Wind and Structures, *Vol. 23, No. 1 (2016) 59-73*
DOI: http://dx.doi.org/10.12989/was.2016.23.1.059

An analytical method for free vibration analysis of functionally graded sandwich beams

K. Bouakkaz[1,2], L. Hadji*[1,2], N. Zouatnia[3] and E.A. Adda Bedia[2]

[1]*Département de Génie Civil, Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algérie*
[2]*Laboratoire des Matériaux & Hydrologie, Université de Sidi Bel Abbes, 22000 Sidi Bel Abbes, Algérie*
[3]*Laboratoire de Structures, Géotechnique et Risques, Département de Génie Civil, Université de Chlef, Algérie*

(Received March 10, 2016, Revised May 20, 2016, Accepted May 26, 2016)

Abstract. In this paper, a hyperbolic shear deformation beam theory is developed for free vibration analysis of functionally graded (FG) sandwich beams. The theory account for higher-order variation of transverse shear strain through the depth of the beam and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. The material properties of the functionally graded sandwich beam are assumed to vary according to power law distribution of the volume fraction of the constituents. The core layer is still homogeneous and made of an isotropic material. Based on the present refined beam theory, the equations of motion are derived from Hamilton's principle. Navier type solution method was used to obtain frequencies. Illustrative examples are given to show the effects of varying gradients and thickness to length ratios on free vibration of functionally graded sandwich beams.

Keywords: functionally graded material; sandwich beam; hamilton's principle; vibration

1. Introduction

In recent years, the application of functionally graded (FG) sandwich structures in aerospace, marine, civil construction is growing rapidly due to their high strength-to-weight ratio. There exist two common types: sandwich structures with FG core and sandwich structures with FG skins. With the wide application of FG sandwich structures, understanding vibration of FG sandwich structures becomes an important task. Aydogdu and Taskin (2007) investigated the free vibration behavior of a simply supported FG beam by using Euler- Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation theory. Sallai *et al.* (2009) investigated the static responses of a sigmoid FG thick beam by using different beam theories. Ying *et al*. (2008) obtained the exact solutions for bending and free vibration of FG beams resting on a Winkler-Pasternak elastic foundation based on the two- dimensional elasticity theory by assuming that the beam is orthotropic at any point and the material properties vary exponentially along the thickness direction. Şimşek (2010) studied the dynamic deflections and the stresses of an FG simply-supported beam subjected to a moving mass by using Euler-Bernoulli, Timoshenko and the higher order shear deformation theories by considering the centripetal, inertia and Coriolis effects

∗Corresponding author, Dr., E-mail: had_laz@yahoo.fr

http://www.techno-press.org/?journal=was&subpage=8 ISSN: 1226-6116 (Print), 1598-6225 (Online)

of the moving mass. Thai and Vo (2012) presented a Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories. Taj *et al*. 2013 conducted static analysis of FG plates using higher order shear deformation theory. Recently, Tounsi and his co-workers (Hadji *et al.* 2011, Houari *et al.* 2011, El Meiche *et al.* 2011, Bourada *et al.* 2012, Tounsi *et al*. (2013), Klouche Djedid *et al.* 2014, Nedri *et al.* 2014, Ait Amar Meziane *et al.* 2014, Draiche *et al.* 2014, Sadoune *et al.* 2014, Zidi *et al.* (2014), Ait Yahia *et al.* 2015, Belkorissat *et al.* 2015) developed a new shear deformation plates theories involving only four unknown functions. Vo *et al.* (2014) investigated the finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory. Bennai *et al.* (2015) presented a new higher-order shear and normal deformation theory for functionally graded sandwich beams. Vo *et al.* (2015) developed a quasi-3 D theory for vibration and buckling of functionally graded sandwich beams. Kar *et al.* (2015) investigated the large deformation bending analysis of functionally graded spherical shell using FEM. Gan *et al.* (2015) studied the dynamic response of non-uniform Timoshenko beams made of axially FGM subjected to multiple moving point loads. Hassaine Daouadji *et al.* (2015) developed an analytical solution of nonlinear cylindrical bending for functionally graded plates. Bouderba *et al.* (2013) studied the thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations. Hebali *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Bousahla *et al.* (2014) used a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Larbi Chaht *et al.* (2015) presented the bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Al-Basyouni *et al.* (2015) investigated a size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Bourada *et al.* (2015) used a new simple shear and normal deformations theory for functionally graded beams. Hamidi *et al.* (2015) used a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Mahi *et al.* (2015) investigated a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Attia *et al.* (2015) studied the free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories. Bennoun *et al.* (2016) developed a novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates. Bellifa *et al.* (2016) developed the bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position. Bounouara *et al.* (2016) used a nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation. Bouderba *et al.* (2016) investigated the thermal stability of functionally graded sandwich plates using a simple shear deformation theory.

In this work, a hyperbolic shear deformation beam theory is presented to study the free vibration response of FG sandwich beams. The most interesting feature of this theory is that it accounts for a parabolic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the beam without using shear correction factors. Material properties of the sandwich beam faces are assumed to vary in the thickness direction only according to power-law form distribution in terms of the volume fractions of the constituents. The core layer is still homogeneous and made of an isotropic material. Then, the present theory together with Hamilton's principle, are employed to extract the motion equations of the functionally graded sandwich beams. Analytical solutions for free vibration are obtained. Numerical examples are presented to verify the accuracy of the present theory.

2. Problem formulation

Consider a FG sandwich beam, composed of ''Layer 1'', ''Layer 2'', and ''Layer 3'', as shown in Fig. 1. The x, y, and z axes are taken along the length (L), width (b), and height (h) of the beam, respectively. The core of sandwich beam is fully metal or ceramic and skins are composed of a FG material across the beam depth. The vertical positions of the bottom and top, and of the two interfaces between the layers are denoted by $h_0 = \frac{h}{2}, h_1, h_2, h_3 = \frac{h}{2}$, respectively.

2.1 Material properties

The effective material properties for each layer, like Young's modulus E and mass density ρ, can be expressed as

$$P^{(n)}(z) = P_2 + (P_1 - P_2)V^{(n)} \tag{1}$$

where P_t and P_b denote the material property located at the skins and at the core, respectively. The volume fraction function $V^{(n)}$ defined by the power-law form as follows

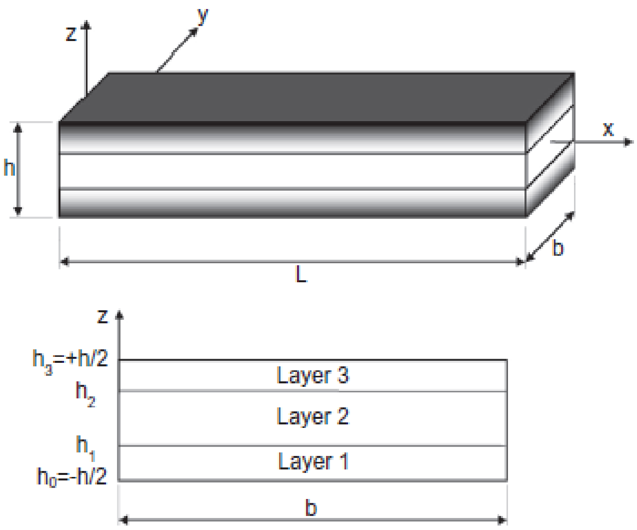

Fig. 1 Geometry and coordinate of a FG sandwich beam.

$$V^{(1)} = \left(\frac{z-h_0}{h_1-h_0}\right)^k, \qquad z \in [h_0, h_1] \tag{2a}$$

$$V^{(2)} = 1, \qquad z \in [h_1, h_2] \tag{2b}$$

$$V^{(3)} = \left(\frac{z-h_3}{h_2-h_3}\right)^k, \qquad z \in [h_2, h_3] \tag{2c}$$

where k is a power-law index which is positive.

2.2 Kinematics and constitutive equations

The displacement field of the present refined beam theory can be obtained as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{3a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{3b}$$

where $f(z)$ is a hyperbolic shape function. The function $f(z)$ is chosen in the form (Kettaf *et al.* 2013)

$$f(z) = z\left[1 + \frac{3\pi}{2}\sec h^2\left(\frac{1}{2}\right)\right] - \frac{3\pi}{2} h \tanh\left(\frac{z}{h}\right) \tag{4}$$

The strains associated with the displacements in Eq. (3) are

$$\varepsilon_x = \varepsilon_x^0 + z\,k_x^b + f(z)\,k_x^s \tag{5a}$$

$$\gamma_{xz} = g(z)\,\gamma_{xz}^s \tag{5b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad k_x^s = -\frac{\partial^2 w_s}{\partial x^2}, \quad \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \tag{5c}$$

$$g(z) = 1 - f'(z) \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \tag{5d}$$

The state of stress in the beam is given by the generalized Hooke's law as follows

$$\sigma_x^{(n)} = Q_{11}(z)\,\varepsilon_x \quad \text{and} \quad \tau_{xz}^{(n)} = Q_{55}(z)\,\gamma_{xz} \tag{6a}$$

where

$$Q_{11}(z) = E^{(n)}(z) \quad \text{and} \quad Q_{55}(z) = \frac{E^{(n)}(z)}{2(1+\nu)} \tag{6b}$$

2.3 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Thai and Vo 2012)

$$\delta \int_{t_1}^{t_2} (U - T) dt = 0 \tag{7}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy and δT is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned} \delta U &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dzdx \\ &= \int_0^L \left(N \frac{d\delta u_0}{dx} - M_b \frac{d^2 \delta w_b}{dx^2} - M_s \frac{d^2 \delta w_s}{dx^2} + Q \frac{d\delta w_s}{dx} \right) dx \end{aligned} \tag{8}$$

where N, M_b, M_s and Q are the stress resultants defined as

$$(N, M_b, M_s) = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f)\, \sigma_x dz \quad \text{and} \quad Q = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} g \tau_{xz} dz \tag{9}$$

The variation of the kinetic energy can be expressed as

$$\begin{aligned} \delta T &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z) [\dot{u} \delta \dot{u} + \dot{w} \delta \dot{w}] dzdx \\ &= \int_0^L \left\{ I_0 [\dot{u}_0 \delta \dot{u}_0 + (\dot{w}_b + \dot{w}_s)(\delta \dot{w}_b + \delta \dot{w}_s)] - I_1 \left(\dot{u}_0 \frac{d\delta \dot{w}_b}{dx} + \frac{d\dot{w}_b}{dx} \delta \dot{u}_0 \right) \right. \\ &\quad + I_2 \left(\frac{d\dot{w}_b}{dx} \frac{d\delta \dot{w}_b}{dx} \right) - J_1 \left(\dot{u}_0 \frac{d\delta \dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx} \delta \dot{u}_0 \right) + K_2 \left(\frac{d\dot{w}_s}{dx} \frac{d\delta \dot{w}_s}{dx} \right) \\ &\quad \left. + J_2 \left(\frac{d\dot{w}_b}{dx} \frac{d\delta \dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx} \frac{d\delta \dot{w}_b}{dx} \right) \right\} dx \end{aligned} \tag{10}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are the mass inertias defined as

$$\left(I_0, I_1, J_1, I_2, J_2, K_2\right) = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(1, z, f, z^2, zf, f^2\right) \rho(z) dz \tag{11}$$

Substituting the expressions for δU and δT from Eqs. (11) and (14) into Eq.(7) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s, the following equations of motion of the functionally graded beam are obtained

$$\delta u_0 : \; \frac{dN}{dx} = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{w}_b}{dx} - J_1 \frac{d\ddot{w}_s}{dx} \tag{12a}$$

$$\delta w_b : \; \frac{d^2 M_b}{dx^2} = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \frac{d\ddot{u}_0}{dx} - I_2 \frac{d^2 \ddot{w}_b}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2} \tag{12b}$$

$$\delta w_s : \; \frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} = I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 \frac{d\ddot{u}_0}{dx} - J_2 \frac{d^2 \ddot{w}_b}{dx^2} - K_2 \frac{d^2 \ddot{w}_s}{dx^2} \tag{12c}$$

Eqs. (12) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (5), (6) and (9) as follows

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_b}{\partial x^3} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{w}_b}{dx} - J_1 \frac{d\ddot{w}_s}{dx} \tag{13a}$$

$$B_{11} \frac{\partial^3 u_0}{\partial x^3} - D_{11} \frac{\partial^4 w_b}{\partial x^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \frac{d\ddot{u}_0}{dx} - I_2 \frac{d^2 \ddot{w}_b}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2} \tag{13b}$$

$$B_{11}^s \frac{\partial^3 u_0}{\partial x^3} - D_{11}^s \frac{\partial^4 w_b}{\partial x^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} + A_{55}^s \frac{\partial^2 w_s}{\partial x^2} = I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 \frac{d\ddot{u}_0}{dx} - J_2 \frac{d^2 \ddot{w}_b}{dx^2} - K_2 \frac{d^2 \ddot{w}_s}{dx^2} \tag{13c}$$

where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) dz \tag{14a}$$

and

$$A_{55}^s = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55} \left[g(z)\right]^2 dz \tag{14b}$$

3. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_b, w_s can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ W_{bm} \sin(\lambda x) e^{i\omega t} \\ W_{sm} \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \tag{15}$$

where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with m th eigenmode, and $\lambda = m\pi / L$.

Substituting the expansions of u_0, w_b, w_s from Eqs. (15) into the equations of motion Eq. (13), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \right) \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

where

$$a_{11} = A_{11}\lambda^2, a_{12} = -B_{11}\lambda^3, \; a_{13} = -B_{11}^s\lambda^3, a_{22} = D_{11}\lambda^4, \quad a_{23} = D_{11}^s\lambda^4 \tag{17a}$$

$$a_{33} = H_{11}^s\lambda^4 + A_{55}^s\lambda^2$$

$$m_{11} = I_1, \quad m_{12} = -I_2\lambda, \quad m_{13} = -I_3\lambda, \quad m_{22} = I_1 + I_4\lambda^2$$

$$m_{23} = I_1 + I_5\lambda^2 \tag{17b}$$

$$m_{33} = I_1 + I_6\lambda^2$$

4. Results and discussion

In this section, various numerical examples are presented and discussed to verify the accuracy of present theories in predicting the free vibration responses of simply supported FG beams. The FG beam is taken to be made of aluminum and alumina with the following material properties:

Ceramic (P_C : Alumina, Al_2O_3): $E_c = 380$ GPa; $\nu = 0.3$; $\rho_c = 3960$ kg/m3.

Metal (P_M : Aluminium, Al): $E_m = 70$ GPa; $\nu = 0.3$; $\rho_m = 2702$ kg/m3.

And their properties change through the thickness of the beam according to power-law. For convenience, the following dimensionless form is used

$$\bar{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$$

4.1 Results for free vibration analysis

For free vibration analysis, different types of FG sandwich beams are considered. Table 1 and 2 shows the nondimensional fundamental frequencies $\bar{\omega}$ of FG beams with both hardcore and softcore. The obtained results are compared with those of of Vo *et al*. (2014) and Bennai *et al*. (2015) for different values of power law index k and span-to-depth ratio L/h.

Table 1 Non-dimensional fundamental natural frequencies of simply supported FG sandwich beams with homogeneous hardcore $(L/h=5)$

k	Theory	$\bar{\omega}$				
		1-0-1	2-1-2	1-1-1	1-2-1	1-8-1
0	CBT	5.3953	5.3953	5.3953	5.3953	5.3953
	Vo *et al*. (2014)	5.1528	5.1528	5.1528	5.1528	5.1528
	Bennai *et al*. (2015)	5.1529	5.1529	5.1529	5.1529	5.1529
	Present	5.1529	5.1529	5.1529	5.1529	5.1529
0.5	CBT	4.2640	4.3772	4.4796	4.6432	5.0469
	Vo *et al*. (2014)	4.1268	4.2351	4.3303	4.4798	4.8422
	Bennai *et al*. (2015)	4.1270	4.2353	4.3305	4.4799	4.8425
	Present	4.1272	4.2354	4.3305	4.4797	4.8419
1	CBT	3.6706	3.8314	3.9859	4.2394	4.8661
	Vo *et al.* (2014)	3.5735	3.7298	3.8755	4.1105	4.6084
	Bennai *et al.* (2015)	3.5730	3.7302	3.8754	4.1108	4.6091
	Present	3.5739	3.7301	3.8757	4.1104	4.6791
2	CBT	3.1377	3.3068	3.4976	3.8322	4.6835
	Vo *et al*. (2014)	3.0680	3.2365	3.4190	3.7334	4.5142
	Bennai *et al*. (2015)	3.0672	3.2368	3.4187	3.7336	4.5151
	Present	3.0687	3.2370	3.4193	3.7333	4.5136
5	CBT	2.8082	2.8953	3.0741	3.4517	4.5031
	Vo *et al*. (2014)	2.7446	2.8439	3.0181	3.3771	4.3501
	Bennai *et al*. (2015)	2.7433	2.8436	3.0178	3.3770	4.3511
	Present	2.7457	2.8447	3.0186	3.3771	4.3495
10	CBT	2.7688	2.7839	2.9306	3.3018	4.4237
	Vo *et al.* (2014)	2.6932	2.7355	2.8808	3.2356	4.2776
	Bennai *et al.* (2015)	2.6918	2.7353	2.8806	3.2353	4. 2782
	Present	2.6945	2.7365	2.8815	3.2358	4.2769

An excellent agreement between the present solutions and results of Vo *et al*. (2014) and Bennai *et al*. (2015) is found. It should be remembered that the frequencies predicted by the shear deformable beam theories are smaller than those predicted by the classical beam theory and the difference between the frequencies of CBT and the shear deformable beam theories decreases as the value of L/h increases (see Tables 1-4).

Figs. 2 and 3 depict the fundamental frequencies parameters versus the material parameter k of simply supported power-law (2-1-2) FG sandwich beams with both homogeneous hardcore and softcore, respectively. It can be seen, that as the power law index increases, the natural frequencies decrease for sandwich beams with hardcore and increase for sandwich beams with softcore.

Table 2 Non-dimensional fundamental natural frequencies of simply supported FG sandwich beams with homogeneous hardcore $(L/h = 20)$

k	Theory	$\bar{\omega}$				
		1-0-1	2-1-2	1-1-1	1-2-1	1-8-1
0	CBT	5.4777	5.4777	5.4777	5.4777	5.4777
	Vo *et al*. (2014)	5.4603	5.4603	5.4603	5.4603	5.4603
	Bennai *et al*. (2015)	5.4603	5.4603	5.4603	5.4603	5.4603
	Present	5.4603	5.4603	5.4603	5.4603	5.4603
0.5	CBT	4.3244	4.4389	4.5429	4.7094	5.1212
	Vo *et al*. (2014)	4.3148	4.4290	4.5324	4.6979	5.1067
	Bennai *et al*. (2015)	4.3148	4.4290	4.5324	4.6979	5.1067
	Present	4.3148	4.4290	4.5325	4.6979	5.1066
1	CBT	3.7214	3.8838	4.0404	4.2979	4.9365
	Vo *et al*. (2014)	3.7147	3.8768	4.0328	4.2889	4.9233
	Bennai *et al.* (2015)	3.7146	3.8768	4.0328	4.2889	4.9233
	Present	3.7147	3.8768	4.0328	4.2889	4.9233
2	CBT	3.1812	3.3514	3.5443	3.8837	4.7501
	Vo *et al*. (2014)	3.1764	3.3465	3.5389	3.8769	4.7382
	Bennai *et al*. (2015)	3.1763	3.3465	3.5389	3.8769	4.7382
	Present	3.1764	3.3466	3.5389	3.8769	4.7381
5	CBT	2.8483	2.9346	3.1149	3.4972	4.5661
	Vo *et al*. (2014)	2.8439	2.9310	3.1111	3.4921	4.5554
	Bennai *et al.* (2015)	2.8438	2.9310	3.1110	3.4921	4.5554
	Present	2.8440	2.9311	3.1111	3.4921	4.5554
10	CBT	2.8094	2.8221	2.9696	3.3451	4.4851
	Vo *et al*. (2014)	2.8439	2.9310	3.1111	3.4921	4.5554
	Bennai *et al*. (2015)	2.8040	2.8188	2.9661	3.3406	4.4749
	Present	2.8042	2.8189	2.9662	3.3406	4.4749

Table 3 Non-dimensional fundamental natural frequencies of simply supported FG sandwich beams with homogeneous softcore $(L/h = 5)$

k	Theory	$\bar{\omega}$				
		1-0-1	2-1-2	1-1-1	1-2-1	1-8-1
0	CBT	2.8034	2.8034	2.8034	2.8034	2.8034
	Vo *et al*. (2014)	2.6773	2.6773	2.6773	2.6773	2.6773
	Bennai *et al*. (2015)	2.6774	2.6774	2.6774	2.6774	2.6774
	Present	2.6774	2.6774	2.6774	2.6774	2.6774
0.5	CBT	4.8058	4.6979	4.5838	4.3727	3.6851
	Vo *et al*. (2014)	4.8683	4.7368	4.6050	4.3814	3.7101
	Bennai *et al*. (2015)	4.4427	4.3046	4.1839	3.9921	3.4342
	Present	4.4386	4.2993	4.1791	3.9898	3.4355
1	CBT	5.2408	5.1686	5.0670	4.8491	4.0231
	Vo *et al*. (2014)	5.1002	5.0012	4.8815	4.6512	3.9296
	Bennai *et al*. (2015)	5.1108	5.0190	4.8984	4.6677	3.9344
	Present	4.8471	4.7090	4.5765	4.3601	3.7081
2	CBT	5.4609	5.4534	5.3881	5.1982	4.3118
	Vo *et al*. (2014)	5.1880	5.1603	5.0703	4.8564	4.1139
	Bennai *et al*. (2015)	5.1916	5.1644	5.0769	4.8646	4.1140
	Present	5.0895	4.9864	4.8608	4.6347	3.9319
5	CBT	5.4992	5.5760	5.5669	5.4353	4.5566
	Vo *et al*. (2014)	5.1880	5.1603	5.0703	4.8564	4.1139
	Bennai *et al*. (2015)	5.1848	5.1966	5.1301	4.9326	4.1855
	Present	5.1851	5.1507	5.0557	4.8405	4.1149
10	CBT	5.4647	5.5814	5.6026	5.5067	4.6537
	Vo *et al*. (2014)	5.1848	5.1966	5.1301	4.9326	4.1855
	Bennai *et al*. (2015)	4.4557	4.3184	4.1968	4.0016	3.4379
	Present	5.1833	5.1886	5.1163	4.9152	4.1859

Table 4 Non-dimensional fundamental natural frequencies of simply supported FG sandwich beams with homogeneous softcore $(L/h = 20)$

k	Theory	$\bar{\omega}$				
		1-0-1	2-1-2	1-1-1	1-2-1	1-8-1
0	CBT	2.8462	2.8462	2.8462	2.8462	2.8462
	Vo *et al*. (2014)	2.8371	2.8371	2.8371	2.8371	2.8371
	Bennai *et al*. (2015)	2.8371	2.8371	2.8371	2.8371	2.8371
	Present	2.8371	2.8371	2.8371	2.8371	2.8371

Continued-

0.5	CBT	4.8854	4.7762	4.6602	4.4453	3.7442
	Vo *et al.* (2014)	4.8579	4.7460	4.6294	4.4160	3.7255
	Bennai *et al.* (2015)	4.8582	4.7465	4.6297	4.4161	3.7257
	Present	4.8576	4.7456	4.6290	4.4158	3.7256
1	CBT	5.3283	5.2564	5.1536	4.9317	4.0889
	Vo *et al.* (2014)	5.2990	5.2217	5.1160	4.8938	4.0648
	Bennai *et al.* (2015)	5.2996	5.2220	5.1165	4.8941	4.0647
	Present	5.2986	5.2209	5.1152	4.8932	4.0649
2	CBT	5.5512	5.5462	5.4811	5.2884	4.3836
	Vo *et al.* (2014)	5.5239	5.5113	5.4410	5.2445	4.3542
	Bennai *et al.* (2015)	5.5244	5.5118	5.4415	5.2448	4.3541
	Present	5.5235	5.5103	5.4398	5.2435	4.3543
5	CBT	5.5873	5.6696	5.6626	5.5303	4.6337
	Vo *et al.* (2011)	5.5645	5.6382	5.6242	5.4843	4.5991
	Bennai *et al.* (2015)	5.5648	5.6387	5.6247	5.4847	4.5991
	Present	5.5643	5.6374	5.6229	5.4828	4.5991
10	CBT	5.5505	5.6739	5.6983	5.6029	4.7329
	Vo *et al.* (2014)	5.5302	5.6452	5.6621	5.5575	4.6960
	Bennai *et al.* (2015)	5.5303	5.6459	5.6627	5.5579	4.6961
	Present	5.5301	5.6445	5.6609	5.5559	4.6960

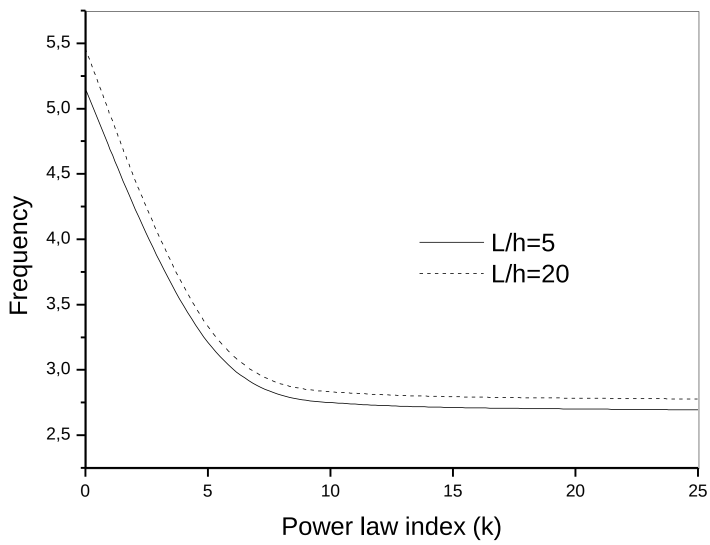

Fig. 2 Variation of fundamental frequencies $\bar{\omega}$ versus the material parameter k for (2-1-2) simply supported FG sandwich beams with homogeneous hardcore

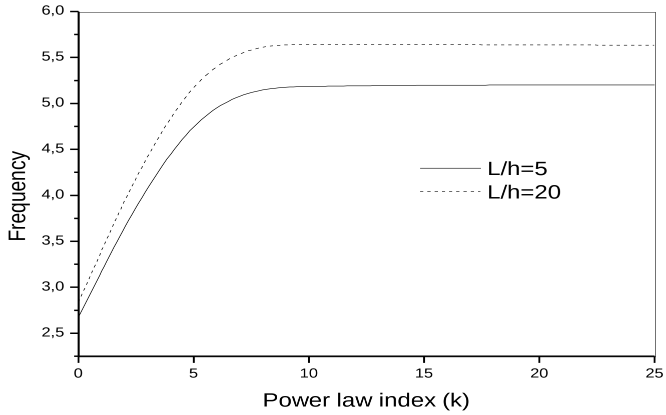

Fig. 3 Variation of fundamental frequencies $\bar{\omega}$ versus the material parameter k for (2-1-2) simply supported FG sandwich beams with homogeneous softcore

In Fig. 4 and 5 the variations of non-dimensional fundamental frequencies of a FG sandwich beams with homogeneous hardcore and softcore, respectively for different power law index k versus the span-to-height ratio using the present theory are given. It is shown that the natural fundamental frequencies decrease with the decrease of the material rigidity, which is due to the increase of k for FG sandwich beams with homogeneous hardcore or the decrease of k for FG sandwich beams with homogeneous.

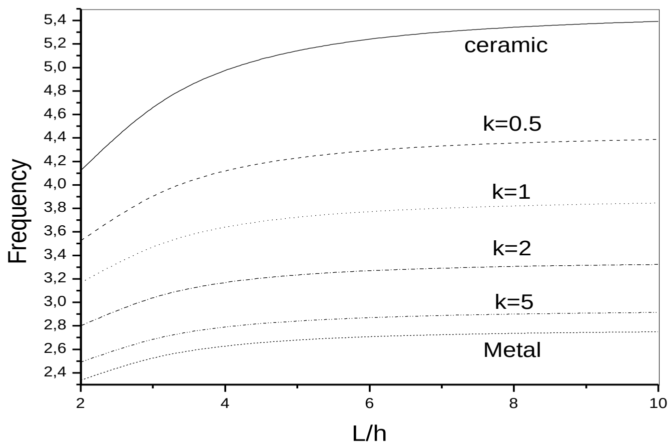

Fig. 4 Fundamental frequency $\bar{\omega}$ as a function of span-to-height ratio L/h of symmetric FGM sandwich beam (2-1-2) with homogeneous hardcore for various values of k

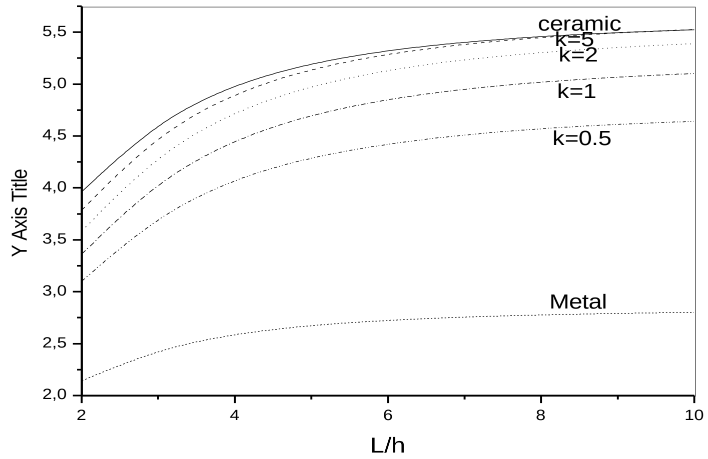

Fig. 5 Fundamental frequency $\bar{\omega}$ as a function of span-to-height ratio L/h of symmetric FGM sandwich beam (2-1-2) with homogeneous hardcore for various values of k

5. Conclusions

A new hyperbolic shear deformation theory for the free vibration analysis of FG sandwich beams is developed. The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. It is based on the assumption that the transverse displacements consist of bending and shear components. Based on the present beam theory, the equations of motion are derived from Hamilton's principle. Numerical examples show that the proposed theory gives solutions which are almost identical with those obtained using other shear deformation theories.

References

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, **53**(6), 1143 – 1165.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Aydogdu, M. and Taskin, V. (2007), "Free vibration analysis of functionally graded beams with simply supported edges", *Mater. Design*, **28**(5), 1651-1656.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct.*, **18**(1), 187-212.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Composites: Part B*, **60**, 274–283.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng.*, **38**, 265-275.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct*., **18**(4), 1063-1081.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams", *Steel Compos. Struct.*, **19**(3), 521-546.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struc.*, **23**(4), 423-431.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.

Bouderba *et al*. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**, 5-33.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18(**2), 409 -423.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth*., **11**(6), 1350082.

Draiche, K., Tounsi, A. and Khalfi, Y. (2014), "A trigonometric four variable plate theory for free vibration of rectangular composite plates with patch mass", *Steel Compos. Struct.*, **17**(1), 69-81.

El Meiche, N., Tounsi, A., Ziane, N., Mechab, I. and Adda Bedia, E.A. (2011), "A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate", *Int. J. Mech. Sci*., **53**, 237-247.

Gan, B.S., Trinh, T.H., L, T.H. and Nguyen, D.K. (2015), "Dynamic response of non-uniform Timoshenko beams made of axially FGM subjected to multiple moving point loads", *Struct. Eng. Mech.*, **53**(5), 981-995.

Hadji, L., Atmane, H.A., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2011), "Free vibration of functionally graded sandwich plates using four variable refined plate theory", *Appl. Math. Mech.*, **32**, 925-942.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 235-253.

Hassaine, D.T. and Hadji, L. (2015), "Analytical solution of nonlinear cylindrical bending for functionally graded plates", *Geomech. Eng.*, **9**(5), 631-644.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech. - ASCE*, **140**, 374-383.

Houari, M.S.A., Benyoucef, S., Mechab, I., Tounsi, A. and Adda Bedia, E.A. (2011), "Two variable refined plate theory for thermoelastic bending analysis of functionally graded sandwich plates", *J. Therm. Stresses*, **34**, 315-334.

Kar, V.R. and Panda, S.K. (2015), "Large deformation bending analysis of functionally graded spherical shell using FEM" , *Struct. Eng. Mech.*, **53**(4), 661-679.
Kettaf, F.Z., Houari, M.S.A., Benguediab, M. and Tounsi, A. (2013), "Thermal buckling of functionally graded sandwich plates using a new hyperbolic shear displacement model", *Steel Compos. Struct.*, **15**(4), 399-423.
Klouche Djedid, I., Benachour, A., Houari, M.S.A., Tounsi, A. and Ameur, M. (2014), "A *n*-order four variable refined theory for bending and free vibration of functionally graded plates", *Steel Compos. Struct.*, **17**(1), 21-46.
Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425-442.
Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Modell.*, **39**, 2489-2508.
Nedri, K., El Meiche, N. and Tounsi, A. (2014), "Free vibration analysis of laminated composite plates resting on elastic foundations by using a refined hyperbolic shear deformation theory", *Mech. Compos. Mater.*, **49**(6), 641-650.
Sadoune, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2014), "A novel first-order shear deformation theory for laminated composite plates", *Steel Compos. Struct*., **17**(3), 321-338.
Sallai, B.O., Tounsi, A., Mechab, I., Bachir, B.M., Meradjah, M. and Adda Bedia, E.A. (2009), "A theoretical analysis of flexional bending of Al/Al_2O_3 S-FGM thick beams", *Comput. Mater. Sci.*, **44**(4), 1344-1350.
Şimşek, M. (2010), "Vibration analysis of a functionally graded beam under a moving mass by using different beam theories", *Compos. Struct*., **92**(4), 904-917.
Taj, M.N.A., Chakrabarti, A. and Sheikh, A. (2013), "Analysis of functionally graded plates using higher order shear deformation theory", *Appl. Math. Modell.*, **30**, 1-11.
Thai, H.T. and Vo, T.P. (2012), "Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories", *Int. J. Mech. Sci*., **62**(1), 57-66.
Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**, 209-220.
Vo, T.P., Thai, H.T., Nguyen, T.K., Maheri, A. and Lee, J. (2014), "Finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory", *Eng. Struct.*, **64**, 12-22.
Vo, T.P., Thai, H.T., Nguyen, T.K., Inam, F. and Lee, J. (2015), "A quasi-3D theory for vibration and buckling of functionally graded sandwich beams", *Compos. Struct.*, **119**, 1-12.
Ying, J., Lü, C.F. and Chen, W.Q. (2008), "Two-dimensional elasticity solutions for functionally graded beams resting on elastic foundations", *Compos. Struct*., **84**(3), 209-219.
Zidi, M., Tounsi, A., Houari, M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

CC