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Analytical solution for scale-dependent static stability analysis of temperature-dependent nanobeams subjected to uniform temperature distributions

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Abstract. In this paper, the thermo-mechanical buckling characteristics of functionally graded (FG) size-dependent Timoshenko nanobeams subjected to an in-plane thermal loading are investigated by presenting a Navier type solution for the first time. Material properties of FG nanobeam are supposed to vary continuously along the thickness according to the power-law form and the material properties are assumed to be temperature-dependent. The small scale effect is taken into consideration based on nonlocal elasticity theory of Eringen. The nonlocal governing equations are derived based on Timoshenko beam theory through Hamilton's principle and they are solved applying analytical solution. According to the numerical results, it is revealed that the proposed modeling can provide accurate critical buckling temperature results of the FG nanobeams as compared to some cases in the literature. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of the several parameters such as material distribution profile, small scale effects and aspect ratio on the critical buckling temperature of the FG nanobeams in detail. It is explicitly shown that the thermal buckling of a FG nanobeams is significantly influenced by these effects. Numerical results are presented to serve as benchmarks for future analyses of FG nanobeams.

Keywords: thermal buckling; Timoshenko beam theory; functionally graded material; nonlocal elasticity theory

1. Introduction

Structural elements such as beams, plates, and membranes in micro or nanolength scale are commonly used as components in micro/nano electromechanical systems (MEMS/NEMS). Therefore understanding the mechanical and physical properties of nanostructures is necessary for its practical applications. Nanoscale engineering materials have attracted great interest in modern science and technology after the invention of carbon nanotubes (CNTs) by Iijima, (1991). They have significant mechanical, thermal and electrical performances that are superior to the conventional structural materials. In recent years, nanobeams and CNTs hold a wide variety of potential applications (Zhang *et al*. 2004, Wang 2005, Wang and Varadan 2006) such as sensors, actuators, transistors, probes, and resonators in NEMSs. For instance, in MEMS/NEMS; nanostructures have been used in many areas including communications, machinery, information technology and biotechnology technologies.

Moreover functionally graded materials (FGMs) are composite materials with inhomogeneous micromechanical structure. They are generally composed of two different parts such as ceramic with excellent characteristics in heat and corrosive resistances and metal with toughness. The material properties of FGMs change smoothly between two surfaces and the advantages of this combination lead to novel structures which can withstand in large mechanical loadings under high temperature environments (Ebrahimi and Rastgoo 2008). Presenting novel properties, FGMs have also attracted intensive research interests, which were mainly focused on their static, dynamic and vibration characteristics of FG structures (Ebrahimi *et al.* 2009, Aghelinejad *et al.* 2011, Ebrahimi and Rastgoo 2008a, b, 2009, 2011).

Since conducting experiments at the nanoscale is a daunting task, and atomistic modeling is restricted to small-scale systems owing to computer resource limitations, continuum mechanics offers an easy and useful tool for the analysis of CNTs. However the classical continuum models need to be extended to consider the nanoscale effects and this can be achieved through the nonlocal elasticity theory proposed by Eringen (1972) which consider the size-dependent effect. According to this theory, the stress state at a reference point is considered as a function of strain states of all points in the body. This nonlocal theory is proved to be in accordance with atomic model of lattice dynamics and with experimental observations on phonon dispersion (Eringen 1983).

Moreover, in recent years the application of nonlocal elasticity theory, in micro and nanomaterials has received a considerable attention within the nanotechnology community. Peddieson *et al*. (2003) proposed a version of nonlocal elasticity theory which is employed to develop a nonlocal Euler beam model. Wang and Liew (2007) carried out the static analysis of micro- and nano-structures based on nonlocal continuum mechanics using Euler-Bernoulli

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beam theory and Timoshenko beam theory. Aydogdu (2009) proposed a generalized nonlocal beam theory to study bending, buckling, and free vibration of nanobeams based on Eringen model using different beam theories. Phadikar and Pradhan (2010) reported finite element formulations for nonlocal elastic Euler–Bernoulli beam and Kirchhoff plate theory. Civalek and Demir (2011) developed a nonlocal beam model for the bending analysis of microtubules based on the Euler–Bernoulli beam theory. The size effect is taken into consideration using the Eringen's nonlocal elasticity theory and the analysis of mechanical characteristics of nanostructures is one of the interesting research topics. (Ebrahimi and Barati 2016f, g, h, Ebrahimi and Barati 2017).

Furthermore, with the development of the material technology, FGMs have also been employed in MEMS/NEMS (Witvrouw and Mehta 2005, Lee *et al.* 2006). Because of high sensitivity of MEMS/ NEMS to external stimulations, understanding mechanical properties and vibration behavior of them are of significant importance to the design and manufacture of FG MEMS/NEMS. Thus, establishing an accurate model of FG nanobeams is a key issue for successful NEMS design. Kiani *et al.* (2011) proposed the critical buckling temperature of Timoshenko FGM beams with surface-bonded piezoelectric layers subjected to both thermal loading and constant electric voltage. It was shown that increasing the thickness of piezoelectric FGM beam, the critical buckling temperature difference increases. The nonlinear static response of FGM beams under in-plane thermal loading is studied by Ma and Lee (2012). Ke and Wang (2011) exploited the size effect on dynamic stability of functionally graded Timoshenko microbeams. Employing modified couple stress theory the nonlinear free vibration of FG microbeams based on von-Karman geometric nonlinearity was presented by Ke *et al.* (2012). It was revealed that both the linear and nonlinear frequencies increase significantly when the thickness of the FGM microbeam was comparable to the material length scale parameter. Eltaher *et al*. (2013) applied a finite element formulation for static-buckling analysis of FG nanobeams based on nonlocal Euler beam theory. Using nonlocal Timoshenko and Euler–Bernoulli beam theory, Simsek and Yurtcu (2013) investigated bending and buckling of FG nanobeam by analytical method. Recently, Ebrahimi and Salari (2015) studied thermal buckling and free vibration of functionally graded nanobeam within the framework of Timoshenko beam model subjected to linear temperature rise. Thermal buckling and free vibration analysis of FG nanobeams subjected to temperature distribution have been exactly investigated by Ebrahimi and Salari (2015a, b, c) and Ebrahimi *et al.* (2015a, b). Ebrahimi and Barati (2016o, p, q) investigated buckling behavior of smart piezoelectrically actuated higher-order size-dependent graded nanoscale beams and plates in thermal environment.

As the common use of FGM beams in high temperature environment leads to considerable changes in material properties. Consequently, thermal effects become important when the FG nanodevice has to operate in either extremely hot or cold temperature environments. Therefore, there is strong scientific need to understand the thermal buckling behavior of graded nanobeams under thermal loading. According to this fact, in this study, thermal buckling characteristics of FG nanobeams considering the effect of uniform temperature rise across the thickness is analyzed. An analytical method called Navier solution is employed for thermal buckling analysis of FG nanobeams for the first time. The thermo-mechanical material properties of the beam is assumed to be graded in the thickness direction according to the power law distribution. Non-classical Timoshenko beam model and Eringen's nonlocal elasticity theory can capture size effect are employed. Governing equations and boundary conditions for the thermal buckling of a nonlocal FG nanobeam have been derived via Hamilton's principle. These equations are solved using Navier type method and numerical solutions are obtained. The detailed mathematical derivations are presented while the emphasis is placed on investigating the effect of several parameters such as power-law index, aspect ratio and length scale parameter on buckling characteristics of size-dependent FG nanobeams. Comparison between results of the present study and those available data in literature shows the accuracy of this model. Due to lack of similar results on the buckling response of FG nanostructure, this study is likely to fill a gap in the state of the art of this problem.

2. Theory and formulation

2.1 Nonlocal power-law FG nanobeam equations

Consider a FG nanobeam of length *L*, width *b* and uniform thickness *h* in the unstressed reference configuration. The coordinate system for FG nanobeam is shown in Fig. 1. The nanobeam is made of elastic and isotropic functionally graded material with properties varying smoothly in the z thickness direction only. The effective material properties of the FG beam such as Young's modulus E_f and shear modulus G_f are assumed to vary continuously in the thickness direction (z-axis direction) according to a power function of the volume fractions of the constituents.

According to the rule of mixture, the effective material properties, P_f , can be expressed as (Simsek and Yurtcu, 2013)

$$P_f = P_c V_c + P_m V_m \tag{1}$$

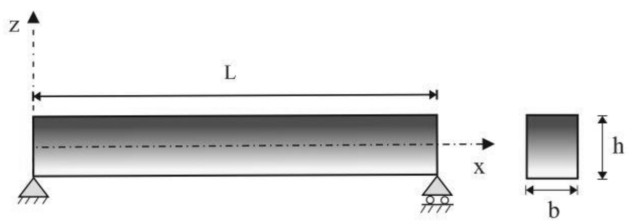


Fig. 1 Geometry and coordinates of FG nanobeam

where P_m, P_c, V_m and V_c are the material properties and the volume fractions of the metal and the ceramic constituents related by

$$V_c + V_m = 1 \tag{2a}$$

The volume fraction of the ceramic constituent of the beam is assumed to be given by

$$V_c = (\frac{z}{h} + \frac{1}{2})^p \tag{2b}$$

Here *p* is the non-negative variable parameter (power-law exponent) which determines the material distribution through the thickness of the beam and *z* is the distance from the mid-plane of the FG beam. The FG beam becomes a fully ceramic beam when *p* is set to be zero. Therefore, from Eqs. (1) and (2), the effective material properties of the FG nanobeam such as Young's modulus (E), thermal expansion (α) and Poisson's ratio (ν) can be expressed as follows

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_m \tag{3}$$

To predict the behavior of FGMs under high temperature more accurately, it is necessary to consider the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in function of temperature $T(K)$ can be expressed as (Touloukian 1967)

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{4}$$

where $T = T_0 + \Delta T$ and $T_0 = 300\,\mathrm{K}$ (ambient or free stress temperature), ΔT is the temperature change, P_0, P_{-1}, P_1, P_2 and P_3 are the temperature dependent coefficients which can be seen in the table of materials properties (Table 1) for Si_3N_4 and SUS304 . The bottom surface (*z = -h/2*) of FG nanobeam is pure metal (SUS304), whereas the top surface (*z = h/2*) is pure ceramics (Si_3N_4).

2.2 Kinematic relations

The equations of motion is derived based on the Timoshenko beam theory according to which the displacement field at any point of the beam can be written as

$$\begin{aligned} u_x(x,z,t) &= u(x,t) + z\,\varphi(x,t), \\ u_z(x,z,t) &= w(x,t) \end{aligned} \tag{5}$$

where t is time, φ is the total bending rotation of the cross-section, u and w are displacement components of the mid-plane along x and z directions, respectively. Therefore, according to the Timoshenko beam theory, the nonzero strains are obtained as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\,\frac{\partial \varphi}{\partial x} \tag{6}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \varphi \tag{7}$$

where ε_{xx} and γ_{xy} are the normal strain and shear strain, respectively. Based on the Hamilton's principle, which states that the motion of an elastic structure during the time interval $t_1 < t < t_2$ is such that the time integral of the total dynamics potential is extremum

$$\int_0^t \delta(U + V)\,dt = 0 \tag{8}$$

Here U is strain energy and V is work done by external forces. The virtual strain energy can be calculated as

$$\begin{aligned} \delta U &= \int_v \sigma_{ij}\,\delta\,\varepsilon_{ij}\,dV = \\ &= \int_v (\sigma_{xx}\,\delta\,\varepsilon_{xx} + \sigma_{xz}\,\delta\,\gamma_{xz})dV \end{aligned} \tag{9}$$

Substituting Eqs. (6) and (7) into Eq. (9) yields

$$\delta U = \int_0^L (N(\delta\frac{\partial u}{\partial x}) + M(\delta\frac{\partial \varphi}{\partial x}) + Q(\delta\frac{\partial w}{\partial x} + \delta\varphi))dx \tag{10}$$

In which N is the axial force, M is the bending moment and Q is the shear force. These stress resultants used in Eq. (10) are defined as

$$\begin{aligned} N &= \int_A \sigma_{xx}\,dA, \quad M = \int_A \sigma_{xx} z\,dA, \\ Q &= \int_A K_s \sigma_{xz}\,dA \end{aligned} \tag{11}$$

where $K_s = 5/6$ is the shear correction factor. For a typical FG nanobeam which has been in high temperature environment for a long period of time, it is assumed that the temperature can be distributed uniformly across its thickness, so that the case of uniform temperature rise is taken into consideration. In this investigation, initial uniform temperature ($T_0 = 300\ K$), which is a stress free state, changes to final temperature with ΔT . Hence, the first variation of the work done corresponding to temperature change can be written in the form (Kim 2005, Mahi *et al.* 2010)

$$\delta V = \int_0^L N^T\,\frac{\partial w}{\partial x}\,\delta(\frac{\partial w}{\partial x})dx \tag{12}$$

where N^T is thermal resultant can be expressed as

$$N^T = \int_{-h/2}^{h/2} E(z,T)\,\alpha(z,T)\Delta T\,dz \tag{13}$$

By Substituting Eqs. (10) and (12) into Eq. (8) and setting the coefficients of $\delta u, \delta w$ and $\delta\varphi$ to zero, the following Euler–Lagrange equation can be obtained

$$\frac{\partial N}{\partial x}=0 \tag{14a}$$

$$\frac{\partial Q}{\partial x}-N^{T}\frac{\partial^2 w}{\partial x^2}=0 \tag{14b}$$

$$\frac{\partial M}{\partial x}-Q=0 \tag{14c}$$

Under the following boundary conditions

$$N=0 \text{ or } u=0 \text{ at } x=0 \text{ and } x=L \tag{15a}$$

$$Q=0 \text{ or } w=0 \text{ at } x=0 \text{ and } x=L \tag{15b}$$

$$M=0 \text{ or } \varphi=0 \text{ at } x=0 \text{ and } x=L \tag{15c}$$

2.3 The nonlocal elasticity model for FG nanobeam

According on Eringen nonlocal elasticity model (Eringen and Edelen 1972), for a homogeneous and isotropic elastic solid the nonlocal stress-tensor components σ_{ij} at any point x in the body may be obtained as

$$\sigma_{ij}(x)=\int_{\Omega}\alpha(|x'-x|,\tau)\,t_{ij}(x')\,d\Omega(x') \tag{16a}$$

$$t_{ij}=C_{ijkl}\varepsilon_{kl} \tag{16b}$$

where t_{ij} are the components of the classical stress tensor at point x which are related to the components of the linear strain tensor ε_{kl} by the conventional constitutive relations. The kernel function $\alpha(|x'-x|,\tau)$ is nonlocal modulus, $|x'-x|$ is the Euclidean distance and τ is a material constant. According to (Eringen 1983) for a class of physically admissible kernel $\alpha(|x'-x|,\tau)$ it is possible to represent the integral constitutive relations given by Eq. (16) in an equivalent differential form as

$$(1-(e_0 a)^2\nabla^2)\sigma_{kl}=t_{kl} \tag{17}$$

where ∇^2 is the Laplacian operator. $(e_0 a)^2$ is the nonlocal parameter, in which, e_0 is a constant appropriate to each material and a is an internal characteristics length. For an elastic material in the one dimensional case, the nonlocal constitutive relations may be simplified as

$$\sigma_{xx}-(e_0 a)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\,\varepsilon_{xx} \tag{18}$$

$$\sigma_{xz}-(e_0 a)^2\frac{\partial^2\sigma_{xz}}{\partial x^2}=G\,\gamma_{xz} \tag{19}$$

where σ and ε are the nonlocal stress and strain, respectively. E is the Young's modulus, $G=E/2(1+\nu)$ is the shear modulus (where ν is the poisson's ratio). For Timoshenko nonlocal FG beam, Eqs. (18) and (19) can be rewritten as

$$\sigma_{xx}-\mu\frac{\partial^2\sigma_{xx}}{\partial x^2}=E(z)\varepsilon_{xx} \tag{20}$$

$$\sigma_{xz}-\mu\frac{\partial^2\sigma_{xz}}{\partial x^2}=G(z)\gamma_{xz} \tag{21}$$

where ($\mu=(e_0 a)^2$). Integrating Eqs. (20) and (21) over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal Timoshenko FG beam theory can be obtained as follows

$$N-\mu\frac{\partial^2 N}{\partial x^2}=A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial\varphi}{\partial x} \tag{22}$$

$$M-\mu\frac{\partial^2 M}{\partial x^2}=B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial\varphi}{\partial x} \tag{23}$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2}=C_{xz}\left(\frac{\partial w}{\partial x}+\varphi\right) \tag{24}$$

in which the cross-sectional rigidities are defined as

$$(A_{xx},B_{xx},D_{xx})=\int_A E(z,T)(1,z,z^2)dA \tag{25}$$

$$C_{xz}=K_s\int_A G(z)\,dA \tag{26}$$

The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq. (14(a)) into Eq. (22) as follows

$$N=A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial\varphi}{\partial x} \tag{27}$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of M from Eq. (14(c)) into Eq. (23) as follows

$$M=B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial\varphi}{\partial x}+\mu\left(N^{T}\frac{\partial^2 w}{\partial x^2}\right) \tag{28}$$

By substituting for the second derivative of Q from Eq. (14(b)) into Eq. (24), the following expression for the nonlocal shear force is derived

$$Q = C_{xz}\left(\frac{\partial w}{\partial x} + \varphi\right) + \mu\left(N^T \frac{\partial^3 w}{\partial x^3}\right) \quad (29)$$

The nonlocal governing equations of Timoshenko FG nanobeam in terms of the displacement can be dreived by substituting for N, M and Q from Eqs. (27)-(29), respectively, into Eq. (14) as follows

$$A_{xx}\frac{\partial^2 u}{\partial x^2} + B_{xx}\frac{\partial^2 \varphi}{\partial x^2} = 0 \quad (30a)$$

$$C_{xz}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x}\right) + \mu\left(N^T \frac{\partial^4 w}{\partial x^4}\right) - N^T \frac{\partial^2 w}{\partial x^2} = 0 \quad (30b)$$

$$B_{xx}\frac{\partial^2 u}{\partial x^2} + D_{xx}\frac{\partial^2 \varphi}{\partial x^2} - C_{xz}\left(\frac{\partial w}{\partial x} + \varphi\right) = 0 \quad (30c)$$

3. Solution procedures

In this section, the analytical solutions of the governing equations for thermal buckling of FG nanobeam with simply supported (S-S) boundary conditions are derived by using Navier method. The displacement functions are expressed as product of undetermined coefficients and known trigonometric functions to satisfy the governing equations and the conditions at $x = 0, L$. The following displacement fields are assumed to be of the form

$$u(x,t) = \sum_{n=1}^{\infty} U_n \cos\left(\frac{n\pi}{L}x\right)e^{i\omega_n t} \quad (31)$$

$$w(x,t) = \sum_{n=1}^{\infty} W_n \sin\left(\frac{n\pi}{L}x\right)e^{i\omega_n t} \quad (32)$$

$$\varphi(x,t) = \sum_{n=1}^{\infty} \varphi_n \cos\left(\frac{n\pi}{L}x\right)e^{i\omega_n t} \quad (33)$$

where (U_n, W_n, φ_n) are the unknown Fourier coefficients to be determined for each n value. Substituting Eqs. (31)-(33) into Eqs. (30(a))-(30(c)) respectively, and setting the determinant of the coefficient matrix, the analytical solutions can be obtained from the following equations

$$\left\{([K] + \Delta T\,[K_T])\right\}\begin{Bmatrix} U_n \\ W_n \\ \phi_n \end{Bmatrix} = 0 \quad (34)$$

where $[K]$ and $[K_T]$ are stiffness matrix and the coefficient matrix of temperature change, respectively. By setting this polynomial to zero, we can find critical buckling temperature ΔT_{cr} .

4. Numerical results and discussions

In this section, the thermal buckling of an FGM nanobeam under uniform thermal loading is investigated through some numerical examples and some comparisons are made between the results obtained from Navier solution method and other numerical technique so that the accuracy of present work is verified. Also, to demonstrate the length-to-thickness ratio and nonlocal parameter effects on the thermal buckling analysis of FG nanobeams, variations of the critical buckling temperatures versus nonlocal parameter, power law index, and aspect ratios of the FG nanobeam, are presented in this section. To this end, the nonlocal FG beam made of SUS304 and Si_3N_4 , with thermo-mechanical material properties listed in Table 1, is considered. The bottom surface of the graded nanobeam is SUS304 rich, whereas the top surface of the beam is Si_3N_4 rich. Also, the beam geometry has the following dimensions: L (length) = 10 nm and h (thickness) = varied.

The numerical or analytical results for the thermal buckling behavior of FG nanobeam based on the nonlocal elasticity theory are not available in the literature. As part of the validation of the present method, a comparison study is performed to check the reliability of the present method and formulation. For this purpose, the FG nanobeam consists of SUS304 and Si_3N_4 is considered. Thus to check the accuracy of the developed model, in Table 2, the critical buckling temperatures of S-S FG nanobeams under linear thermal loading are compared with those of Ebrahimi and Salari (2015) which has been obtained by analytical solution for various values of the gradient index and nonlocality parameter. It is obvious from Table 2 that there is good agreement between the two results.

After extensive validation of the present formulation for S-S FG nanobeams, the effects of different parameters such as aspect ratio, nonlocality parameter and gradient index on the thermal buckling of FG nanobeam are investigated. In Table 3 critical buckling temperature of the simply supported FG nanobeams are presented for various values of the gradient index (p =0,0.2,0.5,1,2,5), nonlocal parameters (μ =0,1,2,3,4) and three different values of aspect ratio (L/h =40, 50, 60) based on analytical Navier solution method. It is evident from the results of the table that increasing the nonlocality parameter yields the reduction in buckling temperature for every material graduation and aspect ratio parameters, which these observations mean that the small scale effects in the nonlocal model make FG nanobeams more flexible. So that by fixing other parameters and increasing nonlocal parameter from 0 to 4 the ΔT_{cr} decreases about 28%. In addition, it is indicated that increase the power indexes and aspect ratio parameter lead to a decrease of the ΔT_{cr} .

Table 1 Physical and mechanical properties of Si_3N_4 and SUS304

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α(K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ν	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α(K^{-1})	12.330e-6	0	8.086e-4	0	0
	ν	0.3262	0	-2.002e-4	3.797e-7	0

Table 2 Comparison of the critical buckling temperature for a S-S FG nanobeam under linear thermal loading with various volume fraction index (h =0.25 nm, L =10 nm)

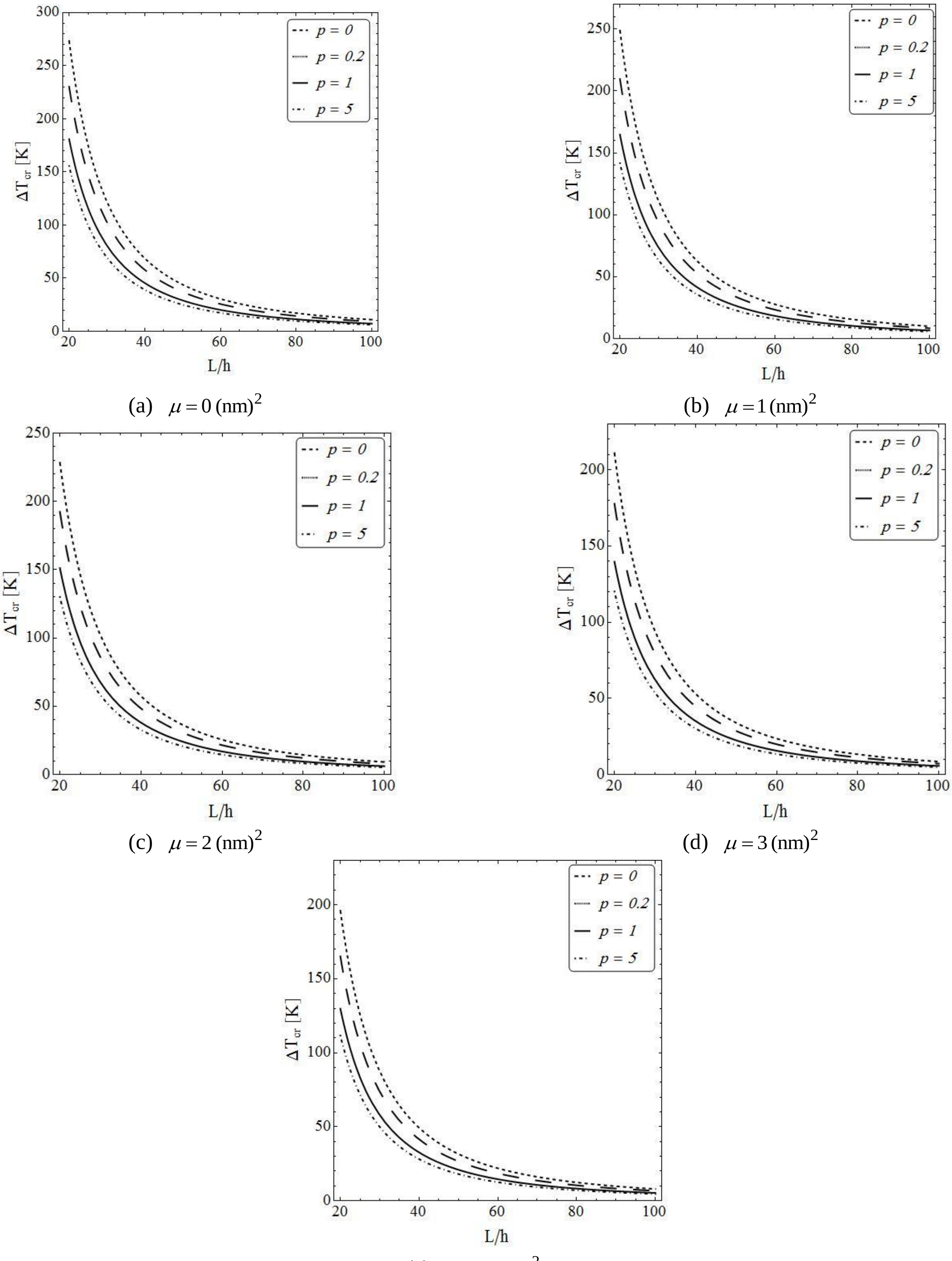
μ (nm)^2	p = 0		p = 0.5		p = 1		p = 5	
	Ebrahimi and Salari (2015b)	Present	Ebrahimi and Salari (2015b)	Present	Ebrahimi and Salari (2015b)	Present	Ebrahimi and Salari (2015b)	Present
0	127.3340	127.334297	95.5739	95.573949	84.6229	84.622945	69.4307	69.430660
1	114.9980	114.997534	86.0456	86.045625	76.0818	76.081774	62.2798	62.279780
2	104.6950	104.694508	78.0881	78.088061	68.9486	68.948630	56.3077	56.307734
3	95.9606	95.960616	71.3424	71.342420	62.9019	62.901852	51.2452	51.245221
4	88.4628	88.462758	65.5514	65.551434	57.7108	57.710823	46.8992	46.899162

Table 3 Material graduation and aspect ratio effect on the critical buckling temperature of a S-S FG nanobeam with different nonlocality parameters

μ (nm)^2	L / h	Gradient index					
		0	0.2	0.5	1	2	5
0	40	68.6671	57.8509	50.5266	45.4570	41.9905	39.1223
	50	43.9712	37.0448	32.3547	29.1086	26.8894	25.0534
	60	30.5447	25.7331	22.4752	20.2204	18.6790	17.4039
1	40	62.4988	52.6542	45.9878	41.3736	38.2185	35.6080
	50	40.0212	33.7170	29.4482	26.4938	24.4739	22.8029
	60	27.8008	23.4215	20.4562	18.4040	17.0011	15.8405
2	40	57.3473	48.3141	42.1972	37.9634	35.0683	32.6729
	50	36.7224	30.9379	27.0209	24.3100	22.4566	20.9233
	60	25.5093	21.4910	18.7701	16.8870	15.5998	14.5349
3	40	52.9803	44.6350	38.9840	35.0725	32.3979	30.1849
	50	33.9261	28.5820	24.9633	22.4588	20.7466	19.3300
	60	23.5668	19.8545	17.3408	15.6011	14.4119	13.4280
4	40	49.2314	41.4766	36.2254	32.5907	30.1054	28.0490
	50	31.5254	26.5595	23.1969	20.8696	19.2785	17.9622
	60	21.8992	18.4496	16.1137	14.4972	13.3921	12.4779

This is because that as increasing the value of gradient index the percentage of SUS304 phase will rise, thus making such FG nanobeams more flexible. At the same time, there is no available data for the critical buckling temperature of FG nanobeams as far as the author knows. Therefore, it is believed that the tabulated results can be useful reference for future studies. Effects of changing length-to-thickness ratio (L/h) on the thermal buckling behavior of FG nanobeam for different values of power index and nonlocal parameter are investigated in Fig. 2. Observing this figure, it can be pointed that the values of critical temperature difference decrease with the increasing value of the aspect ratio at a constant material distribution. That is because a higher length-to-thickness ratio indicates that the FGM nanobeam is thinner with a lower stiffness. In addition, it is deduced that the buckling temperature decreases by increasing nonlocality parameters.

Finally, in order to clarify the effect of the small scale parameter and power indexes on the buckling analysis, Fig. 3 intuitively exhibits the variations of the critical temperature difference of nonlocal FG beam with respect to volume fraction indexes for different values of nonlocal parameter and slenderness ratio. It is easily deduced for every cases of aspect ratio that, the buckling temperature reduce with high rate where the power exponent in range from 0 to 2 than that where power exponent in range



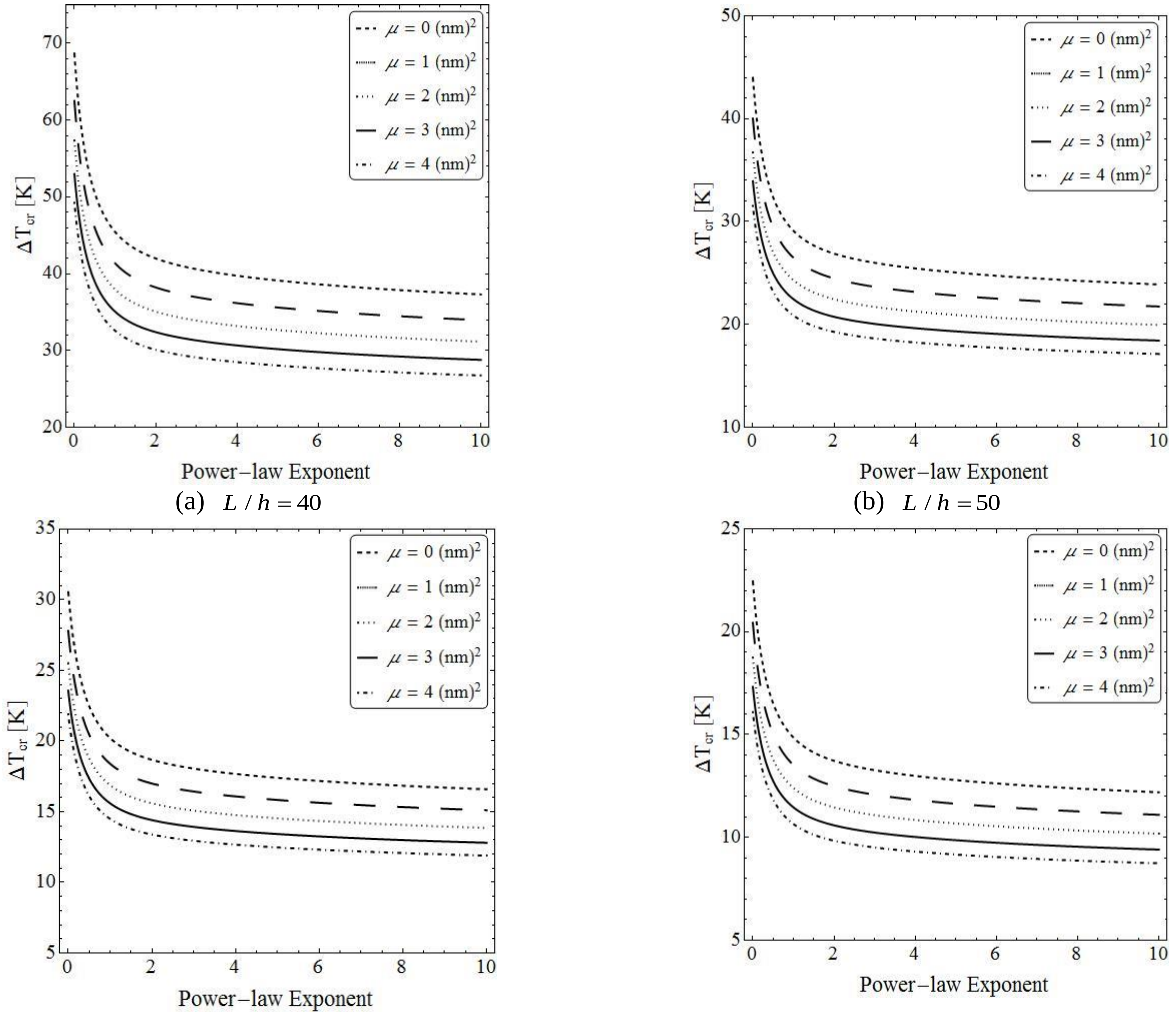
(a) $\mu = 0\,(\text{nm})^2$ (b) $\mu = 1\,(\text{nm})^2$

(c) $\mu = 2\,(\text{nm})^2$ (d) $\mu = 3\,(\text{nm})^2$

(e) $\mu = 4\,(\text{nm})^2$

Fig. 2 The variation of the critical buckling temperature of S-S FG nanobeam with aspect ratios and material graduations for different nonlocality parameters.

between 2 and 10. However, the above results obtained also show that the critical temperature of the nonlocal FG model are always smaller than those of the classical graded beam model. With the increase the nonlocal parameter μ from 0 to 4 $(\text{nm})^2$, the ΔT_{cr} decrease significantly. The results indicate that the nonlocal effect is tending to weaken the stiffnes of nanostructures and hence decreases the buckling temperatures.



(a) $L/h=40$ (b) $L/h=50$

(c) $L/h=60$ (d) $L/h=70$

Fig. 3 The variation of the critical buckling temperature of S-S FG nanobeam with material graduations and nonlocality parameters for different aspect ratios

5. Conclusions

This study focuses on the thermal buckling of a size-dependent FG nanobeam by using Timoshenko beam theory and Eringen's nonlocal elasticity theory. The governing differential equations and related boundary conditions are derived by implementing Hamilton's principle. The Navier solution method is adopted to obtain the analytical solutions of the stability equations. Thermo-mechanical properties of the FG nanobeams are assumed to be function of thickness and based on power-law model. Accuracy of the results is examined using available data in the literature. Finally, through some parametric study and numerical examples, the effect of different parameters are investigated for graded nanobeams. As shown in several numerical exercises, it is revealed that many parameters such as small scale parameter, power-law gradient index and aspect ratio have significant impact on critical buckling temperature of FG nanobeams. As previously specified, increasing the nonlocal parameter yields the decrease in critical temperatures for every graduation index parameter. However, the FG nanobeam model produces smaller buckling temperature than the classical beam model. Therefore, the small scale effects should be considered in the analysis of mechanical behavior of nanostructures. Also, it was observed that the dramatic reduction in critical temperature differences of the nonlocal FG beam is detected as the increase of the power-law index and aspect ratio.

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