0452	0.8196	-0.4785	0.8637	1.6833
	LAD	0.2370	0.7541	-1.5423	5.9031	6.6571
	WLAD	-0.0457	0.5735	-1.6038	6.8717	7.4452
	MMAD	-0.0034	0.5143	-0.0910	0.4692	0.9834
	LMS	0.0352	0.1593	-0.5858	1.6005	1.7598
10	*PE*	0.2141	0.3453	-0.7895	1.7910	2.1363
	ML	0.0417	0.2999	-0.3046	0.3736	0.6735
	LAD	0.1134	0.2220	-0.8206	2.1811	2.4031
	WLAD	0.0226	0.1970	-0.6112	1.2822	1.4792
	MMAD	0.0671	0.1949	-0.0245	0.1620	0.3569
	LMS	0.0286	0.0769	-0.2335	0.2757	0.3526
20	*PE*	0.1478	0.1585	-0.4892	0.8014	0.9599
	ML	0.0202	0.1139	-0.1966	0.1802	0.2940
	LAD	0.0429	0.0970	-0.3132	0.6108	0.7079
	WLAD	-0.0123	0.0718	-0.1668	0.0954	0.1672
	MMAD	0.0843	0.1020	-0.0140	0.0790	0.1810
	LMS	0.0137	0.0346	-0.1341	0.0906	0.1252
30	*PE*	0.1121	0.0958	-0.3532	0.4356	0.5315
	ML	0.0154	0.0634	-0.1554	0.1287	0.1921
	LAD	0.0159	0.0497	-0.1559	0.1316	0.1813
	WLAD	-0.0100	0.0424	-0.1024	0.0452	0.0877
	MMAD	0.0936	0.0684	-0.0096	0.0515	0.1199
	LMS	0.0111	0.0198	-0.0980	0.0541	0.0739
40	*PE*	0.0951	0.0692	-0.2850	0.2914	0.3606
	ML	0.0148	0.0447	-0.1376	0.1068	0.1515
	LAD	0.0085	0.0322	-0.1035	0.0410	0.0733
	WLAD	-0.0078	0.0320	-0.0739	0.0312	0.0631
	MMAD	0.0968	0.0541	-0.0064	0.0392	0.0932
	LMS	0.0119	0.0143	-0.0836	0.0408	0.0551
50	*PE*	0.0823	0.0575	-0.2325	0.2188	0.2763
	ML	0.0157	0.0337	-0.1200	0.0842	0.1180
	LAD	0.0084	0.0251	-0.0789	0.0272	0.0524
	WLAD	-0.0041	0.0250	-0.0580	0.0231	0.0481
	MMAD	0.1011	0.0462	-0.0034	0.0306	0.0768
	LMS	0.0103	0.0116	-0.0689	0.0296	0.0412
100	*PE*	0.0514	0.0264	-0.1357	0.0873	0.1137
	ML	0.0088	0.0141	-0.0777	0.0435	0.0576
	LAD	0.0055	0.0122	-0.0383	0.0100	0.0221
	WLAD	-0.0004	0.0123	-0.0266	0.0096	0.0219
	MMAD	0.1082	0.0295	-0.0025	0.0145	0.0440
	LMS	0.0075	0.0054	-0.0384	0.0138	0.0192

Table 3 Bias, *MSD* and *TMSD* values of estimates for data with two outliers

		$\hat{\phi}$		$\hat{\psi}$		
n		Bias	*MSD*	Bias	*MSD*	*TMSD*
5	*PE*	0.2895	1.3540	-2.1091	8.3666	9.7206
	ML	-0.0131	1.4248	-1.0137	2.0175	3.4423
	LAD	0.3255	1.1998	-2.3403	9.8704	11.0701
	WLAD	-0.0053	1.0128	-2.4441	12.8286	13.8413
	MMAD	0.0699	0.8701	-0.4837	1.1146	1.9847
	LMS	0.0518	0.2844	-1.1407	4.0658	4.3502
10	*PE*	0.3006	0.5107	-1.3035	3.4342	3.9449
	ML	0.0653	0.4546	-0.6498	0.8440	1.2986
	LAD	0.1816	0.3414	-1.3735	4.1329	4.4743
	WLAD	0.0290	0.2680	-1.0483	2.4273	2.6953
	MMAD	0.0644	0.2376	-0.1589	0.2429	0.4805
	LMS	0.0257	0.0969	-0.4031	0.5596	0.6565
20	*PE*	0.2447	0.2443	-0.8315	1.5456	1.7899
	ML	0.0548	0.1623	-0.4167	0.3915	0.5539
	LAD	0.0884	0.1288	-0.5739	1.1349	1.2637
	WLAD	-0.0031	0.0815	-0.2933	0.2068	0.2883
	MMAD	0.0964	0.1088	-0.0693	0.0903	0.1991
	LMS	0.0242	0.0354	-0.1999	0.1405	0.1759
30	*PE*	0.1839	0.1512	-0.5908	0.8686	1.0197
	ML	0.0393	0.0908	-0.3211	0.2649	0.3557
	LAD	0.0328	0.0606	-0.2804	0.2935	0.3541
	WLAD	-0.0104	0.0491	-0.1610	0.0663	0.1154
	MMAD	0.0971	0.0749	-0.0444	0.0558	0.1307
	LMS	0.0145	0.0221	-0.1396	0.0768	0.0989
40	*PE*	0.1573	0.1047	-0.4784	0.5596	0.6642
	ML	0.0341	0.0585	-0.2694	0.1953	0.2538
	LAD	0.0192	0.0352	-0.1776	0.0755	0.1107
	WLAD	-0.0062	0.0333	-0.1129	0.0405	0.0738
	MMAD	0.1011	0.0567	-0.0298	0.0425	0.0992
	LMS	0.0122	0.0143	-0.1078	0.0496	0.0640
50	*PE*	0.1396	0.0888	-0.4055	0.4396	0.5284
	ML	0.0313	0.0452	-0.2404	0.1685	0.2137
	LAD	0.0137	0.0271	-0.1341	0.0454	0.0724
	WLAD	-0.0049	0.0262	-0.0868	0.0278	0.0540
	MMAD	0.1037	0.0477	-0.0233	0.0316	0.0793
	LMS	0.0098	0.0121	-0.0880	0.0352	0.0473
100	*PE*	0.0829	0.0381	-0.2270	0.1607	0.1987
	ML	0.0166	0.0170	-0.1552	0.0841	0.1012
	LAD	0.0045	0.0123	-0.0614	0.0135	0.0258
	WLAD	-0.0030	0.0123	-0.0419	0.0111	0.0234
	MMAD	0.1070	0.0296	-0.0132	0.0151	0.0448
	LMS	0.0054	0.0056	-0.0504	0.0160	0.0216

the smallest *MSD* values, and it also shows a large difference compared to performances of the other estimation methods; (*iii)* For parameter ϕ, although the performance of *WLAD* is not good for $n = 5$, it can be seen that its performance is quite close to the performance of *LMS* as the sample size n increases; *(iv)* For parameter ψ, *MMAD* shows better performance than the others do with the smallest bias of all for all sample sizes. However, almost all of these estimators are asymptotically unbiased when n is large; *(v)* In terms of the *MSD* for parameter ψ, the *MMAD* performs the best of all the estimation methods for $n < 40$. When $n \geq 40$, the *WLAD* shows better performance than the others do; *(vi)* The *MMAD* and *LMS* outperform the other estimation methods in terms of the *TMSD* for $n \leq 10$ and for $n > 10$, respectively.

Additionally, we observe that the bias values of the *MMAD* estimator of ϕ increases, but its *MSD* values decreases as sample size n increases for data with and without outliers. For $n = 5$, *WLAD* and *LAD* estimators demonstrate even poorer performance in terms of both *MSD* of ψ and *TMSD* for all cases (i.e., the data with and without outliers) than *PE* estimator does.

The simulation results presented in Tables 1-3 have been given graphically for ease of interpretation; see Figs. 2-4. From Fig. 2, we can easily see that the *MMAD* estimator is not consistent for location parameter ϕ in all situations, but it is consistent for scale parameter ψ. For the location parameter, the performance of *LMS* is the best of all the estimation methods in terms of the *MSD* for three cases; see Fig. 3. For the scale parameter, the *ML* is the most efficient estimator for data that does not include outliers, and the *MMAD* outperforms the other estimation methods for the same data, and contains one outlier or two outliers, in terms of the *MSD*; see Fig. 3. However, the performance of the considered robust estimators remains almost the same as the sample size gets larger in terms of the *MSD* of ψ. From Fig. 4, similar comments can also be stated for the *TMSD* terms.

5. An example: daily mean wind speed data

In this section, we analyse the daily mean wind speed data for October, taken from the Turkish Meteorological Services for Sinop, Turkey, in 2015. The data is presented in Table 4.

Gumbel distribution is frequently used to model extreme events. In contrast to the literature, we assume that the daily mean wind speed data has Gumbel distribution. On the other hand, the most widely-used statistical distribution to model wind speed data is Weibull in relation to studies of wind energy estimation. However, Weibull distribution may not be suitable for modelling the all wind regimes, such as those having high frequencies of null winds and for short time horizons (Sohoni *et al.* 2016). Therefore, different distributions are used for modelling the wind speed data. In recent years, some studies of other distributions applied to wind speed data contain gamma (Morgan *et al.* 2011), generalized extreme value (Kollu *et al.* 2012), Johnson S_B (Soukissian 2013), Rayleigh (Pishgar-Komleh *et al.* 2015),

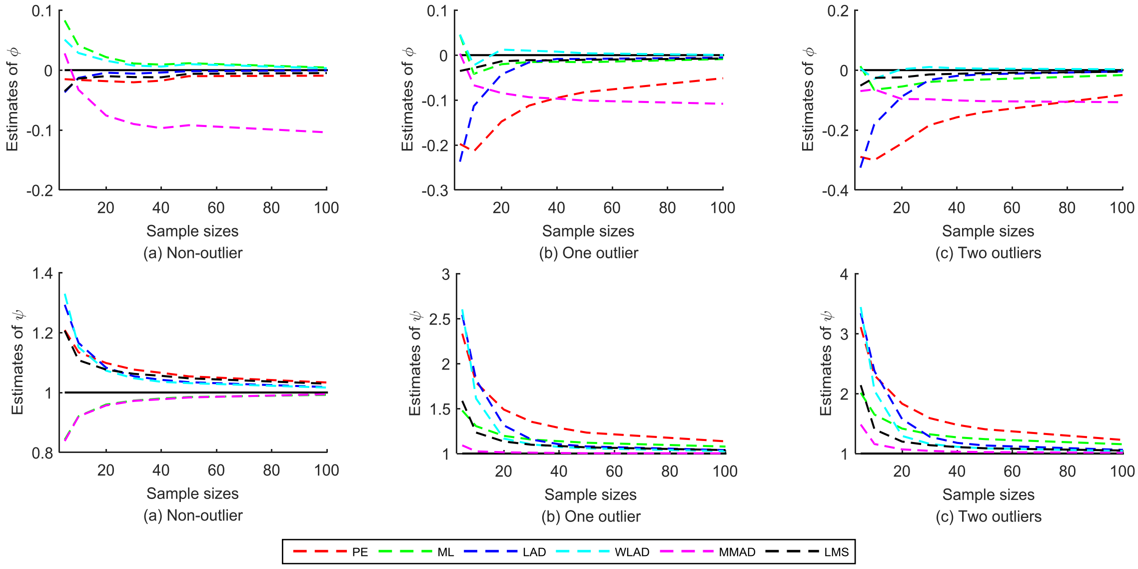


Fig. 2 Plots of the estimates of $\phi = 0$ and $\psi = 1$ for $n = 5, 10, 20, 30, 40, 50, 100$ and the cases of (a) non-outlier, (b) one outlier and (c) two outliers

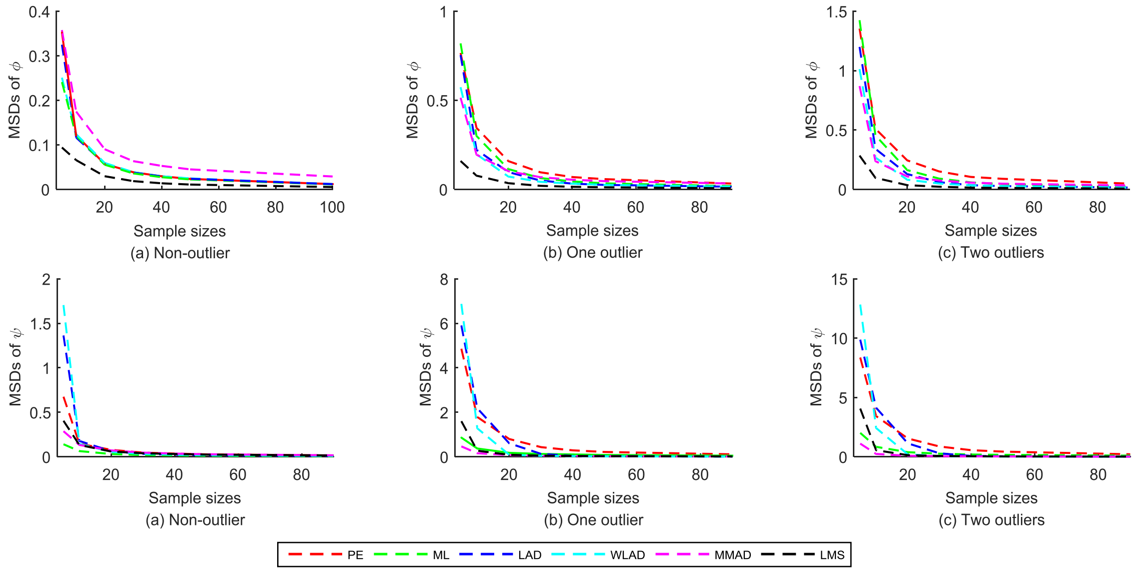


Fig. 3 Plots of the *MSD*s of estimates of $\phi = 0$ and $\psi = 1$ for $n = 5, 10, 20, 30, 40, 50, 100$ and the cases of (a) non-outlier, (b) one outlier and (c) two outliers

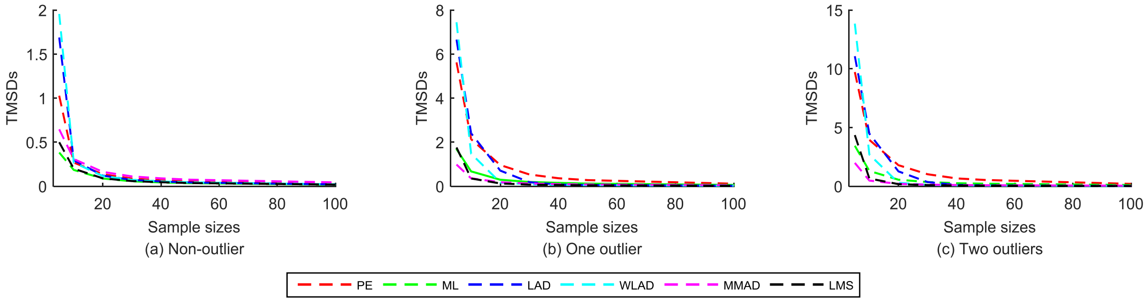


Fig. 4 Plots of the *TMSD* values of estimators for $n = 5, 10, 20, 30, 40, 50, 100$ and the cases of (a) non-outlier, (b) one outlier and (c) two outliers

inverse Weibull (Akgul *et al.* 2016), generalized Lindley and power Lindley (Arslan *et al*. 2017), Birnbaum-Saunders (Mohammadi *et al*. 2017), extended generalized Lindley (Kantar *et al*. 2018).

In this study, we compare the fitting performances of Gumbel and Weibull distributions to the wind speed data set obtained from the Sinop station in Turkey. For this aim, we evaluate the conformity of Gumbel and Weibull (having two parameters, Weibull distribution with shape parameter c and scale parameter σ) to fit the wind speed data by using the Kolmogorov–Smirnov (*KS*) test. The results of the *KS* test (where theoretical value is $KS_{0.05,30} = 0.24$) in Table 5 show that Gumbel and Weibull distributions provide a suitable model for wind speed data since the computed values of the *KS* test for Gumbel and Weibull distributions are less than theoretical value of the *KS* test.

Next, we determine which distribution provides a better fit to the wind speed data by using the root mean square error (*RMSE*) and coefficient of determination (R^2) criteria. The following formulas are used to calculate them

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{F}_i - F_i\right)^2}$$

and

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\hat{F}_i - F_i)^2}{\sum_{i=1}^{n}(\hat{F}_i - \bar{\hat{F}})^2}.$$

Here, $\hat{F}_i$ is the estimated cumulative probability of the corresponding to the distribution, $\bar{\hat{F}}$ is the mean of $\hat{F}_i$ and F_i is the observed cumulative probability.

Table 5 shows that Gumbel distribution provides better modelling than Weibull distribution for wind speed data because the lower value of $RMSE$ and the higher value of R^2 indicate better fit. Furthermore, to identify which distribution provides better fit to the wind speed data visually, Q-Q plots showing fits of the data to Gumbel and Weibull and histograms with fitted Gumbel and Weibull probability plots for the data are obtained, see Figs. 5 and 6.

From Fig. 5, it can be seen that Gumbel distribution provides better fitting to the data than Weibull distribution because the data points in the Q-Q plot for Weibull distribution indicate more deviation from a straight line compared to the data points in the Q-Q plot for Gumbel distribution.

Table 4 Daily mean wind speed data for October

3.8	3.2	2.0	1.7	3.2	1.7	2.8	4.6	2.6	2.4	5.0
2.9	2.0	2.7	3.3	2.1	1.1	1.8	3.6	2.4	2.8	4.1
1.7	2.5	2.4	1.9	1.4	1.4	1.5	2.4	5.1		

Table 5 Estimates of the parameters and computed values of R^2 and *RMSE* based the *ML* of Gumbel and Weibull distributions for the wind speed data

Model	*ML* estimate		*KS*	*RMSE*	R^2
Gumbel	$\hat{\phi} = 2.1765$	$\hat{\psi} = 0.7947$	0.0679	**0.0319**	**0.9886**
Weibull	$\hat{c} = 2.7289$	$\hat{\sigma} = 2.9833$	0.1087	0.0464	0.9721

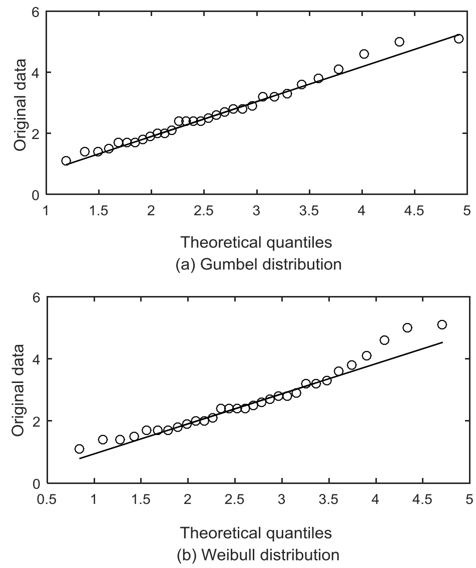


Fig. 5 Q–Q plots showing fits of the data to Gumbel and Weibull obtained from *ML* estimates

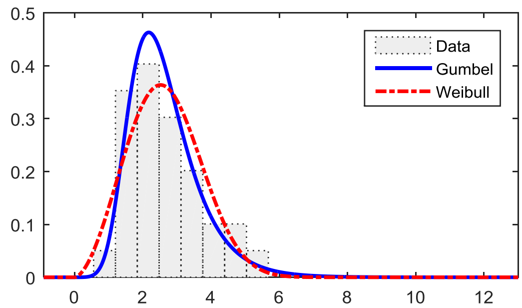
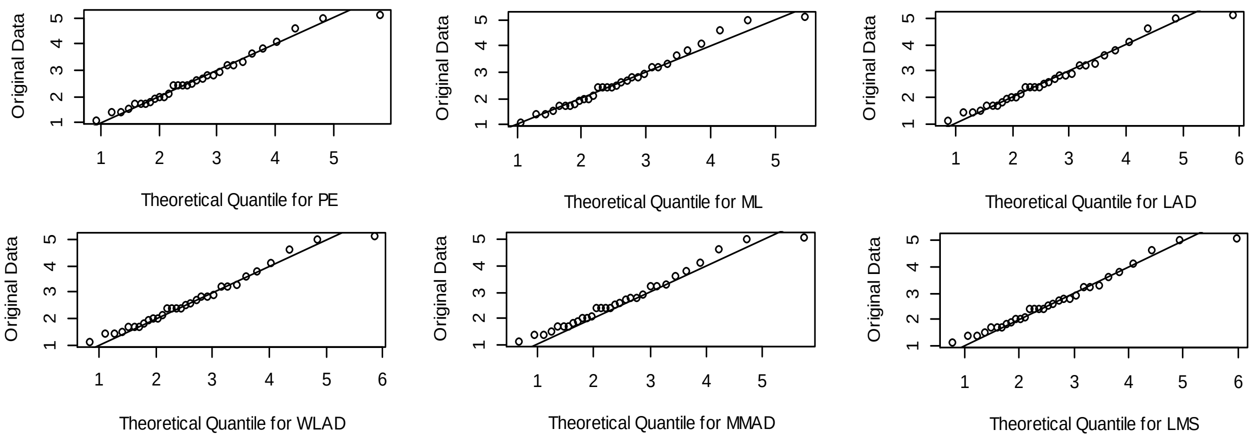


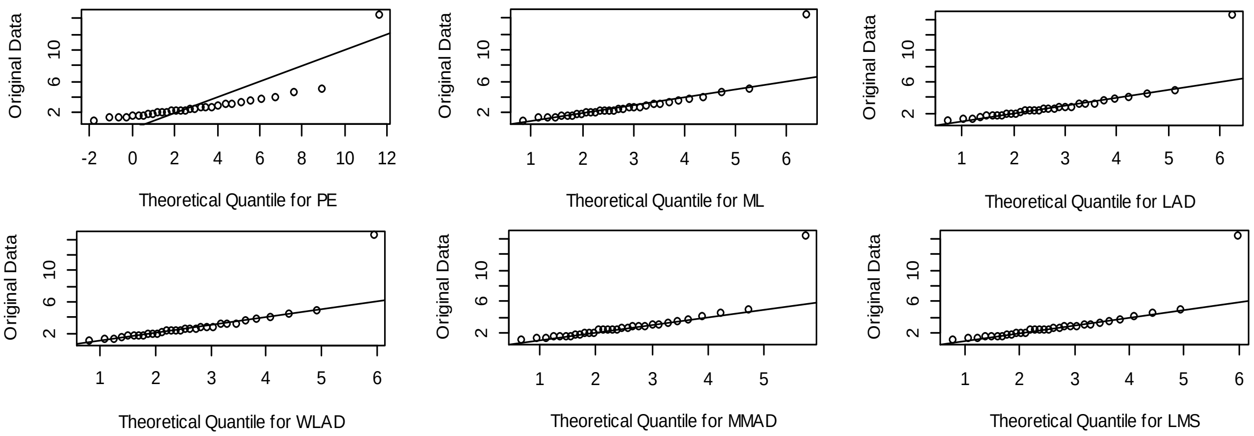
Fig. 6 Histogram with fitted densities for the wind speed data

Additionally, Fig. 6 shows that Gumbel distribution gives better fitting to the wind speed data compared to Weibull distribution since Gumbel fitted density is more compatible throughout the whole histogram of the wind speed data than Weibull fitted density.

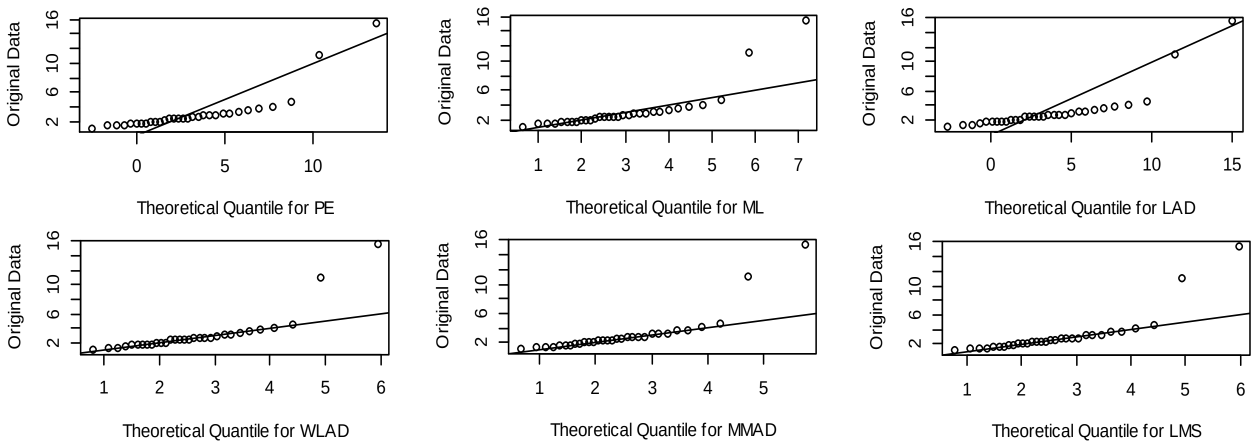
Now, we compare the performances of mentioned the estimation methods for the wind speed data with and without outliers having Gumbel distribution using numerical and graphical methods. The data is modified as a generated large outlier according to $U\left(X_{(n)} + 5S,\ X_{(n)} + 10S\right)$, where $X_{(n)}$ is the maximum value observed in the data set and S is a sample standard deviation of the data set. Next, we replace $X_{(n)}$ with the generated outlier. In order to compare graphically, since the probability density plot and the Q–Q plot use the sample data to show how close a data sample is to a specified distribution, we use them to determine how a model fails to fit due to outliers (Cox 2005). Therefore, Q-Q plots and fitted Gumbel densities superimposed onto a histogram obtained from all considered estimators for the original data and modified data (with one outlier or two outliers) are given in Figs. 7



(a) Non-outlier

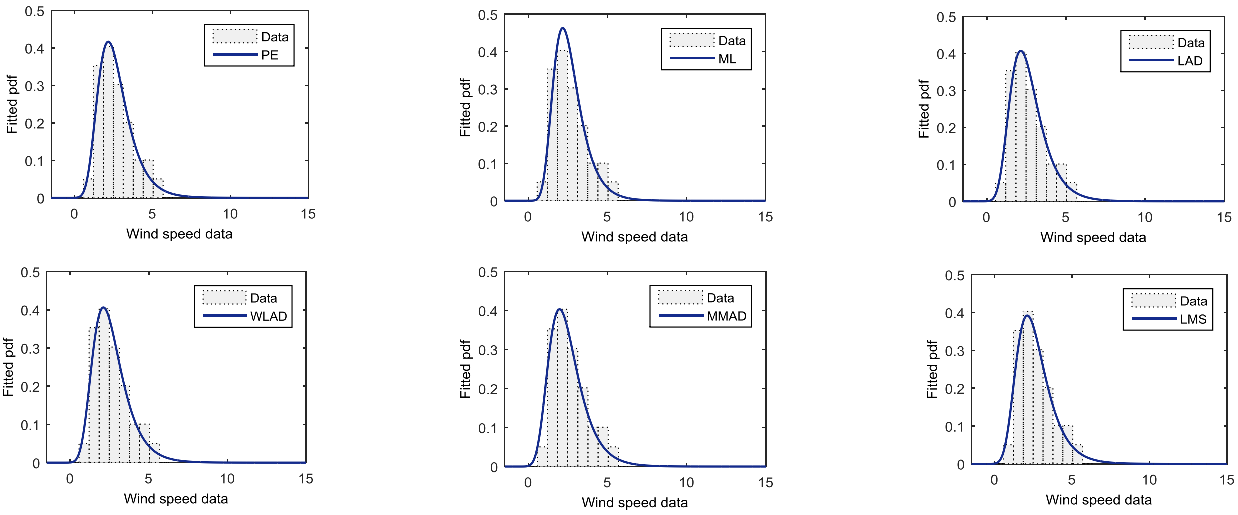


(b) One outlier

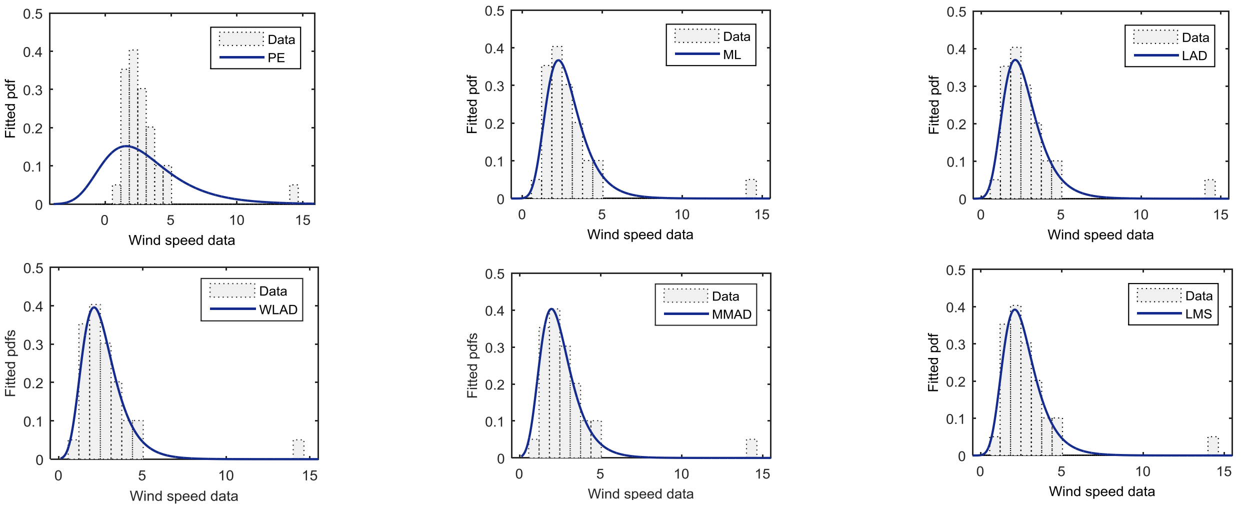


(c) Two outliers

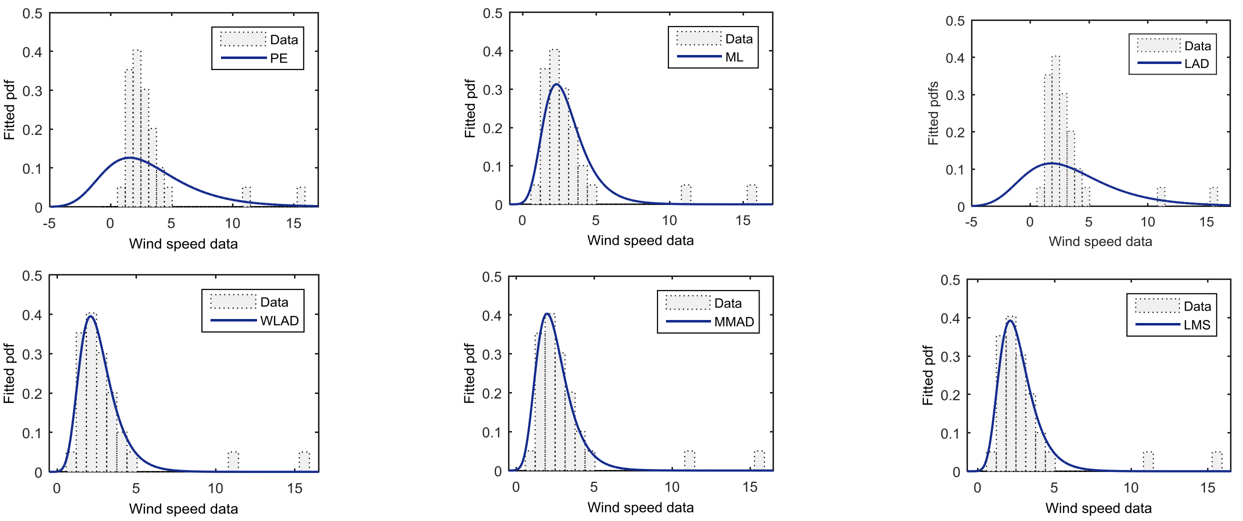
Fig. 7 Q–Q plots showing fits of the data to Gumbel distribution obtained from the *PE*, *ML*, *LAD*, *WLAD*, *MMAD* and *LMS* in the cases of (a) non-outlier, (b) one outlier and (c) two outliers



(a) Non-outlier



(b) One outlier



(c) Two outliers

Fig. 8 Histograms with *pdf* plots showing fits of the data to Gumbel distribution obtained from the *PE*, *ML*, *LAD*, *WLAD*, *MMAD* and *LMS* in the cases of (a) non-outlier, (b) one outlier and (c) two outliers and 8, respectively. Because the outlier pulls the line toward itself, Fig. 7 shows that the Q–Q plot based on *PE* provides a very poor model fit for modelling the wind speed probability distribution in the case of one outlier or two outliers and the Q–Q plot based on *LAD* gives a very poor model fit to model the data including two outliers when compared with the other models (which are based on the *ML*, *MMAD*, *LMS* and *WLS*). On the other hand, Q–Q plots based on the *MMAD*, *LMS* and *WLS* offer a very good performance to model the wind speed in the case of one outlier or two outliers. From Fig. 8, in the case of one outlier or two outliers, the fitted densities obtained from the *MMAD* and *LMS* are better than the other fitted densities in terms of modelling the data, and the *WLAD* follows them. A density probability plot obtained from the *PE* provides a very poor model fit to the distribution of wind speeds for the presence of one outlier or two outliers. Furthermore, the fitted density obtained from the *LAD* is similar to the fitted density obtained from the *PE* in terms of modelling the data in the case of two outliers. Fitting of the Gumbel distribution obtained from the *ML* decreases slowly compared to the *PE* and *LAD* as the number of outlier increases; see Fig. 8.

After this, we obtain estimates of the ϕ and ψ parameters of Gumbel distribution using the methodologies of the *PE*, *ML*, *LAD*, *WLAD*, *MMAD* and *LMS* for the original data and the modified data, i.e., data including outliers. The results are presented in Table 6: (i) Almost all of the considered estimators of ϕ are not affected by outliers as the estimators of ψ are; (ii) For the scale parameter, the most affected estimator by one outlier is the *PE* among the others; the next is the *ML*. The *LAD* estimator is slightly affected in the case of one outlier; however, in the case of two outliers, it is affected as much as *PE* is. These results are confirmed by the fitted Gumbel *pdf* plot for *LAD* in Fig. 8.

Table 6 Estimates of parameters for wind speed data with and without outliers

	PE		*ML*		*LAD*	
Cases	$\hat{\phi}$	$\hat{\psi}$	$\hat{\phi}$	$\hat{\psi}$	$\hat{\phi}$	$\hat{\psi}$
Non-outlier	2.1594	0.8831	2.1765	0.7947	2.1456	0.9045
One outlier	1.6084	2.4258	2.2502	1.0029	2.1255	0.9951
Two outliers	1.5651	2.9137	2.3006	1.1758	1.8025	3.1946

	WLAD		*MMAD*		*LMS*	
	$\hat{\phi}$	$\hat{\psi}$	$\hat{\phi}$	$\hat{\psi}$	$\hat{\phi}$	$\hat{\psi}$
Non-outlier	2.1085	0.9061	1.9640	0.9126	2.1035	0.9386
One outlier	2.1046	0.9301	1.9640	0.9126	2.1035	0.9386
Two outliers	2.1051	0.9316	1.9640	0.9126	2.1035	0.9386

It should be noted that the *MMAD* and *LMS* are robust against outliers since their estimates do not change for both the location parameter ϕ and the scale parameter ψ with the *WLAD* following them, since it is slightly affected by outliers; see Table 6. The results of the study show a parallelism with numerical results obtained by Kantar and Yıldırım (2015) for certain robust estimators of the parameters of the extended Burr type III distribution.

6. Conclusions

In this paper, alternative robust estimation methods are considered to obtain unknown parameters of Gumbel distribution. The performances of the estimators are compared with respect to bias, mean square deviation and total mean square deviation comparison criteria using a Monte-Carlo simulation with the different cases, i.e., the data with and without outliers. Simulation results have shown that the *LMS* estimator is the most efficient in estimating the location parameter of Gumbel distribution for data with and without outliers in terms of the *MSD*. The *MMAD* estimator of the scale parameter has the smallest *MSD* values for small ($n \leq 10$) and moderate ($n = 20$) sample sizes in the cases of one outlier or two outliers. Furthermore, the *LAD* is a robust estimator, however it has larger *MSD* of scale parameter and *TMSD* values compared to *PE* estimator for $n \leq 10$ in all cases.

In application, it is shown that Gumbel distribution provides better fitting to daily mean wind speed data than Weibull distribution. Finally, the data set modified by outliers is analysed to demonstrate the suitability of the robust estimators. Consequently, the results obtained from the simulations and real data example show that the considered robust estimators for Gumbel distribution may be preferred as plausible alternative estimators to deal with outliers.

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