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An efficient method for universal equivalent static wind loads on long-span roof structures

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Abstract. Wind-induced response behavior of long-span roof structures is very complicated, showing significant contributions of multiple vibration modes. The largest load effects in a huge number of members should be considered for the sake of the equivalent static wind loads (ESWLs). Studies on essential matters and necessary conditions of the universal ESWLs are discussed. An efficient method for universal ESWLs on long-span roof structures is proposed. The generalized resuming forces including both the external wind loads and inertial forces are defined. Then, the universal ESWLs are given by a combination of eigenmodes calculated by proper orthogonal decomposition (POD) analysis. Firstly, the least squares method is applied to a matrix of eigenmodes by using the influence function. Then, the universal ESWLs distribution is obtained which reproduces the largest load effects simultaneously. Secondly, by choosing the eigenmodes of generalized resuming forces as the basic loading distribution vectors, this method becomes efficient. Meanwhile, by using the constraint equations, the universal ESWLs becomes reasonable. Finally, reproduced largest load effects by load-response-correlation (LRC) ESWLs and universal ESWLs are compared with the actual largest load effects obtained by the time domain response analysis for a long-span roof structure. The results demonstrate the feasibility and usefulness of the proposed universal ESWLs method.

Keywords: universal equivalent static wind loads; long-span roof structures; generalized resuming forces; POD analysis; constraint equations

1. Introduction

Wind load cannot be applied separately in structural design. The combined effects between wind load and many other loads such as snow load, dead load, live load should be considered. In addition, the procedures for wind-induced dynamic response analysis may be too complicated for structural design. Therefore, it is convenient and necessary to determine a so-called equivalent static wind loads (ESWLs) for combinations with other loads. The ESWLs are the static loads providing the “largest load effect” due to dynamic wind excitations. In this paper, the term “largest load effect” is used to denote either the maximum (i.e., positive peak) or minimum (i.e., negative peak) load effect when it is not necessary to clearly indicate them.

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The ESWLs reproducing the largest load effects of a structure were first introduced by Davenport (1967) using the gust loading factor (GLF) method. This original method focused on the largest response displacement of the structure. Accurate evaluation of the largest load effects is very important in the structural design. Many studies on the ESWLs for one specified target response have been reported (e.g., Simiu 1976, Solari 1982, Zhang 1988, Kasperski and Niemann 1992, Holmes 2002, Tamura et al. 2002, Chen and Kareem 2004).

There are various wind load distributions that can reproduce the target largest load effect, if the target is only one load effect. However, the wind-induced response behavior of long-span roof structures is very complicated, showing significant contributions of multiple vibration modes. The largest load effects in a huge number of members should be considered while their largest values should never happen simultaneously. Some scholars have carried out the related research to deal with this situation. Katsumura *et al*. (2004, 2007) and Tamura *et al*. (2012) chose the eigenmodes of the fluctuating wind loads as the basic loading distribution vectors in order to determine the most reasonable coefficients of combination for different responses.

Katsumura *et al.* (2004, 2007) introduced the POD eigenmodes as one of the possible examples, these basic loading distribution vectors are efficient for a long-span cantilevered roof. For the general long-span roof structures, there are more suitable basic vectors exist for calculating the universal ESWLs. Hu (2006) calculated several ESWLs corresponding to some key responses and chose those wind loads as the basic loading distribution vectors. Then, the universal ESWLs was obtained by the least squares method. However, there is no detailed rule to judge how many and which typical ESWLs distributions should be selected, and it will need many ESWLs distributions if the number of target effects is large. Chen *et al.* (2010, 2012) and Yang *et al.* (2011, 2013) chose the eigenmodes of fluctuating wind loads and structural dominant inertia forces as the basic loading distribution vectors so that the ESWLs have a certain physical meaning.

As mentioned before, there are various wind load distributions that can reproduce the single target largest load effect. Under ideal conditions, there will be a so-called universal equivalent static wind loads which can reproduce any number of multiple largest load effects for all the target members. For the prospective of mathematics, many kinds of basic loading distribution vectors are feasible to express ESWLs, which can be given by

$$\{F_e\} = c_1\{f\}_1 + c_2\{f\}_2 + \cdots + c_n\{f\}_n \tag{1}$$

where $\{F_e\}$ is the ESWLs distribution which can reproduce the largest load effects; $c_1, c_{2,}, \cdots, c_n$ are the combination factors; $\{f\}_1, \{f\}_2, \cdots, \{f\}_n$ are the basic loading distribution vectors.

On one hand, there will be a universal ESWLs which can almost reproduce any number of multiple largest load effects for the targeted members. Definitely, there are some basic loading distribution vectors which will be more efficient and more feasible on the basis of the same number of loading distribution vectors. According to the previous research, the external wind loads reflect the structural background responses, while the inertial forces reflect the resonant responses.

If the basic loading distribution vectors are derived from the loads including both the external wind loads and inertial forces, these basic loading distribution vectors will be more efficient than others. In this study, the generalized resuming forces which include both the fluctuating wind loads and inertial forces will be defined. Then, the proper orthogonal decomposition (POD) method is conducted to obtain the eigenmodes and eigenvalues. Thus, this approach will become efficient by applying the dominant eigenmodes of the generalized resuming forces as the basic loading distribution vectors. The method which chose eigenmodes of fluctuating wind loads and structural dominant inertia forces needs to determine the number of modes required for fluctuating wind loads, i.e., N_b, and the number of modes required for structural dominant inertia forces, i.e., N_r. However, there is no detail study on how to choose and balance N_b and N_r. The method proposed in this paper chose the eigenmodes of generalized resuming forces as the basic loading distribution vectors which only needs to select the first few eigenmodes. In addition, this method also can consider the correlation between the fluctuating wind loads and structural inertia forces cross. Therefore, the method of this paper will be more efficient and convenient to apply.

On the other hand, unrealistic concentrated loads were found in the universal ESWLs in many cases, which may reach a very large value up to hundreds or even thousands Pascal. So the next problem is how to obtain the universal ESWLs with reasonable distribution and value. In this study, the constraint equations are used to ensure the universal ESWLs within a reasonable range. Then, the least squares method is employed to calculate the combination factors of these basic loading distribution vectors. Finally, the appropriate constraint factor is suggested to obtain the universal ESWLs which can reproduce the largest load effects for most of the targeted members.

2. Analysis of the basic loading distribution vectors

For a structure system with *n* degrees of freedom, the structural dynamic displacement can be divided into two parts: the first *m* modes where the resonant effect is significant, and modes *m*+1 to *n* where the response is essentially quasi-static, which can be expressed as (Huang and Chen 2007)

$$\{y(t)\}=\sum_{i=1}^{m}\phi_i q_i(t)+\sum_{i=m+1}^{n}\phi_i q_i(t)=\sum_{i=1}^{n}\phi_i q_{b,i}(t)+\sum_{i=1}^{m}\phi_i q_{r,i}(t)=\{y(t)\}_{b,n}+\{y(t)\}_{r,m} \tag{2}$$

where $q_i(t)$ is the *i*-th generalized displacement; ϕ_i is the *i*-th modal shape; $q_{b,i}(t)$ is the *i*-th background generalized displacement; $q_{r,i}(t)$ is the *i*-th resonant generalized displacement; $y(t)\}_{b,n}$ is the background displacement vector including all modal contributions; $\{y(t)\}_{r,m}$ is the resonant displacement vector including first *m* modal contributions.

From Eq. (2), the auto and cross correlation functions of the dynamic displacement can be expressed as

$$\begin{aligned}\{R(\tau)\}_{yy}&=E\left[\left(\{y(t)\}_{b,n}+\{y(t)\}_{r,m}\right)\left(\{y(t+\tau)\}_{b,n}+\{y(t+\tau)\}_{r,m}\right)\right]\\&=\{R(\tau)\}_{y_b y_b}+\{R(\tau)\}_{y_b y_r}+\{R(\tau)\}_{y_r y_b}+\{R(\tau)\}_{y_r y_r}\end{aligned} \tag{3}$$

According to the rotation symmetry of the covariance matrix, the displacement covariance matrix can be calculated as

$$\left[C_{yy}\right]=\left[C_{yy}\right]_b+\left[C_{yy}\right]_r+2\left[C_{yy}\right]_{rb} \tag{4}$$

where $[C_{yy}]$ is the total displacement covariance matrix; $[C_{yy}]_b$, $[C_{yy}]_r$ and $[C_{yy}]_{rb}$ are the background, resonant and cross displacement covariance matrix, respectively.

In this study, the generalized resuming force vector is defined as

$$\{p_e(t)\}=[K]\{y(t)\}=[K]\{y(t)\}_{b,n}+[K]\{y(t)\}_{r,m}=\{p_b(t)\}+\{p_r(t)\} \tag{5}$$

where $\{p_e(t)\}$, $\{p_b(t)\}$ and $\{p_r(t)\}$ are the generalized resuming force vector, external wind load vector and inertial force vector, respectively.

As can be seen, the generalized resuming forces include both the external wind loads and inertial forces. If the eigenmodes of the generalized resuming forces are chosen as the basic loading distribution vectors, this method will become efficient.

The covariance matrix of the generalized resuming forces can be calculated as

$$\begin{aligned}\left[C_{pp}\right] &= \overline{\{P_e(t)\}\{P_e^T(t)\}} = [K]\overline{\{y(t)\}\{y^T(t)\}}[K]^T \\ &= [K]\left[C_{yy}\right]_b[K]^T + [K]\left[C_{yy}\right]_r[K]^T + 2[K]\left[C_{yy}\right]_{rb}[K]^T \\ &= \left[C_{pp}\right]_b + \left[C_{pp}\right]_r + 2\left[C_{pp}\right]_{rb}\end{aligned} \tag{6}$$

where $[C_{pp}]_b$, $[C_{pp}]_r$ and $[C_{pp}]_{rb}$ are the background, resonant and cross covariance matrix of the generalized resuming forces, respectively.

Theoretically, the covariance matrix of the generalized resuming forces should be obtained by using Eq. (6). The cross covariance matrix of the generalized resuming forces mainly comes from the cross correlation between the background and resonance in the resonant interval. In this narrow interval, the resonant response is usually much larger than the background response when the structural damping is small and the natural frequency is low. Generally, long-span roof structures have the characteristics of small damping and low natural frequencies. Thus, the cross covariance matrix is small enough to be ignored compared to the resonant covariance matrix in many cases . The cross-correlation between the background and resonance should be considered only when the structural damping is slightly larger and some of the higher-order modes have effects on the responses. When the cross covariance matrix of the generalized resuming forces is small enough to be ignored, a simplified equation can be expressed as

$$\begin{aligned}\left[C_{pp}\right] &\approx \left[C_{pp}\right]_b + \left[C_{pp}\right]_{\mathrm{r}} = \left[C_{pp}\right]_b + [K]\left[C_{yy}\right]_r[K]^T \\ &= \left[C_{pp}\right]_b + [K][\Phi]\left[C_{qq}\right]_r[\Phi]^T[K]^T \\ &= \left[C_{pp}\right]_b + [Q_0]\left[C_{qq}\right]_r[Q_0]^T\end{aligned} \tag{7}$$

where $[\Phi]$ is the modal shape matrix; $[C_{qq}]_r$ is the resonant modal covariance matrix; $[Q_0]$ can be expressed as

$$[Q_0]=[K][\Phi]=[M][\Phi][\Lambda] \tag{8}$$

where $[M]$ is the mass matrix; $[\Lambda]=diag(\omega_1^2\cdots\omega_m^2)$.

As $[\Phi]$ and $[\Lambda]$ can be obtained by modal analysis which only need main m modes,

$\left[Q_0\right]=\left[M\right]\left[\Phi\right]\left[\Lambda\right]$ can be a known matrix.

Through Eq. (7), the background covariance matrix of generalized resuming forces $[C_{pp}]_b$ can be directly calculated by using the time history of the external wind loads. Once the resonant modal covariance matrix $[C_{qq}]_r$ is given, the resonant covariance matrix of generalized resuming force $[C_{pp}]_r$ is determined. Then, the covariance matrix of generalized resuming force $[C_{pp}]$ can be obtained.

The resonant modal covariance matrix $[C_{qq}]_r$ can be quickly obtained by approximate formula (e.g., Chen and Kareem 2005), the main process is given as follows.

The resonant component of the j-th modal response, $\sigma_{q_{jr}}^2$, is quantified as

$$\sigma_{q_{jr}}^2=\frac{1}{M_j^2\omega_j^3}\frac{\pi}{2\xi_j}S_{Q_{jj}}\left(\omega_j\right) \tag{9}$$

where M_j, ω_j, ξ_j are the j-th generalized mass, frequency, and damping ratio, respectively; $S_{Q_{jj}}\left(\omega\right)$ is the power spectral density (PSD) function of the j-th resonant modal response.

The covariance between the j-th and k-th resonant modal responses, $\sigma_{q_{jkr}}^2=\sigma_{q_{kjr}}^2$, is given by

$$\sigma_{q_{jkr}}^2=r_{jkr}\sigma_{q_{jr}}\sigma_{q_{kr}} \tag{10}$$

where r_{jkr} is the correlation coefficient for the j-th and k-th resonant modal responses, which can be approximated by the following expression when they are close to each other:

$$r_{jkr}=\alpha_{jkr}\rho_{jkr} \tag{11}$$

$$\alpha_{jkr}=\frac{\mathrm{Re}\left[S_{Q_{jk}}\left(\omega\right)\right]}{\sqrt{S_{Q_{jj}}\left(\omega\right)S_{Q_{kk}}\left(\omega\right)}}\bigg|_{\omega=\omega_j\ or\ \omega_k} \tag{12}$$

where $S_{Q_{jk}}\left(\omega\right)$ is the cross power spectral density (XPSD) function between the j-th and k-th generalized forces; ρ_{jkr} is given in Der Kiureghian (1980)

$$\rho_{jkr}=\frac{8\sqrt{\xi_j\xi_k}\left(\beta_{jk}\xi_j+\xi_k\right)\beta_{jk}^{3/2}}{\left(1-\beta_{jk}^2\right)^2+4\xi_j\xi_k\beta_{jk}\left(1+\beta_{jk}^2\right)+4\left(\xi_j^2+\xi_k^2\right)\beta_{jk}^2} \tag{13}$$

where $\beta_{jk}=\omega_j/\omega_k$, with $0\le\rho_{jkr}\le1$ and $-1\le\alpha_{jkr}\le1$. It is clear that the correlation coefficient of modal responses depends not only on the modal frequencies and damping ratios but also on the correlation of the associated generalized forces.

After the above analysis, the covariance matrix of generalized resuming forces $[C_{pp}]$ can be obtained very conveniently. Then the generalized resuming forces is decomposed by POD method, the eigenmodes and eigenvalues of generalized resuming forces are obtained as

$$\left[C_{pp}\right]\{G\}_l = \lambda_l \{G\}_l \tag{14}$$

where $\{G\}_l$ and λ_l are the l-th eigenmode and eigenvalue of the generalized resuming forces. Accordingly, the basic loading distribution vectors are determined by the dominant eigenmodes of the generalized resuming forces.

3. Modeling of the universal equivalent static wind loads

3.1 Universal ESWLs reproducing multiple load effects

By selecting the first N dominant eigenmodes of the generalized resuming forces, the universal ESWLs can be expressed as

$$\{F_e\} = c_1\{G\}_1 + c_2\{G\}_2 + \cdots + c_N\{G\}_N = [F_0]\{c\} \tag{15}$$

where $\{c\}$ is the combination factor vector; $\{G\}_1, \{G\}_2, \cdots, \{G\}_N$ are the basic loading distribution vectors which are derived from the eigenmodes of the generalized resuming forces; $[F_0]$ is the load distribution matrix by the basic loading distribution vectors.

For the equivalent targets, the combination factor vector $\{c\}$ needs to meet

$$\begin{cases} \{\beta\}_1^T\{F_e\} = \{\beta\}_1^T[F_0]\{c\} = \hat{r}_1 \\ \{\beta\}_2^T\{F_e\} = \{\beta\}_2^T[F_0]\{c\} = \hat{r}_2 \\ \cdots \\ \{\beta\}_N^T\{F_e\} = \{\beta\}_N^T[F_0]\{c\} = \hat{r}_N \end{cases} \tag{16}$$

where $\{\beta\}_i$ $(i=1,2,\cdots,N)$ is the i-th influence line vector; $\hat{r}_i$ $(i=1,2,\cdots,N)$ is the i-th largest load effect response, which can be obtained by any kind of dynamic response analysis, usually by the time domain response analysis. The largest load effect response can be expressed as

$$\hat{r}_i = \mathrm{sgn}(\overline{r}_i)g\sigma_{r_i} \tag{17}$$

where $\overline{r}_i$ $(i=1,2,\cdots,N)$ is the i-th mean load effect response; σ_{r_i} $(i=1,2,\cdots,N)$ is the i-th root mean square (RMS) load effect response; g is the peak factor generally ranging from 3 to 4; $\mathrm{sgn}(\cdot)$ is the sign function. When the sign of the mean load effect response is selected as that of the largest load effect response, the fluctuating ESWLs thus can be directly combined with the mean wind loads.

Eq. (16) can be expressed in matrix form as

$$[\beta]\{F_e\} = [\beta][F_0]\{c\} = \{\hat{r}\} \tag{18}$$

where $[\beta]$ is the influence matrix; $\{\hat{r}\}$ is the largest load effect response vector.

The influence function matrix $[\beta]$ can be obtained accordingly from the structural system. Once the matrix $[F_0]$ is given, the remain problem is to solve the combination coefficient vector $\{c\}$ based on the target largest load effect vector $\{\hat{r}\}$.

If the number of the dominant eigenmodes is equal to the number of the target largest load effects, i.e., $N = M$, Eq. (18) can be solved uniquely. If $N > M$, N can be appropriately reduced to M, thus Eq. (18) can also be solved uniquely. However, due to the huge number of structural members, $N < M$ is always valid in many cases. There will be no solution for Eq. (18). In this paper, the least squares solution technique will be used to search for the most appropriate solutions. It is a matter of course that any mathematical technique can be used to obtain the appropriate solutions.

3.2 Constraint equations

The ESWLs are not deeply discussed and analyzed in Eq. (18). In many cases, unrealistic concentrated loads were found in the universal ESWLs, which may reach a very large value up to hundreds or even thousands Pascal. In this study, the constraint equations are used to improve this situation. The absolute value of the ESWLs will be constrained within a reasonable range by the constraint factor and the extreme generalized restoring forces, which can be expressed as

$$\begin{cases} |F_{e1}| \le \alpha \cdot \hat{p}_{e1} \\ |F_{e2}| \le \alpha \cdot \hat{p}_{e2} \\ \quad \cdots \\ |F_{eN}| \le \alpha \cdot \hat{p}_{e2} \end{cases} \tag{19}$$

where α is the constraint factor; $\hat{p}_{e1}, \hat{p}_{e2}, \cdots, \hat{p}_{eN}$ are the extreme generalized restoring forces which can be expressed as

$$\{\sigma_{p_e}\} = \sqrt{diag([C_{pp}])} \tag{20}$$

$$\hat{p}_{ei} = g\{\sigma_{p_e}\}_i \qquad (i = 1, 2, \cdots, N) \tag{21}$$

where $\{\sigma_{p_e}\}$ is the RMS value vector of the generalized restoring forces; $diag(\cdot)$ is the mathematical procedure to take the diagonal elements of a matrix to a column vector; g is the peak factor generally ranging from 3 to 4.

For single target ESWLs method, such as LRC method, the maximal ESWLs will be up to generalized restoring forces, which can be expressed as

$$F_{ej} = \rho_{r_i p_{ej}} \hat{p}_{ej} \tag{22}$$

$$\left|\rho_{r_i p_{ej}}\right| \le 1 \tag{23}$$

where F_{ej} and p_{ej} are the *j*-th ESWL and generalized restoring force, respectively; r_i is the *i*-th load effect response; $\rho_{r_i p_{ej}}$ is the load response correlation coefficient between r_i and p_{ej}.

An appropriate constraint factor should be chose for analyzing the ESWLs for multiple targets. On one hand, the constraint factor should not be too small. Otherwise it is difficult to obtain a reasonable solution for Eq. (18). Since the constraint factor of the single-objective equivalent static wind loads will close to 1, then the lower bounds of the constraint factor needs to be greater than or equal to 1. On the other hand, the constraint factor should not be too large. Otherwise the constraint equation will be ineffective. The upper bounds of the constraint factor can be determined according to the actual model. When the constraint factor is so large that the concentrated load still appears, the constraint factor should be reduced appropriately. In this paper, the concentrated loads are defined as the maximum load in a small area of the roof, and the loads of the rest area are much smaller than it. When the constraint factor is so small that the target deviation is still large, the constraint factor should be enlarged appropriately. It is recommended to take the range from 1 to 2 for the constraint factor in this paper. With a suitable constraint factor, the value of the universal ESWLs can be well controlled by the constraint equations. Accordingly, the distribution of the universal ESWLs will become reasonable .

In summary, this efficient method for universal ESWLs on long-span roof structures require the following procedures:

- Getting the target largest load effects by dynamic analysis, usually by the time domain response analysis;
- Acquiring the eigenmodes of the generalized restoring forces by POD analysis and choosing the first *N* dominant eigenmodes as the basic loading distribution vectors;
- Establishing the constraint equations by selecting a suitable constraint factor;
- Calculating the optimal combination factor vector $\{c\}$ through Eqs. (18) and (19) and obtaining the universal ESWLs based on given target load effects;
- Utilizing the ESWL distributions to create database for structural design.

4. Application

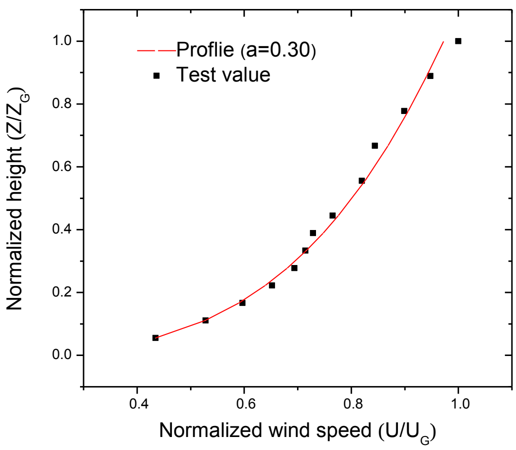
4.1 Wind tunnel experiments

The applicability of this method to long-span roof structures is investigated. Since the boundary layer wind tunnel has became the basic tool of wind engineering, it would be quite useful to conduct a wind tunnel investigation to evaluate wind effects on long-span roof structures. Fluctuating pressure acting on a long-span roof structure was measured in wind tunnel experiments. The long-span roof structure is a novel and unique shell structure, which has a long span of 143 m, a short span of 80 m and a height of 24 m. The geometric scale is 1:75. A total of 621 wind pressure measurement points were arranged on the roof, and fluctuating wind pressures were measured simultaneously at all measurement points. The wind tunnel test was carried out using a boundary layer wind tunnel (a width of 22.5 m, a height of 4.5 m, a length constituting a boundary layer of 36 m) of the Research Center for Wind Engineering at Southwest Jiaotong University, as shown in Fig. 1.

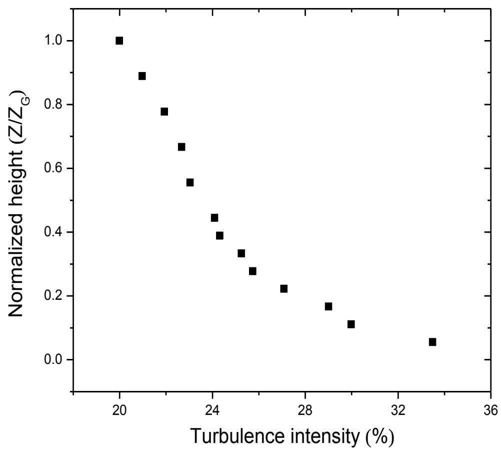
With regard to the oncoming flow in the experiments, spires and roughness elements were used to simulate a typical boundary layer wind flow. The power law exponent of the vertical profile for the mean wind speed was 0.3. The turbulence intensity at the height of the roof was 24%. The measured mean wind speeds and turbulence intensities at various heights over the test section are illustrated in Fig. 2.

The structure model was established to estimate the load effect caused by the fluctuating wind loads, as shown in Fig. 3. Dynamic responses analyses of all members were carried out using the fluctuating wind loads obtained from wind tunnel experiments, and the largest load effects of the equivalent targets are obtained.

Fig. 1 Rigid model for wind tunnel tests



(a) Mean wind speed profile



(b) Turbulence intensity profile

Fig. 2 Oncoming flow in the experiments

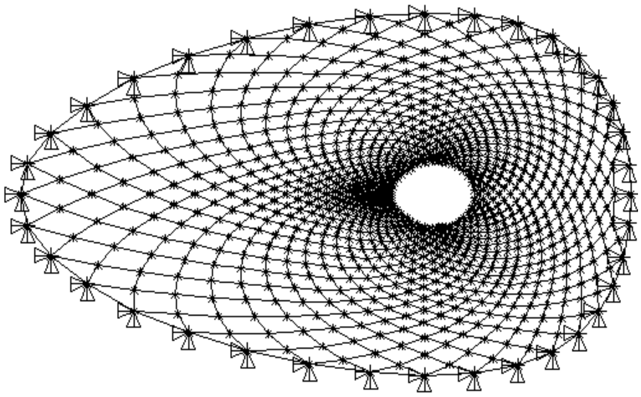


Fig. 3 Finite element model

4.2 Analyses of ESWLs

4.2.1 ESWLs obtained by LRC method

Fig. 4(a) shows the ESWLs obtained by LRC method in terms of the largest vertical displacement response. The LRC-ESWLs distribution is a realistic distribution, which most probably generates the quasi-static component of the largest vertical displacement of a target member.

Fig. 4(b) compares the largest vertical displacements in members obtained by the time domain response analysis with those reproduced by the LRC-ESWLs. The abscissa indicates 50 main members numbered 1 to 50. The maximum displacement of member No.50 obtained by the time domain dynamic response analysis showed the largest value of all members, and it was selected as the target largest load effect in the calculations of ESWLs for the LRC method. Therefore, the largest displacement in Member No.50 reproduced by LRC-ESWLs is exactly the same as the actual largest vertical displacement obtained by the time domain dynamic response analysis as indicated in the figure. However, the reproduced largest vertical displacements in the other members are underestimated by LRC-ESWLs. It is understandable because the vertical displacements in the other members do not necessarily reach their maximum values at the same time that the maximum displacement in Member No.50 appears.

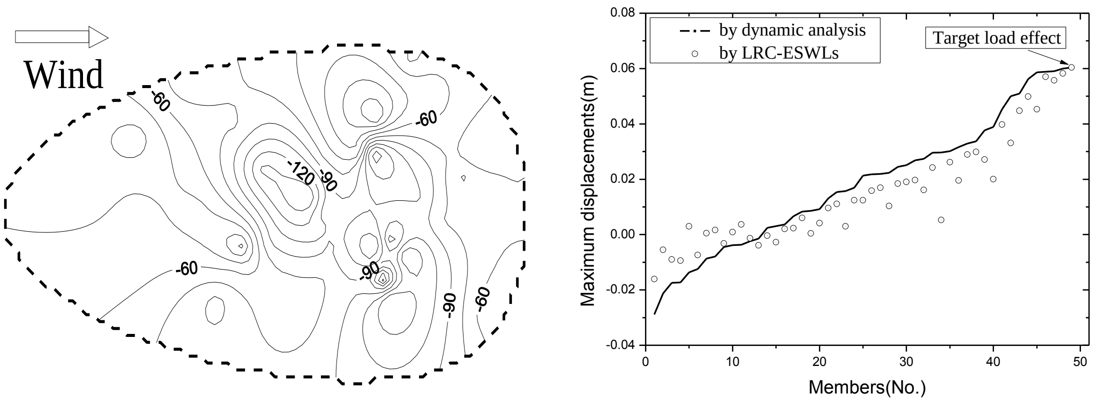
4.2.2 Analysis of constraint equations

As mentioned before, unrealistic concentrated loads can be found in the universal ESWLs in many cases. In order to demonstrate this situation, the universal ESWLs is calculated without the constraint equations in this case.

Fig. 5(a) shows that the universal ESWLs simultaneously reproducing the largest displacement of all members without the constraint equations; Fig. 5(b) compares the largest displacements in members obtained by the time domain response analysis with those reproduced by the universal ESWLs. It can be seen that almost all of the largest displacement responses reproduced by universal ESWLs match well with the actual largest displacement obtained by the time domain dynamic response analysis. However, the distribution of the universal ESWLs is very unreasonable which changes dramatically. And the value of the universal ESWLs is relatively concentrated, up to -700Pa in some places, which is much larger than the result by LRC-ESWLs method.

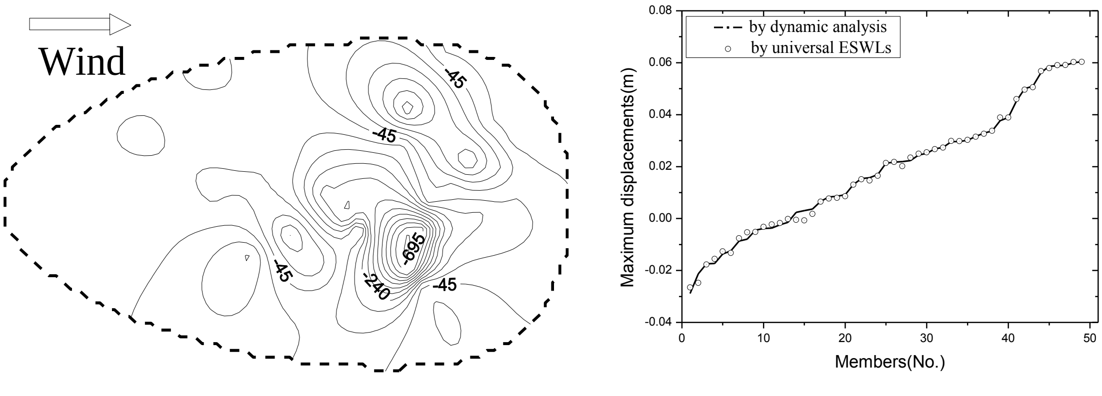
4.2.3 Comparison of the basic loading distribution vectors

In order to compare the calculation efficiency between the proposed method and the latest existing method, two types of basic loading distribution vectors were used to calculate the ESWLs. One is the eigenmodes of the fluctuating wind loads used by Katsumura *et al.* (2004, 2007), the other is the eigenmodes of the generalized resuming forces used in this study.



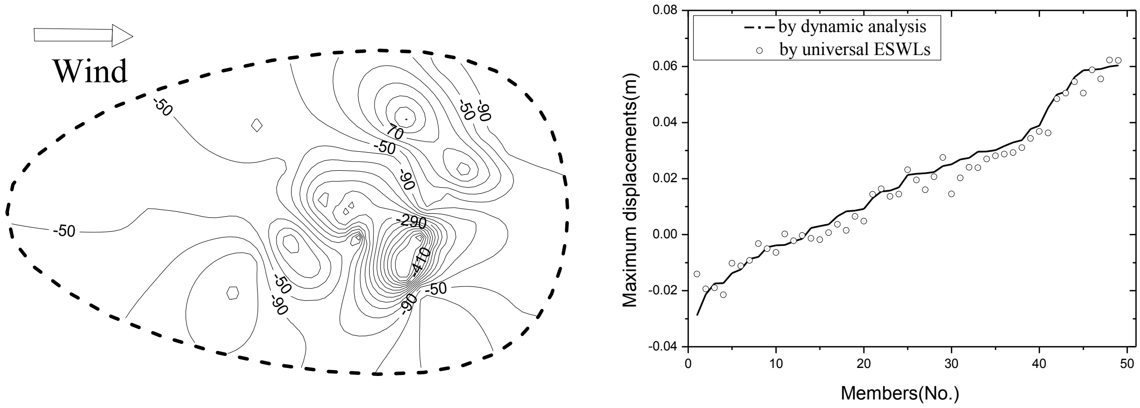
(a) LRC-ESWLs distributions (b) The largest displacement responses

Fig. 4 The results by LRC-ESWLs method



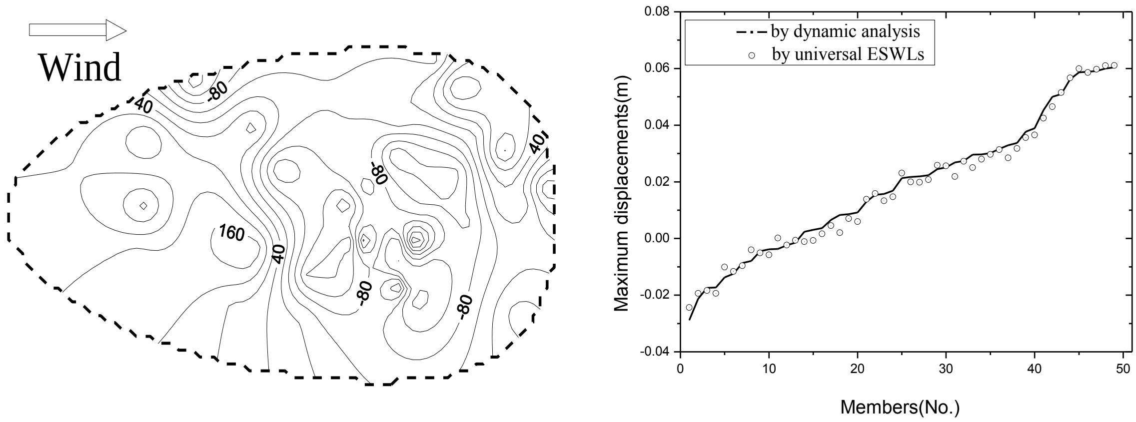
(a) Universal ESWLs distribution (Pa) (b) The largest displacement responses

Fig. 5 The results by universal ESWLs without constraint equations



(a) Universal ESWLs distribution (b) The largest displacement responses

Fig. 6 The results by the eigenmodes of the fluctuating wind loads



(a) Universal ESWLs distribution (b) The largest displacement responses

Fig. 7 The results by the eigenmodes of the generalized resuming forces

By choosing the eigenmodes of the fluctuating wind loads as the basic loading distribution vectors, the universal ESWLs are calculated using Eqs. (18) and (19). Fig. 6(a) shows the universal ESWLs simultaneously reproducing the largest displacement of all members with the constraint equations. Fig. 6(b) compares the largest displacements in members obtained by the time domain response analysis and those reproduced by the universal ESWLs. As can be seen, the majority of the largest displacement responses reproduced by universal ESWLs agree well with the actual largest displacement obtained by the time domain dynamic response analysis. However, there are also deviations in some target responses. The distribution of the equivalent static wind loads is not particularly reasonable, showing concentrated loads in some partial positions.

By choosing the eigenmodes of the generalized resuming forces as the basic loading distribution vectors, the universal ESWLs are also calculated using Eqs. (18) and (19). Fig. 7(a) shows the universal ESWLs simultaneously reproducing the largest displacement of all members with the constraint equations. Fig. 7(b) compares the largest displacements in members obtained by the time domain response analysis with those reproduced by the universal ESWLs. As can be seen, almost all of the largest displacement responses reproduced by universal ESWLs agree well with the actual largest displacement obtained by the time domain dynamic response analysis, and the distribution of the equivalent static wind loads becomes reasonable.

Through the above analysis, these basic loading distribution vectors derived from the eigenmodes of the generalized resuming forces are efficient and feasible on the basis of the same number of loading distribution vectors.

5. Conclusions

In this paper, an efficient method for universal equivalent static wind loads on long-span roof structures was proposed. The conclusions are summarized as follows.

- It is obvious that the traditional single-target approach such as LRC-ESWLs has a limitation for long-span roof structures. Hence, the multiple-target approach such as universal ESWLs is required and has the possibility of further development in its application.
- The method of analyzing the ESWLs for multiple targets is presented by applying the dominant eigenmodes of the generalized resuming forces as the basic loading distribution vectors. Because of the generalized resuming forces including both the external wind loads and inertial forces, this method becomes efficient. In addition, it was clarified that the contribution of the POD primary eigenmode of the generalized resuming forces was highest for the universal ESWLs. However, a lot of POD eigenmodes were required to accurately estimate largest load effects.
- The optimal combination coefficients of these basic vectors are obtained through the least squares method. Then, the universal ESWLs can be obtained. In many cases, unrealistic concentrated loads were shown in the universal ESWLs. However, this situation has been improved by using the constraint equations in this study.
- The ESWLs of a long span roof structure was analyzed by this method. The feasibility and usefulness was demonstrated through comparing the reproduced load effects with the largest load effects obtained by the time domain response analysis.

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References

Chen, B., Yang, Q. and Wu, Y. (2010), "Multi-objective equivalent static wind loads for large-span structures", *China Civil Eng. J.*, **43**(3), 62-67.

Chen, B., Yang, Q. and Wu, Y. (2012), "Wind-induced response and equivalent static wind loads of long span roofs", *Adv. Struct. Eng.*, **15**(2), 1099-1114.

Chen, X. and Kareem, A. (2004), "Equivalent static wind loads on buildings: new model", *J. Struct. Eng. - ASCE*, **130**(10), 1425-1435.

Chen, X. and Kareem, A. (2005), "Coupled dynamic analysis and equivalent static wind loads on buildings with three-dimensional modes", *J. Struct. Eng. - ASCE*, **131**(7), 1071-1082.

Davenport, A.G. (1967), "Gust loading factors", *J. Struct. Div. - ASCE*, **93**(3), 11-34.

Der Kiureghian, A. (1980), "Structural response to stationary excitation", *J. Eng. Mech. - ASCE*, **106**(6), 1195-1213.

Holmes, J.D. (2002), "Effective static load distributions in wind engineering", *J. Wind Eng. Ind. Aerod.*, **90**, 91-109.

Hu, X. (2006), "Wind loading effects and equivalent static wind loading on low rise buildings", Ph.D. Dissertation, Texas Tech University, Lubbock, TX.

Huang, G. and Chen, X. (2007), "Wind load effects and equivalent static wind loads of tall buildings based on synchronous pressure measurements", *Eng. Struct.*, **29**(10), 2641-2653.

Kasperski, M. and Niemann, H.J. (1992), "The LRC (load-response-correlation)-method a general method of estimating unfavourable wind load distributions for linear and non-linear structural behaviour", *J. Wind Eng. Ind. Aerod.*, **43**(1-3), 1753-1763.

Katsumura, A., Tamura, Y. and Nakamura, O. (2007), "Universal wind load distribution simultaneously reproducing largest load effects in all subject members on large-span cantilevered roof", *J. Wind Eng. Ind. Aerod.*, **95**(9-11), 1145-1165.

Nakayama, M., Sasaki, Y., Masuda, K. et al. (1998), "An efficient method for selection of vibration modes contributory to wind response on dome-like roofs", *J. Wind Eng. Ind. Aerod.*, **73**, 31-43.

Simiu, E. (1976), "Equivalent static wind loads for tall buildings design", *J. Struct. Div. - ASCE*, **102**(4), 719-737.

Solari, G. (1982), "Alongwind response estimation: closed form solution", *J. Struct. Div. - ASCE*, **108**(1), 225-244.

Tamura, Y. and Katsumura, A. (2012), "Universal equivalent static wind load for structures", *Proceedings of the 7th International Colloquium on Bluff Body Aerodynamics and Applications* (*BBAA7*), Shanghai, China; September.

Tamura, Y., Kikuchi, H. and Hibi, K. (2002), "Actual extreme pressure distributions and LRC formula", *J. Wind Eng. Ind. Aerod.*, **90**, 1959-1971.

Tamura, Y., Suganuma, S., Kikuchi, H. and Hibi, K. (1999), "Proper orthogonal decomposition of random wind pressure field", *J. Fluids Struct.*, **13**(78), 1069-1095.

Yang, Q. and Tian, Y. (2011), "Wind-induced responses of Beijing National Stadium", *Wind Struct.*, **14**(3), 239-252.

Yang, Q., Chen, B., Wu, Y. and Tamura, Y. (2013), "Wind-induced response and equivalent static wind load of long-span roof structures by combined ritz-proper orthogonal decomposition method", *J. Struct. Eng. - ASCE*, **139**(6), 997-1008.

Zhang, X. (1988), "The current Chinese code on wind loading and comparative study", *J. Wind Eng. Ind. Aerod.*, **30**,133-142.

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